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# SUPPLEMENT TO “FRAMING CONTINGENCIES” (*Econometrica*, Vol. 78, No. 2, March 2010, 655–695)

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This online supplement provides additional material to accompany “Framing Contingencies” by Ahn and Ergin. We characterize monotonicity, subadditivity, and superadditivity of the support function. We connect the case where the available partitions constitute a filtration and the case where all partitions are available. We define a local notion of transparency relative to a description of the state space. This local notion has desirable epistemic properties and naturally extends the structural properties of global transparency.

## S.1. FEATURES OF THE SUPPORT FUNCTION

WE NOW DISCUSS some functional features of $\nu$. Throughout this section, let $\Pi = \Pi^*$, that is, assume all descriptions are part of the model.

### S.1.1. *Monotonicity*

Monotonicity (in the set containment order) of the support function might seem natural. Nonetheless, psychological experiments repeatedly demonstrate violations.[1] Such violations suggest that a particularly likely or salient subcontingency is overlooked unless explicitly mentioned. When the set function $\nu$ is unique up to a scalar multiple, as characterized in Theorem 4 of the main paper (henceforth AE), the following condition guarantees that $\nu$ is monotone.

AXIOM S.1—Monotonicity: *For all $E \subset F$ and $p, q, r, s \in \Delta X$ such that $p \succ q$,*

$$s \succsim \begin{pmatrix} p & F \\ q & F^{\complement} \end{pmatrix} \implies \begin{pmatrix} r & E \\ s & E^{\complement} \end{pmatrix} \succsim \begin{pmatrix} r & E \\ p & F \setminus E \\ q & F^{\complement} \end{pmatrix}.$$

The preference on the left reflects the relative likelihood of $F$ versus $F^{\complement}$, in particular, a willingness to trade the bet on $F$ for some lottery $s$. The preference on the right reveals that the relative likelihood of $F \setminus E$ versus $F^{\complement}$ conditional on $E^{\complement}$ cannot be larger, since the decision maker is still willing to trade the bet on $F$ for the lottery $s$ conditional on $E^{\complement}$.[2] So the support of $F \setminus E$ cannot exceed that of $F$. The following straightforward equivalence is stated without proof.

[1]Two violations are mentioned in AE: The Linda problem of Footnote 8 and the judged frequency of seven-letter words ending with `ing` versus having `n` as the sixth letter, which was mentioned in Example 3.

[2]For example, in the Linda problem, subjects thought Linda was more likely to be a feminist librarian than a librarian. This behavior is excluded because the likelihood ratio of librarian to nonlibrarian must be larger than the likelihood ratio of feminist librarian to nonlibrarian.

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PROPOSITION S.1: *Suppose* $\{\succsim_\pi\}_{\pi\in\Pi^*}$ *admits a unique PDEU representation* $(u,\nu)$. *Then* $\{\succsim_\pi\}_{\pi\in\Pi^*}$ *satisfies monotonicity if and only if* $E\subset F\subsetneq S$ *implies* $\nu(E)\le\nu(F)$.[3]

### S.1.2. *Underpacking and Overpacking*

Subadditivity or superadditivity of the set function $\nu$ determines whether likelihood increases or decreases as an event becomes more finely described.

DEFINITION S.1: A set function $\nu$ is *subadditive* [*superadditive*] if $\nu(E\cup F)\le$ $[\ge]\nu(E)+\nu(F)$ whenever $E\cap F=\emptyset$ and $E\cup F\ne S$.[4]

Tversky and Koehler (1994) originally endorsed subadditivity of the support function, but more recently Sloman, Rottenstreich, Wisniewski, Hadjichristidis, and Fox (2004) presented experimental cases of superadditivity, where explicitly mentioning atypical or unlikely contingencies decreased the subjective probability of an event. Without taking a prior position, either case can be behaviorally verified by examining whether the certainty equivalent for a bet on an event is increasing or decreasing in the fineness of the event's description. The behavioral characterization is again straightforward and its proof is omitted.

