Recent Advances in Understanding Competitive Markets

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1 Introduction

The Arrow-Debreu model of competitive markets has proven to be a remarkably rich and flexible foundation for studying an array of important economic problems, ranging from social security reform to the determinants of long-run growth. Central to the study of such questions are many of the extensions and modifications of the classic Arrow-Debreu framework that have been advanced over the last 40 years. These include dynamic models of exchange over time, as in overlapping generations or continuous time, and under uncertainty, as in incomplete financial markets. Many of the most interesting and important recent advances have been spurred by questions about the role of markets in allocating resources and providing opportunities for risk-sharing arising in finance and macroeconomics. This article is intended to give an overview of some of these more recent developments in theoretical work involving competitive markets, and highlight some promising areas for future work.

I focus on three broad topics. The first is the role of asymmetric information in competitive markets. Classic work in information economics suggests that competitive markets break down in the face of informational asymmetries. Recently these issues have been reinvestigated from the vantage point of more general models of the market structure, resulting in some surprising results overturning these conclusions and clarifying the conditions under which perfectly competitive markets can incorporate informational asymmetries. The second concentrates on the testable implications of competitive markets. Again classic work in general equilibrium theory, going back to the Sonnenschein-Debreu-Mantel theorem, suggests that the competitive model has no refutable implications. More recent work challenges this view, in part by arguing for a broader interpretation of observable data, and seeks to give the theoretical framework of competitive equilibrium an empirical counterpart. Finally, I turn to recent developments in modeling markets with infinitely many goods, which encompasses a range of applications in dynamic choice, choice under uncertainty, continuous-time trading in financial markets, growth and innovation.

Any such survey is bound to represent idiosyncratic tastes and outlooks, and this one is no exception. Another author likely could have written an entirely different and equally compelling overview based on a disjoint set of papers. My approach to narrowing down an impossibly large topic area has been to focus mainly on very recent work, carried out over the last ten years or so. In so doing I have tried to highlight recent work that has some insights of fairly general applicability, and in areas still nascent and ripe for further work. While I have of necessity left out many interesting topics and papers and only briefly touched on others, I simply hope the outline painted spurs further investigation into all I wished to include but could not.

2 Information and Financial Markets

Time and uncertainty are fundamental aspects of many economic decisions. Consumers and firms trade virtually continuously a wide array of financial instruments such as bonds, options, futures, insurance contracts and a host of derivative securities. One of the major challenges of finance and macroeconomics, as well as theory more broadly, is to model these markets and contracts in a manner that illuminates and explains their role in the allocation of resources over time and the transfer of wealth across states. The classical Arrow-Debreu model, with its assumption that all consumers and producers meet at a single time and trade contingent contracts for all conceivable goods and states and dates, fails to explain much of the rich institutional structure of financial markets for risk-sharing, as well as suggesting a number of implausible conclusions such as the neutrality of financial instruments, the uselessness of money, and the irrelevance of corporate financial and control policies.

The general equilibrium model with incomplete markets addresses many of these criticisms by incorporating a more realistic model of trading in financial markets into the setting of competitive markets. By pairing spot markets for commodities with a system of financial markets which may be incomplete, the incomplete markets model not only provides a closer approximation of the system of financial markets and contracts we observe in reality, but also explains a number of observations contradicted by the classical complete markets Arrow-Debreu model: the non-neutrality of financial instruments, a role for money, the relevance of corporate financial and control policies, the existence of default and bankruptcy in an equilibrium framework, the possibility that such default and bankruptcy are actually Pareto improving, and the possibility of a positive role for government intervention in financial markets. The development of the incomplete markets model, surveyed by Magill and Shafer (1991), Geanakoplos (1990), and Magill and Quinzii (1996), has been one of the most important theoretical advances of the last decade, yet the basic model still retains a number of assumptions at odds both with the workings of actual financial markets, and with many of the most important models used in finance and macroeconomics.

In particular, this work takes much about the structure of financial markets as exogenous, including the assets that are marketed. This work leaves no room for financial innovation to respond to the demands of investors for more efficient means of risk sharing. This work also assumes that agents always deliver all of their promised obligations, leaving no possibility of default as anything but a disequilibrium phenomenon. Excluding these and other institutional features of financial markets precludes the study of many of the most important institutions underlying them. Moreover, explicit modeling of such institutional details and reasons for market incompleteness may drastically change some of the conclusions coming from the literature on incomplete markets. Many of the important recent advances in modeling competitive markets, as I discuss in this section, have been motivated by an attempt to incorporate innovation, default, and asymmetric information more generally.

2.1 Default in Financial Markets

Individuals and firms default. Contrary to the implicit assumption of standard Arrow-Debreu and incomplete markets models of financial markets, penalties for default are not exorbitant. In addition to a host of non-economic justifications for milder punishments, many economic rationales for allowing default have been advanced. A large body of work dating back many years studies innumerable aspects of default and bankruptcy, including their causes and consequences and the design of optimal rules for dealing with them.

Much of this work is cast in a partial equilibrium framework, or in a strategic setting in which parties are not anonymous. Neither of these frameworks is appropriate for modeling a variety of issues surrounding default in large financial markets. Such markets are more appropriately modeled as anonymous competitive markets. Moreover, the general equilibrium framework also captures the possibility of chain reactions, that is, defaults by some agents on promises to deliver to other agents might force them to default on further promises, and so on, in a chain reaction of defaults throughout the economy.

A growing body of recent work casts questions concerning default in a general equilibrium framework. This work highlights some of the features of default that may be desirable with incomplete opportunities for risk sharing in financial markets. If markets are complete, then all contingencies can be included in contracts agents write, and thus prohibiting default is always optimal, just as when contracts can be made complete it is always optimal to prohibit renegotiation. With incomplete markets, however, agents cannot contract over all possible contingencies, so there might be efficiency gains from allowing default, just as with incomplete contracts renegotiation might result in ex post gains in efficiency. Default allows agents to tailor asset payoffs to suit their needs, effectively replacing existing assets with others that are more personalized. Default may also endogenously increase the asset span, as one existing asset can be transformed into as many assets as there are agents in the economy. Thus it may be natural to observe relatively lenient penalties for default in financial markets that are relatively incomplete.

Many of these themes are explored in the recent work on default in financial markets. This work differs mainly in the institutions surrounding default that are modeled, and in the methods of penalizing agents who default. There are two main strands in this literature. The first follows the seminal work of Kehoe and Levine (1993) in focusing on perfect information, complete markets environments.

Kehoe and Levine (1993) consider an infinite-horizon model in which agents trade both in intertemporal financial markets and in spot markets. They imagine that agents might default on their obligations in financial markets. Agents who default are penalized by being barred from future trading in financial markets and by seizure of their portfolios, while physical endowments cannot be confiscated and trade in spot markets – presumably large and anonymous – cannot be prohibited. Kehoe and Levine (1993) assume a framework of complete markets and perfect information; thus no agent can undertake liabilities on which he would choose to default. That is, agents' portfolio and consumption choices are constrained by an additional individual rationality constraint specifying that chosen strategies must yield at least as much utility at each point in time and in each state as the agent could secure solely by trading on spot markets from then on. In this way the possibility of default in their model generates endogenous limits on debt and portfolio holdings.

