Optimal Taxation with Endogenous Default under Incomplete Markets

Demian Pouzo and Ignacio Presno *

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Abstract

In a dynamic economy, we characterize the fiscal policy of the government when it levies distortionary taxes and issues defaultable bonds to finance its stochastic expenditure. Default may occur in equilibrium as it prevents the government from incurring in future tax distortions that would come along with the service of the debt. Households anticipate the possibility of default generating endogenous credit limits. These credit limits hinder the government's ability to smooth taxes using debt, rendering more volatile and less serially correlated fiscal policies, higher borrowing costs and lower levels of indebtness. Also, the near-random walk behavior of debt and taxes with risk-free debt under incomplete markets is altered once default risk is incorporated.

In order to exit temporary financial autarky following a default event, the government has to repay a random fraction of the defaulted debt. We show theoretically that our debt restructuring process has implications for haircuts and duration of renegotiation episodes that are aligned with the data.

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Keywords: Optimal Taxation, Government Debt, Incomplete Markets, Default, Secondary Markets.

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*Pouzo: UC Berkeley, Dept. of Economics, 530-1 Evans # 3880, Berkeley CA 94720, Email: dpouzo@econ.berkeley.edu; Presno: Universidad de Montevideo, Dept. of Economics, 2544 Prudencio de Pena St., Montevideo, Uruguay 11600. Email: jipresno@um.edu.uy. We are deeply indebted to Xiaohong Chen, Ricardo Lagos and Tom Sargent for their thoughtful advice and insightful discussions. We are also grateful to Arpad Abraham, Mark Aguiar, David Ahn, Andy Atkeson, Marco Basetto, Hal Cole, Jonathan Halket, Greg Kaplan, Juanpa Nicolini, Anna Orlik, Nicola Pavoni, Andres Rodriguez-Clare, Ana Maria Santacreu, Ennio Stacchetti, and especially to Ignacio Esponda and Constantino Hevia. We thank for comments and suggestions to seminar participants at Berkeley, CEF 2013, CREI-UPF, Federal Reserve Board of Governors, FRB of Chicago, Cleveland, Philadelphia and NY, LBS, NYU, North American Summer Meeting 2013, Rochester, SED 2013, U of Chicago, UC Davis, UCL, UG, UIUC and UPenn. We are grateful to Nan Lu and Sandra Spirovska for excellent research assistance. Usual disclaimer applies.

1 Introduction

As originally indicated by Barro (1979), tax smoothing motives play a fundamental role in the design of optimal fiscal policies. The ability of the Ramsey planner to smooth taxes across states and over time relies significantly on the market structure for government debt. Relative to the complete market economy of Lucas and Stokey (1983), Aiyagari et al. (2002) show that taxes typically display higher variability and lower serial correlation under incomplete markets, for sufficiently stringent government debt and asset limits. In this latter economy, the government is assumed to have access to one-period risk-free bonds. But how are optimal tax and debt policies affected if the government is given the option to default and restructure its debt, as we have observed several times throughout history? In this paper we answer this question. We show how the presence of default risk and the actual default contingency gives rise to endogenous credit limits that hinder the government's ability to smooth shocks using debt. As a result, taxes become even more volatile, and less serially correlated than in the benchmark incomplete market framework.

In this paper we analyze the dynamic taxation problem of a benevolent government with access to distortionary labor taxes and non-state-contingent debt in a closed economy. We assume, however, that the government cannot commit to pay back the debt. In case the government defaults, the economy enters temporary financial autarky wherein it faces exogenous random offers to repay a fraction of the defaulted debt that arrive at a given rate.[1] The government has the option to accept the offer — and thereby exit financial autarky — or to stay in financial autarky awaiting new offers. During temporary financial autarky, the defaulted debt still has some value as the recovery rate is positive; a fraction of it will be eventually repaid in the future. Hence, households can trade the defaulted debt in a secondary market from which the government is excluded, giving rise to an equilibrium price of the debt during the period of default. Finally, in line with the aforementioned optimal taxation literature, we assume that the government commits itself to its optimal path of taxes as long as the economy is not in financial autarky.

In the model, the government has three policy instruments: (1) distortionary taxes, (2) government debt, and (3) default decisions that consist of: (a) whether to default on the outstanding debt and (b) whether to accept the offer to exit temporary financial autarky.

In order to finance the stochastic process of expenditures, the government faces a trade-off between levying distortionary taxes and not defaulting, or issuing debt and thereby increasing

[1] While in our model we allow only for outright default on government bonds, governments in practice could liquidate the real value of the debt and repayments through inflation risk, which could be viewed as a form of partial default. In several economies, however, this second option may not available, either because the country has surrendered the control over its monetary policy (for example, as in the eurozone, Ecuador, and Panama), or a significant portion of the government debt is either foreign-currency denominated, or local-currency denominated but indexed to the CPI or a similar index. We see our environment particularly appropriate for this class of economies.

the exposure to default risk. Defaulting introduces some degree of state contingency on the payoff of the debt since the financial instrument available to the government becomes an option, rather than a non-state-contingent bond. In equilibrium, the government may optimally decide not to honor its debt contracts —even though the bondholders are the households whose welfare it cares about— because default would prevent the government from incurring in the future tax distortions that would come along with the service of the debt. We believe this is a novel motive to default on government debt which, to our knowledge, had not been explored before in the literature.

The option to default, however, does not come free of charge: in equilibrium households anticipate the possibility of default, demanding a compensation for it embedded in the pricing of the bond; this originates a "Laffer curve" type of pattern for the bond proceedings, thereby implying endogenous credit limits. In this sense, our model generates "debt intolerance" endogenously. This is different from Aiyagari et al. (2002) wherein credit limits are imposed ad hoc. In our framework, the possibility of default introduces a trade-off between the cost of the lack of commitment to repay the debt, partly reflected in the price of the debt, and the flexibility that comes from the option to default and partial payments, as captured by the payoff of the bond.

Our theoretical model is motivated by some observations of tax and debt dynamics along with episodes of domestic defaults and debt restructuring throughout the history for a number of economies. First, we view our model as a suitable framework to study the government policies and fiscal accounts for France in the 100 years preceding the French Revolution of 1789, in accord with Sargent and Velde (1995). Whereas Great Britain honored all its debt contracts through this period, France defaulted recurrently and no tax-smoothing features stand out from its debt and tax dynamics.[2] Second, our model can be used to examine the debt restructuring plan proposed by Secretary of the Treasury Alexander Hamilton for the U.S. economy in 1790 and the policy debates around it.[3] Finally, we see our model as able to capture some empirical regularities observed nowadays for emerging economies, where default is a recurrent event, taxes are more volatile, borrowing costs are sizable, and indebtness levels are significantly lower than for developed economies.[4]

[2]In contrast, time-series for Great Britain debt broadly resemble Barro's random walk behavior.

[3]As Hall and Sargent (2014) state, through renegotiating the U.S. debt with no discrimination scheme across creditors, Hamilton was hoping to improve the federal government access to credit markets, which in turn would eventually allow for lower borrowing costs to finance temporary increases in government spending and thereby smooth out taxes. Few years later, in 1807, in his report to Congress, Secretary of Treasury Albert Gallatin was advocating for a fiscal policy largely in accord with Barro's tax smoothing idea.

[4]Several domestic defaults for emerging economies were also external. In any case, empirical evidence seems to suggest that government default has a significant direct impact on domestic residents; either because a considerable portion of the foreign debt is in the hands of local investors, or because the government also defaults on domestic debt. For example, for Argentina's default in 2001, about 60 percent of the defaulted debt is estimated to have been in the hands of Argentinean residents; local pension funds alone held almost 20 percent of the total defaulted

In a benchmark case, with quasi-linear utility and i.i.d. government expenditure but allowing for offers of partial payments to exit financial autarky, we characterize analytically the determinants of the optimal default decision and its effects on the optimal taxes, debt and allocations. In particular, we first show that default is more likely when the government expenditure or debt is higher, and that the government is more likely to accept any given offer to pay a fraction of the defaulted debt when the level of defaulted debt is lower. These theoretical results have implications for haircuts and duration of debt restructuring processes that are aligned with the data. Second, we find that prices — both outside and during financial autarky — are non-increasing on the level of debt, thus implying that spreads are non-decreasing and also implying the existence of endogenous borrowing limits. Third, we demonstrate that our model is isomorphic to the endowment economy with sovereign default of Arellano (2008) and Aguiar and Gopinath (2006), but with a non-standard per-period payoff exhibiting a satiation point; section 4 elaborates on this point.

Fourth, we prove that the law of motion of the optimal government tax policy departs from the standard martingale-type behavior found in the standard incomplete market framework. Barro (1979) conjectured that optimal debt and taxes should exhibit a random walk behavior by laying out an analogy to the permanent income model of consumption. In a general equilibrium setup, Aiyagari et al. (2002) reaffirmed Barro's result by showing that the Ramsey plan prescribes a near-random walk component in taxes, regardless of the stochastic process of government spending, if government asset limits are sufficiently stringent. In our paper we show how this result is altered once default risk is incorporated. More specifically, the law of motion of the optimal government tax policy will be affected, on the one hand, by the benefit from having more state-contingency on the payoff of the bond, but, on the other hand, by the cost of having the option to default (manifested in higher borrowing costs).

Finally, we conduct a series of numerical exercises to assess the quantitative performance of our model along the aforementioned dimensions. In terms of welfare, when there are ad hoc output costs of default, the numerical simulations suggest a monotonic relationship between household utility and the probability of receiving an offer of partial payments. In particular, increasing the probability of receiving offers for exiting autarky decreases welfare.

The paper is organized as follows. We first present the related literature. Section 2 introduces the model. Section 3 presents the competitive equilibrium, and section 4 presents the government's problem. Section 5 derives analytical results. Section 6 contains some numerical exercises. Section 7 briefly concludes. All proofs are gathered in the appendices.

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debt. For Russia's default in 1998 about 60 percent of the debt was held by residents. For Ukraine's default in 1997-98, residents — Ukrainian banks and the National Bank of Ukraine (NBU) among others — held almost 50 percent of the outstanding stock of T-bills. See Sturzenegger and Zettelmeyer (2006).

1.1 Related Literature

A growing literature has emerged from the seminal work of Barro (1979) highlighting the role of tax-smoothing motives in the design of optimal fiscal and debt policy. In a partial equilibrium deterministic framework, Barro (1979) assumes that the government needs to finance an exogenous sequence of public spending either by levying distortionary taxes or issuing non-state-contingent debt. To model distortionary taxes, a convex deadweight cost of output is adopted. Barro (1979) shows that the government wants to smooth tax distortions across periods by recurring to debt issuance to finance temporary increases in public spending. In a stochastic environment, the model predicts a random walk response of debt and taxes to public spending.

Lucas and Stokey (1983) show that this result of Barro (1979) does not survive in an environment with complete markets. In particular, in a general equilibrium setup where a Ramsey planner disposes of distortionary labor taxes and a complete set of Arrow-Debreu securities, Lucas and Stokey (1983) prove that optimal taxes (and debt) follow no random walk process but roughly inherits the stochastic properties from the government spending dynamics.

By extending Lucas and Stokey (1983) general equilibrium framework to incomplete markets, Aiyagari et al. (2002) revitalize Barro (1979) to some extent by demonstrating that the Ramsey plan prescribes a near-random walk component into debt and taxes. Our model builds on Aiyagari et al. (2002) by incorporating two key ingredients. First, we give the government the option to default on its public debt. By doing this, we endogenize the ad hoc government credit limits imposed in Aiyagari et al. (2002). Additionally, the interest rate on government bonds may significantly differ from the risk-free rate charged to the government in Aiyagari et al. (2002). Second, in our model the government is confronted with an exogenous debt restructuring process following a default event.

Farhi (2010) extends the setting of Aiyagari et al. (2002) by introducing capital accumulation and letting the government to levy capital taxes in addition to labor ones. Shin (2006) studies the Ramsey fiscal policy in an environment akin to Aiyagari et al. (2002) but with heterogeneous households that face idiosyncratic efficiency risk in labor markets.[5] We see these papers as complementary to ours. A recent paper by Bhandari et al. (2013) builds on Aiyagari et al. (2002) by allowing the government to trade a single possibly risky asset. While in Bhandari et al. (2013) the asset payoff is risky as it obeys a Markov structure optimally chosen by the benevolent government in the initial period, in our model it is risky due to the option to default

The paper also contributes to the literature on quantitative default models. We model the strategic default decision of the government as in Arellano (2008) and Aguiar and Gopinath (2006), who first adapted the theoretical framework of Eaton and Gersovitz (1981) to study

[5] Angeletos (2002) and Buera and Nicolini (2004) study the optimal maturity structure of government debt and show how noncontingent bonds of different maturities can be used to implement the allocations with state-contingent debt.

sovereign default risk and its interaction with the business cycles in emerging economies.[6] From this strand of literature, our paper is closely related to Doda (2007) and Cuadra et al. (2010). Both papers analyze the procyclicality of fiscal policy in developing countries by solving an optimal taxation problem of a government with distortionary labor taxes and incomplete financial markets.[7] Their models, however, differ from ours along several dimensions. First, they consider a small open economy with foreign lenders, while we assume a closed economy.[8] Since bondholders are the domestic households whose welfare our benevolent government wants to maximize, in our economy tax-smoothing concerns are the dominant determinant in the decision to default or not. Second, we assume a debt restructuring process while in Doda (2007) and Cuadra et al. (2010) the government exits autarky exempt of any repayment of the defaulted debt. Finally, our paper addresses a different class of question. In particular, our work is rather centered on the normative analysis of optimal taxation in the context of government default and provides an analytical characterization of optimal fiscal and debt policies.[9]

Benjamin and Wright (2009), Pitchford and Wright (2008), Yue (2010) and Bai and Zhang (2012) propose alternative ways of modeling the entire debt restructuring process. Although our mechanism to reach a debt settlement is not fully endogenous as theirs, it is sufficiently reach to replicate key features of debt renegotation episodes.

From a more technical perspective, we assume that the government has the ability to commit to a tax policy at any time, except in the periods of debt renegotiation when it regains access to financial markets, in which case it can revise and reset its fiscal policy. This assumption is to some extent similar to Debortoli and Nunes (2010). Debortoli and Nunes (2010) studies the dynamics of debt in a setting similar to Lucas and Stokey (1983) but with the peculiarity that at each time t, with some given probability, the government can lose its ability to commit to taxes and reoptimize; a feature labeled by the authors as "loose commitment." Thus, our model

[6]Chatterjee and Eyingungor (2012) extends this setup by incorporating long-term debt.

[7]Aguiar et al. (2009) also allow for default in a small open economy with capital where households do not have access to neither financial markets nor capital and provide labor inelastically. The authors' main focus is on capital taxation and the debt "overhang" effect.

[8]From our viewpoint, little attention has been put on quantitative models with domestic default. A notable exception is D'Erasmo and Mendoza (2015), where, in an economy with wealth inequality across domestic agents, redistributional motives influence the incentives to default. Similarly, Dovis et al. (2015) study optimal policies for taxes, transfers, and both domestic and external debt in an open economy with competition between political parties.

[9]There are additional differences between our paper and theirs. For example, in Doda (2007) and Cuadra et al. (2010) the public spending is a control variable for the government, which provides direct utility to the household. In contrast, we assume an exogenous stochastic process for government spending, as in Lucas and Stokey (1983) and Aiyagari et al. (2002) economies. Furthermore, the source of uncertainty in these two papers is the productivity shock. In our model instead, the uncertainty comes only from the government spending and stochastic factors embedded in the debt restructuring process. Finally, these two papers assume that the government cannot commit to fiscal policies, while we do, in line with Aiyagari et al. (2002).

provides a mechanism that "rationalizes" this probability of "loosing commitment" by allowing for endogenous default, and resetting of fiscal policy when a debt settlement is reached.

2 The Economy

In this section we describe the stochastic structure of the model, the timing and policies of the government and present the household's problem.

2.1 The Setting

Let time be indexed as $t = 0, 1, ...$ Let (g_t, δ_t) be the vector of government expenditure at time t and the fraction of the defaulted debt which is to be repaid when exiting autarky, respectively. If the economy is not in financial autarky, δ_t is either one or zero in order to model the option of the government to repay the totality of the debt or to default. These are the exogenous driving random variables of this economy. Let $\omega_t \equiv (g_t, \delta_t) \in \mathbb{G} \times \bar{\Delta}$, where $\mathbb{G} \subset \mathbb{R}$, $\bar{\Delta} \equiv \Delta \cup \{1\} \cup \{\bar{\delta}\}$ and $\Delta \subset [0, 1)$, and in order to avoid technical difficulties, we assume $|\mathbb{G}|$ and $|\Delta|$ are finite.[10] The set Δ models the offers — as fractions of outstanding debt — to repay the defaulted debt; and $\bar{\delta}$ is designed to capture situations where the government does not receive any offer to repay.[11]

For any $t \in \{1, ..., \infty\}$, let $\Omega^t = (\mathbb{G} \times \bar{\Delta})^t$ be the space of histories of exogenous shocks up to time t, a typical element is $\omega^t = (\omega_0, \omega_1, ..., \omega_t)$.

2.2 The Government Policies and Timing

In this economy, the government finances exogenous government expenditures by levying labor distortionary taxes and trading one-period, discount bonds with households. The government, however, cannot commit to repay and may default on the bonds at any point in time.

Let $\mathbb{B} \subseteq \mathbb{R}$ be compact. Let $B_{t+1} \in \mathbb{B}$ be the quantity of bonds issued at time t to be paid at time $t + 1$ so that $B_{t+1} > 0$ means that the government is borrowing at time t from households. Let τ_t be the linear labor tax. Also, let d_t be the default decision, which takes value 1 if the government decides to default and 0 otherwise. Finally, let a_t be the decision of accepting an offer to repay the defaulted debt. It takes value 1 if the offer is accepted and 0 otherwise.

Also, for any t, let ϕ_t be the variable that takes value 0 if at time t the government cannot issue bonds during this period, and value 1 if it can. The implied law of motion for ϕ_t is $\phi_t \equiv \phi_{t-1}(1 - d_t) + (1 - \phi_{t-1})a_t$. That is, if at time $t - 1$, the government could issue bonds, then $\phi_t = (1 - d_t)$, but if it was in financial autarky, then $\phi_t = a_t$, reflecting the fact that the

[10] For a given set, $|S|$ is the cardinal of the set.

[11] An alternative way of modeling this situation is to work with $\bar{\Delta} \equiv \Delta \cup \{1\} \cup \{\emptyset\}$ where $\emptyset$ indicates no offer. Another alternative way is to add an additional random variable, $\iota \in \{0, 1\}$ that explicitly indicates if the government received an offer ($\iota = 1$) or not ($\iota = 0$) and let $\bar{\Delta} \equiv \Delta \cup \{1\}$.

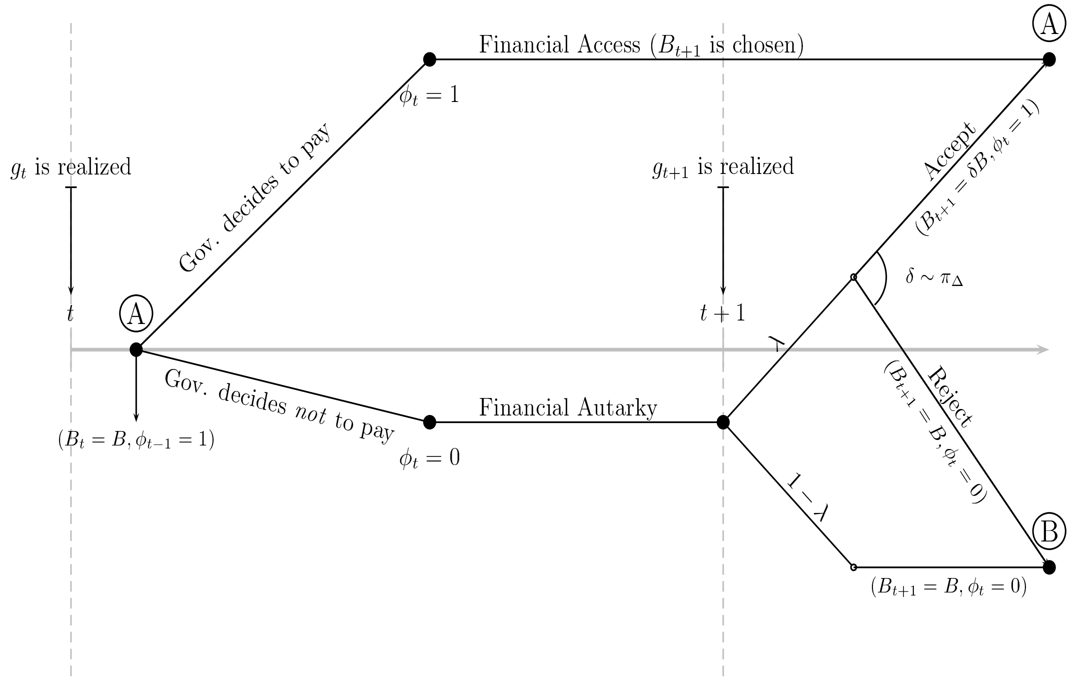

Figure 2.1: Timing of the Model

government regains access to financial markets only if the government decides to renegotiate the defaulted debt.

The timing for the government is as follows. Following a period with financial access, after observing the current government expenditure, the government has the option to default on the totality of the outstanding debt carried from last period, B_t.

As shown in figure 2.1, if the government opts to exercise the option to default at time t, it cannot issue bonds in that period and runs a balanced budget, i.e., tax revenues equal government expenditure. At the beginning of next period, time $t+1$, with probability $1-\lambda$, the government remains in temporary financial autarky for that period (node B). With probability λ, the government receives a random offer to repay a fraction δ of the debt, and has the option to accept or reject it. If the government accepts the offer, it pays the restructured amount (the outstanding defaulted debt times the fraction δ), and it is able to issue new bonds for the following period (node A). If the government rejects the offer, it stays in temporary financial autarky (node B).

Finally, if the government decides not to default, it levies distortionary labor taxes, and allocates discount bonds to the households to cover the expenses g_t and liabilities carried from last period. Next period, it has again the option to default, for the new values of outstanding debt and government expenditure (node A).

As it will become clear later, default on bonds can be seen as a negative lump-sum transfer to households, but a costly one. Default will turn to be costly for two reasons. First, households anticipate the government default strategies and demand higher returns to bear the bond. Second, default is assumed to be followed by temporary financial autarky. During autarky, the government is not only unable to issue debt but also could be subject to an ad hoc output cost, as shown later.

We now formalize the probability model. Let $\pi_{\mathbb{G}} : \mathbb{G} \to \mathcal{P}(\mathbb{G})$ be the Markov transition probability function for the process of government expenditures and let $\pi_{\Delta} \in \mathcal{P}(\Delta)$ be the probability measure over the offer space Δ.[12]

Assumption 2.1. *For any* (t, ω^t), $\Pr(g_t = g|\omega^{t-1}) = \pi_{\mathbb{G}}(g|g_{t-1})$ *for any* $g \in \mathbb{G}$ *and*

$$\Pr(\delta_t = \delta|g_t, \omega^{t-1}) = \begin{cases} \mathbf{1}_{\{1\}}(\delta) & if\ \phi_{t-1} = 1 \\ (1-\lambda)\mathbf{1}_{\{\bar{\delta}\}}(\delta) + \lambda\pi_{\Delta}(\delta) & if\ \phi_{t-1} = 0 \end{cases}$$

for any $\delta \in \bar{\Delta}$.[13]

Essentially, this assumption imposes a Markov restriction on the probability distribution over government expenditures and also additional restrictions over the probability of offers. In particular, this assumption implies that in financial autarky with probability $1-\lambda$, $\delta = \bar{\delta}$ (i.e., receiving no offer) and with probability λ, an offer from the offer space is drawn according to π_{Δ}. Also, if $\phi_{t-1} = 1$ (i.e., the government was not in financial autarky at period $t-1$), then $\delta_t = 1$ with probability one, which implies that if the government decides *not to default* at time t, it will pay the totality of the outstanding debt.

Finally, we use Π to denote the probability distribution over Ω^{∞} generated by assumption 2.1, and $\Pi(\cdot|\omega^t)$ to denote the conditional probability over Ω, given ω^t.

The next definitions formalize the concepts of government policy, allocation, prices of bonds and the government budget constraint. In particular, it formally introduces the fact that taxes, default decisions and debt depend on histories of past realizations of shocks, and in particular that debt is non-state contingent (i.e., B_{t+1} only depends on the history up to time t, ω^t).

Definition 2.1. *A government policy is a collection of stochastic processes* $\boldsymbol{\sigma} = (B_{t+1}, \tau_t, d_t, a_t)_{t=0}^{\infty}$, *such that for each* t, $(B_{t+1}, \tau_t, d_t, a_t) \in \mathbb{B} \times [0,1] \times \{0,1\}^2$ *are measurable with respect to* ω^t *and* (B_0, ϕ_{-1}).

Definition 2.2. *An allocation is a collection of stochastic processes* $(g_t, c_t, n_t)_{t=0}^{\infty}$ *such that for each* t, $(g_t, c_t, n_t) \in \mathbb{G} \times \mathbb{R}_+ \times [0,1]$ *are measurable with respect to* ω^t *and* (B_0, ϕ_{-1}).

[12] For a finite set $\mathbb{X}$, $\mathcal{P}(\mathbb{X})$ is the space of all probability measures defined over $\mathbb{X}$. Also, for any $A \subseteq \mathbb{X}$, the function $\mathbf{1}_A(\cdot)$ takes value 1 over the set A and 0 otherwise.

[13] It is easy to generalize this to a more general formulation such as λ and π_{Δ} depending on g. For instance, we could allow for, say, $\pi_{\Delta}(\cdot|g_t, B_t, d_t, d_{t-1}, ..., d_{t-K})$ some $K > 0$, denoting that possible partial payments depend on the credit history and level of debt. See Reinhart et al. (2003), Reinhart and Rogoff (2008) and Yue (2010) for an intuition behind this structure.

Given a government policy, we say an allocation is *feasible* if for any (t, ω^t)

$$c_t(\omega^t) + g_t = \kappa_t(\omega^t) n_t(\omega^t), \tag{2.1}$$

where $\kappa_t : \Omega^t \to \mathbb{R}_+$ is such that $\kappa_t(\omega^t)$ is the productivity at period t, given history ω^t. For simplicity, we set $\kappa_t(\omega^t) = \phi_t(\omega^t) + \kappa(1 - \phi_t(\omega^t))$ with $\kappa \in (0, 1]$. The parameter κ represents direct output loss following a default event, associated for example with financial disruption in the banking sector, limited insurance against idiosyncratic risk, among others.