DEFINITION S.2: $\{\succsim_\pi\}_{\pi\in\Pi^*}$ *underpacks* [overpacks] if, for any nonempty event $E$, $\pi'_E\in\Pi^*_E$, $p,q,r\in\Delta X$ with $q\succ r$, and $\pi$ with $E\in\pi$ implies

$$\begin{pmatrix} q & E\\ r & E^{\complement}\end{pmatrix}\succsim_\pi[\precsim_\pi]p\quad\Longrightarrow\quad\begin{pmatrix} q & E\\ r & E^{\complement}\end{pmatrix}\succsim_{\pi\vee\pi'_E}[\precsim_{\pi\vee\pi'_E}]p.$$

PROPOSITION S.2: *Suppose* $\{\succsim_\pi\}_{\pi\in\Pi^*}$ *admits a PDEU representation* $(u,\nu)$ *and satisfies strict admissibility. Then* $\{\succsim_\pi\}_{\pi\in\Pi^*}$ *underpacks* [overpacks] *if and only if* $\nu$ *is subadditive* [superadditive].

## S.2. FILTRATION INVARIANCE

AE considers two structural assumptions on the behavioral data available to the analyst. In the first, only progressively finer descriptions can be presented.

[3]Event reachability is indispensable in Proposition S.1. In general, there could exist one representation where $\nu$ is monotone, but another where $\nu'$ is not. This is because without uniqueness of $\nu$, the likelihood ratio of $\nu(E)/\nu(F)$ when $E\subset F$ is not fixed and can be allowed to be larger than 1. Without event reachability, we can only conclude that all subevents of null events are null.

[4]Note that superadditivity is strictly weaker than convexity: a set function $\nu$ is convex if $\nu(E\cup F)+\nu(E\cap F)\ge\nu(E)+\nu(F)$ for all $E,F\subset S$. Convexity is commonly assumed for value functions in cooperative games or for capacities in Choquet integration, but carries little behavioral significance beyond the implied superadditivity in our model of framing.

In the second, all descriptions can be presented in any order. Here, we provide a theoretical connection between the two cases. This connection yields two converse insights.

First, a potential concern with the filtration case is that the relative support of two events might depend on the particular filtration $\Pi$.[5] This concern begs what counterfactual assumption about the unobserved descriptions outside of $\Pi$ is required to eliminate this sensitivity. The required assumption is exactly binary bet acyclicity. Some experimental papers attempt to estimate support from laboratory choices. Our point is that to take estimates seriously, one must accept binary bet acyclicity on unobserved choices in counterfactual elicitations.

Second, it provides behavioral insight on the representation in the rich case. PDEU demands two kinds of invariance: the invariance implied by the sure-thing principle and an invariance to the filtration used to present the partitions. For some, the latter invariance might be more compelling or intuitive than binary bet acyclicity.

For any $\Pi \subset \Pi^*$, let $\mathcal{C}_\Pi = \bigcup_{\pi \in \Pi} \pi$ denote the collection of cells of partitions of $\Pi$.

ASSUMPTION S.1: *$\{\succsim_\pi\}_{\pi\in\Pi^*}$ satisfies strict admissibility and there exists a nonconstant affine vNM utility function $u : \Delta X \to \mathbb{R}$ and a collection of support functions $\nu_\Pi : \mathcal{C}_\Pi \to \mathbb{R}_+$ such that, for every filtration $\Pi \subset \Pi^*$, $(u, \nu_\Pi)$ is a PDEU representation of $\{\succsim_\pi\}_{\pi\in\Pi}$.*

The existence of the $\nu_\Pi$ in Assumption S.1 follows if the sure-thing principle is imposed on all filtrations. We write a filtration $\Pi = \{\pi_t\}_{t=0}^T$ is *gradual* if $\pi_t \cap \pi_{t+1} \neq \emptyset$ for all $t = 1, \dots, T-1$.[6]

AXIOM S.2—Filtration Invariance: *For all gradual filtrations $\Pi$ and $\Pi'$, there exists $\alpha > 0$ such that $\nu_\Pi(E) = \alpha \nu_{\Pi'}(E)$ for all $E \in (\mathcal{C}_\Pi \cap \mathcal{C}_{\Pi'}) \setminus \{S\}$.*

That is, the likelihood ratio $\nu_\Pi(E)/\nu_\Pi(F)$ of $E$ to $F$ does not depend on the filtration $\Pi$ used for elicitation, since the resulting supports will differ only by a shared coefficient $\alpha > 0$. Note that we require filtration invariance only on gradual filtrations because the PDEU representation of a nongradual filtration is not unique. Given Assumption S.1, the following theorem shows that filtration invariance is equivalent to binary bet acyclicity.