This work is motivated by the widely-observed fact that changes in individual consumption are imperfectly correlated with changes in aggregate consumption. Debt constraints, such as those arising endogenously in their model, can provide one explanation for this imperfect risk-sharing, even in the presence of complete markets. They show that these endogenous debt constraints can result in interest rates lower than agents' subjective discount rates. They also investigate versions of the welfare theorems and show, not surprisingly, that whether or not constrained versions of the welfare theorems hold (constrained by individual rationality constraints) depends essentially on whether there are effects from spot market price fluctuations. More precisely, they identify a set of restrictions, which they label the "identically homothetic" case, under which constrained versions of the welfare theorems are valid. These assumptions include the case of a single consumption good each period, and more generally require that in each state agents have identical homothetic preferences. In the general multi-good case, equilibria may be Pareto ranked, mirroring the distinction between the constrained efficiency of equilibria in the one-good incomplete markets model and the generic constrained inefficiency of equilibria in the many-good incomplete markets model. With many goods, prices must simultaneously equilibrate supply and demand and keep agents from defaulting; thus failures of the welfare theorems should be expected.

Many other explanations can be given for the observed patterns of imperfect risk sharing in individual consumption data. One such standard explanation is the presence of asymmetric information, necessitating mechanisms that involve correlation between individual consumption and income in order to induce optimal actions. Kocherlakota (1996) considers this justification, and argues instead that the informational asymmetry is often not among "insiders" – such as within a village – but between "insiders" and "outsiders" – such as between villagers and a lending agency. This renders the environment one of perfect information but no commitment, as third parties cannot verify the terms of contracts, providing a motivation for the environment Kehoe and Levine study. Kocherlakota examines the connection between efficient allocations and subgame perfect equilibria in a repeated transfer game. Kocherlakota also shows that it is possible to distinguish between the case of asymmetric information with commitment and perfect information without commitment.

Alvarez and Jermann (2000a) consider the asset pricing implications of default in a simplified version of the models of Kehoe-Levine and Kocherlakota. They consider a one-good version of this model with identical agents in which the participation constraints of Kehoe-Levine are replaced by "solvency" constraints on portfolios that limit agents' holdings of assets. They formalize the idea implicit in Kehoe-Levine that their debt constraints act as endogenous limits on portfolio holdings by showing that for a particular choice of these portfolio constraints, their equilibria with solvency constraints are equivalent to Kehoe and Levine's equilibria with participation constraints. Given this correspondence, they are able to appeal to results in Kehoe and Levine for versions of the welfare theorems in these cases. Alvarez-Jermann also explore the asset pricing implications of this model. Like Kehoe and Levine, they show that when compared to a model with unconstrained trading, interest rates are lower than subjective discount rates. Moreover, pricing kernels are more volatile, and asset prices are determined by agents with substantial individual risk. A companion paper Alvarez and Jermann (2000b) shows that large equity premia and risk premia may arise for long term bonds in this model.

In a closely related paper, Kehoe and Levine (2000) compare the predictions from their model with participation constraints and Alverez-Jermann's portfolio constraints in a simplified model, and find that the dynamic properties of equilibria in stochastic settings are quite different: with participation constraints the steady states are relatively simple while with portfolio constraints the dynamics are much more complicated and shocks are persistent.

This work is motivated largely by the desire for a tractable model in which imperfect risk sharing occurs in equilibrium together with interest rates below discount rates and more volatile pricing kernels. While debt constraints produce these effects in a model of complete markets and perfect information, either incomplete markets or asymmetric information provides an alternative explanation. Compared to incomplete markets models, Kehoe-Levine and Alvarez-Jermann argue that their models are simpler and yield predictions independent of "arbitrary" assumptions about the set of assets available. Moreover, they argue that a second advantage, precisely because markets are assumed complete, any asset can be priced in their models. In the large financial markets they seek to model, however, the assumptions of perfect information and complete markets are not always plausible. In addition the assumption of perfect information means that there is no default in equilibrium. The only explanation for the large volume of default actually observed within the context of their models is as a disequilibrium phenomenon. The assumption of complete markets also means there is no economic rationale for allowing default in their models, as outcomes are always Pareto dominated by the equilibria in a standard Arrow-Debreu framework with no possibility of default.

A second strand of work instead examines default in settings that allow for imperfect information and potentially incomplete markets. In this vein, by contrast, default may be consistent with equilibrium and the orderly functioning of markets, and hence may be viewed as a robust equilibrium phenomenon, rather than arising solely out of equilibrium. Indeed, much of this work is motivated by the large volume of observed default and the attempt to reconcile this observation with equilibrium theories of market functions. Only by allowing for incomplete markets and imperfect information, and hence default to emerge in equilibrium, can questions regarding the optimality of default and various institutions surrounding credit markets and debt recovery be addressed.

The work of Dubey, Geanakoplos, and Shubik (1990) is some of the earliest foundational work advancing this argument in a general equilibrium setting. They consider a model of financial markets in which assets are bought and sold but sellers can default on delivery of these assets if they choose. A number of potentially surprising results emerge in their framework. First, they give a general existence result in which equilibrium may involve non-trivial levels of trade and default.[1] Default is then consistent with equilibrium and the orderly functioning of competitive markets, which might come as a surprise given the presence of asymmetric information and the well-known difficulties of incorporating asymmetric information in competitive markets (e.g. Helpman and Laffont (1975); see also the discussion in section 2.2).

Special cases of their model generate both Akerlof's lemons model and the insurance model of Rothschild and Stiglitz (1976), which may make it seem even more surprising that they are able to give a general existence result given the breakdowns in markets characteristic of both Akerlof (1970) and Rothschild and Stiglitz (1976). This apparent discrepancy can be resolved in the case of Akerlof's model essentially in terms of semantics: Dubey, Geanakoplos, and Shubik (1990) simply argue that the "equilibrium" is one involving zero trade at a price of zero.

The difference between the functioning of markets in Dubey, Geanakoplos, and Shubik (1990) and the failure of markets in Rothschild and Stiglitz (1976) is more fundamental, arising from a difference in how Dubey, Geanakoplos, and Shubik (1990) model agents' expectations regarding delivery rates on "new" assets currently available but not traded. Since the equilibrium notion involves rational expectations regarding repayment rates, there is always a "trivial" pessimistic equilibrium in such a model, in which agents

[1] There is always a "trivial" pessimistic equilibrium in such a model, in which agents expect complete default, and there is no trade in any asset markets.

expect complete default and thus there is no trade in any asset markets. Dubey, Geanakoplos, and Shubik (1990) adopt a refinement of their equilibrium notion by refining admissable beliefs regarding repayment rates in a manner reflecting cautious expectations, under which agents expect delivery rates consistent with buyers and sellers with highest valuations for the asset. Rothschild and Stiglitz adopt a different convention regarding expected deliveries on untraded assets, assuming instead that agents expect that the distribution of buyers and sellers will be equal to the average in the population. Under these expectations, expected delivery rates may be too low to support trade and there may be no equilibria, which is not the case for Dubey, Geanakoplos, and Shubik (1990).

A second important feature of their work is that the set of marketed assets arises endogenously, through the combination of default penalties and expectations about repayment rates. Since the equilibrium asset structure need not involve complete markets even when a complete set of assets is available for trade, the possibility of default leads to an endogenous explanation for incomplete markets and potentially falsifiable predictions regarding which markets should be active in equilibrium. Replicating the standard Arrow-Debreu model, Arrow securities do emerge when they are offered in complete markets with infinite default penalties, and these are the only assets traded under these circumstances. Comparative statics results link default penalties to the span of actively traded assets: as default penalties decrease the span of traded assets decreases as well.