Definition 2.3. *A price process is an stochastic process $(p_t)_{t=0}^{\infty}$ such that for each t, $p_t \in \mathbb{R}_+$ is measurable with respect to ω^t and (B_0, ϕ_{-1}).*

Note that p_t denotes the price of one unit of debt in any state of the world, both with access to financial markets and during autarky, where it represents the price of defaulted debt in secondary markets. Finally, we introduce the government budget constraint.

Definition 2.4. *A government policy $\boldsymbol{\sigma}$ is attainable, if for all (t, ω^t),*

$$g_t + \phi_t(\omega^t)\delta_t B_t(\omega^{t-1}) \leq \kappa_t(\omega^t)\tau_t(\omega^t)n_t(\omega^t) + \phi_t(\omega^t)p_t(\omega^t)B_{t+1}(\omega^t), \tag{2.2}$$

and $d_t(\omega^t) = 1$ if $\phi_{t-1}(\omega^{t-1}) = 0$ and $a_t(\omega^t) = 0$ if $\phi_{t-1}(\omega^{t-1}) = 1$ or $\delta_t = \bar{\delta}$.[14]

Observe that in equation 2.2, if the government is in financial autarky ($\phi_t(\omega^t) = 0$), its budget constraint boils down to $g_t \leq \kappa_t(\omega^t)\tau_t(\omega^t)n_t(\omega^t)$. On the other hand, if the government has access to financial markets ($\phi_t(\omega^t) = 1$), then it has liabilities to be repaid for $\delta_t B_t$ and can issue new debt.[15] The final restriction on $d_t(\omega^t)$ and $a_t(\omega^t)$ simply states that if last period the government was in financial autarky, then it trivially cannot choose to default at time t, and if $\delta_t = \bar{\delta}$ or if last period the government had access to financial markets $a_t(\omega^t)$ is set to 0.

A few final remarks about the "debt-restructuring process" are in order. This process is parameterized by (λ, π_Δ). These parameters capture the fact that debt restructuring is time-consuming but, generally, at the end a positive fraction of the defaulted debt is honored.This debt-restructuring process intends to capture the fact that after defaults (on domestic or international debt, or both), economies see their access to credit severely hindered.[16]

[14]The inequality in equation 2.2 implies that the government can issue lump-sum transfers to the households. Lump-sum taxes are not permitted.

[15]If the government had access to financial markets at time $t-1$ ($\phi_{t-1} = 1$), then by assumption 2.1, $\delta_t = 1$ and the outstanding debt if simply B_t.

[16]The duration of debt restructuring after sovereign defaults in particular on external debt has received considerable attention in the literature. For instance, for Argentina's default in 2001 the settlement with the majority of the creditors was reached in 2005. In the default episodes of Russia (1998), Ecuador (1999) and Ukraine (1998), the renegotiation process lasted 2.3, 1.7 and 1.4 years, respectively, according to Benjamin and Wright (2009). In general, domestic debt restructuring periods tend to be not as long as in the case of external debt. For example, as documented by Sturzenegger and Zettelmeyer (2006), after the default by Russia in 1998 it took six months to restructure the domestic GKO bonds.

2.3 The Household's Problem

There is a continuum of identical households, that are price takers and have time-separable preferences for consumption and labor processes. They also make debt/savings decisions by trading government bonds. Formally, we define a household debt process as a stochastic process given by $(b_{t+1})_{t=0}^{\infty}$ where $b_{t+1} : \Omega^t \to [\underline{b}, \bar{b}]$ is the household's savings in government bonds at time $t+1$ for any history ω^t.[17]

For convenience, let q_t denote the price of defaulted debt at time t, i.e., $q_t = p_t$ if $\phi_t = 0$. Given a government policy $\boldsymbol{\sigma}$, for each t, let $\varrho_t : \Omega^t \to \mathbb{R}$ be the payoff of a government bond at period t; i.e.,

$$\varrho_t(\omega^t) = \phi_t(\omega^t)\delta_t + (1 - \phi_t(\omega^t))q_t(\omega^t). \tag{2.3}$$

A few remarks about ϱ are in order. First, since the household takes government actions as given, from the point of view of the households the government debt is an asset with payoff that depends only on the state of the economy, and this dependence clearly illustrates that default decisions add certain degree of state contingency to the government debt. In particular, if $\phi_t(\omega^t) = 1$, then $\varrho_t(\omega^t) = \delta_t$ denoting the fact that the government pays a fraction δ_t. If the government defaults or rejects the repayment option, the household can sell each unit of government debt in the secondary market at a price $\varrho_t(\omega^t) = q_t(\omega^t)$.

The household's problem consists of choosing consumption, labor and debt processes in order to maximize the expected lifetime utility. That is, given (ω_0, b_0) and $\boldsymbol{\sigma}$,

$$\sup_{(c_t,n_t,b_{t+1})_{t=0}^{\infty} \in \mathbb{C}(g_0,b_0;\boldsymbol{\sigma})} E_{\Pi(\cdot|\omega_0)} \left[\sum_{t=0}^{\infty} \beta^t u(c_t(\omega^t), 1 - n_t(\omega^t)) \right]$$

where $\beta \in (0,1)$ is the discount factor, $E_{\Pi(\cdot|\omega_0)}[\cdot]$ is the expectation using the conditional probability $\Pi(\cdot|\omega_0)$, and $\mathbb{C}(g_0, b_0; \boldsymbol{\sigma})$ is the set of household's allocations and debt process, given government policy $\boldsymbol{\sigma}$, such that for all t and all $\omega^t \in \Omega^t$,

$$c_t(\omega^t) + p_t(\omega^t)b_{t+1}(\omega^t) = (1 - \tau_t(\omega^t))\kappa_t(\omega^t)n_t(\omega^t) + \varrho_t(\omega^t)b_t(\omega^{t-1}) + T_t(\omega^t),$$

where $T_t(\omega^t) \geq 0$ are lump-sum transfers from the government. The previous equation indicates that after-tax labor income, proceedings from bond holdings and government transfers have to be sufficient to cover consumption and new purchases of government bonds.

3 Competitive Equilibrium

We now define a competitive equilibrium for a given government policy and derive the equilibrium taxes and prices.

[17] We assume $b_{t+1} \in [\underline{b}, \bar{b}]$ with $[\underline{b}, \bar{b}] \supset \mathbb{B}$ so in equilibrium these restrictions will not be binding.

Definition 3.1. *Given $\omega_0, B_0 = b_0$ and ϕ_{-1}, a competitive equilibrium is a government policy, $\boldsymbol{\sigma}$, an allocation, $(g_t, c_t, n_t)_{t=0}^{\infty}$, a household debt process, $(b_{t+1})_{t=0}^{\infty}$, and a price process $(p_t)_{t=0}^{\infty}$ such that:*

1. *Given the government policy and the price process, the allocation and debt process solve the household's problem.*
2. *The government policy, $\boldsymbol{\sigma}$, is attainable.*
3. *Given $\boldsymbol{\sigma}$, the allocation is feasible.*
4. *For all (t, ω^t), $B_{t+1}(\omega^t) = b_{t+1}(\omega^t)$, and $B_{t+1}(\omega^t) = B_t(\omega^{t-1})$ if $\phi_t(\omega^t) = 0$.*

Observe that the market clearing for debt indicates that $B_{t+1}(\omega^t) = b_{t+1}(\omega^t)$. In addition, if the economy is in financial autarky — where the government cannot issue debt, and thus agents can only trade among themselves —, imposing $B_{t+1}(\omega^t) = B_t(\omega^{t-1})$ implies, since agents are identical, that in equilibrium $b_t(\omega^{t-1}) = b_{t+1}(\omega^t)$, i.e., agents do not change their debt positions.

3.1 Equilibrium Prices and Taxes

In this section we present the expressions for equilibrium taxes and prices of debt. The former quantity is standard (e.g. Aiyagari et al. (2002) and Lucas and Stokey (1983)); the latter quantity, however, incorporates the possibility of default of the government. The following assumption is standard and ensures that u is smooth enough to compute first order conditions.

Assumption 3.1. *$u \in \mathbb{C}^2(\mathbb{R}_+ \times [0,1], \mathbb{R})$ with $u_c > 0$, $u_{cc} < 0$, $u_l > 0$ and $u_{ll} > 0$, and $\lim_{l \to 0} u_l(l) = \infty$.*[18]

Henceforth, for any (t, ω^t), we use $u_c(\omega^t)$ as $u_c(c_t(\omega^t), 1 - n_t(\omega^t))$ and proceed similarly for other derivatives and functions.

From the first order conditions of the optimization problem of the households (assuming an interior solution) the following equations hold for any (t, ω^t),[19]

$$\frac{u_l(\omega^t)}{u_c(\omega^t)} = (1 - \tau_t(\omega^t))\kappa_t(\omega^t), \tag{3.4}$$

and

$$\begin{aligned} p_t(\omega^t) =& E_{\Pi(\cdot|\omega^t)}\left[\beta \frac{u_c(\omega^{t+1})}{u_c(\omega^t)} \varrho_{t+1}(\omega^{t+1})\right] \\ =& \beta E_{\Pi(\cdot|\omega^t)}\left[\frac{u_c(\omega^{t+1})}{u_c(\omega^t)} \phi_{t+1}(\omega^{t+1})\delta_{t+1}\right] + \beta E_{\Pi(\cdot|\omega^t)}\left[\frac{u_c(\omega^{t+1})}{u_c(\omega^t)}(1 - \phi_{t+1}(\omega^{t+1}))q_{t+1}(\omega^{t+1})\right] \end{aligned} \tag{3.5}$$

[18] $\mathbb{C}^2(X, Y)$ is the space of twice continuously differentiable functions from X to Y. The assumption $u_{cc} < 0$ could be relaxed to include $u_{cc} = 0$ (see the section 5 below).

[19] See appendix B for the derivation.

Given the definition of ϱ and the restrictions on Π, equation 3.5 implies for $\phi_t(\omega^t) = 1$,[20]

$$\begin{aligned} p_t(\omega^t) =& \beta \int_{\mathbb{G}} \left(\frac{u_c(\omega^t, g', 1)}{u_c(\omega^t)} (1 - d_{t+1}(\omega^t, g', 1)) \right) \pi_{\mathbb{G}}(dg'|g_t) \\ &+ \beta \int_{\mathbb{G}} \frac{u_c(\omega^t, g', 1)}{u_c(\omega^t)} d_{t+1}(\omega^t, g', 1) q_{t+1}(\omega^t, g') \pi_{\mathbb{G}}(dg'|g_t), \end{aligned} \tag{3.6}$$

and for $\phi_t(\omega^t) = 0$[21]

$$\begin{aligned} q_t(\omega^t) =& \beta\lambda \int_{\mathbb{G}} \int_{\Delta} \left(\frac{u_c(\omega^t, g', \delta')}{u_c(\omega^t)} \delta' a_{t+1}(\omega^t, g', \delta') \right) \pi_{\Delta}(d\delta') \pi_{\mathbb{G}}(dg'|g_t) \\ &+ \beta\lambda \int_{\mathbb{G}} \left\{ \int_{\Delta} \left(\frac{u_c(\omega^t, g', \delta')}{u_c(\omega^t)} (1 - a_{t+1}(\omega^t, g', \delta')) \pi_{\Delta}(d\delta') \right) \right\} q_{t+1}(\omega^t, g') \pi_{\mathbb{G}}(dg'|g_t) \\ &+ \beta(1-\lambda) \int_{\mathbb{G}} \left(\frac{u_c(\omega^t, g', \bar{\delta})}{u_c(\omega^t)} \right) q_{t+1}(\omega^t, g') \pi_{\mathbb{G}}(dg'|g_t). \end{aligned} \tag{3.7}$$

Equation 3.5 reflects the fact that in equilibrium households anticipate the default strategies of the government and demand higher returns to compensate for the default risk. The second line in the Euler equation 3.6 shows that, due to the possibility of partial repayments in the future, defaulted debt has positive value and agents can sell it in a secondary market at price $q_{t+1}(\omega^{t+1})$. Equation 3.7 characterizes this price. Each summand in the right hand side corresponds to a "branch" of the tree depicted in figure 2.1. The first line represents the value of one unit of debt when an offer arrives and the government decides to repay the realized fraction of the defaulted debt next period. The second and third lines capture the value of one unit of debt when either the government decides to reject the repayment offer, or it does not receive one. A final observation is that, as it will become clear later on, when $\phi_{t+1}(\omega^{t+1}) = 0$, $u_c(\omega^{t+1})$ is actually only a function of g_{t+1} (not the entire past history ω^{t+1}) because in equilibrium the government runs a balanced budget.

To shed some more light on equations 3.6 and 3.7, consider the case where $u_c = 1$, $\lambda = 0$. In this case, for any (t, ω)

$$p_t(\omega) = \beta \int_{\mathbb{G}} (1 - d_{t+1}(\omega^t, g')) \pi_{\mathbb{G}}(dg'|g_t).$$

Also observe that, since $\lambda = 0$, it follows that $q_t(\omega^t) = \int_{\mathbb{G}} q_{t+1}(\omega^t, g') \pi_{\mathbb{G}}(dg'|g_t)$, which by substituting forward and invoking standard transversality conditions, yields $q_t(\omega^t) = 0$. Thus, the bond price p_t, is simply the discounted one-period ahead probability of not defaulting. These pricing equations are analogous to those in Arellano (2008) and Aguiar and Gopinath (2006) and references therein.

The novelty of these pricing equations with respect to the standard sovereign default model is the presence of secondary market prices, q_t. By imposing a positive recovery rate (with some

[20] The notation (ω^t, g, δ) denotes the partial history ω^{t+1} where $(g_{t+1}, \delta_{t+1}) = (g, \delta)$.

[21] As it will become clear below, the price q_t does not depend on δ_t, so we omit it from the notation.

probability), the model is able to deliver a positive price of defaulted debt during the financial autarky period. In sections 5 and 6, we shed some light on the pricing implications of this model.

3.2 Characterization of the Competitive Equilibrium

In this environment, the set of competitive equilibria can be characterized by a sequence of non-linear equations which impose restrictions on $(d_t, a_t, B_{t+1}, n_t)_{t=0}^{\infty}$ and are derived from the first order conditions of the household, the budget constraint of the government and the feasibility condition. The next theorem formalizes this claim.

Henceforth, we call $(d_t, a_t, B_{t+1}, n_t)_{t=0}^{\infty}$ an *outcome path* of allocations. We say an outcome path is *consistent with a competitive equilibrium* if the outcome path and $(c_t, p_t, b_{t+1}, \tau_t, g_t)_{t=0}^{\infty}$, derived using the market clearing, feasibility and first order conditions, is a competitive equilibrium. Also, let

$$Z_t(\omega^t) \equiv z(\kappa_t(\omega^t), n_t(\omega^t), g_t) = \left(\kappa_t(\omega^t) - \frac{u_l(\omega^t)}{u_c(\omega^t)}\right) n_t(\omega^t) - g_t \tag{3.8}$$

be the primary surplus (if it is negative, it represents a deficit) at time t given history $\omega^t \in \Omega^t$.

Theorem 3.1. *Given $\omega_0, B_0 = b_0$ and ϕ_{-1}, the outcome path $(d_t, a_t, B_{t+1}, n_t)_{t=0}^{\infty}$ is consistent with a competitive equilibrium iff for all $(t, \omega^t) \in \{0, 1, 2, ...\} \times \Omega^t$, the following holds:*

$$\begin{aligned} Z_t(\omega^t)u_c(\omega^t) + \phi_t(\omega^t)\{p_t(\omega^t)u_c(\omega^t)B_{t+1}(\omega^t) - \delta_t u_c(\omega^t)B_t(\omega^{t-1})\} &\geq 0, \\ B_{t+1}(\omega^t) = B_t(\omega^{t-1}) \text{ if } \phi_t(\omega^t) &= 0, \end{aligned} \tag{3.9}$$

and $c_t(\omega^t) = \kappa_t(\omega^t)n_t(\omega^t) - g_t(\omega^t)$ and equations 3.4 and 3.6 hold.

For any $(\omega, B, \phi) \in (\mathbb{G} \times \overline{\Delta}) \times \mathbb{B} \times \{0, 1\}$, let $CE_\phi(\omega, B)$ denote the set of all outcome paths that are consistent with competitive equilibria, given $\omega_0 = \omega$, $\phi_0(\omega_0) = \phi$ and where B is the outstanding debt of time 0, after any potential debt restructuring in that period.[22] We observe that by setting $\phi_0(\omega_0) = \phi$ we are implicitly imposing restrictions on a_0, d_0, ϕ_{-1} and δ_0.[23]

Equation 3.9 summarizes the budget constraint of the government but replacing prices and taxes by the first order conditions, as in the "primal approach" used by Lucas and Stokey (1983) and Aiyagari et al. (2002).

[22]Constructing the set $CE_\phi(\omega, B)$ is useful since, in order to make a default/repayment decision, the default authority evaluates alternative utility values both for repayment and for autarky that are sustained by competitive equilibrium allocations.

[23]For example, if $\phi_0 = 1$ we could only arrive to it because either $\phi_{-1} = 1$ and $d_0 = 0$, given $(g_0, B_0) = (g, B)$, or because $\phi_{-1} = 0$ with defaulted debt $\widetilde{B}_0 = B_0/\delta_0$ – and offer δ_0 is accepted ($a_0 = 1$).

4 The Government Problem

The government is benevolent and maximizes the welfare of the representative household by choosing policies. The government, however, cannot commit to repaying the debt, but commits to previous tax promises until a debt restructuring takes place. That is, as long as the government keeps access to financial markets, it honors past promises of taxes. This assumption facilitates the comparison to the optimal taxation literature in the spirit of Lucas and Stokey (1983) and Aiyagari et al. (2002).

For autarky states, the government chooses taxes that balance its budget. Once the government accepts an offer to restructure the debt, it regains access to financial markets and starts anew, without any outstanding tax promises, by assumption. A similar feature is present in Debortoli and Nunes (2010), where the government can randomly re-optimize and reset fiscal policies with a given exogenous probability.[24]

The government problem can thus be viewed as a problem involving two types of authorities: a *default authority* and a *fiscal authority*. On the one hand, the default authority can be seen as comprised by a sequence of one-period administrations, where the time-t administration makes the default and repayment decision in period t, taking as given the behavior of all the other agents including the fiscal authority. On the other hand, the *fiscal authority* can be viewed as a sequence of consecutive administrations, each of which stays in office until there is a debt renegotiation. While ruling, a fiscal administration has the ability to commit, and chooses the optimal fiscal and debt processes, taking as given the behavior of the default authority. When debt is renegotiated, the fiscal administration is replaced by a new one, which is not bound by previous tax promises, and is free to reset the fiscal and debt policy.[25]

4.1 The Government Policies

For any $t \in \{0, 1, ...\}$, let $h_t \equiv (\phi_{t-1}, B_t, \omega_t)$ and $h^t \equiv (h_0, h_1, ..., h_t)$ be the *public history* until time t.[26] We use $\mathbb{H}^t$ to denote the set of all public histories until time t.

A *government strategy* is given by a strategy for the default and fiscal authorities, $\boldsymbol{\gamma} \equiv (\boldsymbol{\gamma}^D, \boldsymbol{\gamma}^F)$. The strategy for the default authority $\boldsymbol{\gamma}^D$ specifies a default and a repayment de-

[24]In our model, however, the *resetting* event, given by the debt restructuring, is an equilibrium outcome that emerges endogenously.

[25]We focus exclusively on symmetric strategies for households, where all of them take the same decisions along the equilibrium path. Similarly, we assume that all default and fiscal administrations choose identical actions conditional on the same state of the economy, thereby introducing a Markovian structure for optimal strategies.

[26]In our economy an individual household cannot alter prices and faces a (strictly) concave optimization problem. Any deviation from the equilibrium path determined by the Euler equation and the consumption-labor optimality condition, taking prices and policies as given, cannot be profitable from the household's perspective. Hence, there is no need to specify the household's behavior off the equilibrium path as well as to make households' strategies depend on private histories but only on public ones.

cision for any period t and any public history $h^t \in \mathbb{H}^t$, i.e., $\boldsymbol{\gamma}^D = (\gamma_t^D(\cdot))_{t=0}^{\infty}$ with $\gamma_t^D(h^t) \equiv (d_t(h^t), a_t(h^t))$ for any $h^t \in \mathbb{H}^t$. The strategy for the fiscal authority, $\boldsymbol{\gamma}^F$, specifies next period's debt level for any public history $h^t \in \mathbb{H}^t$ and any ϕ_t, i.e., $\boldsymbol{\gamma}^F = (\gamma_t^F(\cdot,\cdot))_{t=0}^{\infty}$ with $\gamma_t^F(h^t,\phi_t) \equiv B_{t+1}(h^t,\phi_t)$ for any $(h^t,\phi_t) \in \mathbb{H}^t \times \{0,1\}$. The fact that $\gamma_t^F(h^t,\phi_t)$ depends on ϕ_t reflects our assumption on the timing protocol by which the default authority moves first in each period.[27] Finally, note that any strategy $\boldsymbol{\gamma}$ jointly with $(\omega_t)_{t=0}^{\infty}$ generates an *outcome path* of allocations $(d_t, a_t, B_{t+1}, n_t)_{t=0}^{\infty}$. To stress that a particular policy action, say $B_{t+1}(h^t,\phi_t)$, belongs to a given strategy we use $B_{t+1}(\boldsymbol{\gamma})(h^t,\phi_t)$.

Let $\boldsymbol{\gamma}|_{(h^t,\phi_t)}$ denote the continuation of strategy $\boldsymbol{\gamma}$ after history $(h^t,\phi_t) \in \mathbb{H}^t \times \{0,1\}$.[28] We say a strategy $\boldsymbol{\gamma}$ is *consistent with a competitive equilibrium*, if after any $(h^t,\phi_t) \in \mathbb{H}^t \times \{0,1\}$, the outcome path generated by $\boldsymbol{\gamma}|_{(h^t,\phi_t)}$ belongs to $CE_{\phi_t}(\omega_t, B)$ with outstanding debt $B = (\delta_t\phi_t + (1-\phi_t))B_t(\boldsymbol{\gamma})(h^{t-1}, \phi_{t-1}(\boldsymbol{\gamma})(h^{t-1}))$. That is, if there is full repayment (i.e. no default) or not borrowing at all in the current period t, the debt level B is given by the bond holdings carried over from last period. Otherwise, if the government just regained access by accepting an offer δ_t, B is the restructured debt level.

For any $h_0 \in \mathbb{H}$ and $\phi_0 \in \{0,1\}$, we use $\mathcal{S}(h_0,\phi_0)$ to denote the set of such strategies; see appendix D for the formal expression of $\mathcal{S}(h_0,\phi_0)$. Henceforth, we only consider strategies that are consistent with competitive equilibrium.

Finally, for any public history $h^t \in \mathbb{H}^t$, $\phi \in \{0,1\}$ and $\boldsymbol{\gamma} \in \mathcal{S}(h_0,\phi)$, let

$$V_t(\boldsymbol{\gamma})(h^t,\phi) = E_{\Pi(\cdot|\omega_t)}\left[\sum_{j=0}^{\infty} \beta^j u(\kappa_{t+j}(\omega^{t+j})n_{t+j}(\boldsymbol{\gamma})(\omega^{t+j}) - g_{t+j}, 1 - n_{t+j}(\boldsymbol{\gamma})(\omega^{t+j}))\right] \quad (4.10)$$

be the expected lifetime utility of the representative household at time t, given strategy $\boldsymbol{\gamma}|_{(h^t,\phi)}$.

4.2 Default and Renegotiation Policies

As mentioned before, the default authority can be viewed as comprised by a sequence of one-period administrations, each of which makes the default and renegotiation decision in its respective period, taking as given the behavior of all the other agents including the other default and fiscal administrations. It is easy to see that, for each public history $h^t \in \mathbb{H}^t$, the default authority will optimally choose as follows: if $\phi_{t-1} = 1$

$$d_t^*(\boldsymbol{\gamma})(h^t) = \begin{cases} 0 & if\ V_t(\boldsymbol{\gamma})(h^t,1) \geq V_t(\boldsymbol{\gamma})(h^t,0) \\ 1 & if\ V_t(\boldsymbol{\gamma})(h^t,1) < V_t(\boldsymbol{\gamma})(h^t,0) \end{cases} \quad (4.11)$$

[27]We omit labor taxes (or labor directly) as part of the government strategy because, given (h^t,ϕ_t) and $\gamma_t^F(h^t,\phi_t)$, labor taxes are obtained from the budget constraint. For this reason we do not include them as part of the public history.

[28]Observe that while a strategy prescribes that the default authority moves first at $t=0$, with the continuation strategy, as we defined, the fiscal authority is moving first at t and then the default authority moves at $t+1$.

and if $\phi_{t-1} = 0$

$$a_t^*(\boldsymbol{\gamma})(h^t) = \begin{cases} 1 & if\ V_t(\boldsymbol{\gamma})(h^t, 1) \geq V_t(\boldsymbol{\gamma})(h^t, 0) \\ 0 & if\ V_t(\boldsymbol{\gamma})(h^t, 1) < V_t(\boldsymbol{\gamma})(h^t, 0) \end{cases} \tag{4.12}$$

The dependence on $\boldsymbol{\gamma}$ denotes the fact that d_t^* and a_t^* are associated with the strategy of the fiscal authority $\boldsymbol{\gamma}^F$. Indeed, to specify the optimal default and repayment decisions at any history $h^t \in \mathbb{H}^t$ we need to know the value of repayment and the value of default, $V_t(\boldsymbol{\gamma})(h^t, 1)$ and $V_t(\boldsymbol{\gamma})(h^t, 0)$, respectively, which are evidently functions of $\boldsymbol{\gamma}^F$.[29]

4.3 Recursive Representation of the Government Problem

Taking as given the optimal decision rules 4.11 and 4.12 for the default authority, we now turn to the optimization problem of the fiscal authority and the recursive representation of the government problem. To do so, we adopt a recursive representation for the competitive equilibria by introducing an adequate state variable. Following Kydland and Prescott (1980) and Chang (1998) among others, it follows that the relevant (co-)state variable is the "promised" marginal utilities of consumption.[30]

For any $h_0 = (\phi_{-1}, B_0, g_0, \delta_0) \in \mathbb{H}$ and $\phi \in \{0, 1\}$, let $\Omega(h_0, \phi)$ be the set of all marginal utility values (μ) and lifetime utilities at time zero (v) that can be sustained in a competitive equilibrium, wherein the default authority reacts optimally from next period on; see appendix D for the formal expression of $\Omega(h_0, \phi)$.