THEOREM S.1: *Suppose $\{\succsim_\pi\}_{\pi\in\Pi^*}$ satisfies Assumption S.1. Then the following statements are equivalent:*

[5]We thank a referee for pointing this out.

[6]Note that given a $\{\succsim_\pi\}_{\pi\in\Pi^*}$ satisfying strict admissibility, the filtration $\Pi$ is gradual if and only if it is gradual with respect to $\{\succsim_\pi\}_{\pi\in\Pi}$ in the sense of Definition 6 in AE.

(i) $\{\succsim_\pi\}_{\pi\in\Pi^*}$ *satisfies filtration invariance*.
(ii) $\{\succsim_\pi\}_{\pi\in\Pi^*}$ *satisfies binary bet acyclicity*.
(iii) $\{\succsim_\pi\}_{\pi\in\Pi^*}$ *admits a PDEU representation*.

For the proof, see the Appendix.

## S.3. LOCAL TRANSPARENCY

The decision maker's understanding of the state space depends on how it is described. We now characterize how understanding changes over descriptions. This involves defining an operator that takes partitions to families of events.

DEFINITION S.3: Fix $\{\succsim_\pi\}_{\pi\in\Pi^*}$ and $\pi\in\Pi^*$. An event $A$ is *transparent at* $\pi$ if for any $\rho\in\Pi^*$ such that $\rho\geq\pi$ and for any $f,g\in\mathcal{F}_\rho$,

$$f\succsim_\rho g \quad\Longleftrightarrow\quad f\succsim_{\rho\vee\{A,A^\complement\}} g.$$

Let $\mathbf{A}(\pi)$ denote the family of all transparent events at $\pi$.

If an event $A$ is understood when $\pi$ is described, then mentioning $A$ certainly should not reverse preferences $\succsim_\pi$, that is, $f\succsim_\pi g$ if and only if $f\succsim_{\pi\vee\{A,A^\complement\}} g$. However, this condition is too weak, because the restriction to $\succsim_\pi$ ignores all acts that are not $\pi$-measurable. For example, using this criterion, every event would be understood at the vacuous description $\{S\}$, because no nontrivial acts can be expressed! So, we also require that no preference is reversed for any description $\rho$ that is finer than $\pi$. If mentioning $A$ reverses a preference at a finer $\rho$, then the fact it does not reverse preference at $\pi$ is a coincidence, not a function of the decision maker's understanding of $A$.[7] Using this stronger notion, $\mathbf{A}(\{S\})=\mathcal{A}$. The following proposition summarizes general properties of the operator $\mathbf{A}:\Pi^*\to 2^S$ without assuming a PDEU representation for $\{\succsim_\pi\}_{\pi\in\Pi^*}$.

PROPOSITION S.3: *For any* $\{\succsim_\pi\}_{\pi\in\Pi^*}$, $\mathbf{A}(\pi)$ *satisfies the following properties*:
(i) *Reflexivity*: $\pi\subset\mathbf{A}(\pi)$, *for all* $\pi\in\Pi^*$.
(ii) *Monotonicity*: $\pi\leq\pi'$ *implies* $\mathbf{A}(\pi)\subset\mathbf{A}(\pi')$ *for all* $\pi,\pi'\in\Pi^*$.
(iii) *Positive introspection*: $\pi'\subset\mathbf{A}(\pi)$ *implies* $\mathbf{A}(\pi')\subset\mathbf{A}(\pi)$ *for all* $\pi,\pi'\in\Pi^*$.