A key feature of the asymmetric information default introduces is that it is one-sided. While sellers may default individually on delivery in different states and different proportions depending on their idiosyncratic preferences and endowments, buyers instead buy slices of an aggregate pool, and hence face only the average default risk. A prime example of such a market is the market for collateralized mortgage obligations (CMO), as Geanakoplos (1996) argues. This feature of these markets is central for the existence of market clearing prices. As a simple example illustrating this fact, taken from Dubey, Geanakoplos, and Shubik (1990), consider a produce stand selling fruit from a number of different suppliers. Suppose each individual farmer knows exactly the proportion of bad fruit in his lot, but fruit from all suppliers is sold together. If customers are not allowed to choose their own fruit, instead ordering by quantity that is then randomly selected by the seller, then there will be a single price clearing the market for this "pooled" good. On the other hand, if customers are allowed to select their own fruit there may be no single equilibrium price for the fruit.[2]

Their work also provides an interesting illustration of various economic rationales for allowing default. Clearly there are a variety of drawbacks to imposing sufficiently lenient penalties on default such that some agents rationally choose to default, including the reduction in lending spurred by creditors' rational expectations that they might not be repaid fully, for either direct or indirect reasons, the deadweight loss of penalties imposed in response to default, and the externality imposed on "reliable" agents by "unreliable" agents, since they must then borrow on less favorable terms than those they would face in the absence of default. Balancing these drawbacks are a number of benefits, chief among them the fact that agents who default in some states are essentially able to replace the existing assets with alternative ones tailored to meet their needs and reflect their preferences and endowments, at the cost of incurring the default penalty, thereby increasing the asset span in exactly the way they prefer. Through examples Dubey, Geanakoplos, and Shubik (1990) show that the absence of default may be optimal when markets are complete, yet with incomplete markets default may yield Pareto improvements (and positive default rates) in equilibrium.

Striking confirmation of the role default can play in promoting efficiency when markets are incomplete is given by Zame (1993). He argues that default can affect efficiency in a way that simply opening new markets cannot, precisely because default allows the creation of new assets tailored to agents' needs. Using a version of the Dubey, Geanakoplos, and Shubik (1990) model with infinitely many possible states of nature, Zame (1993) shows that if markets are initially incomplete, then "randomly" opening markets while prohibiting default may result in equilibrium allocations that are bounded away from Pareto optimal allocations, while with default equilibria become approximately efficient as default penalties go to infinity and the number of securities traded goes to infinity.

The importance and preponderance of default raises a number of questions concerning the institutions surrounding credit markets and default – both financial and legal – involving regulation of borrowing and lending,

[2]See the next section for further discussion of this point.

protection of creditors and punishment for delinquent borrowers. Dubey, Geanakoplos, and Shubik (1990) take a simple, tractable approach to modeling the penalties associated with default by assuming these are assessed directly in terms of agents' utility. Agents in their model incur a penalty in terms of a decrease in utility that is a linear function of the amount by which they default. This provides a simple reduced form version of more complicated repercussions from default such as reputation effects, limitations in future access to credit markets, forgone collateral, and so on. This model is robust enough to yield a number of important insights into general equilibrium effects of default, while remaining simple enough to analyze easily. Addressing the array of questions surrounding the institutions involved requires models in which these institutions appear more explicitly, however, and in which some of these repercussions are modeled directly.

One prominent institution by which loans are secured and default punished is collateral. Collateral is a central component in countless lending transactions, from sophisticated financial markets to routine home mortgages to Las Vegas pawn shops. Indeed, Geanakoplos (1996) argues that one of the primary functions of Wall Street is to stretch the amount of available collateral to cover a wider array of transactions and thereby increase the liquidity of credit markets. Geanakoplos (1996) and Geanakoplos and Zame (2000) develop a general equilibrium model of collateralized borrowing and lending. They argue that the reliance on collateral has a profound impact on the allocation of commodities, the efficiency of markets, and on prices and volatility. This impact is driven in part by the scarcity of collateral, as well as by the limits collateral requirements place on borrowing – since borrowers must have the requisite collateral to secure their loans – and on lending – since in the case of default the lender receives only the value of the collateral in exchange for his loan.

They adapt a standard intertemporal, multi-good incomplete markets model by allowing for durable goods, which may then serve as collateral backing assets. Assets are characterized both by their payoffs and by the collateral requirement associated with purchasing the asset, which is denominated in terms of physical goods. Assets become options in this framework, as buyers default if and only if delivery on the asset is more costly than the value of the collateral that backs it.[3] When assets are collateralized in this way there is no problem of adverse selection, since the identity of the borrower is immaterial to the eventual return to the seller (the value of the collateral is independent of the identity of the buyer). A collateral equilibrium always exists under standard assumptions on fundamentals, demonstrating again that default can be consistent with the functioning of competitive markets. Indeed, collateralized markets are better behaved than standard incomplete financial markets, as the collateral requirements work as endogenous bounds on short and long sales of assets. By endogenously bounding asset trades, collateral thus ensures the existence of equilibrium, along the lines of Radner's original construction, while without collateral requirements equilibria need not exist in a model of incomplete markets with multiple goods (see Duffie and Shafer (1985)).

This work illustrates some of the important roles collateral can play in enhancing the efficiency of financial markets. For example, if assets are not collateralized and there are no other penalties for default, then trade in assets should break down. The introduction of collateral requirements in such an environment then makes these asset markets potentially viable. The resulting collateral equilibria may be inefficient, yet Pareto dominate outcomes in economies with no trade in assets. Moreover, collateral equilibria have redistributive effects which may result in equilibria that are not Pareto dominated by standard Arrow-Debreu equilibria. For example, using homes as collateral in mortgage markets works to increase the price of homes, benefitting sellers at the expense of buyers, even though both may be better off than if lending is unsecured and hence markets break down under the possibility of default. Not surprisingly, such markets may exhibit a tension between making loans safer by increasing collateral requirements, which lenders may prefer, and making loans more readily available by decreasing collateral requirements, which benefits buyers. Nonetheless, higher collateral requirements may result in Pareto inferior equilibria.

Collateral requirements may also provide another partial explanation for endogenous breakdowns in some markets. In contrast with standard results regarding trade in incomplete markets, in which (at least generically) all

[3]Note, however, that the amount delivered on an asset is endogenous, so an equivalent economy cannot be constructed simply by respecifying the payoffs of the assets.

marketed assets will be traded, equilibrium with collateral requirements may result in robust breakdowns in markets for some assets.[4]

The model Geanakoplos and Zame (2000) introduce can also be used to give a general equilibrium foundation to a classic partial equilibrium argument regarding the effects of collateral requirements in fueling market crashes and enhancing price volatility. A standard story would start with optimistic speculators buying on margin, after which bad news regarding stock fundamentals would cause prices to fall, resulting in a margin call forcing speculators to sell their stock when they cannot meet the margin call. These sales in turn push stock prices even lower, furthering the downward swing. Collateral requirements magnify shifts in the wealth distribution, and these shifts provide the general equilibrium link between collateral requirements and market crashes. See Kiyotaki and Moore (1997) for a study of the dynamics of credit cycles in a different setting.