It is worth to point out that this set differs from the standard set of equilibrium promised marginal utilities in Kydland and Prescott (1980) along some dimensions. In particular, in an standard Ramsey problem it would suffice to only specify the set of promised marginal utilities, but in our framework with endogenous default decisions we find it necessary to also specify continuation values to evaluate alternative courses of action of the default authority. By the same token, we compute this set for any ϕ, even for the value of ϕ not optimally chosen by the default authority through its policy action.

For any $(g, B, \mu) \in \mathbb{G} \times \mathbb{B} \times \mathbb{R}_+$, let $V_1^*(g, B, \mu)$ be the value of a fiscal authority that had access to financial markets last period and continue to have it this current period (i.e., $\phi_{-1} = \phi = 1$) and that takes as given the optimal behavior of the default and subsequent fiscal authorities, with outstanding debt B and a promised marginal utility of μ and government expenditure g. Similarly, let $V_0^*(g, B)$ be the value of a fiscal authority that does not have access to financial

[29] Also, recall that by assumption $a_t^*(\gamma)(h^t) = 0$ if $\phi_{t-1} = 1$ or $\delta_t = \bar{\delta}$ and $d_t^*(\gamma)(h^t) = 1$ if $\phi_{t-1} = 0$.

[30] By keeping track of the profile of "promised" marginal utilities of consumption, we ensure that the fiscal authority commits to deliver the "promised" marginal utility —as long as the default authority does not restructure the debt— for each realization of g; thereby guaranteeing that the last-period households' Euler equation is satisfied after each possible history. If the debt is restructured and a new fiscal administration takes power, it sets the current marginal utility at its convenience, which in equilibrium is anticipated by the households.

markets (i.e., $\phi = 0$). Observe that since in financial autarky the government ought to run a balanced budget, V_0^* does not depend on μ.

Finally, let $\overline{V}_1^*(g, \delta B)$ be the value of a "new" fiscal authority (i.e., when $\phi_{-1} = 0$ and $\phi = 1$) that takes as given the optimal behavior of the default and subsequent fiscal authorities, when an offer δ is accepted, given government spending g and outstanding defaulted debt B. Note that in this case the fiscal authority does not have any outstanding "promised" marginal utility and thus it sets the current marginal utility at its convenience. By definition of Ω, it follows that

$$\overline{V}_1^*(g, \delta B) = \max\{v | (\mu, v) \in \Omega(0, B, g, \delta, 1)\}, \tag{4.13}$$

as the government maximizes the households' utility without any attached promise of marginal utility to be delivered.[31] Let $\overline{\mu}(g, \delta, B) = \{\mu | (\mu, \overline{V}_1^*(g, \delta B)) \in \Omega(0, B, g, \delta, 1)\}$ be the associated marginal utility.

Given the aforementioned value functions, the optimal policy functions of the default authority in expressions 4.11-4.12 become[32,33]

$$\mathbf{d}^*(g, B, \mu) = \begin{cases} 0 & if\ V_1^*(g, B, \mu) \geq V_0^*(g, B) \\ 1 & if\ V_1^*(g, B, \mu) < V_0^*(g, B) \end{cases} \tag{4.14}$$

and

$$\mathbf{a}^*(g, \delta, B) = \begin{cases} 1 & if\ \overline{V}_1^*(g, \delta B) \geq V_0^*(g, B) \\ 0 & if\ \overline{V}_1^*(g, \delta B) < V_0^*(g, B) \end{cases} \tag{4.15}$$

The next theorem presents a recursive formulation for the value functions. In what follows, we denote the marginal utility of consumption in financial autarky as $m_A(g)$ for all $g \in \mathbb{G}$.[34]

Theorem 4.1. *The value functions V_0^* and V_1^* satisfy the following recursions*

$$V_1^*(g, B, \mu) = \max_{(n, B', \mu'(\cdot)) \in \Gamma(g, B, \mu)} \left\{ u(n - g, 1 - n) + \beta \int_{\mathbb{G}} \max\{V_1^*(g', B', \mu'(g')), V_0^*(g', B')\} \pi_{\mathbb{G}}(dg'|g) \right\}, \tag{4.16}$$

and

$$\begin{aligned} V_0^*(g, B) =& u(\kappa \mathbf{n}_0^*(g) - g, 1 - \mathbf{n}_0^*(g)) + \beta\lambda \int_{\mathbb{G}} \int_{\Delta} \max\{\overline{V}_1^*(g', \delta' B), V_0^*(g', B)\} \pi_{\Delta}(d\delta') \pi_{\mathbb{G}}(dg'|g) \\ & + \beta(1 - \lambda) \int_{\mathbb{G}} V_0^*(g', B) \pi_{\mathbb{G}}(dg'|g) \end{aligned} \tag{4.17}$$

[31] We are implicitly assuming that the maximum is achieved. This assumption is imposed to ease the exposition and could be relaxed by defining $\overline{V}_1^*$ in terms of a supremum and approximate maximizers.

[32] Implicit in both definitions is the refinement that in case of indifference, the government decides to accept/not default on the debt. Without this refinement, the optimal decisions will be correspondences that take any value between 0 and 1 in case of indifference.

[33] As indicated before, by assumption, $\mathbf{d}^*(g, B, \mu) = 1$ if $\phi_{-1} = 0$ and $\mathbf{a}^*(g, \delta, B) = 0$ if $\phi_{-1} = 1$ or $\delta = \bar{\bar{\delta}}$.

[34] Formally, $m_A(g) = u_c(\kappa \mathbf{n}_0^*(g) - g, 1 - \mathbf{n}_0^*(g))$ where $\mathbf{n}_0^*(g) = \arg\max_{n \in [0,1]} \{u(\kappa n - g, 1 - n) : z(\kappa, n, g) = 0\}$. See appendix D for details.

where, for any (g, B, μ),

$$\Gamma(g,B,\mu) = \Big\{(n, B', \mu'(\cdot)) \in [0,1] \times \mathbb{B} \times \mathbb{R}^{|\mathbb{G}|} : \\ (B', \mu'(g'), V_1^*(g', B', \mu'(g'))) \in Graph(\Omega(1, \cdot, g', 1, 1)),\ \forall g' \in \mathbb{G} \\ \mu = u_c(n-g, 1-n) \ and \ z(1,n,g)\mu + \mathcal{P}_1^*(g, B', \mu'(\cdot))B' - B\mu \geq 0\} \tag{4.18}$$

and, for any $(B', \mu'(\cdot))$,

$$\mathcal{P}_1^*(g, B', \mu'(\cdot)) = \beta \int_{\mathbb{G}} \left((1 - \mathbf{d}^*(g', B', \mu'(g')))\mu'(g') + \mathbf{d}^*(g', B', \mu'(g'))m_A(g')\mathcal{P}_0^*(g', B')\right) \pi_{\mathbb{G}}(dg'|g)$$

$$\mathcal{P}_0^*(g, B') = \beta \int_{\mathbb{G}} \left(\int_{\Delta} \overline{\mu}(g', \delta', B')\delta' \mathbf{a}^*(g', \delta', B')\pi_{\Delta}(d\delta') + \pi_A^*(g', B')m_A(g')\mathcal{P}_0^*(g', B')\right) \pi_{\mathbb{G}}(dg'|g)$$

where $\pi_A^*(g,B) \equiv \{(1-\lambda) + \lambda \int_{\Delta}(1 - \mathbf{a}^*(g, \delta, B))\pi_{\Delta}(d\delta)\}$ *for any* (g, B).

Below we present some particular cases of special interest where the recursive representation of the government problem gets simplified.

Example 4.1 (nondefaultable debt)**.** *Consider an economy with (ad hoc) risk-free debt. The value function* V_0^* *is irrelevant and* V_1^* *boils down to*

$$V_1^*(g, B, \mu) = \max_{(n, B', \mu'(\cdot)) \in \Gamma(g,B,\mu)} \left\{u(n-g, 1-n) + \beta \int_{\mathbb{G}} V_1^*(g', B', \mu'(g'))\pi_{\mathbb{G}}(dg'|g)\right\}$$

where

$$\Gamma(g, B, \mu) = \left\{ \begin{array}{l} (n, B', \mu'(\cdot)) : z(1,n,g)\mu + \beta E_{\pi_{\mathbb{G}}(\cdot|g)}[\mu'(g')]B' - B\mu \geq 0 \\ where \ \mu = u_c(n-g, 1-n) \end{array} \right\}.$$

In addition, since there is no "re-setting" time, $\bar{V}_1^*$ *coincides with the value function at time 0 with* μ *chosen optimally. This case is precisely the type of model studied in Aiyagari et al. (2002).*□

Example 4.2 (quasi-linear per-period payoff, $\lambda \geq 0$, and $\pi_{\Delta} = \mathbf{1}_{\{0\}}$)**.** *Assume that* $u(c, 1-n) = c + H(1-n)$ *for some function* H *consistent with assumption 3.1. Under this assumption,* μ *can be dropped as a state variable since* $u_c = 1$ *and thus it does not affect the pricing equation. In this case, the value function during financial autarky is given by*

$$V_0^*(g) = \kappa \boldsymbol{n}_0^*(g) - g + H(1 - \boldsymbol{n}_0^*(g)) + \beta \int_{\mathbb{G}} \left(\lambda V_1^*(g', 0) + (1-\lambda)V_0^*(g')\right) \pi_{\mathbb{G}}(dg'|g).$$

This expression follows from the fact that there is no need to keep the debt B *as part of the state during financial autarky since none of the defaulted debt is ever repaid, and all the offers of zero repayment are accepted by the government. The value function during financial access is given by*

$$V_1^*(g, B) = \max_{(n, B') \in \Gamma(g,B)} \left\{n - g + H(1-n) + \beta \int_{\mathbb{G}} \max\{V_1^*(g', B'), V_0^*(g')\}\pi_{\mathbb{G}}(dg'|g)\right\},$$

where $\Gamma(g,B) \equiv \{(n,B') \ : \ z(1,n,g) + \beta E_{\pi_{\mathbb{G}}(\cdot|g)}[\mathbf{1}_{\{g':V_1^*(g',B') \geq V_0^*(g')\}}(g')]B' - B \geq 0\}$.

The expression for the price function highlights an important difference between our default model and a model with risk-free debt such as Aiyagari et al. (2002) (henceforth, AMSS). Since $u_c = 1$*, the market stochastic discount factor is equal to* β*, and thus in the AMSS model the government cannot manipulate the return of the discount bond. In our economy with defaultable debt, however, while not being able to influence the risk-free rate, the government is still able to manipulate the return of the discount bond by altering its payoff through the decision of default.*

Moreover, assuming H *is increasing and strictly concave with* $H'(1) < 1$ *and* $2H''(l) < H'''(l)(1-l)$*, we can view the government problem as directly choosing tax revenues* R *with a per-period payoff given by* $W_\kappa(R) = \kappa n_\kappa(R) + H(1-n_\kappa(R))$ *where* $n_\kappa(R)$ *is the amount of labor needed to collect revenues equal to* R*, given* κ*. Under our assumptions,* W_κ *is non-increasing and concave function. The Bellman equation of the value of repayment is given by*

$$V_1^*(g,B) = \max_{(R,B')} \left\{ W_1(R) - g + \beta \int_{\mathbb{G}} \max\{V_1^*(g',B'), V_0^*(g')\} \pi_{\mathbb{G}}(dg'|g) \right\}, \tag{4.19}$$

subject to $R + \beta E_{\pi_{\mathbb{G}}(\cdot|g)}[\mathbf{1}_{\{g':V_1^*(g',B') \geq V_0^*(g')\}}(g')]B' \geq g + B$*, and*

$$V_0^*(g) = W_\kappa(g) - g + \beta \int_{\mathbb{G}} \left(\lambda V_1^*(g',0) + (1-\lambda) V_0^*(g')\right) \pi_{\mathbb{G}}(dg'|g). \tag{4.20}$$

The previous equations imply that this government problem is analogous to that studied in Arellano (2008) and Aguiar and Gopinath (2006) among others, where the government chooses how much to "consume", captured by $-R$*, given an exogenous process of "income",* $-g$*. An important difference, however, is the non-standard per-period payoff which reflects the distortive nature of labor taxes. In particular in our model the per-period payoff has a satiation point at* $R = 0$ *(i.e., zero distortive taxes).*[35]

We think this last observation is relevant because it allows us to extend some of our results to general sovereign debt models with endogenous default, especially those results regarding the impact of the debt restructuring in debt prices (both, before and during financial autarky). □

5 Analytical Results

In this section we present analytical results for a benchmark model characterized by quasi-linear per-period utility, i.i.d. government expenditure shocks and debt repayments for exiting financial autarky. The proofs for the results are gathered in appendix E.

Assumption 5.1. *(i)* $\kappa = 1$*; (ii)* $u(c,n) = c + H(1-n)$ *where* $H \in \mathbb{C}^2((0,1),\mathbb{R})$ *with* $H'(0) = \infty$*,* $H'(l) > 0$*,* $H'(1) < 1$*,* $H''(l) < 0$ *and* $2H''(l) < H'''(l)(1-l)$

[35] Another subtle difference with the standard sovereign default literature is that while in our economy government and bondholders share the same preference, in this literature they do not. In particular, the government tends to be more impatient than (foreign) investors, thus bringing about incentives to front-load consumption through borrowing.

Part (i) implies that there are no direct cost of defaults in terms of output. Part (ii) of this assumption imposes that the per-period utility of the households is quasi-linear and it is analogous to assumption in p. 10 in AMSS. As noted above, under this assumption, μ can be dropped as a state variable. This implies that the value functions V_0^*, V_1^* are only functions of (g, B) and the same holds true for the optimal policy functions.

We also assume that government expenditure are i.i.d., formally

Assumption 5.2. *For any $g' \neq g$, $\pi_{\mathbb{G}}(\cdot|g) = \pi_{\mathbb{G}}(\cdot|g')$.*

With a slight abuse of notation and to simplify the exposition we use $\pi_{\mathbb{G}}(\cdot)$ to denote the probability measure of g. Finally, to further simplify the technical details, we assume that $\mathbb{B}$ has only finitely many points, unless stated otherwise.[36]

For the rest of the section, we proceed as if these assumptions 5.1 - 5.2 hold and will not be referenced explicitly.

5.1 Characterization of Optimal Default Decisions

The next proposition characterizes the optimal decisions to default and to accept offers to repay the defaulted debt as "threshold decisions". These results are analogous to Arellano (2008) but extended to this setting, in particular we allow for partial repayments of government debt. Recall that $\mathbf{d}^*(g, B)$ and $\mathbf{a}^*(g, \delta, B)$ are the optimal decision of default and of renegotiation, respectively, given the state (g, δ, B).

Proposition 5.1. *There exists $\bar{\lambda}$ such that for all $\lambda \in [0, \bar{\lambda}]$, the following holds:*

1. *There exists a $\hat{\delta} : \mathbb{G} \times \mathbb{B} \to \Delta$ such that $\mathbf{a}^*(g, \delta, B) = \mathbf{1}_{\{\delta : \delta \leq \hat{\delta}(g,B)\}}(\delta)$ and $\hat{\delta}$ non-increasing as a function of B.*[37]

2. *There exists a $\bar{g} : \mathbb{B} \to \mathbb{G}$ such that $\mathbf{d}^*(g, B) = \mathbf{1}_{\{g : g \geq \bar{g}(B)\}}(g)$ and $\bar{g}$ non-increasing for all $B > 0$.*

This result shows that for a (non-trivial) range of probabilities of receiving outside offers, $\lambda \in [0, \bar{\lambda}]$, default is more likely to occur for high levels of debt, and so are rejections of offers to exit financial autarky. The latter result implies that the average recovery rate, $E_{\pi_{\mathbb{G}}}[\int_{\delta' \in \Delta} \delta' \mathbf{1}_{\{\delta \,:\, \delta \leq \hat{\delta}(g,B)\}}(\delta') \pi_{\Delta}(d\delta')]$, is decreasing in the level of debt, as documented by Yue (2010) in the data. It also follows that other things equal, higher debt levels are on average associated with longer financial autarky periods. Thus, these two results imply a positive co-movement between the (observed) average haircut and the average length of financial autarky.

[36] This assumption is made for simplicity. It can be relaxed to allow for general compact subsets, but some of the arguments in the proofs will have to be changed slightly. Also, the fact that $\mathbb{B} \equiv \{B_1, ..., B_{|\mathbb{B}|}\}$ is only imposed for the government; the households can still choose from convex sets; only in equilibrium we impose $\{B_1, ..., B_{|\mathbb{B}|}\}$.

[37] It turns out that the first part of the statement holds for any λ.

This last fact seems to be consistent with the data; see fact 3 in Benjamin and Wright (2009). Cruces and Trebesch (2013) found a similar relationship for 180 sovereign debt restructuring cases of 68 countries between 1970 and 2010.[38]

5.2 Implications for Equilibrium Prices and Taxes

We now study the implications of the above results on equilibrium prices and taxes.

Equilibrium prices and endogenous debt limits. Under assumption 5.2 equilibrium prices do not depend on g, i.e., $\mathcal{P}^*_\phi(\cdot) \equiv \mathcal{P}^*_\phi(g,\cdot)$ for any $g \in \mathbb{B}$. By proposition 5.1 it follows that, for any $B' \in \mathbb{B}$,

$$\mathcal{P}^*_1(B') = \beta \int_{\mathbb{G}} \mathbf{1}_{\{g' \leq \bar{g}(B')\}}(g')\pi_{\mathbb{G}}(dg') + \left(\beta \int_{\mathbb{G}} \mathbf{1}_{\{g' > \bar{g}(B')\}}(g')\pi_{\mathbb{G}}(dg')\right)\mathcal{P}^*_0(B') \tag{5.21}$$

and[39]

$$\mathcal{P}^*_0(B) = \frac{\beta\lambda \int_\Delta \left(\int_{\mathbb{G}} \mathbf{1}_{\{\delta:\delta \leq \hat{\delta}(g',B)\}}(\delta)\pi_{\mathbb{G}}(dg')\right)\delta\pi_\Delta(d\delta)}{1-\beta+\beta\lambda\int_\Delta\int_{\mathbb{G}} \mathbf{1}_{\{\delta:\delta \leq \hat{\delta}(g',B)\}}(\delta)\pi_{\mathbb{G}}(dg')\pi_\Delta(d\delta)}. \tag{5.22}$$

A key feature of endogenous default models is the existence of endogenous borrowing limits. A necessary condition for this result is that, due to the possibility of default, equilibrium prices are non-increasing as a function of debt, thus implying a "Laffer-type curve" for the revenues coming from selling bonds. In an economy without debt repayment (e.g., $\pi_\Delta = \mathbf{1}_{\{0\}}$), it follows that $\mathcal{P}^*_0 = 0$ and $\mathcal{P}^*_1(B') = \beta\int_{\mathbb{G}} \mathbf{1}_{\{g:g\leq\bar{g}(B')\}}(g')\pi_{\mathbb{G}}(dg')$ which is non-increasing in B' by proposition 5.1. Moreover, it takes value zero for sufficiently high B'. Therefore, there exists an endogenous debt limit, i.e., finite value of B' that maximize the debt revenue $\mathcal{P}^*_1(B')B'$.

In an economy where we allow for debt repayments, by inspection of equation 5.21 and the fact that $\mathcal{P}^*_0 \geq 0$, it is easy to see that, other things equal, the previous result is attenuated by the presence of (potential) defaulted debt payments and secondary markets. The next proposition shows that when repayment offers exist but are non-random, the price is non-increasing on the level of debt and there are endogenous borrowing limits.

Proposition 5.2. *Suppose $\pi_\Delta(\cdot) = \mathbf{1}_{\delta_0}(\cdot)$ for some $\delta_0 \in [0,1]$. Then there exists a $\bar{\lambda} > 0$, such that for all $\lambda \in [0,\bar{\lambda}]$, $\mathcal{P}^*_i(\cdot)$ is non-increasing for $B > 0$ and for $i = 0,1$.*

This proposition shows that high levels of debt are associated with higher return on debt, both before and during financial autarky. This result is consistent with the evidence of the

[38]It is important to note, however, that we derived the implications by looking at *exogenous* variations of the debt level; in the data this quantity is endogenous and, in particular, varies with g. This endogeneity issue taken into account in the numerical simulations, wherein we perform a more thorough test of the aforementioned implications.

[39]See lemma E.5(3) in the appendix for the derivation.

positive relationship between debt-to-output levels and default risk measures; see section F in the supplementary material.

In addition, the existence of endogenous borrowing limits implies that the ability to roll over high levels of debt is hindered. Since the primary surplus function $z(1, \cdot, g)$ is concave in n, as shown in lemma D.1 of appendix D, labor is more "sensitive" to fluctuations in government expenditure when the indebtness level is high. This feature is consistent with the stylized fact that on average higher volatility of tax revenue-to-output ratios is observed when debt and default risk are high, as documented in section F. We further explore this mechanism in the numerical simulations.

Default risk and the law of motion of equilibrium taxes. In order to analyze the ex-ante effect of default risk on the law of motion of taxes, we look at the case $\lambda = 0$ (i.e., autarky is an absorbing state) to simplify the analysis. We also strengthen assumption 5.1 by requiring that $H''(l) < H'''(l)(1-l)$. By proposition 5.1, the default decision is a threshold decision, so for each history $\omega^{\infty} \in \Omega^{\infty}$ we can define $T(\omega^{\infty}) = \inf\{t : g_t \geq \bar{g}(B_t(\omega^{t-1}))\}$ (it could be infinity) as the first time the economy enters in default. For all $t \leq T(\omega^{\infty})$ the economy is not in financial autarky, and the implementability constraint is given by

$$B_t(\omega^{t-1}) + g_t \leq \Big(1 - H'(1 - n_t(\omega^t))\Big)\, n_t(\omega^t) + \mathcal{P}_1^*(B_{t+1}(\omega^t))B_{t+1}(\omega^t),$$

where $\mathcal{P}_1^*(g_t, B_{t+1}(\omega^t)) \equiv E_{\pi_{\mathbb{G}}}[1 - \mathbf{d}^*(g', B_{t+1}(\omega^t))]$. Let $\nu_t(\omega^t)$ be the Lagrange multiplier associated to this restriction in the optimization problem of the government, given $\omega^t \in \Omega^t$. In appendix E.2 we derive the FONC of the government and provide a closed form expression for $\nu_t(\omega^t)$ as a *decreasing* nonlinear function of $n_t(\omega^t)$; see equation E.87.[40] Hence, as noted by AMSS, by studying the law of motion of ν_t we can shed light on the law of motion of labor and taxes.

From the FONC of the government it follows (see appendix E.2 for the derivation)[41,42]

$$\nu_t(\omega^t)\left(1 + \frac{d\mathcal{P}_1^*(B_{t+1}(\omega^t))}{dB_{t+1}} \frac{B_{t+1}(\omega^t)}{\mathcal{P}_1^*(B_{t+1}(\omega^t))}\right) = \int_{\mathbb{G}} \nu_{t+1}(\omega^t, g') \frac{1\{g' \leq \bar{g}(B_{t+1}(\omega^t))\}}{\int_{\mathbb{G}} 1\{g' \leq \bar{g}(B_{t+1}(\omega^t))\}\pi_{\mathbb{G}}(dg')} \pi_{\mathbb{G}}(dg'). \tag{5.23}$$

The Lagrange multiplier associated with the implementability condition is constant in Lucas and Stokey (1983) and, thus, trivially a martingale. In Aiyagari et al. (2002) the Lagrange

[40] The strengthening of assumption 5.1 is only needed to show that $\nu_t(\omega^t)$ is an decreasing function of $n_t(\omega^t)$; the proof is in appendix E.2.

[41] This derivation assumes that $\mathbb{B}$ is a convex set and $\pi_{\mathbb{G}}$ has a density with respect to the Lebesgue measure, so as to make sense of differentiation. It also assumes differentiability of V^*.

[42] The martingale property is also preserved if capital is added to the economy; see Farhi (2010). If we allow for ad hoc borrowing/savings limits, the equality has to be replaced by the corresponding inequality; see Aiyagari et al. (2002).

multiplier associated with the implementability condition is a martingale with respect to the probability measure $\pi_{\mathbb{G}}$ away from the asset limits. Equation 5.23 implies that the law of motion of the Lagrange multiplier differs in two important aspects. First, the expectation is computed under the so-called *default-adjusted probability measure*, given by $\frac{1\{\cdot\leq\bar{g}(B_{t+1}(\omega^t))\}}{\int_{\mathbb{G}}1\{g'\leq\bar{g}(B_{t+1}(\omega^t))\}\pi_{\mathbb{G}}(dg')}\pi_{\mathbb{G}}(\cdot)$. The wedge between $\pi_{\mathbb{G}}$ and this new probability stems from the fact that the option to default adds "some" degree of state-contingency to the payoff of the government debt making it lower for high values of government expenditure, and implies that the default-adjusted probability measure is first order dominated by $\pi_{\mathbb{G}}$. In fact, it implies that only the states tomorrow in which there is repayment, are relevant for the law of motion of ν_t.[43] It follows that, in the case that $\nu_{t+1}(\omega^t,\cdot)$ is increasing, the presence of the default-adjusted probability lowers $\nu_t(\omega^t)$ and, consequently, lowers also the tax distortions.[44] Second, $\nu_t(\omega^t)$ in the left-hand side is multiplied by $\left(1+\frac{d\mathcal{P}_1^*(B_{t+1}(\omega^t))}{dB_{t+1}}\frac{B_{t+1}(\omega^t)}{\mathcal{P}_1^*(B_{t+1}(\omega^t))}\right)$, which can be interpreted as the "markup" that the government has to pay for having the option to default; this effect increases $\nu_t(\omega^t)$ and the tax distortions. These two forces act in opposite directions, and is not clear which one will prevail. In order to shed more light on this issue, in section 6 we explore quantitatively this trade-off by plotting the impulse responses for $\nu_t(\omega^t)$ delivered by our model and a version of Aiyagari et al. (2002).

6 Numerical Results

Throughout this section, we run a battery of numerical exercises in order to assess the impact of endogenous default risk on fiscal policy and the overall economy's dynamics. We compare our findings with an economy in which the option to default is not present—precisely the model considered in Aiyagari et al. (2002). We denote the variables associated with this model with a (sub)superscript "AMSS"; variables associated to our economy are denoted with a (sub)superscript "ED" (short for Economy with Default).