[7]For example, consider the principle of insufficient reason over $S=\{a,b,c,d\}$. At $\pi=\{\{a,b\},\{c,d\}\}$, the weights of $\{a,b\}$ and $\{c,d\}$ are equal. Now, suppose $\{b,c\}$ is mentioned. Then the new description is $\pi\vee\{\{b,c\},\{a,d\}\}=\{\{a\},\{b\},\{c\},\{d\}\}$, and the weights of $\{a,b\}$ and $\{c,d\}$ stay equal. The decision maker had not been aware of $\{b,c\}$ at $\pi$, but coincidentally had each cell split equally by mentioning $\{b,c\}$. Now consider the partition $\pi'=\{\{a\},\{b\},\{c,d\}\}$, where $\{a,b\}$ is twice as likely as $\{c,d\}$. Then mentioning $\{b,c\}$ will split only the $\{c,d\}$ cell and change the likelihood ratio so $\{a,b\}$ and $\{c,d\}$ are equally likely.

PROOF: Reflexivity and monotonicity of $\mathbf{A}$ are immediate. We only prove that $\mathbf{A}$ satisfies positive introspection. Let $\pi' = \{E_1, \dots, E_n\} \subset \mathbf{A}(\pi)$. Let $\rho \in \Pi^*$, $f, g \in \mathcal{F}_\rho$, and $\rho \geq \pi$. For any $i = 1, \dots, n-1$, we have $E_i \in \mathbf{A}(\pi)$, $\rho \vee \{E_1, E_1^{\mathsf{C}}\} \vee \cdots \vee \{E_{i-1}, E_{i-1}^{\mathsf{C}}\} \geq \pi$, and $f, g \in \mathcal{F}_{\rho \vee \{E_1, E_1^{\mathsf{C}}\} \vee \cdots \vee \{E_{i-1}, E_{i-1}^{\mathsf{C}}\}}$. Therefore,

$$f \succsim_{\rho \vee \{E_1, E_1^{\mathsf{C}}\} \vee \cdots \vee \{E_{i-1}, E_{i-1}^{\mathsf{C}}\}} g \iff f \succsim_{\rho \vee \{E_1, E_1^{\mathsf{C}}\} \vee \cdots \vee \{E_{i-1}, E_{i-1}^{\mathsf{C}}\} \vee \{E_i, E_i^{\mathsf{C}}\}} g.$$

Since the above equation holds for $i = 1, \dots, n-1$, we have

$$\text{(S.1)} \qquad f \succsim_\rho g \iff f \succsim_{\rho \vee \{E_1, E_1^{\mathsf{C}}\} \vee \cdots \vee \{E_n, E_n^{\mathsf{C}}\}} g \iff f \succsim_{\rho \vee \pi'} g.$$

Take any $A \in \mathbf{A}(\pi')$. Let $\rho \in \Pi^*$, $f, g \in \mathcal{F}_\rho$, and $\rho \geq \pi$. Then

$$\begin{aligned} f \succsim_\rho g &\iff f \succsim_{\rho \vee \pi'} g \iff f \succsim_{\rho \vee \{A, A^{\mathsf{C}}\} \vee \pi'} g \\ &\iff f \succsim_{\rho \vee \{A, A^{\mathsf{C}}\}} g, \end{aligned}$$

where the first and last equivalences follow from Equation (S.1), the middle equivalence follows from $A \in \mathcal{A}(\pi')$, and $\rho \vee \pi' \geq \pi'$. Therefore, $A \in \mathcal{A}(\pi)$, which proves positive introspection. *Q.E.D.*

The next proposition generalizes the properties of $\mathcal{A}^*$ to the operator $\mathcal{A}$ when $\{\succsim_\pi\}_{\pi \in \Pi^*}$ has a PDEU representation.