While collateral of many forms is clearly a fundamental part of the functioning of financial markets and many other credit markets, a rich array of other institutions have evolved surrounding credit markets. For example, Sabarwal (2000) focuses on a variety of institutions surrounding bankruptcy in competitive markets with unsecured lending, in particular on the interaction between default and credit limits over time. His is a model in which trades are subject to exogenously set credit limits, and defaults result in seizure of assets up to some exemption levels. The resulting model is a reduced form general equilibrium model of the interaction between banks or credit agencies and trade in financial markets. Again equilibria will exist under standard assumptions (in particular a continuum of agents to mitigate the non-convexity in the budget constraint resulting form the possibility of default), and bankruptcy may be a feature of equilibrium for some agents. Araujo and Pascoa (2000) develop a similar model, in which they establish a weak constrained efficiency result (similar in spirit to what holds in Geanakoplos and Zame (2000)). Among all feasible allocations satisfying agents' budget constraints at given equilibrium prices the equilibrium allocations are Pareto optimal. Among all allocations satisfying feasibility and budget constraints for *some* prices, however, the equilibrium allocations are

[4]Equilibria may depend on price levels here, however, because asset payoffs are nonlinear in prices, so should expect large degree of nominal indeterminacy in equilibria.

generically inefficient, which is not surprising.[5]

2.2 Asymmetric Information and Competitive Markets

The frictions created by default and bankruptcy in competitive markets result in part from the asymmetric information generated regarding agents' promises when default is possible. An equilibrium analysis of default gives a particular illustration of the effects of asymmetric information on competitive markets more generally. Defaulting is but one way an agent might exert some control over the payoffs of the securities he trades. The broader effects of asymmetric information on competitive markets is a classic question, going back to the foundations of information economics, e.g. Akerlof (1970), Rothschild and Stiglitz (1976), Spence, and to the more abstract formulation in the seminal work of Prescott and Townsend (1984).

The more recent work on default discussed above has produced some particular predictions that would be unexpected from general intuitions concerning effects of asymmetric information on market outcomes, in particular compatibility with equilibrium. Several special features of the asymmetry created by default serve to explain this. Bisin and Gottardi (1999) study a broad class of economies in which agents trade non-exclusive financial contracts. This trading is subject to asymmetric information due to the dependence of contract payoffs on both idiosyncratic and aggregate shocks, which is of a form general enough to encompass both adverse selection and moral hazard. In this general framework Bisin and Gottardi (1999) focus on the issue of the existence of equilibria, and identify two main problems in reconciling equilibria in competitive markets with asymmetric information. The first problem has to do with feasibility: due to the dependence of payoffs on both idiosyncratic and aggregate shocks, market clearing in securities holdings doesn't automatically imply that the aggregate payoff to a security is 0. The second problem is that agents may have arbitrage opportunities linked to their private information at *any* prices, that is, there may be no non-arbitrage prices.

[5]See also Araujo, Barbachan, and Pascoa (2000) for connection to arbitrage pricing with collateral, Duffie (1998), Duffie and Singleton (1997) for asset pricing with default.

To overcome these two problems and obtain general conditions under which equilibrium is consistent with the presence of asymmetric information, Bisin and Gottardi (1999) modify the trading requirements in two ways. To address the issues surrounding feasibility, they introduce a "one-sided" constraint whereby agents can only take long positions in the securities depending on their idiosyncratic shocks. Agents can, however, take short (or long) positions in "pool" securities aggregating these idiosyncratic securities that depend only on aggregate shocks, as in the model of Dubey, Geanakoplos, and Shubik (1990). Alternatively, short sales of idiosyncratic securities can be incorporated as long as they allow for a simple non-linearity in prices by which a bid-ask spread can be introduced. To address the issues surrounding arbitrage prices, they impose bounds on trades in individual securities (i.e. those depending on idiosyncratic shocks).

This work also points to the importance of the exclusive versus non-exclusive nature of contracts in mitigating asymmetric information. If exclusive contracts are traded, so that agents are offered a menu of contracts and can choose only one, and the quantities traded fully reveal agents' types, then the equilibrium will be a separating equilibrium and the problems with feasibility will not arise. In many settings including large financial markets, however, the assumptions that contracts are non-exclusive appears more plausible, as the informational requirements of verifying every transaction of every agent are prohibitive. Also many contracts observed in such markets have terms independent of all other transactions. Some of the contrasts between the conclusions drawn by Bisin and Gottardi (1999) and others can then be traced to this assumption of non-exclusive contracts.[6]

This taxonomy is very helpful in framing and synthesizing other work on asymmetric information in competitive markets. Their work can be viewed as an extension of Dubey, Geanakoplos, and Shubik (1990) that distills the basic elements of their default model. For example, Dubey, Geanakoplos, and Shubik (1990) focus on non-exclusive contracts in which the option of default by the seller creates exactly the idiosyncratic securities and "one-sided" constraint Bisin and Gottardi (1999) posit, while buyers instead purchase the pool security depending only on aggregate repayment rates. The pioneering

[6]Bisin, Geanakoplos, Gottardi, Minelli, and Polemarchakis (2001) consider a more general model of markets for contracts.

work of Prescott and Townsend (1984) instead considers exclusive contracts, and argues that the difference between economies with moral hazard and adverse selection is that under moral hazard agents' trades fully reveal their types, hence equilibria exist and are separating and efficient, while under adverse selection trades do not fully reveal types, leading to breakdowns in existence and efficiency. (See also Kehoe, Levine, and Prescott (1998)). Similarly, Gale (1992) constructs a model in which agents are exogenously specified ex ante to be either buyers or sellers for one given security, and hence can take only one side of a position on a contract.[7]

When markets do function in the presence of asymmetric information, the resulting equilibrium allocations are typically inefficient. Despite the preponderance of informational asymmetries in virtually any real market transaction, many economists would argue that competitive markets should still produce allocations that are efficient or nearly so, based on the idea that in a sufficiently large market any single agent will have a negligible effect on the equilibrium outcomes despite holding some private information. Gul and Postlewaite (1992) formalize this idea in a natural way. They link the inefficiencies created by private information to the power agents' private information gives them. Under suitable conditions they show that when the economy is replicated, the incremental impact of each agent's information on the demand for each good decreases, in turn guaranteeing that allocations are asymptotically efficient. The recent work of McLean and Postlewaite (1999) develops a more general, formal notion of informational size measuring the extent to which an agent's information affects the ability to predict which state will occur. They show that competitive markets can achieve approximate efficiency provided agents are sufficiently informationally small. In a similar vein, Krasa and Shafer (1999) show that informational size in the sense defined by McLean and Postlewaite (1999) is critical to the robustness of equilibria in complete information economies to small amounts of incomplete information. Krasa and Shafer (1999) show that provided agents' informational size goes to zero along with the amount of incomplete information, equilibria in the incomplete information economy approximate equilibria in the economy with complete information.

[7]For related work, see Bisin and Guaitoli (1999), Kahn and Mookherjee (1998), and Magill and Quinzii (1996).

This work lays a foundation for pursuing a host of other important questions regarding the functioning of financial markets. Some of the most compelling among these involve the endogenous emergence of various institutions supporting credit markets and financial markets in the presence of asymmetric information. Rather than being exogenously fixed, there is significant competition among banks and credit providers for many of these services, for example. Under such competition what sort of institutions should we expect to emerge in equilibrium? What sort of institutions are optimal in these settings, and will competitive markets select such institutions as the result of strategic interactions between sufficiently large numbers of firms and consumers? These and related questions suggest there is a rich area at the nexus of equilibrium analysis, contract theory and finance ripe for further study.