A natural question that arises in this context is what characteristics of an economy will prompt it to behave as the AMSS- or ED-type models prescribe, in particular when it comes to the propensity or willingness to honor debt contracts. Several reasons have been put forth to explain why some governments always repay while some others do not. One reason is attributed to factors unrelated to the model, such as political instability and polarization, which could result in lower discounting of future consumption by the incumbent ruling party and hence stronger incentives to default.[45] An alternative explanation, in line with our model, is that for

[43] If default occurs, the link between multipliers today and tomorrow is disrupted and tax rates stop exhibiting persistence, as they are set to balance the budget inheriting the stochastic properties of the government spending process.

[44] By calculations in the appendix E.2, $\nu_{t+1}(\omega^t,\cdot)$ is increasing iff $\mathbf{n}_1^*$ is decreasing in g.

[45] See for example Cuadra and Sapriza (2008) and D'Erasmo (2011). In a small open political economy where

AMSS-type economies, default is more costly because they are financially more integrated, and the inability to borrow from capital markets after a default could have a more severe impact on financing of the firms, thus lowering their productivity (in our model represented by a lower κ).[46] A third explanation is connected to income inequality and redistributional motives. High income dispersion across domestic households provides stronger incentives to default to the government, expropriate wealth and redistribute those resources to reduce inequality.[47] Although we consider these issues important, we think they are out of the scope of the present paper and do not explore them herein. We take them as given and proceed to characterize the optimal government policies in this economic environment.

While the main focus of the numerical results is to describe some features pointed out in the normative analysis, we will see that the dynamics of the our economy with default risk replicates qualitatively some interesting empirical facts observed in the times of the debt restructuring plan of the U.S. economy in the 18th century and in several emerging markets nowadays.

Benchmark calibration. For all the simulations the utility function is given by $u(c, 1-n) = c + C_1 \frac{(1-n)^{1-\sigma}}{1-\sigma}$. For this preference specification, it is easy to see that whether the fiscal authority has the ability to commit to tax policies or not renders the same equilibrium results. In this parametrization, we assume that the government expenditure g_t follows an AR(1) log-normal process

$$\log g_t = (1-\rho)\mu + \rho \log g_{t-1} + \sigma_\varepsilon \varepsilon_t$$

with $\varepsilon_t \sim N(0,1)$, which is approximated by an 11-state Markov chain using Tauchen (1986) procedure.

The debt state space $\mathbb{B}$ is constructed by discretizing $[0, 0.4]$ into 800 gridpoints. Finally, as in AMSS for this quantitative section we rule out negative lump-sum transfers to the households.

We choose the parameters of the model as follows. We set $\beta = 0.97$, $\psi = 2$, $\kappa = 0.998$ and $C_1 = 0.15$.[48] The parameter values for the stochastic process of the government expenditure are $\mu = 0.114$, $\rho = 0.56$ and $\sigma_\varepsilon = 0.037$. For the debt restructuring process we choose a

different political groups compete with each other for access to government resources, Amador (2011) arrives to a different conclusion. The same arguments why political interactions lead to over-spending actually provide incentives not to repudiate sovereign debt contracts.

[46] Along this line, in a general equilibrium setup Mendoza and Yue (2012) generate an endogenous loss of output resulting from the substitution of imported inputs by less-efficient domestic ones as firms' credit lines are cut during default episodes. Instead, in Sosa-Padilla (2014) a financial disruption in the banking sector occurs after a default event, as banks' balance sheets deteriorate, causing a domestic credit crunch followed by an output drop.

[47] D'Erasmo and Mendoza (2015) and Dovis et al. (2015) elaborate along this dimension.

[48] While the exogenous ad hoc output cost of default is only 0.2 percent, the endogenous drop of consumption is significantly larger. Indeed, if the government runs a balanced budget, the autarkic level of consumption is over 1 percent lower than its counterpart in repayment for high government spending.

probability of receiving an offer $\lambda = 0.47$, and consider ten equally-probable renegotiation offers with equidistant haircuts ranging from 0.45 to 0.9. A period corresponds to a year in this calibration.

In this environment, the one-period gross risk-free rate $1 + r^f$ is equal to the reciprocal of the households' discount factor β. Bond spreads are computed as the differential between bond returns and the risk-free rate. The gross return of the government bond is given by $1/p_t$.

To compute the statistics, we perform 5,000 Monte Carlo (MC) iterations, each consisting of sample paths of 2,500 observations for which the first 500 observations were disregarded in order to eliminate the effect of the initial conditions. We then compute the mean statistics across MC simulations.[49]

Table 6.1: MC Statistics for the whole sample for our model and the risk-free debt model (AMSS).

Statistic	ED	AMSS
Mean(debt/y)(%)	5.2	27.2
Mean(τ)	0.207	0.212
Std.dev.(τ)	0.078	0.053
Autocor.(τ)	0.640	0.831
Mean(y)	0.563	0.563
Mean(c)	0.449	0.449
Std.dev.(y)	0.022	0.015
Std.dev.(c)	0.066	0.057
Mean($r - r^f$) (%)	1.59	0
Mean(default spell)	5.62	NA
Mean(recovery rate) (%)	58.7	NA
Failed reneg. freq. (%)	53.30	NA
Default frequency (%)	1.78	0
20th-percentile(τ)	0.147	0.181
80th-percentile(τ)	0.263	0.246

The results for the whole sample for our model and the risk-free debt model (AMSS) are reported in Table 6.1. In our economy default occurs with an annual frequency of around 1.78 percent. Bond holders anticipate the default strategies in equilibrium and charge higher bond

[49] The unconditional default frequency is computed as the sample mean of the number of default events in the simulations. The model is solved numerically using value function iterations with a discrete state space and an "outer" loop that iterates on prices until convergence.

returns to bear the bond. In our model bond spreads are sizable reaching over 1.5 percent. Facing higher borrowing costs, the government responds by issuing less bonds. Consequently, the average level of indebtedness is significantly lower in our environment than in AMSS model: it is 5.2 percent of output in the former, while 27.2 percent in the latter economy.[50]

The presence of endogenous borrowing limits, arising from the possibility of default, hinders the government's ability to smooth taxes. As a result, taxes are more volatile: the standard deviation of tax rates is 47 percent higher in our model.[51] Also, note that the 20th-80th percentile interval for the tax distribution in the economy with defaultable bonds contains its counterpart in the AMSS model, which reflects the fact that the latter distribution is relatively more spread out. Not surprisingly, taxes are less persistent in our environment. The lower autocorrelation is attributed to two main factors. First, a lower persistence of taxes in borrowing states due to the incidence of endogenous default, manifested in the law of motion of the Lagrange multiplier of the implementability constraint; see subsection 5.2. Second, the fact that if default occurs, the economy switches to autarky and the tax rate inherits the stochastic properties of the government spending, which exhibits a relatively low persistence. Finally, volatile taxes in our model lead more variable labor supply.

Our model also generates a frequency of renegotiation failures of 53.30 percent, which means that on average roughly one out of two offers are rejected by the government in equilibrium. In addition, an average recovery rate is around 59 percent and the mean autarky spell is between 5 and 6 years.[52]

While in table 6.1 we focus on the entire sample, in table 6.2 we show the results for two subsamples: "financial autarky" and "financial access". To construct these subsamples, we split each MC simulation into the periods in which the ED economy is in autarky and those in which

[50] In contrast with sovereign default models, our benevolent government shares the same preferences as bond holders, and in particular is not relatively more impatient than them. Consequently, there is no motive to front-load consumption that would expand the level of indebtness further.

[51] Our model is therefore consistent with several features of the business cycles for emerging economies. First, it can spawn recurrent default episodes. Second, our model is able to generate considerable levels of "debt intolerance" for economies prone to default. Finally, our model is consistent with the empirical regularity of higher volatility of tax revenues-to-output ratios observed in emerging economies, as reported for example by Bauducco and Caprioli (2014).

[52] These numbers are broadly in line with data from debt restructuring episodes after *domestic* default events. As an example, consider the U.S. debt restructuring plan of 1790 implemented by Hamilton. According to Grubb (2005), the US government was in default for almost a decade for its domestic debt incurred by the Continental Congress during the Revolutionary War and the interregnum of the Articles of the Confederation. The final recovery rates for domestic junior US debt were about 49 in 1790 and 63 percent in 1791, as documented by Garber (1991). The exchange package for state debt was worth 49, 63 and 90 percent in the respective 1790, 1791 and 1792 conversion offers. The most senior foreign claims were serviced without interruptions.

it is not.[53]

Table 6.2: MC Statistics for the "financial autarky" sample and "financial access" sample.

	Financial Access		Financial Autarky	
Statistic	ED	AMSS	ED	AMSS
Mean(debt/y)(%)	2.9	24.8	33.3	59.9
Mean(g)	0.112	0.112	0.142	0.142
Std.dev.(g)	0.044	0.044	0.042	0.042
Mean(τ)	0.202	0.207	0.267	0.284
Std.dev.(τ)	0.075	0.049	0.096	0.055
Mean(y)	0.565	0.565	0.543	0.541
Mean(c)	0.453	0.453	0.401	0.399
std.dev.(y)	0.020	0.013	0.032	0.020
std.dev.(c)	0.064	0.055	0.074	0.058
Mean($r - r^f$) (%)	1.72	0	NA	NA
corr(y, $r - r^f$)	-0.255	0	NA	NA
corr(τ, $r - r^f$)	0.232	0	NA	NA
20th-percentile(τ)	0.138	0.178	0.192	0.242
80th-percentile(τ)	0.260	0.240	0.346	0.317

During financial autarky, the average debt-to-output ratio is actually the *defaulted* debt-to-output ratio and is around 33 percent. The fact that it is several times larger than in financial access indicates that the government defaults for high debt levels. Before defaulting, the government optimally chooses to increase dramatically the indebtness level since the price schedule is basically debt inelastic for high debt levels, which in turn is attributed to the lack of sensitivity of the renegotiation strategy to debt variations.

Similarly, note that the average government spending is clearly higher in autarky than with access to financial markets since the government tends to default for high realizations of g_t while renegotiations usually occur for relatively low g_t.

As shown in the table, bond spreads are not only sizable but countercyclical as well.[54] As

[53] Since the "financial autarky" and "financial access" subsamples may contain nonconsecutive periods by construction, the autocorrelation of tax rates is not computed for them.

[54] Another well-documented regularity in the data for emerging economies. See for example Neumeyer and Perri (2005) and Arellano (2008).

indicated by the positive correlation between tax rates and spreads, taxes tend to be higher precisely when borrowing is more expensive. Labor supply is optimally lower in those states leading to a negative co-movement between bond returns and output.

The findings for taxes in financial access and financial autarky echo the results in Table 6.1. When borrowing is allowed, taxes are relatively more volatile in the economy with the defaultable bond due to the endogenous credit limits. Furthermore, when our economy is in financial autarky, the government is precluded from issuing debt, rendering more volatile taxes that in financial access. The 20th-80th percentile intervals for taxes are consistent with the same patterns.

In both economies, taxes are higher in autarky because government spending is relatively higher in those states. Should the government had not defaulted in our economy, taxes would have been even higher. Not surprisingly, autarkic tax rates in the AMSS economy display a larger jump from their repayment counterpart than in the ED economy as the government has to service the outstanding debt.[55] Because of the concavity of the primary surplus function z, this also implies that the volatility of taxes is higher in autarky for AMSS as well.

Both the average output and consumption are similar in both economies in the financial access subsample and always higher than their counterparts in autarky. Higher autarkic levels of labor in both economies results from the fact that the primary surplus function z is decreasing in n and the government outlays are typically higher. Higher average tax rates, and in turn lower after-tax real wages, induce a substitution from labor to leisure. For the same token, autarkic output is slightly higher in our economy than in AMSS model, despite of the ad hoc output cost of default.

At the same time, both economies exhibit more volatile output and consumption in the financial autarky subsample. The fact that the primary surplus function is concave implies a higher sensitivity of these variables to changes in government spending and thus higher volatility during financial autarky. Finally, both output and consumption are more volatile in our economy than in AMSS, portraying the differences in the dynamics of their taxes.[56]

Behavior of taxes. As mentioned in section 5, we observe in the data that default risk and tax volatility are positively correlated and that both are higher for high levels of debt-to-output ratio. Figure 6.2 shows that our model is able to generate this pattern. The blue (red) dots

[55] The discrepancy would have been even more pronounced between the two economies if there were no output cost in our model. This ad hoc drop of output forces the government to raise taxes further in our economy to finance the same public spending flow.

[56] In spite of observing higher autarkic tax rates in the AMSS model, the tax variability there is relatively lower. While it is true that we move along a more concave part of the primary surplus function on average in the AMSS economy, the volatility of the primary surplus is lower as the government disposes of an additional instrument, that is, borrowing, to smooth out taxes. This second effect dominates rendering a lower volatility of taxes in autarky in the AMSS economy.

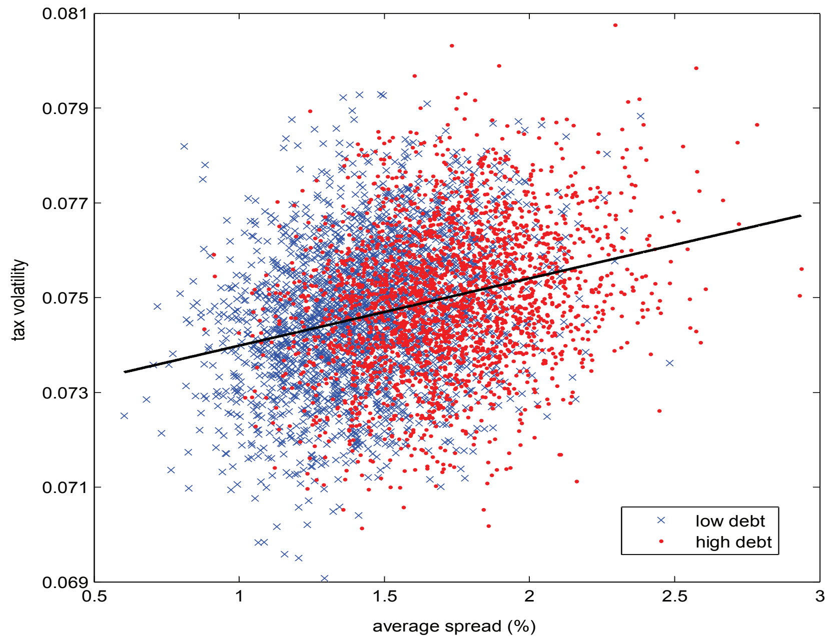

Figure 6.2: Standard deviation of tax rates and mean bond spreads in financial access for low debt (blue) and high debt (red), and fitted OLS line between them (black line).

show the standard deviation of taxes and spreads for low (high) levels of debt, respectively. Each dot corresponds to the financial access subsample in a MC simulation. The solid black line corresponds to the fitted OLS line between these two variables. For low (high) debt we consider debt-to-output ratios below (over) the median of its distribution. For both cases we can see a positive relationship between spreads and tax volatility. This result follows from the fact that higher spreads are caused by higher risk of default which in turn limits the ability of the government to use debt to smooth taxes when financing government shocks. A second noteworthy observation is that the red cloud is shifted to the upper right corner of the graph with respect to the blue cloud, thus indicating that both spreads and tax volatility are higher for higher level of debts.

In order to shed more light regarding the behavior of taxes when there is risk of default, in figure 6.3 we compare, for different values of government expenditure, the histograms (which were smoothed using kernel methods) of taxes in our model (solid red) with that in AMSS (dotted blue).

First, as observed in the bottom panels, for high values of g the distribution of taxes in our model is shifted to the right compared to that in AMSS model. This difference between the two models arises from the fact that due to default risk debt is too costly for our government to finance high government expenditure. In contrast, in the top panels, when the g realization is

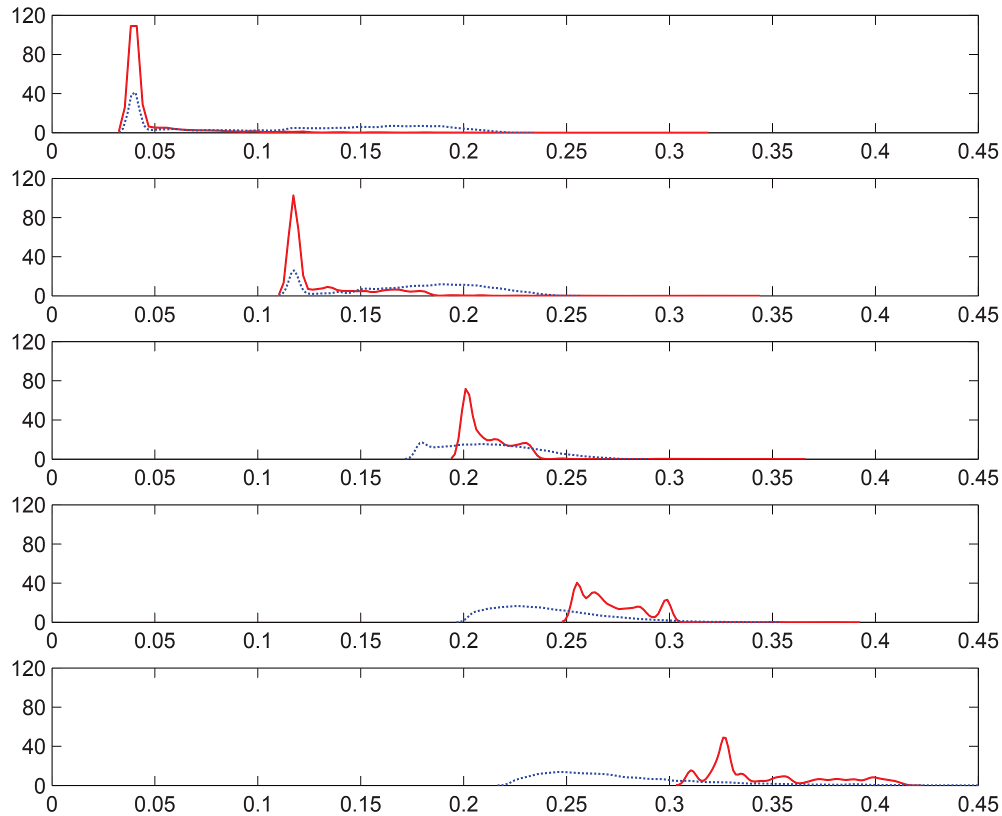

Figure 6.3: Histograms of tax rates in financial access for our model (solid red) and AMSS (dotted blue) conditioned on current g realization. From top to bottom, the five panels correspond to the second, fourth, sixth, eighth and tenth gridpoint of g.

low, the situation is reversed and now the distribution of taxes in AMSS model is shifted to the right relative to our model. During those states, the government repays the outstanding debt, which typically is higher in AMSS than in our credit-constrained economy. A final noteworthy observation is that for our model taxes are more concentrated around a single peak (which shifts to the right with the level of government expenditure), reflecting more limited borrowing. In contrast, in AMSS the distribution of taxes is more spread-out for each g realization but at the same time more "stable" regarding changes in g, a clear reflection of more tax smoothing.

Table 6.3: MC Statistics for debt renegotiation for different values of λ.

λ	0.2	0.4	0.6	0.8	1.0
Avg. offer accepted	0.60	0.59	0.59	0.58	0.57
Avg. duration | High debt	10.08	6.69	6.03	5.19	5.06
Avg. duration | Low debt	9.46	5.82	3.42	3.16	2.92

Debt renegotiation. In table 6.3 we present some statistics regarding the debt renegotiation process for different values of λ. From the first row we see that as the probability of receiving an offer increases, the average offer accepted decreases. This result and the fact that the frequency of rejected offers increases monotonically with λ (bottom panel in figure 6.6), follows because as λ increases the option value of staying in financial autarky increases and thus the government becomes more selective regarding which offers it accepts. The last two rows in the table shows the average duration, conditioning on the fact that the defaulted debt is "high" (second row) and "low" (third row).[57] We can see that for both cases, it decreases as the probability of receiving an offer increases, but more importantly it shows that for "high" levels of debt we have, on average, longer financial autarky spells. In fact, the difference can be as large as 75 percent higher for intermediate values of λ. This result coincides with the implications of proposition 5.1. Moreover, as we see from the table, differences in the duration can be non-negligible.

Impulse responses. Figure 6.4 plots the impulse response for debt and taxes for our model and AMSS. The path of government expenditure is plotted in the first panel: the government expenditure is low and equal to 0.0915 except for $t = 2, 3, 4$ where it is high and equal to 0.159. The initial debt level is set to zero. To finance high government expenditure during the first periods, the government makes use of both instruments: bond issuance and taxes. While in both economies the government accumulates debt in those periods, in our model it does so to a lesser extent due to the presence of endogenous borrowing limits. From $t = 5$ onwards, when government expenditure becomes low, the level of debt decreases, eventually reaching zero.

[57] As low (high) defaulted debt we consider debt-to-output ratios in the default episodes below (above) the unconditional median.

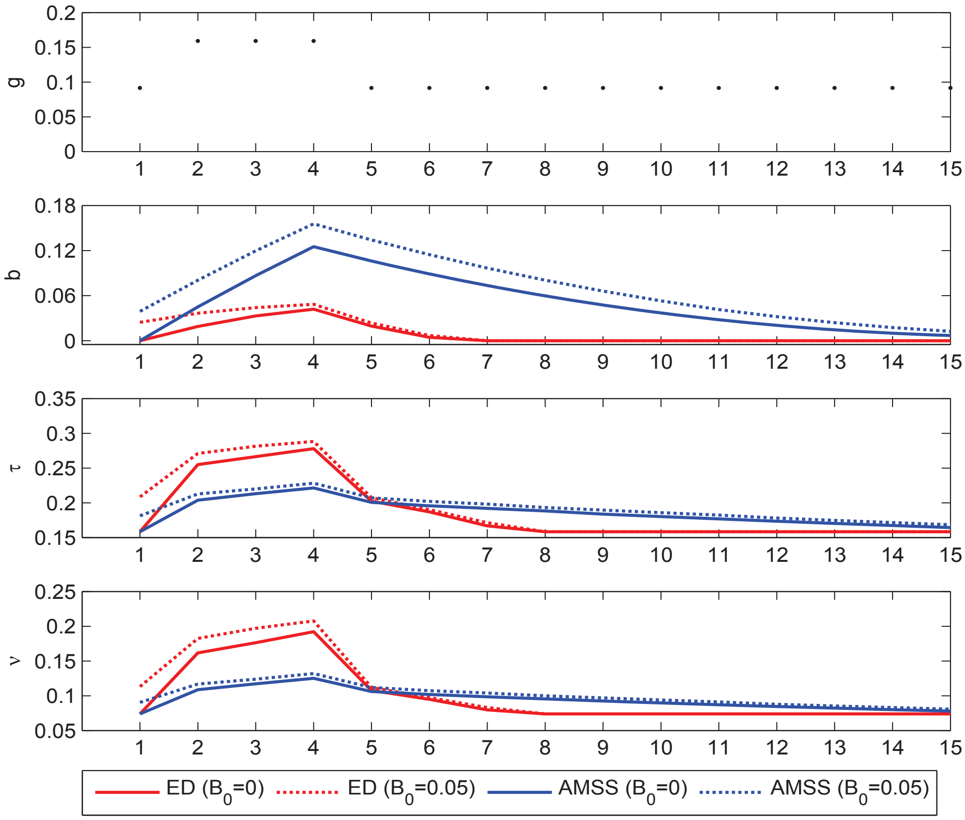

Figure 6.4: Impulse responses for our model (red) and AMSS (blue). Realization of government expenditure (first panel); debt path (second panel); tax path (third panel); and Lagrange multiplier path (fourth path).

Taxes behave analogously.

In our economy taxes are higher than AMSS during the periods of high government expenditure since borrowing is more limited, but they decrease more rapidly when the realization of government expenditure becomes lower (see the third panel). Overall, not surprisingly, one can see a smoother behavior for taxes in AMSS than in our economy. The last panel plots the behavior of the Lagrange multiplier ν_t studied in subsection 5.2. The fact that ours is above the one of AMSS for periods of high government expenditure reflects the "mark-up" effect mentioned in subsection 5.2. Also note that the Lagrange multiplier increases during these periods, reflecting the fact that the marginal cost of debt is increasing in the level of debt. From $t = 4$ onwards, as debt decreases, the Lagrange multiplier in our model falls and eventually converges to the one of AMSS.

Welfare analysis. As a measure of welfare we use the compensation in terms of initial consumption that would make the household indifferent between our economy and AMSS. Formally,

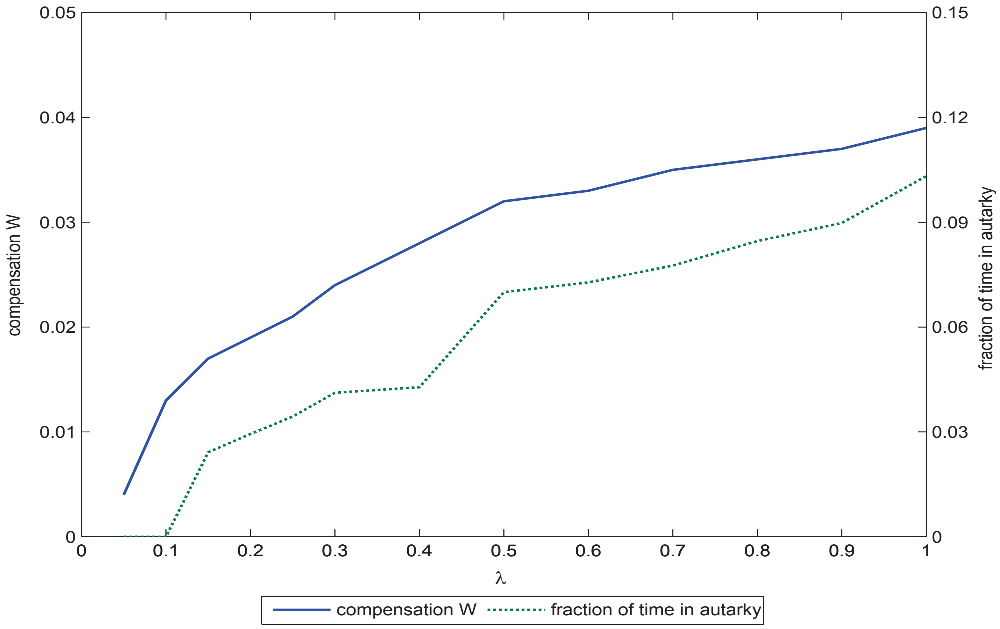

Figure 6.5: Consumption compensation W (solid blue) and fraction of time in autarky under ergodic distribution (dashed green) for different values of arrival probability of offers λ.

this compensation denoted by W is computed as

$$W \equiv \frac{\int V^{AMSS}(g,B)\mu_{AMSS}(dg,dB) - \int V_{\phi}^{*}(g,B)\mu(dg,dB,d\phi)}{\int(\mathbf{n}_{\phi}^{*}(g,B)-g)\mu(dg,dB,d\phi)}$$

where μ_{AMSS} and μ are the ergodic distributions generated by the AMSS and our model respectively and V^{AMSS} is the value function corresponding to the AMSS economy. That is, W measures the increase in initial consumption that would make the household indifferent between our economy and AMSS, under the ergodic distribution.[58]

Figure 6.5 plots this measure for different values of the arrival probability of offers λ with our, otherwise, benchmark parametrization. As we can see, for low values of λ the compensation is almost zero, which is consistent with the fact that in these cases autarky is very costly bringing down the default frequency and thereby implying that our economy and AMSS are very similar. Note as well that even if the time spent in autarky is zero as no default occurs in equilibrium, the compensation is anyhow strictly positive due to the presence of endogenous credit limits. As λ increases, default occurs more often and welfare decreases in our economy relative to AMSS. Welfare in our economy decreases monotonically with λ, resulting in a compensation of almost 4

[58]The ergodic distribution is constructed by collecting the last observation from each of the 5,000 MC simulated paths. For low values of λ we require to consider more MC simulations as default frequency is extremely low and close to zero.