PROPOSITION S.4: *Assume $\{\succsim_\pi\}_{\pi \in \Pi^*}$ satisfies strict admissibility and admits a PDEU representation $(u, \nu)$. Then for any $\pi \in \Pi^*$, the following statements hold*:

(i) *$A \in \mathbf{A}(\pi)$ if and only if $\nu(E) = \nu(E \cap A) + \nu(E \cap A^{\mathsf{C}})$ for all events $E$ such that $E \neq S$ and $E \subset F$ for some $F \in \pi$.*

(ii) *$\mathbf{A}(\pi)$ is an algebra.*

(iii) *Let $F \in \pi$. Then $\nu$ is additive on $\mathbf{A}(\pi) \setminus \{S\}$ restricted to $F$, that is, for all disjoint $A, B \in \mathbf{A}(\pi)$ such that $A, B \subset F$ and $A \cup B \neq S$,*

$$\nu(A \cup B) = \nu(A) + \nu(B).$$

PROOF: To see the $\Rightarrow$ part of (i), assume that $A \in \mathbf{A}(\pi)$, $E \neq S$, and $E \subset F$ for some $F \in \pi$. Assume without loss of generality that $E \neq \emptyset$. Define $\rho \in \Pi^*$ by

$$\rho = \{E, (F \setminus E) \cap A, (F \setminus E) \cap A^{\mathsf{C}}\} \cup \left( \bigcup_{G \in \pi: G \neq F} \{G \cap A, G \cap A^{\mathsf{C}}\} \right).$$

Then $\rho \geq \pi$. Since $E \neq S$, by strict admissibility,

$$\begin{aligned} \kappa := {} & \nu((F \setminus E) \cap A) + \nu((F \setminus E) \cap A^{\mathsf{C}}) \\ & + \sum_{G \in \pi: G \neq F} (\nu(G \cap A) + \nu(G \cap A^{\mathsf{C}})) > 0. \end{aligned}$$

Assume without loss of generality that $[0,1] \subset u(\Delta X)$ and let $p, q, r \in \Delta X$ be such that $u(p) = 1$, $u(q) = 0$, and

$$u(r) = \frac{\nu(E)}{\nu(E) + \kappa}. \tag{S.2}$$

Define the act $f$ by

$$f = \begin{pmatrix} p & E \\ q & E^{\mathsf{C}} \end{pmatrix}.$$

Then $f \in \mathcal{F}_\rho$ and $f \sim_\rho r$. Since $A \in \mathbf{A}(\pi)$, we have $f \sim_{\rho \vee \{A, A^{\mathsf{C}}\}} r$, implying

$$u(r) = \frac{\nu(E \cap A) + \nu(E \cap A^{\mathsf{C}})}{\nu(E \cap A) + \nu(E \cap A^{\mathsf{C}}) + \kappa}. \tag{S.3}$$

By Equations (S.2), (S.3), and $\kappa > 0$, we conclude that $\nu(E) = \nu(E \cap A) + \nu(E \cap A^{\mathsf{C}})$.

The proofs of the "$\Leftarrow$" part of (i), (ii), and (iii) can be straightforwardly adapted from the proof of Proposition 2 in AE. *Q.E.D.*

To see why defining transparency across partitions is potentially useful, suppose $\pi' > \pi$ and $\mathbf{A}(\pi') = \sigma(\mathbf{A}(\pi) \cup \pi')$, the coarsest algebra including $\mathbf{A}(\pi)$ and $\pi'$. Then mentioning the cells of $\pi'$ only provides the immediate description in $\pi'$ for the agent's subjective understanding. On the other hand, one might imagine that mentioning $\pi'$ might induce inferences by the decision maker, which helps her understand events that are not explicitly mentioned in $\pi'$. More specifically, suppose $\mathbf{A}(\pi')$ is strictly larger than the algebra induced by $\mathbf{A}(\pi)$ and $\pi'$. Then, as reflected in the "jump" at $\mathbf{A}(\pi')$, mentioning $\pi'$ induces additional understanding beyond its own description. For example, when asked to list surgeries, a doctor might forget to recall laminotomies, but when asked to list spinal surgeries, she might remember laminotomies, being primed by the more detailed request. In this case, "spinal surgery" triggers the recall of the more specific "laminotomy."