3 Testable Implications of Competitive Markets

The major theoretical questions concerning competitive equilibria in the classical Arrow–Debreu model — existence, uniqueness, comparative statics, and stability of price adjustment processes — have been largely resolved over the last forty years. With the exception of existence, however, this resolution has been fundamentally negative. The conditions under which equilibria can be shown to be unique, comparative statics globally determinate or tâtonnement price adjustment globally stable are quite restrictive. Moreover, the Sonnenschein–Debreu–Mantel theorem shows in striking fashion that no behavior implied by individual utility maximization beyond homogeneity and Walras' Law is necessarily preserved by aggregation in market excess demand. This arbitrariness of excess demand implies that properties such as monotone equilibrium comparative statics and global stability of equilibria under tâtonnement will only result from the imposition of a limited set of conditions on preferences and endowments. Based on these results, many economists conclude that the general equilibrium model has no refutable implications or empirical content.[8]

[8]Hansen and Heckman (1996) give a recent argument along these lines.

No statement concerning refutable implications is meaningful without first specifying what information is observable and what is unobservable. If only market prices are observable, and all other information about the economy such as individual incomes, individual demands, individual endowments, individual preferences and aggregate endowment or aggregate consumption is unobservable, then indeed the general equilibrium model has no testable restrictions. This is essentially the content of Mas-Colell's version of the Sonnenschein–Debreu–Mantel theorem. Mas-Colell (1977) shows that given an arbitrary nonempty compact subset C of strictly positive prices in the simplex, there exists an economy $\mathcal{E}$ composed of consumers with continuous, monotone, strictly convex preferences such that the equilibrium price vectors for the economy $\mathcal{E}$ are given exactly by the set C.

In many instances, however, it is unreasonable to think that only market prices are observable; other information such as individual incomes and aggregate consumption may be observable in addition to market prices. Brown and Matzkin (1996) show that if such additional information is available, then the Arrow-Debreu model does have refutable implications. They demonstrate by example that with a finite number of observations — in fact two — on market prices, individual incomes and aggregate consumptions, the hypothesis that these data correspond to competitive equilibrium observations can be rejected. They also give conditions under which this hypothesis is accepted and there exists an economy rationalizing the observed data.

The starting point for the work of Brown and Matzkin is Afriat's work on rationalizing individual choice. Afriat (1967) poses the following problem. Given a finite number of observations $(p^r, x^r), r = 1, \ldots, N$, on prices and quantities, when is this data consistent with utility maximization by some consumer with a non-satiated utility function? Say that a utility function $U : \mathbf{R}^\ell_+ \to \mathbf{R}$ *rationalizes* the data $(p^r, x^r), r = 1, \ldots, N$, if for each observation r

$$p^r \cdot x^r \geq p^r \cdot x \Rightarrow U(x^r) \geq U(x) \quad \text{for each } x \in \mathbf{R}^\ell_+.$$

Using this terminology, the question above can be restated as follows: given a finite data set, when does there exist a non-satiated utility function which rationalizes these observations? The classic answer to this question was given by Afriat (1967). He shows that there exists a non-satiated utility function that rationalizes the data if and only if there exist solutions U^i and $\lambda^i > 0$

for $i = 1, ..., N$ solving the system of inequalities:

$$U^i - U^j \leq \lambda^j p^j \cdot (x^i - x^j), \quad i, j = 1, ..., N$$

Thinking of U^i as the utility associated with the bundle x^i and λ^i as the Lagrange multiplier associated with the choice problem given prices p^i, these inequalities can be easily understood as resulting from the corresponding first-order conditions x^i must satisfy to be optimal given prices p^i. Afriat (1967) shows in addition that this is equivalent to a condition on the data called "cyclical consistency", which Varian (1982) establishes to be equivalent to what he calls the Generalized Axiom of Revealed Preference (GARP). Thus the rationalizability of a given data set can be checked by directly verifying this condition in terms of the observable data alone. Finally, Afriat (1967) exhibits a particular utility function rationalizing the data when one of these equivalent conditions is satisfied,[9] and thereby demonstrates that there exists a non-satiated utility function rationalizing a given set of data only if in fact there exists a concave, monotone, and continuous, non-satiated utility function rationalizing this data. In particular, this observation implies that the hypothesis that preferences are represented by a concave utility function can never be refuted based on a finite data set, since if the data is rationalizable by any non-satiated utility function then it is rationalizable by a concave, monotone, and continuous one.

Brown and Matzkin build on this framework and develop an equilibrium version of Afriat's analysis. Consider a finite number of observations $\langle p^r, \omega^r, \{I_t^r\}_{t=1}^T \rangle$, $r = 1, \ldots, N$ of prices, aggregate consumption, and income levels for each of T consumers. When can these observations be rationalized as competitive equilibria? Since we do not observe utilities *or* individual consumption bundles, the question becomes whether it is possible to disaggregate aggregate consumption in a way consistent with individual rationality. Thus say these observations are *rationalizable as competitive equilibria* if for each observation $r = 1, \ldots, N$ there exist consumption bundles x_t^r for

[9] Afriat's construction is as follows: for each $x \in \mathbf{R}_+^\ell$, define

$$U(x) = \min_r \{U^r + \lambda^r p^r \cdot (x - x^r)\}.$$

This utility function is easily seen to be continuous, monotone and concave, and rationalizes the data by construction.

each consumer $t = 1, \ldots, T$ such that the individual observations (p^r, x_t^r), $r = 1, \ldots, N$, are rationalizable for each consumer, $p^r \cdot x_t^r = I_t^r$ for each r and t, and such that markets clear in each observation, that is, $\sum_{t=1}^{T} x_t^r = \omega^r$ for each r. Brown and Matzkin follow Afriat's construction to formulate equilibrium versions of Afriat's inequalities, and to derive an equilibrium version of GARP that provides necessary and sufficient conditions on the observable data equivalent to competitive equilibrium rationalizability.

Building on this work, Brown and Shannon (2000) considers the extent to which qualitative features of competitive equilibria are refutable given a finite data set. In particular, they consider the hypothesis that the observed data are competitive equilibria in which each price vector is locally stable under tâtonnement. Based on the Sonnenschein–Debreu–Mantel results and the well-known examples of global instability of tâtonnement such as Scarf's (1960), it may seem at first glance that this hypothesis will be easily refuted in the Arrow-Debreu model. Perhaps surprisingly, however, they show that it is not. Instead they show that a finite set of observations of prices, individual incomes and aggregate consumption vectors is rationalizable in an economy with smooth characteristics if and only if it is rationalizable in a distribution economy in which each observed price is locally stable under tâtonnement. Moreover, the equilibrium correspondence is locally monotone in a neighborhood of each observed equilibrium in these economies, and the equilibrium price vector is locally unique.[10]

Mirroring Afriat's results on individual rationalizability, rationalizable data is always rationalizable in an economy in which market excess demand has a very particular structure. Using recent results of Quah (1997), Brown and Shannon show that if the data is rationalizable then it is rationalizable in

[10]The conclusion that if the data is rationalizable then it is rationalizable in a distribution economy, i.e., an economy in which individual endowments are collinear, is easy to understand: If individual endowments are unobserved and only prices and income levels are observed, then one set of individual endowments consistent with this data is collinear, with shares given by the observed income distribution. Since distribution economies by definition have a price-independent income distribution, this observation may suggest that these results about stability and comparative statics derive simply from this fact. Kirman and Koch (1986) show that this intuition is false, however. They show that the additional assumption of a fixed income distribution places no restrictions on excess demand, hence any dynamic on the price simplex can be replicated by some distribution economy.

an economy in which each individual demand function is locally monotone at each observation.[11] The strong properties of local monotonicity, in particular the fact that local monotonicity of individual demand is preserved by aggregation in market excess demand and the fact that local monotonicity implies local stability in distribution economies, as well as local uniqueness and determinate local comparative statics, imply that if the data is rationalizable in a competitive setting, then it is rationalizable in an economy in which each observation is locally stable under tâtonnement, locally unique, and in which equilibrium comparative statics are locally monotone. Thus global instability, for example, while clearly a theoretical possibility in competitive markets, cannot be detected in a finite data set consisting of observations on prices, incomes, and aggregate consumption.