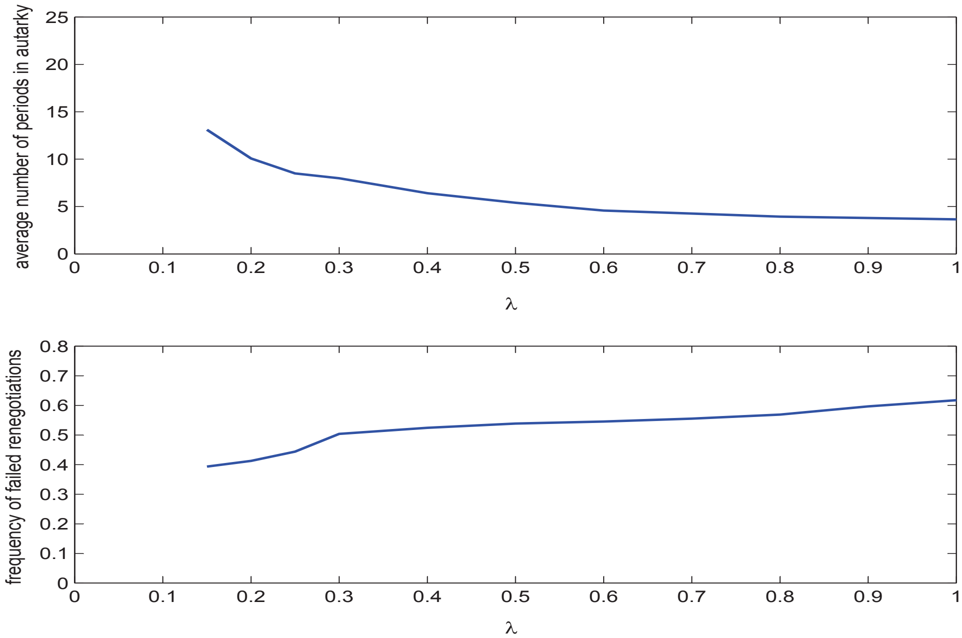

Figure 6.6: Average number of periods in autarky (top panel) and frequency of rejected offers (bottom panel) for different values of arrival probability of offers λ.

percent when offers to renegotiate arrive every period after the default event. For high values of λ, the government is frequently confronted with the *option* to reject a repayment offer and stay in financial autarky, whereas for low values of λ, this option is presented more infrequently. In other words, for high values of λ, the time the economy spends in financial autarky is typically driven by choice, whereas for low values of λ it is not. As shown in figure 6.6, however, the frequency of rejecting offers does not increase significantly with λ. This feature is mainly attributed to the fact that the economy is subject to an ad hoc output cost if waiting and staying in autarky.[59]

Robustness check: aversion to consumption risk. In previous calibrations we assumed that households were consumption-risk neutral by endowing them with quasi-linear preferences. In what follows we relax this assumption by considering a balanced growth preference specification, as in Aiyagari et al. (2002) and Farhi (2010). More specifically, we assume

[59]We should make it clear that this result is not robust. A different pattern is observed if no output cost of default is assumed, i.e. $\kappa = 1$. In this case, the curve of welfare compensation exhibits an inverted-U shape. That is, for sufficiently high values of λ, welfare improves with λ. As autarky does not look so bad, the government defaults with a dramatically higher frequency and also exercises its *option* to reject renegotiation offers more recurrently. Indeed, for λ equal 1, the economy spends almost 70 percent of the time in autarky compared to the mere 12 percent observed in the economy with output cost. Additionally, as default becomes more recurrent in the economy with $\kappa = 1$, endogenous credit limits are tighter and hence welfare compensations are higher than in the economy with output loss of default, for all possible values of λ. Results are available upon request.

$u(c, 1-n) = \log c + C_1 \frac{(1-n)^{1-\sigma}}{1-\sigma}$, with $C_1 = 0.02$ and $\sigma = 3$. For computational purposes, in this new calibration, the government spending shock can only take two values: g^H ("expansionary") and g^L ("normal"), with $g^H > g^L$. It is assumed to be i.i.d. with transition probability $\pi_G(g^L) = 0.9$. There is only one debt renegotiation offer $d = 0.7$ with arrival probability $\lambda = 0.15$ and no ad hoc output cost for default. In this setup, with consumption-risk aversion, the low probability of occurrence of g^H together with the relatively low value of λ are sufficient to deter the government from defaulting too often. The time discount factor β is set to 0.983.

Table 6.4: MC Statistics for the whole sample for our model and the risk-free debt model (AMSS).

Statistic	ED	AMSS
Mean(debt/y)(%)	0.49	0.77
Mean(τ)	0.3047	0.3050
Std.dev.(τ)	0.0155	0.0125
Autocor.(τ)	0.2652	0.5576
Mean(y)	0.7530	0.7530
Mean(c)	0.5237	0.5237
Std.dev.(y)	0.0009	0.00154
Std.dev.(c)	0.0152	0.0147
cor(y,$r - r^f$)	0.0607	0
cor(c,$r - r^f$)	-0.2476	0
cor(y,c)	-0.7687	-0.7876
cor(τ,c)	-0.9525	-0.7778
cor(τ,$r - r^f$)	0.2931	0
Mean($r - r^f$) (%)	35.992	0
Mean(r^f) (%)	7.731	7.643
Mean(default spell)	7.67	NA
Mean(recovery rate) (%)	0.70	NA
Failed reneg. freq. (%)	0.00	NA
Default frequency (%)	3.638	0
20th-percentile(τ)	0.2975	0.2975
80th-percentile(τ)	0.3070	0.3182

A novel feature of defaulting in this environment with commitment to fiscal policies is that it allows for a resetting of taxes. While in our economy eventually after every default episode the fiscal authority has the chance to review and re-optimize its tax rates, in the AMSS model this is not the case, as the government is bound by previous marginal utility promises when $u_c \neq 1$. Therefore, in the latter environment, taxes may stay high and persistent in states with high indebtedness as the economy hovers around the *ad hoc* borrowing limits.

Table 6.4 reports the MC statistics for our model with default and the risk-free debt model (AMSS). The main features observed in previous calibrations continue to hold for this case with consumption-risk aversion, namely, taxes are more volatile and less persistent and debt is lower in the presence of default risk. In particular, taxes are approximately 25 percent more volatile in our economy than in the AMSS one. Also, in our model, default occurs with an annual frequency of over 3.64 percent, which is priced in by the household and, consequently, we observe approximately 45 percent less borrowing in our economy.

Under our assumption on preferences, there is a strong negative co-movement between consumption and both labor supply and taxes in the two economies. As the tax rate increases, consumption drops and the household supplies more labor due to the non-zero "wealth" effect. Moreover, as taxes are usually higher in states with default risk, labor supply tends to also be high in those states, which explains the weakly pro-cyclical interest rates observed in this calibration.[60]

7 Conclusion

We view our model as a suitable framework to study government policies for economies that defaulted and restructured their debt either in the past or in recent years. Some examples include France and the U.S. in the 18th century, and emerging economies nowadays.

While the main focus of the paper points to the normative prescriptions of our model, we believe that our environment with domestic default could be enriched in several ways in order to replicate empirical regularities for a number of economies. For instance, so as to better fit the data of peripheral economies in the eurozone, productivity shocks as well as foreign investors holding large fractions of government debt are necessary.

Our model also provides a novel device that allows us to study asset prices of government debt both during periods of financial access and autarky. Further research could fully explore the pricing implications of this device for general sovereign debt held by uncertainty or risk averse creditors. In addition, our debt restructuring process could be used to study more in detail some observed features in recent debt renegotiation episodes.

[60]This implication of the model is clearly not in line with the data for emerging economies. More realistic results along this dimension call for a utility function different from the balanced growth preference specification used by Aiyagari et al. (2002) and Farhi (2010).

Finally, a more theoretical line of research we find interesting to pursue is to study how (a) household's heterogeneity and (b) endogenous debt renegotiation schemes, affect the trade volume in the secondary markets and consequently welfare.

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A Notation and Stochastic Structure of the Model

Throughout the appendix for a generic mapping f from a set S to T, we use $s \mapsto f(s)$ or $f : S \to T$ to denote it. For the case that a mapping depends on many variables, the notation $s_1 \mapsto f(s_1, s_2)$ is used to denote the function f *only* as a function of s_1, keeping s_2 fixed. Also, for a generic set A, $|A|$ denotes the cardinality of A.

B Optimization Problem for the Households

The Lagrangian associated to the household's problem is given by

$$\begin{aligned}\mathcal{L}(\{c_t, n_t, b_{t+1}, \nu_t, \mu_t, \psi_t\}_{t=0}^{\infty}) \equiv \sum_{t=0}^{\infty} \beta^t E_{\Pi(\cdot|\omega_0)} \Big[\{u(c_t(\omega^t), 1 - n_t(\omega^t))\\ -\nu_t(\omega^t)\{c_t(\omega^t) - (1 - \tau_t(\omega^t))\kappa_t(\omega^t)n_t(\omega^t) + p_t(\omega^t)b_{t+1}(\omega^t) - \varrho_t(\omega^t)b_t(\omega^{t-1})\}\\ + \Psi_t(\omega^t)c_t(\omega^t) + \psi_{1t}(b_{t+1}(\omega^t) - \underline{b}) + \psi_{2t}(\overline{b} - b_{t+1}(\omega^t))\Big\}\Big],\end{aligned}$$

where ν_t and Ψ_t are the Lagrange multipliers associated to the budge constraint and to the restrictions that non-negative bound on consumption, and ψ_{it} $i = 1, 2$ are the Lagrange multipliers associated to the debt limits.

Assuming interiority of the solutions, the first order conditions (FONC) are given by:

$$\begin{aligned}c_t(\omega^t)&: u_c(c_t(\omega^t), 1 - n_t(\omega^t)) - \nu_t(\omega^t) = 0\\ n_t(\omega^t)&: -u_l(c_t(\omega^t), 1 - n_t(\omega^t)) + \nu_t(\omega^t)(1 - \tau_t(\omega^t))\kappa_t(\omega^t) = 0\\ b_{t+1}(\omega^t)&: p_t(\omega^t)\nu_t(\omega^t) - E_{\Pi(\cdot|\omega^t)}[\beta\nu_{t+1}(\omega^{t+1})\varrho_{t+1}(\omega^{t+1})] = 0.\end{aligned}$$

Then, using $u_j(\omega^t)$ for $u_j(c_t(\omega), 1 - n_t(\omega))$ with $j \in \{c, l\}$, it follows

$$\frac{u_l(\omega^t)}{u_c(\omega^t)} = (1 - \tau_t(\omega^t))\kappa_t(\omega^t), \tag{B.24}$$

and

$$p_t(\omega^t) = E_{\Pi(\cdot|\omega^t)}\left[\beta\frac{u_c(\omega^{t+1})}{u_c(\omega^t)}\varrho_{t+1}(\omega^{t+1})\right]. \tag{B.25}$$

From the definition of ϱ, equation B.25 implies, for $\phi_t = 1$,

$$p_t(\omega^t) = E_{\Pi(\cdot|\omega^t)}\left[\beta\frac{u_c(\omega^{t+1})}{u_c(\omega^t)}(1 - d_{t+1}(\omega))\right] + E_{\Pi(\cdot|\omega^t)}\left[\beta\frac{u_c(\omega^{t+1})}{u_c(\omega^t)}d_{t+1}(\omega^{t+1})q_{t+1}(\omega^{t+1})\right].$$

For $\phi_t = 0$, (where in this case recall that $p_t = q_t$)

$$\begin{aligned}p_t(\omega^t) = \lambda E_{\Pi(\cdot|\omega^t)}\left[\beta\frac{u_c(\omega^{t+1})}{u_c(\omega^t)}a_{t+1}(\omega^{t+1})\delta_{t+1}\right]\\ + E_{\Pi(\cdot|\omega^t)}\left[\beta\frac{u_c(\omega^{t+1})}{u_c(\omega^t)}\{1 - \lambda + \lambda(1 - a_{t+1}(\omega^{t+1}))\}q_{t+1}(\omega^{t+1})\right].\end{aligned}$$

C Proofs for Section 3.2

The next lemma characterizes the set of competitive equilibria as a sequence of restrictions involving FONC and budget constraints. The proof is relegated to the end of the section.

Lemma C.1. *Suppose assumption 3.1 holds. The tuple $(c_t, g_t, n_t, b_{t+1}, p_t)_{t=0}^{\infty}$ and $\boldsymbol{\sigma}$ is a competitive equilibrium iff given a $B_0 = b_0$, for all $\omega^t \in \Omega^t$, for all t,*

$$c_t(\omega^t) = \kappa_t(\omega^t) n_t(\omega^t) - g_t, \text{ and } B_{t+1}(\omega^t) = b_{t+1}(\omega^t), \tag{C.26}$$

$$\kappa_t(\omega^t)\tau_t(\omega^t) = \left(\kappa_t(\omega^t) - \frac{u_l(\omega^t)}{u_c(\omega^t)}\right); \tag{C.27}$$

and

$$Z_t(\omega^t) + \phi_t(\omega^t)\{p_t(\omega^t)B_{t+1}(\omega^t) - \delta_t B_t(\omega^t)\} \geq 0, \tag{C.28}$$

$$\text{and if } \phi_t(\omega^t) = 0,\ B_{t+1}(\omega^t) = B_t(\omega^{t-1}) \tag{C.29}$$

where

$$p_t(\omega^t) = E_{\Pi(\cdot|\omega^t)}\left[\beta \frac{u_c(\omega^{t+1})}{u_c(\omega^t)} \varrho_{t+1}(\omega^{t+1})\right]. \tag{C.30}$$

Proof of Theorem 3.1. We now show the "⇒" direction. Consider an outcome path $(d_t, a_t, B_{t+1}, n_t)_{t=0}^{\infty}$ that is consistent. This means by lemma C.1 that the tuple $(c_t, g_t, n_t, b_{t+1}, p_t)_{t=0}^{\infty}$ and $\boldsymbol{\sigma}$ is a competitive equilibrium iff given a $B_0 = b_0$, for all $\omega^t \in \Omega^t$, for all t,

$$c_t(\omega^t) = \kappa_t(\omega^t) n_t(\omega^t) - g_t, \text{ and } B_{t+1}(\omega^t) = b_{t+1}(\omega^t), \tag{C.31}$$

$$\kappa_t(\omega^t)\tau_t(\omega^t) = \left(\kappa_t(\omega^t) - \frac{u_l(\omega^t)}{u_c(\omega^t)}\right); \tag{C.32}$$

and

$$Z_t(\omega^t) + \phi_t(\omega^t)\{p_t(\omega^t)B_{t+1}(\omega^t) - \delta_t B_t(\omega^{t-1})\} \geq 0, \tag{C.33}$$

$$\text{and if } \phi_t(\omega^t) = 0,\ B_{t+1}(\omega^t) = B_t(\omega^{t-1})$$

where

$$p_t(\omega^t) = E_{\Pi(\cdot|\omega^t)}\left[\beta \frac{u_c(\omega^{t+1})}{u_c(\omega^t)} \varrho_{t+1}(\omega^{t+1})\right]. \tag{C.34}$$

It is easy to see that equations C.32 and C.34 imply equations 3.4 and 3.5. Equations C.32, C.34 and C.33 imply equation 3.9.

We now show the "$\Leftarrow$" direction. Suppose now that the outcome path satisfies that for all $\omega^t \in \Omega^t$, the following equations holds: 3.4, 3.5, 3.9 and

$$c_t(\omega^t) = \kappa_t(\omega^t) n_t(\omega^t) - g_t. \tag{C.35}$$

By using $B_{t+1}(\omega^t) = b_{t+1}(\omega^t)$, equations 3.4, 3.5 and the feasibility condition, we can enlarge the outcome path by $(c_t, p_t, b_{t+1}, \tau_t, g_t)_{t=0}^{\infty}$. Clearly, restrictions C.31, C.32 and C.34 hold. By replacing equations 3.4 and 3.5 on 3.9, it is easy to see that equation C.33 holds too. □

C.1 Proofs of Supplementary Lemmas

For the proof of Lemma C.1 we need the following lemma (the proof is relegated to the end of the section).

Lemma C.2. *Suppose assumption 3.1 holds. Then first order conditions 3.4 and 3.5 are also sufficient.*

Proof of Lemma C.1. Take $\boldsymbol{\sigma}$ and $(c_t, g_t, n_t, b_{t+1})_{t=0}^{\infty}$, and a price schedule $(p_t)_t$ that satisfy the equations. It is easy to see that feasibility and market clearing holds (conditions 3 and 4). Also, by lemma C.2 optimality of the households is also satisfied.

To check attainability of the government policy (condition 2). Observe that by equations C.26 - C.28 imply for all $\omega^t \in \Omega^t$,

$$g_t + \phi_t(\omega^t)\delta_t B_t(\omega^{t-1}) - \phi_t(\omega^t) p_t(\omega^t) B_{t+1}(\omega^t) \leq \kappa_t(\omega^t)\tau_t(\omega^t) n_t(\omega^t).$$

Finally, we check optimality of the households. We first check that the sequences satisfy the budget constraint. Observe that by equations C.26 - C.28

$$-c_t(\omega^t) + \kappa_t(\omega^t) n_t(\omega^t) + \phi_t(\omega^t)\{\delta_t B_t(\omega^{t-1}) - p_t(\omega^t) B_{t+1}(\omega^t)\} \leq \kappa_t(\omega^t)\tau_t(\omega^t) n_t(\omega^t).$$

If $\phi_t(\omega^t) = 1$, then equation C.28 implies that $b_{t+1}(\omega^t) = B_{t+1}(\omega^t)$ for all t (and for b_0 we assume it is equal to B_0) and thus

$$-c_t(\omega^t) + \kappa_t(\omega^t) n_t(\omega^t) + \delta_t b_t(\omega^{t-1}) - p_t(\omega^t) b_{t+1}(\omega^t) \leq \kappa_t(\omega^t)\tau_t(\omega^t) n_t(\omega^t).$$

This coincides with the budget constraint of the household.

If $d_t(\omega^t) = 1$, but $a_t(\omega^t) = 0$, then equations C.26 and C.28 imply that $b_t(\omega^{t-1}) = b_{t+1}(\omega^t) = 0$ for all t, so

$$-c_t(\omega^t) + \kappa_t(\omega^t) n_t(\omega^t) = \kappa_t(\omega^t)\tau_t(\omega^t) n_t(\omega^t).$$

This coincides with the budget constraint of the household.

Take $\boldsymbol{\sigma}$ and $(c_t, g_t, n_t, b_{t+1}, p_t)_{t=0}^{\infty}$ being a competitive equilibrium. Then it is easy to see that it satisfies the equations. □

Proof of Lemma C.2. Under assumption 3.1 the objective function of the household optimization problem is strictly concave. The budget constraints and debt constraint form a convex set of constraints. Thus, if the transversality condition holds, the FONC are sufficient; this follows from a simple adaptation of the results in Stokey et al. (1989) Ch. 4.5.

In order to verify the transversality condition, it suffices to show that for any $\zeta_t(\omega^t)$ such that $b_t(\omega^t) + \zeta_t(\omega^t) \in \mathbb{B}$,

$$\lim_{T\to\infty} \beta^T E_\Pi[u_c(\kappa_T(\omega^T)n_T(\omega^T) - g_T, 1 - n_T(\omega^T))\varrho_T(\omega^T)\zeta_T(\omega^T)] = 0.$$

Since, by assumption, debt is constrained, this condition follows from Magill and Quinzii (1994) Theorem 5.2. $\square$

D Proofs for Section 4

In this section we provide formal definitions of the sets $\mathcal{S}(h_0, \phi)$ and $\Omega(h_0, \phi)$ introduced in the recursive representation of the government problem. We also provide the proof of Theorem 4.1.

Formal definition of $\mathcal{S}(h_0, \phi_0)$. For any $h_0 \in \mathbb{H}$ and $\phi_0 \in \{0, 1\}$, let

$$\mathcal{S}(h_0,\phi_0) \equiv \big\{\boldsymbol{\gamma} : \forall\ (h^t,\phi_t) \in \mathbb{H}^t \times \{0,1\},\ \boldsymbol{\gamma}|_{(h^t,\phi_t)}\ \textit{renders}\ (d_\tau(\boldsymbol{\gamma}), a_\tau(\boldsymbol{\gamma}), B_{\tau+1}(\boldsymbol{\gamma}), n_\tau(\boldsymbol{\gamma}))_{\tau=t}^{\infty} \in CE_{\phi_t}(\omega_t, B),$$
$$\textit{with}\ B = (\delta_t\phi_t + (1-\phi_t))B_t(\boldsymbol{\gamma})(h^{t-1}, \phi_{t-1}(\boldsymbol{\gamma})(h^{t-1}))\big\}\,.$$

Analogously to Chang (1998), by drawing the strategies γ from $\mathcal{S}(h_0, \phi_0)$ we ensure that after any history following (h_0, ϕ_0) the continuation strategy delivers competitive equilibrium allocations. As it will become clearer later on, when making the default/repayment decision embedded in ϕ_0, the default authority evaluates welfare after his alternative courses of action. To then compute the utility for any ϕ_0, the candidate strategies γ to be considered have to belong to the corresponding $\mathcal{S}(h_0, \phi_0)$.[61]

Formal definition of $\Omega(h_0, \phi)$. Recall that for autarky ($\phi = 0$), the "promised" marginal utilities of consumption are trivially pinned down by the choice of labor that balances the government budget and maximizes the per-period payoff; i.e., for any $g \in \mathbb{G}$, the "promised" marginal utility of consumption equals $m_A(g) \equiv u_c(\kappa \mathbf{n}_0^*(g) - g, 1 - \mathbf{n}_0^*(g))$ where[62]

$$\mathbf{n}_0^*(g) = \arg\max_{n\in[0,1]} \{u(\kappa n - g, 1 - n) : z(\kappa, n, g) = 0\}.$$

[61] Technically, it could be the case that $\phi_t = 1$ but the debt that period is too high to be repaid in the competitive equilibrium. For this case, we simply set the per-period payoff at an arbitrary large negative value and thus ensure that this choice of ϕ_t will never arise as part of the optimal solution of the government problem.

[62] The lemma D.1(1) ensures that $\mathbf{n}_0^*(g)$ exists and is unique for all g.

We now proceed to formally define our object of interest. For any $h_0 = (\phi_{-1}, B_0, g_0, \delta_0) \in \mathbb{H}$ and $\phi \in \{0,1\}$, let

$$\begin{aligned}
\Omega(h_0, \phi) = \{ & (\mu, v) \in \mathbb{R}_+ \times \mathbb{R} : \\
& \exists\, \boldsymbol{\gamma} \in \mathcal{S}(h_0, \phi),\ and\ (V_\tau(h^\tau, 0), V_\tau(h^\tau, 1))_{h^\tau, \tau}\ such\ that: \\
& \mu = m_A(g)\ if\ \phi = 0,\ and\ \mu = u_c(n_0(\boldsymbol{\gamma})(h_0) - g_0, 1 - n_0(\boldsymbol{\gamma})(h_0))\ if\ \phi = 1, \\
& v = V_0(h_0, \phi) \\
& (V_\tau(h^\tau, \phi))_{h^\tau, \tau}\ satisfies\ expression\ 4.10\ for\ any\ \phi \in \{0,1\}, \\
& \boldsymbol{\gamma}^D|_{h_0, \phi_1(\boldsymbol{\gamma})}\ are\ determined\ by\ expressions\ 4.11 - 4.12 \Big\},
\end{aligned}$$

For each initial history h_0 and ϕ, the set $\Omega(h_0, \phi)$ is given by all the values for marginal utility and lifetime utility values at time zero that can be sustained in a competitive equilibrium, wherein the default authority reacts optimally from next period on. Each pair (μ, v) imposes restrictions on the labor allocation at time 0 as well as on the lifetime utility at time 0, given h_0 and ϕ. Finally, note that the set $\Omega(h_0, \phi)$ when ϕ is the value chosen by the government, contains the promised marginal utilities (and utility values) that can be delivered along the equilibrium path, while $\Omega(h_0, \phi)$ when ϕ is the value not chosen by the government, contains off-equilibrium marginal utilities.

The correspondence Ω is an equilibrium object, endogenously determined, that can be computed using numerical methods as the largest fixed point of an appropriately constructed correspondence operator, in the spirit of Abreu et al. (1990). Henceforth, we proceed to formulate and solve the recursive problem of the fiscal authority as if we already know Ω.

Proof of theorem 4.1 The next lemma characterizes the government surplus function, the proof is relegated to the end of this section.

Lemma D.1. *Let* $(\kappa, n, g) \mapsto z(\kappa, n, g) = \left(\kappa - \frac{u_l(n-g, 1-n)}{u_c(n-g, 1-n)}\right) n - g$. *Then:*

1. $\arg\max_{n \in [0,1]} \{u(\kappa n - g, 1 - n)\ :\ z(\kappa, n, g) = 0\}$ *exists and is unique.*
2. *Suppose assumption 5.1 holds and let* $\bar{n}(g) = \arg\max_{n \in [0,1]} z(1, n, g)$. *Then,* $n \mapsto z(1, n, g)$ *is decreasing and strictly concave for all* $n \in [\bar{n}(g), 1]$

To show theorem 4.1 we need the following lemma whose proof is relegated to the end of this section.