## APPENDIX

PROOF OF THEOREM S.1: We first show that Assumption S.1 implies that $\{\succsim_\pi\}_{\pi \in \Pi^*}$ satisfies the Anscombe–Aumann axioms and the sure-thing principle. Note that for each $\pi \in \Pi^*$, the PDEU representation of $\{\succsim_\pi\}_{\pi \in \Pi}$ for the filtration $\Pi = \{\{S\}, \pi\}$ implies that $\succsim_\pi$ has an expected utility representation $(u, \mu_\pi)$ in the sense of Lemma 1 of AE. Therefore, $\{\succsim_\pi\}_{\pi \in \Pi^*}$ satisfies the Anscombe–Aumann axioms (state independence follows from $u$ being common for all $\pi$).

To see that $\{\succsim_\pi\}_{\pi\in\Pi^*}$ satisfies the sure-thing principle, take any event $E$ and acts $f$, $g$, $h$, and $h'$. Define the acts $\hat{f}$, $\hat{g}$, $\hat{f}'$, and $\hat{g}'$ by

$$\hat{f} = \begin{pmatrix} f & E \\ h & E^{\mathsf{C}} \end{pmatrix}, \quad \hat{g} = \begin{pmatrix} g & E \\ h & E^{\mathsf{C}} \end{pmatrix},$$
$$\hat{f}' = \begin{pmatrix} f & E \\ h' & E^{\mathsf{C}} \end{pmatrix}, \quad \hat{g}' = \begin{pmatrix} g & E \\ h' & E^{\mathsf{C}} \end{pmatrix}.$$

Let $\pi = \pi(\hat{f}, \hat{g})$ and $\pi' = \pi(\hat{f}', \hat{g}')$, where the binary operator $\pi(\cdot, \cdot)$ yields the partition generated by the respective acts in $\Pi^*$. Define the filtrations $\Pi' = \{\{S\}, \pi, \pi \vee \pi'\}$ and $\Pi = \{\{S\}, \pi', \pi \vee \pi'\}$. Note that $\hat{f}, \hat{g}, \hat{f}', \hat{g}' \in \mathcal{F}_{\pi\vee\pi'}$; therefore, by the sure-thing principle applied to $\{\succsim_\pi\}_{\pi\in\Pi}$ (since it has a PDEU representation), we have

$$\text{(S.4)} \qquad \hat{f} \succsim_\pi \hat{g} \iff \hat{f}' \succsim_{\pi\vee\pi'} \hat{g}'.$$

Similarly, by the sure-thing principle applied to $\{\succsim_\pi\}_{\pi\in\Pi'}$, we have

$$\text{(S.5)} \qquad \hat{f} \succsim_{\pi\vee\pi'} \hat{g} \iff \hat{f}' \succsim_{\pi'} \hat{g}'.$$

Note that $\hat{f} \succsim_{\pi\vee\pi'} \hat{g} \Leftrightarrow \hat{f}' \succsim_{\pi\vee\pi'} \hat{g}'$ since $\succsim_{\pi\vee\pi'}$ has an expected utility representation $(u, \mu_{\pi\vee\pi'})$. Therefore, by Equations (S.4) and (S.5) we have

$$\hat{f} \succsim_\pi \hat{g} \iff \hat{f}' \succsim_{\pi'} \hat{g}'.$$

Therefore, $\{\succsim_\pi\}_{\pi\in\Pi^*}$ satisfies the sure-thing principle.

Let $u$ be as given by Assumption S.1 and let $(u, (\mu_\pi)_{\pi\in\Pi^*})$ represent $\{\succsim_\pi\}_{\pi\in\Pi^*}$ in the sense of Lemma 1 of AE. We prove (i) $\Rightarrow$ (ii) below. The (ii) $\Rightarrow$ (iii) direction follows from Theorem 3 of AE since $\{\succsim_\pi\}_{\pi\in\Pi^*}$ satisfies the Anscombe–Aumann axioms and the sure-thing principle. The (iii) $\Rightarrow$ (i) direction is immediate.