Chiappori, Ekeland, Kubler, and Polemarchakis (2000) consider a related question in the spirit of the original Sonnenschein-Debreu-Mantel analysis. Given knowledge of the excess demand function, as a function *both* of prices and of the profile of consumers' initial endowments, they ask whether there are any restrictions placed on the local structure of the equilibrium manifold, that is, they seek joint restrictions on prices and endowments derived from the fact that these prices must be equilibrium prices for the economy given this vector of initial endowments and this excess demand function. They find, contrary to the Sonnenschein-Debreu-Mantel conclusions, that there are significant restrictions on the local structure of the equilibrium manifold. They also show that knowledge of the equilibrium manifold suffices, for generic preferences, to identify and uniquely recover the individual preferences underlying the economy.[12]

These results raise a number of immediate questions. This work shows that a finite set of observations of prices, incomes, and aggregate consumption does not provide sufficient information to refute a number of hypotheses concerning local behavior of equilibria.[13] What data are needed to refute

[11] An individual demand function $f(p, I)$ is locally monotone at $(\bar{p}, \bar{I})$ if there exists a neighborhood $\mathcal{W}$ of $(\bar{p}, \bar{I})$ such that $(p - q) \cdot (f(p, I) - f(q, I)) < 0$ for all $(p, I), (q, I) \in \mathcal{W}$ such that $p \neq q$.

[12] Essentially, observing the entire excess demand function as a function of the profile of individual endowments means that individual income effects can be identified, from which generic preferences can be recovered.

[13] The same arguments show that this result is still true if in addition individual con-

such hypotheses? Clearly if consumers' preferences and initial endowments are observed any such hypotheses can be refuted since we can compute the market excess demand function generated by this economy – at least theoretically – and simply verify the properties of its equilibria. Equally clearly, it is unreasonable to assume that preferences are observable, and it is difficult to imagine observing consumers' entire vectors of initial endowments. More limited information regarding preferences and endowments may be available, however, such as the values of net trade vectors, that preferences are drawn from a certain class, or that consumers are endowed with certain goods but not others, even if the quantities of these endowments are unobservable. Such observations may provide sufficient information about preferences or the nature of wealth effects and the dependence of income on prices to refute some local properties of equilibria. A closely related question is whether *any* local properties of equilibria are refutable given a finite set of observations of prices, incomes, and aggregate consumption (or any other set of "reasonable" variables). Answering this question is important for understanding the potential explanatory or predictive power of the assumption of competitive behavior based on a finite set of observations. These results should prove important regardless of whether this conjecture turns out to be true or false: if false, then at least for some data sets the general equilibrium model imposes relatively strong local predictions, while if true this would represent in some sense the "right" version of the Sonnenschein-Debreu-Mantel theorem, at least from the standpoint of the empirical content of general equilibrium models.

Just as the theoretical analysis of the classical Arrow-Debreu model provides a benchmark for more complicated and realistic models of competitive markets, this work should be viewed primarily as a benchmark for building an empirical counterpart to general equilibrium theory, both because the analysis is cast within the framework of the classical Arrow-Debreu model and because it seeks exact refutable conditions. Because the implicit assumption is that there is no randomness in the observations or underlying model, no role is allowed for violations of these conditions arising solely from measurement error or some underlying stochastic shocks. Even in the absence of such an explicitly econometric analysis, these exact conditions can provide important guidelines regarding the scope for refutation and hypothesis testing in a

sumption choices are observed.

general equilibrium framework. Developing an econometric version of these tests is also important. Although Brown and Matzkin (1997) have made recent progress in this direction by studying the identification and estimation of a nonparametric random utility model, which could provide the basis for an econometric version of the equilibrium inequalities, much of interest remains to be done along these lines.

One of the least satisfactory features of the standard Arrow-Debreu model is its essentially static nature, and the same criticism applies to the work described above. The hypothetical data imagined observable above should really be thought of as cross-sectional observations from different locations with the same distribution of consumers' preferences but different income distributions. Although it is possible to interpret the data as a time series, the questions asked then become somewhat unnatural, since they seek a fixed set of preferences, invariant across periods, and a set of initial endowments and individual consumption bundles for each period such that the observations are competitive equilibria each period. In particular, this means markets clear and budgets balance exactly each period, which rules out any borrowing or lending across time or any role for financial markets. In several recent papers, Kubler (2000, 2002) addresses this criticism by studying the implications of competitive markets for fluctuations in macroeconomic time series such as commodity and asset prices and aggregate consumption, and to recast many of the questions considered above in an explicitly dynamic general equilibrium model.

Consider a standard multi-period exchange economy. The first important difference between static and dynamic general equilibrium models in this context is that any finite data set in which asset prices are arbitrage-free is rationalizable in a dynamic model with infinitely lived agents if we allow general intertemporal preferences (see, e.g., Harrison and Kreps (1979)). In a dynamic economy, a sequence of observations of prices, aggregate consumption, and income levels represents a single equilibrium rather than a sequence of equilibria, and it is trivial to find preferences for which a single observation is part of a competitive equilibrium. Any discussion of observable or testable restrictions in a dynamic context must then start with some restriction on the class of preferences.

Kubler (2002) restricts attention to time-separable expected utility and shows that such an economy generates observable restrictions on the joint process of asset prices, dividends, and aggregate endowments. He also shows that separability is crucial for this conclusion. Slight departures from separability seem to destroy the empirical content of the dynamic general equilibrium model. For example, he shows that it is enough that agents have recursive utility to impose virtually no restrictions on aggregate data.

This work suggests a number of further questions. First, by deriving analogues of the Afriat inequalities this work gives conditions which are necessary and sufficient for the data to be rationalizable, but it is unclear how operational these conditions are since they involve both observable and unobservable quantities. One of the most important conclusions from the work of Afriat (1967) and Varian (1982) is that the Afriat inequalities characterizing price-consumption pairs that are rationalizable as utility maximizing, which are stated in terms of observable price and consumption data and unobservable utility levels and marginal utilities of income, are equivalent to conditions stated solely in terms of the observable data. These conditions, Afriat's cyclical consistency and Varian's GARP, then provide an immediate means of verifying whether or not a given data set is rationalizable. Afriat's inequalities, and their equilibrium counterparts, theoretically serve the same function, since the observed data is rationalizable if and only if there exists a solution to these inequalities. Solving these inequalities, even for relatively few observations, may become computationally challenging, however.[14] It would thus be extremely desirable to derive instead an equilibrium version of GARP, as Brown and Matzkin (1996) do for the static Arrow-Debreu model, expressed only in terms of the observable data characterizing those observations which are rationalizable in a competitive equilibrium framework. Such a condition would be similar in spirit to the nonparametric volatility bounds Hansen and Jagannathan (1991) derive for asset prices consistent with competitive equilibrium.