Lemma D.2. *If, for any* $h_0 = (1, B, g, \delta) \in \mathbb{H}$ *and* $\phi_0 \in \{0,1\}$, $\boldsymbol{\gamma} \in \mathcal{S}(h_0, \phi_0)$, *then* $\boldsymbol{\gamma}|_{h^t, \phi} \in \mathcal{S}(h_t, \phi)$ *for any* $h^t \in \mathbb{H}^t$ *and* $\phi \in \{0,1\}$. *Moreover,*

$$z(\kappa_{\phi_0}, n_0(\boldsymbol{\gamma})(h_0), g)\mu_0(\boldsymbol{\gamma})(h_0) + \phi_0\{\mathcal{P}_{\phi_0}(g, B_1(\boldsymbol{\gamma})(h_0, \phi_0), \mu_1(\boldsymbol{\gamma})(h_0, h_1(\cdot)))B_1(\boldsymbol{\gamma})(h_0, \phi_0) - \delta\mu_0(\boldsymbol{\gamma})(h_0)B\} \geq 0$$

where $\kappa_\phi = \kappa(1 - \phi) + \phi$ *and* $h_1(\cdot) \equiv (1, B_1(\boldsymbol{\gamma})(h_0, 1), \cdot, 1)$ *and for* $t = 0, 1$

$$\mu_{t+1}(\boldsymbol{\gamma})(h^t, h_{t+1}(g')) = u_c(n_{t+1}(\boldsymbol{\gamma})(h^t, h_{t+1}(g')) - g', 1 - n_{t+1}(\boldsymbol{\gamma})(h^t, h_{t+1}(g')))$$

The proof of Theorem 4.1 is analogous to the standard proof of the principle of optimality for the single agent case, e.g. Theorem 9.2 in Stokey et al. (1989).

Proof of Theorem 4.1. By definition of V_1^*, V_0^* and $\overline{V}_1^*$, it follows that with $h_0 = (\phi_{-1}, B, g, \bar{\delta})$ and $\phi = 0$

$$V_0^*(g, B) = \sup_{\boldsymbol{\gamma}} V_0(\boldsymbol{\gamma})(h_0, 0) \tag{D.36}$$

$$\textit{subject to } \boldsymbol{\gamma} = (\boldsymbol{\gamma}^F, \boldsymbol{\gamma}^D) \in \mathcal{S}(h_0, 0) \tag{D.37}$$

$$\boldsymbol{\gamma}^D|_{h_0,\phi=0} \textit{ are determined by } (4.12) - (4.11) \tag{D.38}$$

$$u_c(\kappa n_0(\boldsymbol{\gamma})(h_0) - g, 1 - n_0(\boldsymbol{\gamma})(h_0)) = m_A(g) \tag{D.39}$$

and similarly, with $h_0 = (1, B, g, 1)$ and $\mu \in \mathbb{R}_+$ and $\phi_0 = 1$

$$V_1^*(g, B, \mu) = \sup_{\boldsymbol{\gamma}} V_0(\boldsymbol{\gamma})(h_0, 1) \tag{D.40}$$

$$\textit{subject to } \boldsymbol{\gamma} = (\boldsymbol{\gamma}^F, \boldsymbol{\gamma}^D) \in \mathcal{S}(h_0, 1) \tag{D.41}$$

$$\boldsymbol{\gamma}^D|_{h_0,\phi=1} \textit{ are determined by } (4.12) - (4.11) \tag{D.42}$$

$$u_c(n_0(\boldsymbol{\gamma})(h_0) - g, 1 - n_0(\boldsymbol{\gamma})(h_0)) = \mu. \tag{D.43}$$

Finally, with $h_0 = (0, B, g, \delta)$ and $\phi_0 = 1$

$$\overline{V}_1^*(g, \delta B) = \sup_{\boldsymbol{\gamma}} V_0(\boldsymbol{\gamma})(h_0, 1) \tag{D.44}$$

$$\textit{subject to } \boldsymbol{\gamma} = (\boldsymbol{\gamma}^F, \boldsymbol{\gamma}^D) \in \mathcal{S}(h_0, 1) \tag{D.45}$$

$$\boldsymbol{\gamma}^D|_{h_0,\phi=1} \textit{ are determined by } (4.12) - (4.11). \tag{D.46}$$

The first (sequential) problem consists of selecting $\boldsymbol{\gamma}$, consistent with competitive equilibrium and optimality for the default authority from $t = 1$ on, to maximize the lifetime utility of households, conditional on $h_0 = (\phi_{-1}, B, g, \delta)$ and $\phi = 0$. The solution is given by $V_0^*(g, B)$, which does not depend on δ nor μ. Condition D.39 ensures that the current marginal utility is equal to the autarkic value defined before.

Problem D.40 is analogous to Problem D.36 with $\phi = 1$ and $\phi_{-1} = 1$ instead. In this case, we impose through condition D.43 that the current marginal utility is μ.

Henceforth, we refer to strategies that satisfy the restrictions on the above programs as *admissible*. We also assume that the suprema are achieved; this assumption is to ease the exposition, if this were not the case the proof still goes through by exploiting the definition of the supremum.

By definition, $\overline{V}_1^*(g, \delta B) \geq V_0(\boldsymbol{\gamma})(0, B, g, \delta, 1)$ for all $\boldsymbol{\gamma} \in \mathcal{S}(0, B, g, \delta, 1)$ and $\boldsymbol{\gamma}^D|_{h_0,\phi=1}$ are determined by (4.12)-(4.11). By definition of $\Omega(0, B, g, \delta, 1)$, this implies that for all $(\mu, v) \in \Omega(0, B, g, \delta, 1)$, $\overline{V}_1^*(g, \delta B) \geq v$. On the other hand, assuming that the there exists a strategy $\boldsymbol{\gamma}$ that achieves the supremum, it has to be true that there exists a μ such that $(\mu, \overline{V}_1^*(g, \delta B)) \in \Omega(0, B, g, \delta, 1)$. Therefore,

$$\overline{V}_1^*(g, \delta B) = \max\{v | (\mu, v) \in \Omega(0, B, g, \delta, 1)\}. \tag{D.47}$$

It is easy to see that the same result applies for any time t and any history $(h^t, 0, B, g, \delta)$ (not just $t = 0$ and $h_0 = (0, B, g, \delta)$).

Let $h_0 \equiv (\phi_{-1}, B, g, \delta)$ and $\phi = 0$. Suppose that there exists a strategy $\hat{\boldsymbol{\gamma}}$ that achieves the supremum in program D.36. Then,[63]

$$\begin{aligned} V_0^*(g,B) =& u(\kappa n_0(\hat{\boldsymbol{\gamma}})(g) - g, 1 - n_0(\hat{\boldsymbol{\gamma}})(g)) \\ &+ \beta\lambda \int_{\mathbb{G}} \int_{\Delta} \max\{V_1(\hat{\boldsymbol{\gamma}})(h_0, 0, B, (g', \delta'), 1), V_1(\hat{\boldsymbol{\gamma}})(h_0, 0, B, (g', \delta'), 0)\} \pi_{\Delta}(d\delta') \pi_{\mathbb{G}}(dg'|g) \\ &+ \beta(1-\lambda) \int_{\mathbb{G}} V_1(\hat{\boldsymbol{\gamma}})(h_0, 0, B, (g', \bar{\delta}), 0) \pi_{\mathbb{G}}(dg'|g). \end{aligned}$$

Observe that, for any $g' \in \mathbb{G}$, $V_1(\hat{\boldsymbol{\gamma}})(h_0, 0, B, (g', \delta'), 0)$ is constant with respect to δ'. Also, note that $\hat{\boldsymbol{\gamma}}|_{h^1,\phi}$ is admissible by lemma D.2. It also follows that $V_1(\boldsymbol{\gamma})(h_0, 0, B, (g', \delta'), 0) = V_0(\boldsymbol{\gamma})(0, B, (g', \delta'), 0)$ for any strategy $\boldsymbol{\gamma}$ and any (h_0, g', δ'). Thus

$$V_1(\hat{\boldsymbol{\gamma}})(h_0, 0, B, (g', \bar{\delta}), 0) = V_0^*(g', B), \ \forall g' \in \mathbb{G}. \tag{D.48}$$

Therefore,

$$V_0^*(g,B) = u(\kappa n_0(\hat{\boldsymbol{\gamma}})(g) - g, 1 - n_0(\hat{\boldsymbol{\gamma}})(g)) \tag{D.49}$$

$$+ \beta\lambda \int_{\mathbb{G}} \int_{\Delta} \max\{V_1(\hat{\boldsymbol{\gamma}})(h_0, 0, B, (g', \delta'), 1), V_0^*(g', B)\} \pi_{\Delta}(d\delta') \pi_{\mathbb{G}}(dg'|g) \tag{D.50}$$

$$+ \beta(1-\lambda) \int_{\mathbb{G}} V_0^*(g', B) \pi_{\mathbb{G}}(dg'|g). \tag{D.51}$$

By construction, $n_0(\hat{\boldsymbol{\gamma}})(g) = \mathbf{n}_0^*(g)$ and thus,

$$\begin{aligned} V_0^*(g,B) =& u(\kappa \mathbf{n}_0^*(g) - g, 1 - \mathbf{n}_0^*(g)) \\ &+ \beta\lambda \int_{\mathbb{G}} \int_{\Delta} \max\{V_1(\hat{\boldsymbol{\gamma}})(h_0, 0, B, (g', \delta'), 1), V_0^*(g', B)\} \pi_{\Delta}(d\delta') \pi_{\mathbb{G}}(dg'|g) \\ &+ \beta(1-\lambda) \int_{\mathbb{G}} V_0^*(g', B) \pi_{\mathbb{G}}(dg'|g). \end{aligned}$$

Observe that at $(h_0, \phi_0 = 0, B, g', \delta', \phi_1 = 1)$ a "new" fiscal authority begins at time $t = 1$. By construction, this fiscal authority starts without binding promises regarding the marginal utility of consumption. Since $\hat{\boldsymbol{\gamma}}$ is optimal, it follows that $V_1(\hat{\boldsymbol{\gamma}})(h_0, 0, B, g', \delta', 1) = \overline{V}_1^*(g', \delta' B)$. Therefore,

$$\begin{aligned} V_0^*(g,B) =& u(\kappa \mathbf{n}_0^*(g) - g, 1 - \mathbf{n}_0^*(g)) + \beta\lambda \int_{\mathbb{G}} \int_{\Delta} \max\{\overline{V}_1^*(g', \delta' B), V_0^*(g', B)\} \pi_{\Delta}(d\delta') \pi_{\mathbb{G}}(dg'|g) \\ &+ \beta(1-\lambda) \int_{\mathbb{G}} V_0^*(g', B) \pi_{\mathbb{G}}(dg'|g). \end{aligned} \tag{D.52}$$

We now consider program D.40. With an slight abuse of notation, let $\hat{\boldsymbol{\gamma}}$ be the strategy that achieves the supremum in program D.40. Henceforth, let $\mu_t(\boldsymbol{\gamma})(h^t) \equiv u_c(n_t(\boldsymbol{\gamma})(h^t) - g_t, 1 -$

[63]Henceforth we abuse notation and use $n_0(\boldsymbol{\gamma})(g)$ instead of $n_0(\boldsymbol{\gamma})(h_0)$.

$n_t(\boldsymbol{\gamma})(h^t))$ for any strategy $\boldsymbol{\gamma}$ and history $h^t \in \mathbb{H}^t$. Observe that, for $h_1 = (1, B_1(\hat{\boldsymbol{\gamma}})(h_0, 1), (g', 1))$, due to lemma D.2, $\hat{\boldsymbol{\gamma}}|_{h^1,\phi}$ is admissible (taking μ as $\mu_1(\hat{\boldsymbol{\gamma}})(h^1)$), because $\hat{\boldsymbol{\gamma}}|_{h^1,\phi} \in \mathcal{S}(h_1, \phi)$, and also $\boldsymbol{\gamma}^D|_{h^1,\phi=1}$ are determined by expressions 4.12-4.11. Thus

$$\begin{aligned} V_1(\hat{\boldsymbol{\gamma}})(h_0, h_1, 1) &\leq V_1^*(g', B_1(\hat{\boldsymbol{\gamma}})(h_0, 1), \mu_1(\hat{\boldsymbol{\gamma}})(h^1)) \\ \text{and } V_1(\hat{\boldsymbol{\gamma}})(h_0, h_1, 0) &\leq V_0^*(g', B_1(\hat{\boldsymbol{\gamma}})(h_0, 1)). \end{aligned}$$

Therefore, letting $h^1(g') = (1, B_1(\hat{\boldsymbol{\gamma}})(h_0, 1), (g', 1)))$,

$$\begin{aligned} V_1^*(g, \delta B, \mu) \leq & u(n_0(\hat{\boldsymbol{\gamma}})(g) - g, 1 - n_0(\hat{\boldsymbol{\gamma}})(g)) \\ & + \beta \int_{\mathbb{G}} \max\{V_1^*(g', B_1(\hat{\boldsymbol{\gamma}})(h_0, 1), \mu_1(\hat{\boldsymbol{\gamma}})(h^1(g'))), V_0^*(g', B_1(\hat{\boldsymbol{\gamma}})(h_0, 1))\}\pi_{\mathbb{G}}(dg'|g). \end{aligned}$$

By lemma D.2, $(n_0(\hat{\boldsymbol{\gamma}})(g), B_1(\hat{\boldsymbol{\gamma}})(h_0, 1), \mu_1(\hat{\boldsymbol{\gamma}})(h^1(\cdot))))$ are such that $u_c(n_0(\hat{\boldsymbol{\gamma}})(g) - g, 1 - n_0(\hat{\boldsymbol{\gamma}})(g)) = \mu$ and

$$z(1, n_0(\hat{\boldsymbol{\gamma}})(g), g)\mu + \mathcal{P}_1^*(g, B_1(\hat{\boldsymbol{\gamma}})(h_0, 1), \mu_1(\hat{\boldsymbol{\gamma}})(h^1(\cdot))B_1(\hat{\boldsymbol{\gamma}})(h_0, 1) \geq B\mu.$$

Therefore,

$$V_1^*(g, B, \mu) \leq \max_{(n', B', \mu'(\cdot)) \in \Gamma(g, B, \mu)} u(n - g, 1 - n) + \beta \int_{\mathbb{G}} \max\{V_1^*(g', B', \mu'(g')), V_0^*(g', B')\}\pi_{\mathbb{G}}(dg'|g).$$

We now show that the reversed inequality holds:

$$\begin{aligned} V_1^*(g, B, \mu) =& u(n_0(\hat{\boldsymbol{\gamma}})(g) - g, 1 - n_0(\hat{\boldsymbol{\gamma}})(g)) \\ & + \beta \int_{\mathbb{G}} \max\{V_1(\hat{\boldsymbol{\gamma}})(h_0, 1, B_1(\hat{\boldsymbol{\gamma}})(h_0, 1), (g', 1), 1), V_1(\hat{\boldsymbol{\gamma}})(h_0, 1, B_1(\hat{\boldsymbol{\gamma}})(h_0, 1), (g', 1), 0)\}\pi_{\mathbb{G}}(dg'|g) \\ \geq & u(n_0(\boldsymbol{\gamma})(g) - g, 1 - n_0(\boldsymbol{\gamma})(g)) \\ & + \beta \int_{\mathbb{G}} \max\{V_1(\boldsymbol{\gamma})(h_0, 1, B_1(\boldsymbol{\gamma})(h_0, 1), (g', 1), 1), V_1(\boldsymbol{\gamma})(h_0, 1, B_1(\boldsymbol{\gamma})(h_0, 1), (g', 1), 0)\}\pi_{\mathbb{G}}(dg'|g) \end{aligned} \tag{D.53}$$

where $h_0 = (1, B, g, 1)$ and the second line holds for any $\boldsymbol{\gamma}$ admissible.

For this we construct the following strategy $\tilde{\boldsymbol{\gamma}}$: (1) $\tilde{\boldsymbol{\gamma}}^D$ are determined by expressions 4.12-4.11; (2) for any ϕ and h_1, $\tilde{\boldsymbol{\gamma}}^F(h_0, \phi) = B_1(\tilde{\boldsymbol{\gamma}})(h_0, \phi)$ and $\mu_1(\tilde{\boldsymbol{\gamma}})(h^1)$ are such that

$$z(1, n_0(\tilde{\boldsymbol{\gamma}})(g), g)\mu + \phi\{\mathcal{P}_1^*(g, B_1(\tilde{\boldsymbol{\gamma}})(h_0, 1), \mu_1(\tilde{\boldsymbol{\gamma}})(h_0, \circ))B_1(\tilde{\boldsymbol{\gamma}})(h_0, 1) - B\mu\} \geq 0, \tag{D.54}$$

where $\circ$ stands for $(1, B_1(\tilde{\boldsymbol{\gamma}})(h_0, \phi), (\cdot, \phi))$, and $B_1(\tilde{\boldsymbol{\gamma}})(h_0, \phi) = B$ and $\mu_1(\tilde{\boldsymbol{\gamma}})(h^1) = m_A(g')$ if $\phi = 0$; (3) the remaining components of the strategy $\tilde{\boldsymbol{\gamma}}^F$ agree with $\hat{\boldsymbol{\gamma}}^F$, i.e., $\tilde{\boldsymbol{\gamma}}^F|_{h^1,\phi} = \hat{\boldsymbol{\gamma}}^F|_{h^1,\phi}$ for all history $h^1 \in \mathbb{H}^1$ and $\phi \in \{0, 1\}$.

We now verify that $\tilde{\boldsymbol{\gamma}}$ is admissible, which boils down to proving that $\tilde{\boldsymbol{\gamma}} \in \mathcal{S}(h_0, 1)$. Observe that by our construction $(n_0(\tilde{\boldsymbol{\gamma}})(g), B_1(\tilde{\boldsymbol{\gamma}})(h_0, 1))$ satisfy the implementability constraint (equation D.54) at time $t = 0$ for a price given by $\mathcal{P}_1^*(g, B_1(\tilde{\boldsymbol{\gamma}})(h_0, 1), \mu_1(\tilde{\boldsymbol{\gamma}})(h_0, \circ))$ and it satisfies that

$B_1(\tilde{\boldsymbol{\gamma}})(h_0,0) = B$. Additionally, from lemma D.2, $\hat{\boldsymbol{\gamma}}|_{h^1,\phi} \in \mathcal{S}(h_1,\phi)$, so these two results imply that $\tilde{\boldsymbol{\gamma}} \in \mathcal{S}(h_0,1)$.

Also for any $h^1 \in \mathbb{H}^2$ and $\phi \in \{0,1\}$ with $h_1 = (1, B_1(\tilde{\gamma})(h_0,1), g', 1)$, since $\hat{\boldsymbol{\gamma}}|_{h^1,\phi} \in \mathcal{S}(h_1,\phi)$, it follows that $V_1(\hat{\boldsymbol{\gamma}})(h^1,1) = V_1^*(g', B_1(\tilde{\gamma})(h_0,1), \mu_1(\tilde{\gamma})(h^1))$ and $V_1(\hat{\boldsymbol{\gamma}})(h^1,0) = V_0^*(g', B_1(\tilde{\gamma})(h_0,1))$, otherwise there would be an admissible strategy that achieves a higher value for $V_0(\cdot)(h_0,\phi)$ than $\hat{\boldsymbol{\gamma}}$.

Hence, evaluating display D.53 at $\tilde{\boldsymbol{\gamma}}$, it follows that

$$\begin{aligned} V_1^*(g,B,\mu) \geq & u(n_0(\tilde{\boldsymbol{\gamma}})(g) - g, 1 - n_0(\tilde{\boldsymbol{\gamma}})(g)) \\ & + \beta \int_{\mathbb{G}} \max\{V_1^*(g', B_1(\tilde{\boldsymbol{\gamma}})(h_0,1), \mu_1(\tilde{\boldsymbol{\gamma}})(h_0, h_1(g'))), V_0^*(g', B_1(\tilde{\boldsymbol{\gamma}})(h_0,1))\}\pi_{\mathbb{G}}(dg'|g) \end{aligned}$$

where $h_1(g')$ stands for $(1, B_1(\tilde{\boldsymbol{\gamma}})(h_0,1), (g',1))$. Since $(n_0(\tilde{\boldsymbol{\gamma}})(h_0), B_1(\tilde{\boldsymbol{\gamma}})(h_0,1), \mu_1(\tilde{\boldsymbol{\gamma}})(h^1))$ are arbitrary (other than the fact that they belong to $\Gamma(g,B,\mu)$), it follows that

$$V_1^*(g,B,\mu) \geq \max_{(n,B',\mu'(\cdot)\in\Gamma(g,\delta B,\mu)} u(n-g, 1-n) + \beta \int_{\mathbb{G}} \max\{V_1^*(g', B', \mu'(g')), V_0^*(g', B')\}\pi_{\mathbb{G}}(dg'|g).$$

□

D.1 Proofs of Supplementary Lemmas

Proof of Lemma D.1. (1) Under assumption assumption 3.1, $n \mapsto u'(\kappa n - g, 1-n) = u_c(\kappa n - g) - u_l(1-n) = 1 - (1-\tau)\kappa$ and since $\kappa < 1$ and $\tau \in [0,1]$ it implies that $u'(\kappa n - g, 1-n) > 0$. Also, $n \mapsto u(\kappa n - g, 1-n)$ is continuous. Moreover, $\{n : z(\kappa,n,g) = 0\} = \{n : \kappa(u_c(n-g,1-n) - u_l(n-g,1-n))n - u_c(n-g,1-n)g = 0\}$. Under assumption 3.1, u_c and u_l are continuous, and thus this set is closed (and bounded). Therefore it is compact. By the theorem of the maximum $\arg\max_{n\in[0,1]}\{u(\kappa n - g, 1-n) : z(\kappa,n,g) = 0\}$ exists. Uniqueness follows from the fact that $n \mapsto u(\kappa n - g, 1-n)$ is increasing.

(2) First observe that $n \mapsto z(1,n,g) = (1 - H'(1-n))n - g$ (with $u_c = 1$) is continuous and thus $\bar{n}(g)$ exists for all $g \in \mathbb{G}$ ($\mathbb{G}$ is such that for all $g \in \mathbb{G}$, $\max_{n\in[0,1]} z(1,n,g) \geq g$). Observe that $n \mapsto z'(1,n,g) = (1 - H'(1-n)) + H''(1-n)n$ and $n \mapsto z''(1,n,g) = 2H''(1-n) - H'''(1-n)n$. By assumption 5.1, $z''(1,n,g) < 0$ and thus is strictly concave. We now show that z is decreasing. If $\bar{n}(g) = 1$ then the statement is vacuous, so consider $\bar{n}(g) < 1$. Since $\bar{n}(g)$ is the "argmax", $z'(1,\bar{n}(g),g) \leq 0$. Since z is strictly concave, z' is a decreasing, hence $z'(1,n,g) < z'(1,\bar{n}(g),g) \leq 0$ for all $n > \bar{n}(g)$, and the result follows. □

Proof of Lemma D.2. If $\boldsymbol{\gamma} \in \mathcal{S}(h_0,\phi_0)$ it follows that, for any public history h^t with $h_t = (\phi_{t-1}, B_t, \omega_t = (g_t, \delta_t))$ with $B_t = B_t(\boldsymbol{\gamma})(h^{t-1},\phi)$ and any $\phi \in \{0,1\}$,

$$z(\kappa_\phi, n_t(\boldsymbol{\gamma})(h^t), g_t)u_c(\omega^t) + \phi\{p_t(\omega^t)u_c(\omega^t)B_{t+1}(\boldsymbol{\gamma})(h^t,\phi) - \delta_t u_c(\omega^t)B_t\} \geq 0$$

and $B_{t+1}(\boldsymbol{\gamma})(h^t,0) = B_t$,

$$
\begin{aligned}
p_t(\omega^t)u_c(\omega^t) =& \beta \int_{\mathbb{G}} d_{t+1}(\boldsymbol{\gamma})(h^t,h_{t+1}(g'))\mu_{t+1}(\boldsymbol{\gamma})(h^t,h_{t+1}(g'))\pi_{\mathbb{G}}(dg'|g_t) \\
&+ \beta \int_{\mathbb{G}} (1 - d_{t+1}(\boldsymbol{\gamma})(h^t,h_{t+1}(g')))m_A(g')q_{t+1}(\omega^t,\bar{\delta},g')\pi_{\mathbb{G}}(dg'|g_t)
\end{aligned} \tag{D.55}
$$

where $h_{t+1}(g') \equiv (1, B_{t+1}(\boldsymbol{\gamma})(h^t,1), g', 1)$ and

$$
\mu_{t+1}(\boldsymbol{\gamma})(h^t,h_{t+1}(g')) = u_c(n_{t+1}(\boldsymbol{\gamma})(h^t,h_{t+1}(g')) - g', 1 - n_{t+1}(\boldsymbol{\gamma})(h^t,h_{t+1}(g')))
$$

and q_t is the "secondary market" price at time t, i.e.,

$$
\begin{aligned}
q_{t+1}(\omega^{t+1},\bar{\delta},g) \equiv& \beta\lambda \int_{\mathbb{G}}\int_{\Delta} a_{t+1}(\boldsymbol{\gamma})(h^t,h_{t+1}(g',\delta'))\mu_{t+1}(\boldsymbol{\gamma})(h^t,h_{t+1}(g',\delta'))\delta'\pi_{\Delta}(d\delta')\pi_{\mathbb{G}}(dg'|g) \\
&+ \beta \int_{\mathbb{G}} \left(1 - \lambda + \lambda \int_{\Delta} (1 - a_{t+1}(\boldsymbol{\gamma})(h^t,h_{t+1}(g',\delta')))\pi_{\Delta}(d\delta')\right) m_A(g')q_{t+2}(\omega^{t+1},\bar{\delta},g')\pi_{\mathbb{G}}(dg'|g)
\end{aligned} \tag{D.56}
$$

with $h_{t+1}(g',\delta') = (0, \delta' B_{t+1}(\boldsymbol{\gamma})(h^t,1), g', \delta')$.