(i) $\Rightarrow$ (ii) Consider nonempty events $E_1$, $E_2$, $E_3$, and $E_4$ such that $[E_1 \cup E_3] \cap [E_2 \cup E_4] = \emptyset$. We prove that

$$\frac{E_1}{E_2} \times \frac{E_2}{E_3} = \frac{E_1}{E_4} \times \frac{E_4}{E_3}.$$

This implies, by Theorem 5(ii) of AE, that $\{\succsim_\pi\}_{\pi\in\Pi^*}$ satisfies binary bet acyclicity. Let $i$, $-i$, $j$, and $-j$ be such that $\{i, -i\} = \{1, 3\}$ and $\{j, -j\} = \{2, 4\}$. By strict admissibility, $E_i \cup E_{-i}$ and $E_j \cup E_{-j}$ are nonnull. We next prove that

$$\text{(S.6)} \qquad \frac{E_i}{E_j \cup E_{-j}} \times \frac{E_j \cup E_{-j}}{E_i \cup E_{-i}} \times \frac{E_i \cup E_{-i}}{E_j} \times \frac{E_j}{E_i} = 1.$$

Clearly Equation (S.6) holds if $E_i = E_i \cup E_{-i}$ or $E_j = E_j \cup E_{-j}$. Therefore, without loss of generality, let $E_i \subsetneq E_i \cup E_{-i}$ or $E_j \subsetneq E_j \cup E_{-j}$. Consider the filtrations $\Pi = \{\pi_0, \pi_1, \pi_2, \pi_3\}$ and $\Pi' = \{\pi_0, \pi_1, \pi_2', \pi_3\}$, where $\pi_0 = \{S\}$, $\pi_1 = \{E_i \cup E_{-i}, E_j \cup E_{-j}\}$, $\pi_2 = \{E_i, E_{-i} \setminus E_i, E_j \cup E_{-j}\}$, $\pi_2' = \{E_i \cup E_{-i}, E_j, E_{-j} \setminus E_j\}$, and $\pi_3 = \{E_i, E_{-i} \setminus E_i, E_j, E_{-j} \setminus E_j\}$. Note that both $\Pi$ and $\Pi'$ are gradual. Therefore, by filtration invariance there is $\alpha > 0$ such that $\nu_\Pi(E) = \alpha \nu_{\Pi'}(E)$ for all $E \in (C_\Pi \cap C_{\Pi'}) \setminus \{S\}$. Note that

$$\frac{E_i}{E_j \cup E_{-j}} = \frac{\mu_{\pi_2}(E_i)}{\mu_{\pi_2}(E_j \cup E_{-j})} = \frac{\nu_\Pi(E_i)}{\nu_\Pi(E_j \cup E_{-j})},$$

$$\frac{E_j \cup E_{-j}}{E_i \cup E_{-i}} = \frac{\mu_{\pi_1}(E_j \cup E_{-j})}{\mu_{\pi_1}(E_i \cup E_{-i})} = \frac{\nu_\Pi(E_j \cup E_{-j})}{\nu_\Pi(E_i \cup E_{-i})},$$

$$\frac{E_i \cup E_{-i}}{E_j} = \frac{\mu_{\pi_2'}(E_i \cup E_{-i})}{\mu_{\pi_2'}(E_j)} = \frac{\nu_{\Pi'}(E_i \cup E_{-i})}{\nu_{\Pi'}(E_j)}$$

$$= \frac{\nu_\Pi(E_i \cup E_{-i})/\alpha}{\nu_\Pi(E_j)/\alpha} = \frac{\nu_\Pi(E_i \cup E_{-i})}{\nu_\Pi(E_j)},$$

$$\frac{E_j}{E_i} = \frac{\mu_{\pi_3}(E_j)}{\mu_{\pi_3}(E_i)} = \frac{\nu_\Pi(E_j)}{\nu_\Pi(E_i)},$$

implying Equation (S.6). For each $i \in \{1, 3\}$ and $j \in \{2, 4\}$, solving for $E_j/E_i$ from Equation (S.6) implies the desired conclusion

$$\frac{E_1}{E_2} \times \frac{E_2}{E_3} = \frac{E_1}{E_4} \times \frac{E_4}{E_3}.$$

*Q.E.D.*

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