Finally, the preponderance of observational equivalences among these models only enhances the importance of sensitivity analysis in these settings. When even in the most idealized setting hypothetical (or real) observations

[14]Specifically, when individual consumption is unobservable, these inequalities are polynomial in the unknowns. The Tarski-Seidenberg algorithm provides a finite algorithm for deciding the solvability of such a system, but in the worst case this algorithm is doubly exponential. See Brown and Matzkin (1996) for a further discussion of this point.

cannot distinguish between various models, any measurement of the welfare costs and benefits of policies relies crucially on being able to assess as accurately as possible how sensitive model predictions are to features of the model that can never be determined from a finite data set (see Hansen and Heckman (1996) for more on this argument). Some of these issues, further challenges for theory, are discussed in more detail in the following section.

4 Markets for Many Commodities

One of the major advances in understanding competitive markets over the last 15 years has been the extensive work on equilibrium analysis in economies with infinitely many goods. Motivated by a host of economic issues in dynamic choice, choice under uncertainty, continuous-time trading in financial markets, growth and innovation and commodity differentiation, this work has uncovered the sometimes subtle ways these economies differ from standard finite models. This work also provides deeper insight into large finite markets. As an illustrative example along these lines, borrowed from Anderson and Zame, consider a model of labor supply and human capital in which each agent is endowed with his own labor, which is differentiated from the labor of every other agent. While the labor of some agents may be close substitutes in performing some tasks, the labor of others may be drastically different. This feature is difficult to express in a finite model in which each agent's labor is a distinct commodity, yet naturally modeled in an appropriate infinite-dimensional model of differentiated commodities. Aliprantis, Brown, and Burkinshaw (1989), Mas-Colell and Zame (1991), and Jones (1992) survey much of this work and provide cogent and insightful discussions of some of the sometimes delicate issues surrounding equilibrium analysis in these settings.

This work studied issues surrounding the existence and welfare properties of equilibria extensively, but left open many important questions regarding the qualitative properties of equilibria. Part of the value of the Arrow-Debreu model derives from its role as a benchmark for describing competitive markets. The value of this benchmark can be seen in part through the fact that the Arrow-Debreu model passes a variety of compelling tests of the notion of perfect competition it is designed to approximate as the number of agents becomes sufficiently large. These include Edgeworth's classic equivalence between the core and the set of competitive equilibrium allocations, the correspondence between competitive equilibrium outcomes and non-cooperative solutions such as Cournot equilibrium (e.g. Novshek and Sonnenschein (1978)), and the no-surplus condition of Ostroy (1980). Essentially these results flow from the fact that as the number of agents grows, no agent retains monopoly or monopsony power in classical exchange problems.

Anderson and Zame (1997) investigate the validity of one such test in markets with infinitely many goods, Edgeworth's conjecture that core allocations approximate competitive equilibria as the number of agents grows sufficiently large. Their work reveals a much subtler connection between the number of agents and the retention of monopoly or monopsony power in the presence of many markets. Earlier work by Aliprantis, Brown, and Burkinshaw (1987) had established a general version of the Debreu-Scarf theorem for economies with infinitely many commodities, which shows that core allocations converge to competitive equilibrium allocations as a fixed finite-agent economy is replicated sufficiently often. It may seem that whether the number of agents grows by replicating a fixed set of agents or through an arbitrary sequence is largely a technical issue and should not make an important difference in the limiting behavior of core allocations.[15] Anderson and Zame lend some credence to this intuition by giving a general version of Edgeworth's conjecture for this setting. At the same time they give robust examples in which this difference is important economically and has critical implications for the validity of Edgeworth's conjecture.[16] For example, agents' endowments may be drawn from a bounded set yet each agent's endowment may be uniformly bounded away from every other agent's endowment. This is impossible if there are only finitely many commodities, but not if there are infinitely many. In such an example, each agent may retain monopoly power regardless of the number of agents in the economy, and core allocations may fail to be approximately competitive. If instead endowments are drawn from a compact set, only finitely many agents can retain any monopoly power as

[15] Roughly this is the case for models with finitely many goods.

[16] More precisely, their examples illustrate the restrictiveness of the requisite compactness assumptions on the sequence of allocations and agents' characteristics in infinite-dimensional economies, which are satisfied trivially both in the replica case and in the finite-dimensional case for arbitrary sequences.

the number of agents increases, which will not prevent core allocations from being approximately competitive. Anderson and Zame discuss this and other examples in more detail.

A second argument in favor of the role of the Arrow-Debreu model as a benchmark for perfect competition is it predictive power. In a fundamental sense, the Arrow-Debreu model yields sharp predictions: almost all smooth economies have only finitely many equilibria, and perhaps more importantly, comparative statics in such regular economies are locally determinate, since equilibria can be expressed locally as smooth functions of underlying exogenous parameters. Furthermore, the restrictions on preferences and production technologies needed to generate a smooth economy are relatively mild. For example, if preferences in an exchange economy are smooth in the sense of Debreu (1972), then the resulting economy is smooth and equilibria are generically determinate. The question of determinacy must be resolved if these models are to provide useful or reliable predictions about equilibrium behavior. Indeed, if equilibria are indeterminate, then slight changes in exogenous parameters in the economy or small amounts of measurement error can lead to large changes in equilibria, resulting in equilibria which may then have vastly different properties from the original prediction, invalidating any comparative statics results or policy analysis.

A straightforward extension of Debreu's (1970) seminal work on determinacy in economies with finitely many goods is impossible, however, for a number of reasons. Perhaps the most obvious reason is that in infinite-dimensional commodity spaces demand fails to exist for most prices because budget sets are unbounded. Provided there are finitely many consumers and producers and no market distortions, as in the classical Arrow-Debreu economy, using the welfare theorems and Negishi's (1960) argument provides an alternative characterization of competitive equilibria as the Pareto optimal allocations and supporting prices at which each consumer's budget constraint is exactly satisfied. This characterization of equilibria is independent from the notions of demand or excess demand, and thus provides the framework for much of the existing equilibrium analysis in economies with infinitely many commodities.

Following along these lines, Kehoe and Levine (1985) pioneered an approach to determinacy in the infinite-dimensional setting that relies on the

Negishi characterization of equilibrium as a zero of the excess spending map. Because they consider consumption over a discrete infinite horizon and assume that utility functions are additively separable across time, they can decompose the infinite-dimensional planner's problem into an independent sequence of finite-dimensional planning problems. Because they assume period utility functions are differentiably strictly concave and satisfy Inada conditions, the solutions to these finite-dimensional planning problems are smooth. From this it follows that the excess spending map is smooth, so the machinery of smooth analysis again applies to yield generic determinacy in a fairly straightforward fashion.

Much of the relatively small body of existing work on determinacy in infinite-dimensional models adopts both Kehoe and Levine's approach of using the excess spending map and their assumption of additively separable preferences (see Kehoe, Levine, and Romer (1990), Balasko (1997), and Chichilnisky and Zhou (1998)). Additive separability is clearly economically restrictive, ruling out habit formation, any disentangling of risk aversion and intertemporal substitution, or interpretation of nearby characteristics as close substitutes for example, yet it is crucial for their results.