From equation D.55 it follows that $p_t(\omega^t)u_c(\omega^t) = \mathcal{P}_1(g_t, B_{t+1}(\boldsymbol{\gamma})(h^t,1), \mu_{t+1}(\boldsymbol{\gamma})(h^t,h_{t+1}(\cdot)))$ and from equation D.56 $q_{t+1}(\omega^t,\bar{\delta},g') = \mathcal{P}_0(g', B_{t+1}(\boldsymbol{\gamma})(h^t,1))$. Also, from these equations and the first display it is clear that if $\boldsymbol{\gamma} \in \mathcal{S}(h_0,\phi_0)$, then $\boldsymbol{\gamma}|_{h^t,\phi} \in \mathcal{S}(\phi_{t-1}, B_t, \omega_t, \phi)$. $\square$

E Proofs for Section 5

In order to show proposition 5.1, we need the following lemmas (whose proofs are relegated to the end of this section). Throughout this section we assume that assumption 5.1 holds.

Throughout this section, let

$$
\Gamma_{\phi}(g,B) = \{(n,B') : z(\kappa_{\phi},n,g) + \phi(\mathcal{P}^*_{\phi}(g,B')B' - B) \geq 0 \text{ } and \text{ } B' = B \text{ } if \text{ } \phi = 0\}
$$

with $\kappa_{\phi} \equiv \phi + \kappa(1-\phi)$.

Lemma E.1. *There exists a constant $\infty > C > 0$, such that $|V^*_{\phi}(g,B)| \leq C$ for all (ϕ,g,B) such that $\Gamma_{\phi}(g,B) \neq \{\emptyset\}$.*

Lemma E.2. *$B \mapsto V^*_1(g,B)$ is non-increasing for all $g \in \mathbb{G}$.*[64]

Lemma E.3. *There exists a $C > 0$, such that*

$$
\max_{g' \in \mathbb{G}} \max_{B_1,B_2 \in \mathbb{B}^2} |V^*_0(g',B_1) - V^*_0(g',B_2)| \leq \lambda \frac{\beta C}{1-\beta}.
$$

The previous lemma implies that, for any $\epsilon > 0$, there exists a $\lambda(\epsilon) > 0$ such that, for any $\lambda \in [0,\lambda(\epsilon)]$

$$
\max_{g' \in \mathbb{G}} \max_{B_1,B_2 \in \mathbb{B}^2} |V^*_0(g',B_1) - V^*_0(g',B_2)| \leq \epsilon. \tag{E.57}
$$

[64] This result clearly implies that $\delta \mapsto V^*_1(g,\delta B)$ is non-decreasing for all $g \in \mathbb{G}$ and $B > 0$.

Lemma E.4. *There exists $\bar{\lambda} > 0$ such that for all $\lambda \in [0, \bar{\lambda}]$, the following holds: For all (g, B) such that $B > 0$ and $\mathbf{d}^*(g, B) = 1$, $\mathcal{P}_1^*(g, B')B' \leq B$ for all $B' \in \mathbb{B}$.*

We observe that for each $B \in \mathbb{B}$, $\mathcal{P}_0^*$ is the fixed point of the following mapping

$$\begin{aligned} q \mapsto & T_B^*[q](\cdot) \\ = & \lambda\beta \int_{\mathbb{G}\times\Delta} \mathbf{a}^*(g', \delta', B)\delta'\pi_\Delta(d\delta')\pi_\mathbb{G}(dg'|\cdot) + \beta \int_\mathbb{G} \left((1-\lambda) + \lambda \int_\Delta (1 - \mathbf{a}^*(g', \delta', B))\pi_\Delta(d\delta') \right) q(g')\pi_\mathbb{G}(dg'|\cdot) \\ = & \lambda\beta \int_{\mathbb{G}\times\Delta} \mathbf{a}^*(g', \delta', B)\delta'\pi_\Delta(d\delta')\pi_\mathbb{G}(dg'|\cdot) + \beta \int_\mathbb{G} \left(1 - \lambda \int_\Delta \mathbf{a}^*(g', \delta', B)\pi_\Delta(d\delta') \right) q(g')\pi_\mathbb{G}(dg'|\cdot) \end{aligned}$$

for any $B \in \mathbb{B}$, and $q \in \{f : \mathbb{G} \to \mathbb{R} \mid f \text{ } uniformly\ bounded\}$. We use this insight to derive properties of $\mathcal{P}_0^*$.

Lemma E.5. *Suppose assumption 5.1 holds. Then:*

1. *For each $B \in \mathbb{B}$, T_B^* is a contraction.*
2. *For any $(g, B) \in \mathbb{G} \times \mathbb{B}$, $\mathcal{P}_0^*(g, B) \in \left[0, \lambda \frac{\beta}{1-\beta} E_{\pi_\Delta}[\delta]\right]$.*
3. *If g is iid (distributed according to $\pi_\mathbb{G}(\cdot)$), then $\mathcal{P}_0^*(g, B)$ is constant in g and given by*

$$\mathcal{P}_0^*(g, B) = \frac{\lambda\beta \int_{\mathbb{G}\times\Delta} \mathbf{a}^*(g', \delta', B)\delta'\pi_\Delta(d\delta')\pi_\mathbb{G}(dg')}{1 - \beta + \beta\lambda \int_{\mathbb{G}\times\Delta} \mathbf{a}^*(g', \delta', B)\pi_\Delta(d\delta')\pi_\mathbb{G}(dg')}$$

   and in this case $|\mathcal{P}_0^*(g, B)| \leq \frac{\beta\lambda}{1-\beta+\beta\lambda} < 1$.

Proof of Proposition 5.1. **Part (1).** By lemma E.2, $\delta \mapsto V_1^*(g, \delta B)$ is non-increasing, provided $B > 0$ (but this is the only case it matters since the government will never default on savings $B < 0$). On the other hand $V_0^*(g, B)$ is constant with respect to δ. Therefore if for some $\delta \in \Delta$, $\mathbf{a}^*(g, \delta, B) = 1$, then for all $\delta_1 \leq \delta$ the same must hold. Thus, there exists a $\hat{\delta} : \mathbb{G} \times \mathbb{B} \to [0, 1]$ such that

$$\mathbf{a}^*(g, \delta, B) = \mathbf{1}_{\{\delta:\delta\leq\hat{\delta}(g,B)\}}(\delta). \tag{E.58}$$

We now show that $B \mapsto \hat{\delta}(g, B)$ is non-increasing, for all $g \in \mathbb{G}$. It suffices to show that for any δ such that $\delta > \hat{\delta}(g, B_1)$ then $\delta > \hat{\delta}(g, B_2)$ for any $B_1 < B_2$.

Since $\delta > \hat{\delta}(g, B_1)$, it follows that $V_1^*(g, \delta B_1) < V_0^*(g, B_1)$. Let $\epsilon(g, B_1, \delta) \equiv V_0^*(g, B_1) - V_1^*(g, \delta B_1)$. It is easy to see that $\epsilon(g, B_1, \delta) > 0$ for any (g, B_1, δ) such that $\delta > \hat{\delta}(g, B_1)$. Moreover, since g, B_1 and δ belong to discrete sets, there exists a $\epsilon > 0$ such that $\epsilon \leq \epsilon(g, B_1, \delta)$ for all (g, B_1, δ) such that $\delta > \hat{\delta}(g, B_1)$.

Since $B \mapsto V_1^*(g, B)$ is non-increasing (see lemma E.2) for any $g \in \mathbb{G}$, it follows that $V_1^*(g, \delta B_2) \leq V_1^*(g, \delta B_1)$ for all $(g, \delta) \in \mathbb{G} \times \Delta$ (observe that $\delta > 0$ always). Therefore,

$$\begin{aligned} V_1^*(g, \delta B_2) - V_0^*(g, B_2) \leq & V_1^*(g, \delta B_1) - V_0^*(g, B_2) \\ \leq & V_1^*(g, \delta B_1) - V_0^*(g, B_1) + \{V_0^*(g, B_1) - V_0^*(g, B_2)\}, \end{aligned}$$

for all (g, B_1, B_2, δ) such that $\delta > \hat{\delta}(g, B_1)$.

Hence, if $|V_0^*(g, B_1) - V_0^*(g, B_2)| < \epsilon$ for any (g, B_1, B_2), the previous display implies that $V_1^*(g, \delta B_2) - V_0^*(g, B_2) < 0$ and the desired result follows. We now show that $|V_0^*(g, B_1) - V_0^*(g, B_2)| < \epsilon$ for any (g, B_1, B_2). By lemma E.3, there exists a $C > 0$

$$|V_0^*(g, B_1) - V_0^*(g, B_2)| < \lambda \frac{\beta C}{1-\beta}, \ \forall (B_1, B_2, g)$$

Thus for any $\varepsilon > 0$, there exists a $\lambda(\varepsilon)$, such that

$$|V_0^*(g, B_1) - V_0^*(g, B_1)| < \varepsilon$$

for all $\lambda \in [0, \lambda(\varepsilon)]$. By setting $\varepsilon = \epsilon$ and $\bar{\lambda} = \lambda(\epsilon)$, the desired result follows.

Part (2). Following Arellano (2008) we show the result in two steps. Throughout the proof $\mathbf{n}_\phi^*$ and $\mathbf{B}^*$ are the optimal policy functions for labor and debt.

Step 1. We show that for any $B_1 < B_2$, $\mathbb{S}(B_1) \subseteq \mathbb{S}(B_2)$ where $\mathbb{S}(B) = \{g : \mathbf{d}^*(g, B) = 1\}$. If $\mathbb{S}(B_1) = \{\emptyset\}$ the proof is trivial, so we proceed with the case in which this does not hold and let $\bar{g} \in \mathbb{S}(B_1)$. If B_2 is not feasible, in the sense that there does not exist any B' such that $B_2 - \mathcal{P}_1^*(g; B')B' - \max_{n \in [0,1]} z(1, n, \bar{g}) \leq 0$, then $\mathbb{S}(B_2) = \mathbb{G}$. And the result holds trivially, so we proceed with the case that B_2 is feasible, given $\bar{g}$.

It follows (since we assume that under indifference, the government chooses not to default) $V_1^*(\bar{g}, B_1) < V_0^*(\bar{g}, B_1)$. Since $B \mapsto V_1^*(\bar{g}, B)$ is non-increasing (see lemma E.2), it follows that

$$V_1^*(\bar{g}, B_2) \leq V_1^*(\bar{g}, B_1), \ for \ all \ B_1 < B_2.$$

Therefore,

$$\begin{aligned} V_1^*(\bar{g}, B_2) - V_0^*(\bar{g}, B_2) \leq & V_1^*(\bar{g}, B_1) - V_0^*(\bar{g}, B_2) \\ \leq & V_1^*(\bar{g}, B_1) - V_0^*(\bar{g}, B_1) + \{V_0^*(\bar{g}, B_1) - V_0^*(\bar{g}, B_2)\}. \end{aligned}$$

Let $\epsilon(\bar{g}, B_1) \equiv -\{V_1^*(\bar{g}, B_1) - V_0^*(\bar{g}, B_1)\}$, observe that $\epsilon(\bar{g}, B_1) > 0$ for any $(B_1, \bar{g}) \in Graph\{\mathbb{S}\}$. Thus, if $V_0^*(\bar{g}, B_1) - V_0^*(\bar{g}, B_2) < \epsilon(\bar{g}, B_1)$, then $V_1^*(\bar{g}, B_2) < V_0^*(\bar{g}, B_2)$ and the desired result follows.

Observe that $|\mathbb{B} \times Graph(\mathbb{S})| < \infty$, so there exists $\epsilon > 0$ such that $\epsilon \leq \epsilon(\bar{g}, B_1)$ for any $\bar{g}$ and B_1 in $Graph(\mathbb{S})$. By lemma E.3 and our derivations in part (1), there exists a $\lambda(\epsilon) > 0$ such that

$$|V_0^*(g, B_1) - V_0^*(g, B_2)| < \epsilon, \ \forall \lambda \in [0, \lambda(\epsilon)] \ and \ (g, B_1, B_2) \in \mathbb{G} \times \mathbb{B}^2.$$

Hence, $V_1^*(\bar{g}, B_2) - V_0^*(\bar{g}, B_2) < 0$, thereby implying that $\bar{g} \in \mathbb{S}(B_2)$.

Step 2. We show that, for any $B \in \mathbb{B}$ and any $g_1 < g_2$ in $\mathbb{G}$, if $\mathbf{d}^*(g_1, B) = 1$, then $\mathbf{d}^*(g_2, B) = 1$. That is, we want to show that $V_1^*(g_2, B) < V_0^*(g_2, B)$. Since default occurs for g_1, it suffices to show that

$$V_1^*(g_2, B) - V_0^*(g_2, B) < V_1^*(g_1, B) - V_0^*(g_1, B) \tag{E.59}$$

or equivalently, $V_1^*(g_2,B) - V_1^*(g_1,B) < V_0^*(g_2,B) - V_0^*(g_1,B)$. Observe that

$$V_0^*(g_2,B) - V_0^*(g_1,B) = r(\mathbf{n}_0^*(g_2)) - r(\mathbf{n}_0^*(g_1)) - (g_2 - g_1). \tag{E.60}$$

where $n \mapsto r(n) = n + H(1-n)$. And now take $\tilde{n}$ such that

$$z(1,\tilde{n},g_1) = B - \mathcal{P}_1^*(\mathbf{B}^*(g_2,B))\mathbf{B}^*(g_2,B);$$

i.e., $\tilde{n}$ is such that $(\tilde{n}, \mathbf{B}^*(g_2,B))$ are feasible choices given the state (g_1,B), and recall $z(1,n,g) \equiv (1 - H'(1-n))n - g$ and $(g,B) \mapsto \mathbf{B}^*(g,B)$ is the optimal policy function for debt, when the government has access to financial markets. Observe that if no such choice exists, then, since $z(1,\tilde{n},g_1) \geq z(1,\tilde{n},g_2)$, trivially $\mathbf{d}^*(g_2,B) = 1$. Also, $\mathcal{P}_1^*$ does not depend on g because of the i.i.d. assumption. Given this construction,

$$\begin{aligned}
&V_1^*(g_2,B) - V_1^*(g_1,B)\\
\leq& r(\mathbf{n}_1^*(g_2,B)) - g_2 + \beta\int_{\mathbb{G}} \mathbb{V}^*(g',\mathbf{B}^*(g_2,B))\pi_{\mathbb{G}}(dg') - \left\{ r(\tilde{n}) - g_1 + \beta\int_{\mathbb{G}} \mathbb{V}^*(g',\mathbf{B}^*(g_2,B))\pi_{\mathbb{G}}(dg') \right\}\\
=& r(\mathbf{n}_1^*(g_2,B)) - r(\tilde{n}) - (g_2 - g_1)
\end{aligned}$$

where $(g,B) \mapsto \mathbb{V}^*(g,B) \equiv \max\{V_1^*(g,B), V_0^*(g,B)\}$. Given this display and E.60, it suffices to show that

$$r(\mathbf{n}_1^*(g_2,B)) - r(\tilde{n}) \leq r(\mathbf{n}_0^*(g_2)) - r(\mathbf{n}_0^*(g_1)). \tag{E.61}$$

We now show this inequality. By construction of $\tilde{n}$,

$$z(1,\tilde{n},g_1) = z(1,\mathbf{n}_1^*(g_2,B),g_2) \tag{E.62}$$

where $(g,B) \mapsto \mathbf{n}_1^*(g,B)$ is the optimal policy function for labor, when the government has access to financial markets. Since $n \mapsto z(1,n,g)$ is non-increasing in the relevant domain (by relevant domain we mean the interval of n which are in "correct side of the Laffer curve"; see lemma D.1(2)) and $g_1 < g_2$, $\tilde{n} \geq \mathbf{n}_1^*(g_2,B)$. By analogous arguments, it follows that $\mathbf{n}_0^*(g_1) > \mathbf{n}_0^*(g_2)$.

Also, note that

$$z(1,\tilde{n},g_1) - z(1,\mathbf{n}_0^*(g_1),g_1) = \mathcal{P}_1^*(\mathbf{B}^*(g_2,B))\mathbf{B}^*(g_2,B) = z(1,\mathbf{n}_1^*(g_2,B),g_2) - z(1,\mathbf{n}_0^*(g_2),g_2), \tag{E.63}$$

or equivalently, with $n \mapsto \rho(n) = (1 - H'(1-n))n$

$$\rho(\tilde{n}) - \rho(\mathbf{n}_0^*(g_1)) = \rho(\mathbf{n}_1^*(g_2,B)) - \rho(\mathbf{n}_0^*(g_2)). \tag{E.64}$$

Since $n \mapsto z(1,n,g)$ (and thus ρ) is concave and non-increasing (see lemma D.1(2)), it follows $\tilde{n} > (<)\mathbf{n}_0^*(g_1)$ iff $\mathbf{n}_1^*(g_2,B) > (<)\mathbf{n}_0^*(g_2)$.

Putting all these observations together, we have the following possible orders

$$
\begin{aligned}
(I):&\ \ \mathbf{n}_0^*(g_1) \geq \tilde{n} \geq \mathbf{n}_0^*(g_2) \geq \mathbf{n}_1^*(g_2,B) \\
(II):&\ \ \mathbf{n}_0^*(g_1) \geq \mathbf{n}_0^*(g_2) \geq \tilde{n} \geq \mathbf{n}_1^*(g_2,B) \\
(III):&\ \ \tilde{n} \geq \mathbf{n}_0^*(g_1) \geq \mathbf{n}_1^*(g_2,B) \geq \mathbf{n}_0^*(g_2) \\
(IV):&\ \ \tilde{n} \geq \mathbf{n}_1^*(g_2,B) \geq \mathbf{n}_0^*(g_1) \geq \mathbf{n}_0^*(g_2).
\end{aligned}
$$

Moreover, since in (g_1, B) the government defaults, it follows from the proof of lemma E.4 that $B - \mathcal{P}_1^*(B')B' \geq 0$ for any $B' \in \mathbb{B}$, in particular for $B' = \mathbf{B}^*(g_2, B)$. Therefore, $z(1, \tilde{n}, g_1) > z(1, \mathbf{n}_0^*(g_1), g_1)$, and thus $\tilde{n} \leq \mathbf{n}_0^*(g_1)$, and consequently $\mathbf{n}_1^*(g_2, B) \leq \mathbf{n}_0^*(g_2)$. Hence, cases (III) and (IV) are ruled out.

We now study cases (I) and (II). Since $n \mapsto z(1, n, g)$ is strictly concave and non-increasing (see lemma D.1), equation E.64 and (I) and (II) imply

$$\mathbf{n}_0^*(g_1) - \tilde{n} \leq \mathbf{n}_0^*(g_2) - \mathbf{n}_1^*(g_2, B). \tag{E.65}$$

Since $n \mapsto r(n) \equiv n + H(1-n)$ is concave and increasing under our assumptions, the previous inequality implies that

$$r(\mathbf{n}_0^*(g_1)) - r(\mathbf{n}_0^*(g_2)) \leq r(\tilde{n}) - r(\mathbf{n}_1^*(g_2, B)) \tag{E.66}$$

for both case (I) and (II), or equivalently

$$r(\mathbf{n}_1^*(g_2, B)) - r(\tilde{n}) \leq r(\mathbf{n}_0^*(g_2)) - (\mathbf{n}_0^*(g_1)).$$

Which is precisely equation E.61.

Hence, step 2 establishes that $\mathbf{d}^*$ is of the threshold type, since it shows that, for any B, if $\mathbf{d}^*(g, B) = 1$, the same is true for any $g' > g$. That is $\{g : \mathbf{d}^*(g, B) = 1\}$ is of the form $\{g : g \geq \bar{g}(B)\}$. Step 1 shows that the $\bar{g}$ ought to be non-increasing. $\square$

Proof of Proposition 5.2. We first establish the result for $i = 0$. From lemma E.5(3), observe that

$$\mathcal{P}_0^*(B) = \frac{\beta\lambda \int_{\mathbb{G}} \int_{\Delta} \mathbf{1}_{\{\delta \leq \hat{\delta}(g',B)\}}(\delta)\pi_\Delta(d\delta)\pi_{\mathbb{G}}(dg')}{1 - \beta + \beta\lambda \int_{\mathbb{G}} \int_{\Delta} \mathbf{1}_{\{\delta \leq \hat{\delta}(g',B)\}}(\delta)\pi_\Delta(d\delta)\pi_{\mathbb{G}}(dg')} \frac{\int_{\mathbb{G}} \int_{\Delta} \mathbf{1}_{\{\delta \leq \hat{\delta}(g',B)\}}(\delta)\delta\pi_\Delta(d\delta)\pi_{\mathbb{G}}(dg')}{\int_{\mathbb{G}} \int_{\Delta} \mathbf{1}_{\{\delta \leq \hat{\delta}(g',B)\}}(\delta)\pi_\Delta(d\delta)\pi_{\mathbb{G}}(dg')}.$$

Note that the first term in the RHS is an increasing function (namely $x \mapsto \frac{x}{1-\beta+x}$) of $\beta\lambda \int_{\mathbb{G}} \int_{\Delta} \mathbf{1}_{\{\delta \leq \hat{\delta}(g',B)\}}(\delta)\pi_\Delta(d\delta)\pi_{\mathbb{G}}(dg')$. Since $B \mapsto \hat{\delta}(g, B)$ is non-increasing (proposition 5.1), it follows that $B \mapsto \int_{\Delta} \mathbf{1}_{\{\delta \leq \hat{\delta}(g',B)\}}(\delta)\pi_\Delta(d\delta)$ is also non-increasing, this in turn implies that the first term in the RHS is also non-increasing as a function of B.

By our assumption $\pi_\Delta(\cdot) = 1_{\delta_0}(\cdot)$, the second term in the RHS is given by

$$\frac{\int_{\mathbb{G}} \int_{\Delta} \mathbf{1}_{\{\delta \leq \hat{\delta}(g',B)\}}(\delta)\delta\pi_\Delta(d\delta)\pi_{\mathbb{G}}(dg')}{\int_{\mathbb{G}} \int_{\Delta} \mathbf{1}_{\{\delta \leq \hat{\delta}(g',B)\}}(\delta)\pi_\Delta(d\delta)\pi_{\mathbb{G}}(dg')} = \delta_0 \frac{\int_{\mathbb{G}} \mathbf{1}_{\{\delta_0 \leq \hat{\delta}(g',B)\}}(\delta)\pi_{\mathbb{G}}(dg')}{\int_{\mathbb{G}} \mathbf{1}_{\{\delta_0 \leq \hat{\delta}(g',B)\}}(\delta)\pi_{\mathbb{G}}(dg')} = \delta_0$$

and thus constant. Hence, $B \mapsto \mathcal{P}_0^*(B)$ is non-increasing.

For $i = 1$, observe that for any $B_1 \leq B_2$,

$$\mathcal{P}_1^*(B_1) = \beta \int_{\mathbb{G}} \mathbf{1}_{\{g' \leq \bar{g}(B_1)\}}(g')\pi_{\mathbb{G}}(dg') + \beta \int_{\mathbb{G}} \mathbf{1}_{\{g' > \bar{g}(B_1)\}}(g')\pi_{\mathbb{G}}(dg')\mathcal{P}_0^*(B_1) \tag{E.67}$$

$$\geq \beta \int_{\mathbb{G}} \mathbf{1}_{\{g' \leq \bar{g}(B_2)\}}(g')\pi_{\mathbb{G}}(dg') + \beta \int_{\mathbb{G}} \mathbf{1}_{\{g' > \bar{g}(B_2)\}}(g')\pi_{\mathbb{G}}(dg')\mathcal{P}_0^*(B_1) \tag{E.68}$$

$$\geq \beta \int_{\mathbb{G}} \mathbf{1}_{\{g' \leq \bar{g}(B_2)\}}(g')\pi_{\mathbb{G}}(dg') + \beta \int_{\mathbb{G}} \mathbf{1}_{\{g' > \bar{g}(B_2)\}}(g')\pi_{\mathbb{G}}(dg')\mathcal{P}_0^*(B_2) \tag{E.69}$$

$$= \mathcal{P}_1^*(B_2) \tag{E.70}$$

where the first inequality follows from the fact that $B \mapsto \bar{g}(B)$ is non-increasing (proposition 5.1) and $\mathcal{P}_0^*(B) < 1$ for any $B \in \mathbb{B}$ (see lemma E.5(3)) ; the second inequality follows from the fact that $\mathcal{P}_0^*$ is non-increasing. $\square$

E.1 Proofs of Supplementary Lemmas

Proof of Lemma E.1. For any $(\phi_-, g, \delta, B) \in \{0,1\} \times \mathbb{G} \times \Delta \times \mathbb{B}$, and any function $(\phi_-, g, \delta, B) \mapsto F(\phi_-, g, \delta, B)$ we define the following operator

$$T[F](\phi_-, g, \delta, B) = \max_{(a,d) \in D(\phi_-, \delta)} T_1[F](\phi_-(1-d) + a(1-\phi_-), g, \delta, \varphi(B, \delta, a, d)) \tag{E.71}$$

with $D(0, \delta) = \{0,1\} \times \{1\}$ if $\delta \neq \bar{\delta}$ and $D(0, \bar{\delta}) = \{0\} \times \{1\}$, also $D(1, \delta) = \{1\} \times \{0,1\}$; $\varphi(B, \delta, a, 1) = \delta B a + (1-a)B$ and $\varphi(B, \delta, 0, d) = B$; and

$$T_1[F](\phi, g, \delta, B) = \max_{(n, B') \in \Gamma_\phi(g,B)} \left\{ \kappa_\phi n - g + H(1-n) + \beta \int_{\mathbb{G}} \int_{\bar{\Delta}} F(\phi, g', \delta', B')\pi_{\bar{\Delta}}(d\delta'|\phi)\pi_{\mathbb{G}}(dg') \right\}, \tag{E.72}$$

where $\pi_{\bar{\Delta}}(\cdot|\phi) = \mathbf{1}_{\{1\}}(\cdot)$ if $\phi = 1$ and $\pi_{\bar{\Delta}}(\cdot|\phi) = (1-\lambda)\mathbf{1}_{\{\bar{\delta}\}}(\cdot) + \lambda \pi_\Delta(\cdot)$ if $\phi = 0$.