Simply replacing excess demand with the excess savings equations arising from Negishi's approach still does not lead immediately to a satisfactory theory of determinacy in infinite-dimensional models. In many of these settings, standard assumptions of convexity and continuity of preferences do not guarantee the existence of prices supporting Pareto optimal allocations, and thus the validity of the second welfare theorem and the existence of competitive equilibria. Additional restrictions on preferences and production sets are then necessary for conclusions regarding the determinacy of equilibria to be non-vacuous, as Mas-Colell's (1986) seminal work demonstrates. These assumptions have the effect of bounding consumers' marginal rates of substitution, so are often incompatible with the survival conditions or Inada conditions underlying Debreu's analysis. This observation is important for determinacy analysis because when consumers can shift from consuming a good to not consuming it in response to changes in prices or welfare weights or other parameters, consumption will not vary smoothly with these parameters but will exhibit kinks.[17] Studying determinacy in economies with infinitely

[17]See Shannon (1999) for an example illustrating this point.

many commodities will then require a different mode of analysis, one not predicated on the differential techniques pioneered by Debreu (1970). To address this problem, Shannon (1999) introduces techniques from nonsmooth analysis and demonstrates that Lipschitz continuity of the excess spending map is sufficient to yield generic determinacy.

The usefulness of these intermediate results as building blocks for a theory of determinacy in infinite-dimensional models then relies on finding conditions on the economic primitives – preferences, endowments, and production sets – under which they apply, which raises a more serious substantive question. When goods are perfect substitutes, there are robust indeterminacies in the set of equilibria: over a non-empty open set of endowments the economy has a continuum of equilibria, which are typically extremely responsive to small perturbations in the economy. With a finite number of goods, this is essentially the only situation in which robust indeterminacies can arise. As long as goods are never perfect substitutes locally, formalized by the condition of strict differential concavity, then equilibria are generically determinate. Intuitively, an analogous condition ruling out local perfect substitutes by requiring indifference curves to have positive curvature or imposing some stronger form of concavity should be required to guarantee that equilibria are generically determinate when there are infinitely many goods. The difficulty in verifying this simple intuition lies in defining such a restriction strong enough to prevent robust indeterminacies yet consistent with the other assumptions necessary even for the existence of equilibria, such as uniform properness and continuity. To see this point in a simple discrete-time, infinite-horizon example, consider additively separable preferences of the form $U(x) = \sum \beta^t u(x_t)$, where x_t is consumption in period t, $u : \mathbf{R}_+ \to \mathbf{R}$ is the smooth utility function for consumption, and $\beta \in (0,1)$ is the discount factor. For such preferences, for any consumption plan x such that $x_t > 0$ for each t, $\frac{\partial^2 U}{\partial x_t^2}(x) = \beta^t u''(x_t)$, which goes to 0 as $t \to \infty$ even if $u''(x_t)$ is uniformly bounded away from 0.

The problem then becomes one of disentangling discounting or myopia from the underlying subsitutability between goods. For a simple example in which this problem is apparent, taken from Shannon (1999), consider a discrete-time infinite horizon exchange economy with two consumers in which two goods are available each period. Suppose consumers have identical preferences represented by the utility function

$$U(x_i, y_i) = \sum_{t=1}^{\infty} \beta^t [x_i(t)^{\frac{t-1}{t}} + y_i(t)^{\frac{t-1}{t}}]$$

and initial endowments $(\bar{x}_1, \bar{y}_1), (\bar{x}_2, \bar{y}_2)$ such that $\bar{x}_1(t)+\bar{x}_2(t) = 1$ and $\bar{y}_1(t)+\bar{y}_2(t) = 1$ for each t. In this example it is relatively straightforward to show that although there is a unique equilibrium for every such specification of initial endowments, this allocation is extremely sensitive to changes in the initial endowments.

This robust indeterminacy arises because the goods in this economy are quite literally asymptotic perfect substitutes. This asymptotic substitutability and the indeterminacy it generates do not arise merely because the second derivative is not uniformly bounded away from 0: if instead consumers have utility functions of the form

$$U_i(x, y) = \sum_{t=1}^{\infty} \beta^t \left(\sqrt{x_i(t)} + \sqrt{y_i(t)}\right)$$

then the equilibria are generically determinate, as the results of Shannon (1999) verify, even though "second derivatives" are still not uniformly bounded away from 0.[18] Distinguishing between these two cases, and more generally between determinacy and indeterminacy, must rely on more subtle calculations of curvature involving comparisons between first and second derivatives, even in such simple examples.

Thus one of the major contributions of this paper is to develop a method for measuring curvature in infinite-dimensional economies independent of impatience or myopia. This measure of curvature suggests a natural notion of differential concavity, called uniform concavity, that prohibits goods from becoming perfect substitutes asymptotically. The paper then focuses on economies in which there are countably many commodities, such as discrete time models or markets with countably many assets, and builds on the intuition that these economies can be thought of as the limit of economies

[18] A sufficient condition for generic determinacy in this example is that $U_i(x, y) = \sum_{t=1}^{\infty} \beta^t u_{i_t}(x(t), y(t))$ where u_{i_t} is C^2 and $[D^2 u_{i_t}(c_1, c_2)]^{-1} D u_{i_t}(c_1, c_2)$ is uniformly bounded on bounded sets. See Shannon (1999) for details.

with a large finite number of commodities, suggested by Bewley's (1972) seminal work. The main results of the paper show that the sharp predictions of generic determinacy in economies with finitely many commodities carry over to economies with countably many commodities as long as goods are not perfect substitutes asymptotically.

Shannon (1999) thus gives the first determinacy results in infinite dimensions applicable to a broad class of non-separable preferences. The method Shannon (1999) uses to analyze the planner's problem involves approximation by increasing finite truncations of the commodity space, however, so the results apply only if the number of commodities is countable. Shannon (1999) does not apply to a variety of important commodity spaces, including those that arise in continuous-time models of finance, which have proved to be among the most useful and successful applications of general equilibrium theory, or in models of commodity differentiation. Shannon and Zame (2002) build on this analysis and develop a framework that is sufficiently general to encompass most of the models that have proved important in the study of continuous-time trading in financial markets, trading over an infinite time horizon, and trading of finely differentiated commodities.

Shannon and Zame introduce an alternative set of simple and natural restrictions on utility functions that generalize Debreu's differentiable strict concavity to the infinite-dimensional setting. The most important of these restrictions is a condition called *quadratic concavity*, which requires that near any feasible bundle, utility differs from the linear approximation by an amount that is at least quadratic in the distance to the given bundle. Quadratic concavity provides a quantitative measure of the extent to which distinct commodities are not perfect substitutes — globally, locally, or asymptotically. They use quadratic concavity to give a simple geometric argument showing that the excess spending mapping is Lipschitz. Generic determinacy then follows by arguments similar to those in Shannon (1999). The direct, geometric nature of these arguments means they require neither a countable number of commodities nor separability of preferences. These methods also make it possible to encompass the infinite-dimensional genericity notion developed by Anderson and Zame (2001), Christensen (1974) and Hunt, Sauer, and Yorke (1992). They also discuss several illustrative examples, including models of continuous-time trading, trading in differentiated commodities, and trading over an infinite horizon.

A common thread in almost all of this work is its reliance on frictionless markets in which the welfare theorems hold. This is in part a by-product of some of the technical intricacies of the infinite-dimensional framework. Many of the most important extensions of the competitive framework, including understanding incomplete financial markets and asymmetric information, result in breakdowns in the efficiency of competitive markets. Similarly, promising and challenging research directions for future work involve developing systematic methods for equilibrium analysis with infinitely many goods in the presence of market distortions.

5 Concluding Remarks

This article has given a brief overview of recent developments in the theory of competitive markets. Many of these results lie at the nexus of finance, macroeconomics and theory, which are rich areas for future work that present challenges both for theoretical and applied research. I have tried merely to provide a window into a broad set of topics, with the hope that it will stimulate much further investigation of what lies behind these and related ideas.

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