A fix point of the T operator, is given by

$$\mathbb{V}^*(\phi_-, g, \delta, B) = \max_{(a,d) \in D(\phi_-, \delta)} V^*_{\phi_-(1-d)+a(1-\phi_-)}(g, \varphi(B, \delta, a, d)) \tag{E.73}$$

and for any $\phi \in \{0,1\}$

$$V_\phi^*(g, B) = \max_{(n, B') \in \Gamma_\phi(g,B)} \left\{ \kappa_\phi n - g + H(1-n) + \beta \int_{\mathbb{G}} \int_{\bar{\Delta}} \mathbb{V}^*(\phi, g', \delta', B')\pi_{\bar{\Delta}}(d\delta'|\phi)\pi_{\mathbb{G}}(dg') \right\}. \tag{E.74}$$

In order to verify equation E.74, observe that if $\phi = 0$, $B' = B$ by the restrictions imposed on Γ_0, $\kappa_0 = \kappa$ and

$$\begin{aligned}
\int_{\bar{\Delta}} \mathbb{V}^*(0, g', \delta', B')\pi_{\bar{\Delta}}(d\delta'|0) &= \lambda \int_{\Delta} \mathbb{V}^*(0, g', \delta', B)\pi_{\bar{\Delta}}(d\delta') + (1-\lambda)\mathbb{V}^*(0, g', \bar{\delta}, B) \\
&= \lambda \int_{\Delta} \max_{a \in \{0,1\}} V_a^*(g', B(\delta a + (1-a)))\pi_{\bar{\Delta}}(d\delta') + (1-\lambda)\mathbb{V}^*(0, g', \bar{\delta}, B) \\
&= \lambda \int_{\Delta} \max\{V_1^*(g', B\delta), V_0^*(g', B)\}\pi_{\bar{\Delta}}(d\delta') + (1-\lambda)V_0^*(g', B)
\end{aligned}$$

where the last line follows from the fact that $D(0,\bar{\delta}) = \{0\} \times \{1\}$. If $\phi = 1$, then

$$\begin{aligned}
\int_{\bar{\Delta}} \mathbb{V}^*(1, g', \delta', B')\pi_{\bar{\Delta}}(d\delta'|1) =& \mathbb{V}^*(1, g', 1, B') \\
=& \max_{d\in\{0,1\}} V^*_{(1-d)}(g', \varphi(B', 1, 1, d)) \\
=& \max\{V_1^*(g', \varphi(B', 1, 0, 0)), V_0^*(g', \varphi(B', 1, 0, 1))\} \\
=& \max\{V_1^*(g', B'), V_0^*(g', B')\}.
\end{aligned}$$

Observe that from this fixed point we can derive the functions V^* by using equation E.74.

We now show that the operator T maps bounded functions onto bounded functions. Take F such that $|F(\phi_-, g, \delta, B)| \leq C$ for all (ϕ_-, g, δ, B) and for some finite constant $C > 0$. Then

$$|T[F](\phi_-, g, \delta, B)| = |\max_{(a,d)\in D(\phi_-,\delta)} T_1[F](\phi_-(1-d) + a(1-\phi_-), g, \delta, \varphi(B, \delta, a, d))|.$$

If (g, δ, B) are such that $\Gamma_1(g, \delta B) = \{\emptyset\}$, then by convention, $\phi_-(1-d)+a(1-\phi_-) = 0$ (i.e., there is default/no repayment) and thus $\max_{(a,d)\in D(\phi_-,\delta)} T_1[F](\phi_-(1-d)+a(1-\phi_-), g, \delta, \varphi(B, \delta, a, d)) = F(0, g, \delta, \varphi(B, \delta, 0, 1)) = F(0, g, \delta, B)$ and since by our assumptions over $\mathbb{G}$, $\Gamma_0(g, B) \neq \{\emptyset\}$ for any (g, B), there exists a finite $c' > 0$ such that $|\max_{n\in\Gamma_0(g,B)} \kappa n - g + H(1-n)| \leq c'$. This implies that in this case $|T[F](\phi_-, g, \delta, B)| \leq c' + \beta C$.

Similarly, if (g, δ, B) are such that $\Gamma_1(g, \delta B) \neq \{\emptyset\}$ then $|\max_{n\in\Gamma_1(g,\delta B)} n - g + H(1-n)| \leq c'$ and it follows that $|T[F](\phi_-, g, \delta, B)| \leq c' + \beta C$. Hence, by letting $C = \frac{c'}{1-\beta}$ we showed that T maps bounded functions onto bounded functions.

The fix point $\mathbb{V}^*$ inherits this property, i,.e., $|\mathbb{V}^*(\phi_-, g, \delta, B)| \leq C$ for all (ϕ_-, g, δ, B). This result, the fact that $|\max_{n\in\Gamma_0(g,B)} \kappa n - g + H(1-n)| \leq c'$ and equation E.74 implies that there exists a finite constant $C'' > 0$, such that $|V_0^*(g, B)| \leq C''$. An analogous result holds for $V_1^*(g, B)$ provided that (g, B) are such that $\Gamma_1(g, B) \neq \{\emptyset\}$. □

Proof of Lemma E.2. It is easy to see that $\Gamma_1(g, B_1) \subseteq \Gamma_1(g, B_2)$ for any $B_1 \geq B_2$ and this immediately implies that

$$\begin{aligned}
V_1^*(g, B_1) =& \max_{(n,B')\in\Gamma_1(g,B_1)} \{n - g + H(1-n) + \beta \int_{\mathbb{G}} \max\{V_0^*(g', B'), V_1^*(g', B')\}\pi_{\mathbb{G}}(dg') \\
\leq& \max_{(n,B')\in\Gamma_1(g,B_2)} \{n - g + H(1-n) + \beta \int_{\mathbb{G}} \max\{V_0^*(g', B'), V_1^*(g', B')\}\pi_{\mathbb{G}}(dg') \\
=& V_1^*(g, B_2)
\end{aligned}$$

and the result follows for V_1^*. □

Proof of Lemma E.3. Observe that, for any $(g, B_1, B_2) \in \mathbb{G} \times \mathbb{B}^2$,

$$
\begin{aligned}
|V_0^*(g,B_1) - V_0^*(g,B_2)| \leq & \lambda\beta \int_{\mathbb{G}} \int_{\Delta} \mathbf{a}^*(g,\delta,B)|V_1^*(g',\delta B_1) - V_1^*(g',\delta B_2)|\pi_{\Delta}(d\delta)\pi_{\mathbb{G}}(dg'|g) \\
& + \beta \int_{\mathbb{G}} \{(1-\lambda) + \lambda \int_{\Delta} (1 - \mathbf{a}^*(g,\delta,B))\pi_{\Delta}(d\delta)\}|V_0^*(g',B_1) - V_0^*(g',B_2)|\pi_{\mathbb{G}}(dg'|g) \\
\leq & \lambda\beta \int_{\mathbb{G}} \int_{\Delta} \mathbf{a}^*(g,\delta,B)|V_1^*(g',\delta B_1) - V_1^*(g',\delta B_2)|\pi_{\Delta}(d\delta)\pi_{\mathbb{G}}(dg'|g) \\
& + \beta \max_{g' \in \mathbb{G}} |V_0^*(g',B_1) - V_0^*(g',B_2)| \\
\leq & \lambda\beta C + \beta \max_{g' \in \mathbb{G}} |V_0^*(g',B_1) - V_0^*(g',B_2)|
\end{aligned}
$$

where the last line follows from lemma E.1 and the fact that if (g, δ, B) are such that $\Gamma_1(g, \delta B) = \{\emptyset\}$ then $\mathbf{a}^*(g, \delta, B) = 0$. Therefore,

$$
\max_{g' \in \mathbb{G}} \max_{B_1, B_2 \in \mathbb{B}^2} |V_0^*(g',B_1) - V_0^*(g',B_2)| \leq \lambda \frac{\beta C}{1-\beta}.
$$

□

Proof of Lemma E.4. Suppose not. That is, for any λ, there exists a (g, B) with $B > 0$ such that $\mathbf{d}^*(g, B) = 1$ but there exists a B' such that $\mathcal{P}_1^*(g, B')B' > B$.

First observe that for any (g, B, B') such that $\mathcal{P}_1^*(g, B')B' > B$,

$$
z(1, \mathbf{n}(g, B, B'), g) < z(1, \mathbf{n}_0^*(g), g)
$$

where $\mathbf{n}(g, B, B')$ is the level of labor that solves $z(1, n, g) + \mathcal{P}_1^*(g, B')B' = B$. Since $n \mapsto z(1, n, g)$ is non-increasing in the relevant domain (see lemma D.1(2)), it follows that $\mathbf{n}(g, B, B') > \mathbf{n}_0^*(g)$, thereby implying that the per-period payoff is greater under no default, i.e.,

$$
r(\mathbf{n}(g, B, B')) - g - \{r(\mathbf{n}_0^*(g)) - g\} > 0 \tag{E.75}
$$

where $n \mapsto r(n) = n + H(1-n)$ is increasing by our assumptions. Let $U \equiv \{(g, B, B') \in \mathbb{G} \times \mathbb{B}^2 :$ *equation E.75 holds*$\}$. Under our assumptions $|U| < \infty$, so there exists a $\epsilon' > 0$ such that $r(\mathbf{n}(g, B, B')) - g - \{r(\mathbf{n}_0^*(g)) - g\} \geq \epsilon'$ for all $(g, B, B') \in U$.

Consider any $\lambda \in [0, \lambda(0.5\epsilon')]$ where $\epsilon \mapsto \lambda(\epsilon)$ is such that

$$
\lambda(\epsilon)| \int_{\mathbb{G}} \{ \int_{\Delta} \max\{V_1^*(g', \delta B') - V_0^*(g', B'), 0\}\pi_{\Delta}(d\delta)\}\pi_{\mathbb{G}}(dg'|g)| \leq \epsilon; \tag{E.76}
$$

such λ exists by lemma E.1.

By our hypothesis, there exists a (g, B, B') with $B > 0$ such that $\mathbf{d}^*(g, B) = 1$ and $\mathcal{P}_1^*(g, B')B' > B$. And thus $(g, B, B') \in U$.

By our choice of λ,

$$
\int_{\mathbb{G}} V_0^*(g', B')\pi_{\mathbb{G}}(dg'|g) + 0.5\epsilon' \geq \int_{\mathbb{G}} \{\lambda \int_{\Delta} \max\{V_1^*(g', \delta B'), V_0^*(g', B')\}\pi_{\Delta}(d\delta) + (1-\lambda)V_0^*(g', B')\}\pi_{\mathbb{G}}(dg'|g). \tag{E.77}
$$

By definition of ϵ' and the fact that $(g, B, B') \in U$, it follows that

$$r(\mathbf{n}(g, B, B')) - g + \beta \int_{\mathbb{G}} \max\{V_1^*(g', B'), V_0^*(g', B')\}\pi_{\mathbb{G}}(dg'|g) \tag{E.78}$$

$$>r(\mathbf{n}_0^*(g)) - g + 0.5\epsilon' + \beta \int_{\mathbb{G}} \max\{V_1^*(g', B'), V_0^*(g', B')\}\pi_{\mathbb{G}}(dg'|g) \tag{E.79}$$

$$\geq r(\mathbf{n}_0^*(g)) - g + 0.5\epsilon' + \beta \int_{\mathbb{G}} V_0^*(g', B')\pi_{\mathbb{G}}(dg'|g) \tag{E.80}$$

$$\geq r(\mathbf{n}_0^*(g)) - g + \beta\{\int_{\mathbb{G}} V_0^*(g', B')\pi_{\mathbb{G}}(dg'|g) + 0.5\epsilon'\} \tag{E.81}$$

$$\geq V_0^*(g, B). \tag{E.82}$$

Since $V_1^*(g, B)$ is larger or equal than the LHS, we conclude that for (g, B) the government decides not to default, but this is a contradiction to the fact that $\mathbf{d}^*(g, B) = 1$. □

Proof of Lemma E.5. **Part 1.** To show part 1 we show that for each $B \in \mathbb{B}$, T_B^* satisfies the Blackwell sufficient conditions. Henceforth, consider $B \in \mathbb{B}$ given, observe that T_B^* is of the form

$$T_B^*[q](g) = A_B(g) + \beta \int_{\mathbb{G}} C_B(g')q(g')\pi_{\mathbb{G}}(dg'|g) \tag{E.83}$$

where $A_B(\cdot) \equiv \lambda\,\beta \int_{\mathbb{G}\times\Delta} \mathbf{a}^*(g', \delta', B)\,\delta'\,\pi_\Delta(d\delta')\,\pi_{\mathbb{G}}(dg'|\cdot)$, and $C_B(g) \equiv ((1-\lambda)+\lambda\int_\Delta(1-\mathbf{a}^*(g', \delta', B))\pi_\Delta(d\delta'))$ is non-negative and less than one. Hence for any $g \in \mathbb{G}$ and for any $q \leq q'$, $T_B^*[q](g) \leq T_B^*[q'](g)$ and $T^*[q+a](g) = A_B(g)+\beta\int_{\mathbb{G}} C_B(g')q(g')\pi_{\mathbb{G}}(dg'|g)+\beta\int_{\mathbb{G}} C_B(g')q(g')\pi_{\mathbb{G}}(dg'|g)a \leq A_B(g) + \beta\int_{\mathbb{G}} C_B(g')q(g')\pi_{\mathbb{G}}(dg'|g) + \beta a = T_B^*[q](g) + \beta a$. Therefore T_B^* is a contraction by Blackwell sufficient conditions, see Stokey et al. (1989), moreover its modulus is given by β which does not depend on B.

Part 2. Consider $C \equiv \beta\lambda\frac{E_{\pi_\Delta}[\delta]}{1-\beta}$ such that $|q(g)| \leq C$, then

$$|T_B^*[q](g)| \leq |A_B(g)| + \beta C \leq \beta\lambda E_{\pi_\Delta}[\delta] + \beta C = \beta\lambda E_{\pi_\Delta}[\delta]\{1 + \frac{\beta}{1-\beta}\} = \beta\lambda E_{\pi_\Delta}[\delta]\frac{1}{1-\beta}, \tag{E.84}$$

so in fact T_B^* maps functions bounded by C into themselves; and this holds for any $B \in \mathbb{B}$. Thus the fixed point of T_B^* also satisfies the inequality.

Part 3. Since $\pi_{\mathbb{G}}(\cdot|g)$ are constant with respect to g it follows that

$$\mathcal{P}_0^*(g, B) = \lambda\beta \int_{\mathbb{G}\times\Delta} \mathbf{a}^*(g', \delta', B)\delta'\pi_\Delta(d\delta')\pi_{\mathbb{G}}(dg') + \beta \int_{\mathbb{G}} \left(1 - \lambda \int_\Delta \mathbf{a}^*(g', \delta', B)\pi_\Delta(d\delta')\right) \mathcal{P}_0^*(g', B)\pi_{\mathbb{G}}(dg')$$

and thus $\mathcal{P}_0^*(g, B)$ is constant with respect to g, abusing notation we denote it as $\mathcal{P}_0^*(B)$. From the display above it follows that

$$\begin{aligned}
\mathcal{P}_0^*(B) =& \frac{\lambda\beta\int_{\mathbb{G}\times\Delta} \mathbf{a}^*(g', \delta', B)\delta'\pi_\Delta(d\delta')\pi_{\mathbb{G}}(dg')}{1-\beta\int_{\mathbb{G}}(1-\lambda\int_\Delta \mathbf{a}^*(g', \delta', B))\pi_\Delta(d\delta'))\,\pi_{\mathbb{G}}(dg')} \\
=& \frac{\lambda\beta\int_{\mathbb{G}\times\Delta} \mathbf{a}^*(g', \delta', B)\delta'\pi_\Delta(d\delta')\pi_{\mathbb{G}}(dg')}{1-\beta\,(1-\lambda\int_{\mathbb{G}}\int_\Delta \mathbf{a}^*(g', \delta', B)\pi_\Delta(d\delta')\pi_{\mathbb{G}}(dg'))} \\
=& \frac{\lambda\beta\int_{\mathbb{G}\times\Delta} \mathbf{a}^*(g', \delta', B)\delta'\pi_\Delta(d\delta')\pi_{\mathbb{G}}(dg')}{1-\beta+\beta\lambda\,(\int_{\mathbb{G}}\int_\Delta \mathbf{a}^*(g', \delta', B)\pi_\Delta(d\delta')\pi_{\mathbb{G}}(dg'))}.
\end{aligned}$$

Since $\delta \in \Delta$ is such that $\delta \leq 1$, it is easy to see that $|\mathcal{P}_0^*(B)| \leq \frac{\lambda\beta\left(\int_{\mathbb{G}}\int_{\Delta}\mathbf{a}^*(g',\delta',B)\pi_\Delta(d\delta')\pi_{\mathbb{G}}(dg')\right)}{1-\beta+\beta\lambda\left(\int_{\mathbb{G}}\int_{\Delta}\mathbf{a}^*(g',\delta',B)\pi_\Delta(d\delta')\pi_{\mathbb{G}}(dg')\right)} \leq \frac{\beta\lambda}{1-\beta+\beta\lambda} < 1$.

□

E.2 Derivation of Equation 5.23

By our characterization of the default rule. In this setting, to default or not, boils down to choosing a T (contingent on ω^∞) such that for all $t < T(\omega^\infty)$ there is no default and for $t \geq T(\omega^\infty)$ there is financial autarky. Recall that under our assumptions $u(c,l) = c + H(l)$ and $g_t \sim iid\pi_{\mathbb{G}}$, also $\pi_{\mathbb{G}}$ has a density with respect to Lebesgue, which we denote as $f_{\pi_{\mathbb{G}}}$.

For any $\omega^t \in \Omega^t$ and $t \leq T(\omega^\infty)$,

$$V_1^*(g_t, B_t(\omega^{t-1})) = \max_{(n,B')\in\Gamma(g_t,B_t(\omega^{t-1}),1)} n - g + H(1-n) + \beta \int_{\{g':g'\leq\bar{g}(B')\}} \{V_1^*(g',B') - V_0^*(g')\}\pi_{\mathbb{G}}(dg') \tag{E.85}$$

$$+ \beta \int V_0^*(g')\pi_{\mathbb{G}}(dg') \tag{E.86}$$

and let $\nu_t(\omega^t)$ is the Lagrange multiplier of the restriction, $z(1,n,g_t)+\mathcal{P}_1^*(B')B'-B_t(\omega^{t-1}) \geq 0$. By assumption, the solution of B' is in the interior. So the optimal choice $((n_t(\omega^t))_{t=0}^\infty, (B_{t+1}(\omega^t))_{t=0}^\infty)$ satisfy

$$1 - H'(1-n_t(\omega^t)) + \nu_t(\omega^t)\left(\frac{dz(1,n_t(\omega^t),g_t)}{dn}\right) = 0$$

or equivalently

$$\nu_t(\omega^t) \equiv \boldsymbol{\nu}(n_t(\omega^t)) = -\frac{1-H'(1-n_t(\omega^t))}{1-H'(1-n_t(\omega^t)) + H''(1-n_t(\omega^t))n_t(\omega^t)}, \tag{E.87}$$

and

$$\begin{aligned}
&\nu_t(\omega^t)\left\{\mathcal{P}_1^*(B_{t+1}(\omega^t)) + \frac{d\mathcal{P}_1^*(B_{t+1}(\omega^t))}{dB_{t+1}}B_{t+1}(\omega^t)\right\}\\
=&\beta\frac{d\int_{\{g':g'\leq\bar{g}(B_{t+1}(\omega^t))\}}\{V_1^*(g',B_{t+1}(\omega^t)) - V_0^*(g')\}\pi_{\mathbb{G}}(dg')}{dB_{t+1}}\\
=&\beta\int_{\{g':g'\leq\bar{g}(B_{t+1}(\omega^t))\}}\frac{dV_1^*(g',B_{t+1}(\omega^t))}{dB_{t+1}}\pi_{\mathbb{G}}(dg')\\
&+\beta\{V_1^*(\bar{g}(B_{t+1}(\omega^t)),B_{t+1}(\omega^t)) - V_0^*(\bar{g}(B_{t+1}(\omega^t)))\}f_{\pi_{\mathbb{G}}}(\bar{g}(B_{t+1}(\omega^t)))\frac{d\bar{g}(B_{t+1}(\omega^t))}{dB_{t+1}}.
\end{aligned}$$

Since $V_1^*(\bar{g}(B_{t+1}(\omega^t)),B_{t+1}(\omega^t)) - V_0^*(\bar{g}(B_{t+1}(\omega^t))) = 0$, the last term in the RHS is naught. Also, $\frac{dV_1^*(g_t,B_t(\omega^{t-1}))}{dB_t} = \nu_t(\omega^t)$ and thus

$$\nu_t(\omega^t)\left\{\mathcal{P}_1^*(B_{t+1}(\omega^t)) + \frac{d\mathcal{P}_1^*(B_{t+1}(\omega^t))}{dB_{t+1}}B_{t+1}(\omega^t)\right\} = \beta\int_{\{g':g'\leq\bar{g}(B_{t+1}(\omega^t))\}}\nu_{t+1}(\omega^t,g')\pi_{\mathbb{G}}(dg') \tag{E.88}$$

We now show that $\boldsymbol{\nu}$ is decreasing. For this it is easier to first establish that $\boldsymbol{\nu}^- \equiv 1/\boldsymbol{\nu}$ is increasing. Observe that

$$\boldsymbol{\nu}^-(n) = -1 - \frac{H''(1-n)n}{1-H'(1-n)}$$

and thus

$$\frac{d\boldsymbol{\nu}^-(n)}{dn} = -\frac{-H'''(1-n)n + H''(1-n)}{1-H'(1-n)} - \frac{(H''(1-n))^2 n}{(1-H'(1-n))^2}.$$

Since $-H'''(1-n)n + H''(1-n) < 0$ by assumption and $1 - H'(1-n) = \tau > 0$, then the first term in the RHS is negative; the second term in the RHS is also negative. Hence $\boldsymbol{\nu}^-$ is increasing, which readily implies that $\boldsymbol{\nu}$ is decreasing.

Supplementary Online Material

F Stylized Facts: Emerging vs. Industrialized Economies

Throughout the paper, we mention that our theoretical model is capable of replicating qualitatively several stylized facts observed for a wide range of economies. In this section, we present these stylized facts regarding the domestic government debt-to-output ratio and central government revenue-to-output ratio of several countries: industrialized economies (IND), emerging economies (EME) and a subset of these: Latin American (LAC).[65]

In the dataset set which covers the period 1800-2010, no default event is observed for IND, whereas EME/LAC (LAC in particular) do exhibit several defaults. Thus, we take the former group as a proxy for economies with access to risk-free debt and the latter group as a proxy for economies without commitment to repay. It is worth to point out however that we are not presuming that IND economies are a type of economy that would never never default. In turn, we are just using the fact that in our dataset IND economies do not show default events, to use them as a proxy for the type of economy modeled in AMSS, that is, one with risk-free government debt.

Several stylized facts that stand out in our dataset. First, in EME/LAC economies default is more likely than in IND economies and within the former group, the default risk is much higher for highly indebted economies. Second, EME and LAC economies exhibit tighter debt ceilings than IND, as also reported by Reinhart et al. (2003). Third, economies with higher default risk tend to exhibit more volatile tax revenues than those with low default risk, and this fact is particularly notable for the group of EME/LAC economies. Bauducco and Caprioli (2014) documents a similar finding.

As shown in section 5, our theory predicts that endogenous borrowing limits are more active for a high level of indebtedness. That is, when the government debt is high relative to output, the probability of default next period is higher, thus implying tighter borrowing limits and higher bond spreads. As the government's ability to smooth its needs for funds using debt is hindered, the volatility of taxes turns out to be higher. But when debt is low, default is an unlikely event, thereby implying slacker borrowing limits, lower spreads and therefore lower volatility in taxes. Hence, implications in the upper tail of the domestic debt-to-output ratio distribution can be different from those in the "central part" of it. Therefore, the mean and even the variance of the distribution may not be too informative, as they are affected by the central part of the distribution. Quantiles are better suited for recovering the information in the tails of the distribution.[66]

[65] For government revenue-to-output ratios, we used the data from Kaminsky et al. (2004), and for the domestic government debt-to-output ratios, we used the data from Panizza (2008). We thank Ugo Panizza and Carmen Reinhart for kindly sharing their datasets with us. See appendix G for a detailed description of the data.

[66] We refer the reader to Koenker (2005) for a thorough treatment of quantiles and quantile-based econometric models.

Figure F.7 plots the percentiles of the domestic government debt-to-output ratio and of a measure of default risk for three groups: IND (black triangle), EME (blue square) and LAC (red circle).[67] The X-axis plots the time series averages of domestic government debt-to-output ratio, and the Y-axis plots the values of the measure of default risk.[68] For each group, the last point on the right corresponds to the 95 percentile, the second to last to the 90 percentile and so on; these are comparable between groups as all of them represent a percentile of the corresponding distribution. EME and LAC have lower domestic debt-to-output ratio levels than IND; in fact the domestic debt-to-output ratio value of around 50 percent that pertains to the 95 percentile for EME and LAC, corresponds roughly to only the 85 percentile for IND. *Thus, economies that are prone to default (EME and LAC) exhibit tighter debt ceilings than economies that do not default (in this dataset, represented by IND).*

Figure F.7 also shows that for the IND group, the default risk measure is low and roughly constant for different levels of debt-to-output ratios. On the other hand, the default risk measure for the EME group is not only higher, but increases substantially for high levels of indebtness. We consider this as evidence that for EME economies higher default risk is more prevalent for high levels of debt-to-output ratios.

Table F.5: (A) Measure of default risk (%) for EME and IND groups for different levels of debt-to-output ratio (%); (B) standard deviation of central government revenue over GDP (%) for EME and IND groups for different levels of default risk.

(A)			(B)		
Debt/GDP	EME	IND	Default Risk	EME	IND
25	5.4	2.0	25	0.9	1.4
75	10.7	2.9	75	2.5	1.7

Table F(A) compares the measure of default risk between IND and EME for low and high debt-to-output ratio levels. That is, for both groups (IND and EME) we select economies with debt-to-output ratio below the 25th percentile (low debt-to-output) for which we compute the average risk measure. We proceed analogously with those economies with debt-to-output ratio above the 75th percentile (high debt-to-output). For the case of low debt-to-ouput levels, the

[67]This type of graph is not the conventional QQplot as the axis have the value of the random variable which achieves a certain quantile and not the quantile itself. For our purposes, this representation is more convenient.

[68]The measure of default risk is constructed as the spread using the EMBI+ real index from J.P. Morgan for countries for which it is available and using the 3-7 year real government bond yield for the rest, minus the return of a US Treasury bond of similar maturity. Although bond returns are not entirely driven by default risk but also respond to other factors related to risk appetite, uncertainty and liquidity, for our purpose they constitute a valid conventional proxy of default risk. Furthermore, our spreads are an imperfect measure of default risk for domestic debt since EMBI+ considers mainly foreign debt. However, it is still informative since domestic default are positively correlated with defaults on sovereign debt, at least for the period from 1950's onwards. See figure 10 in Reinhart and Rogoff (2008).

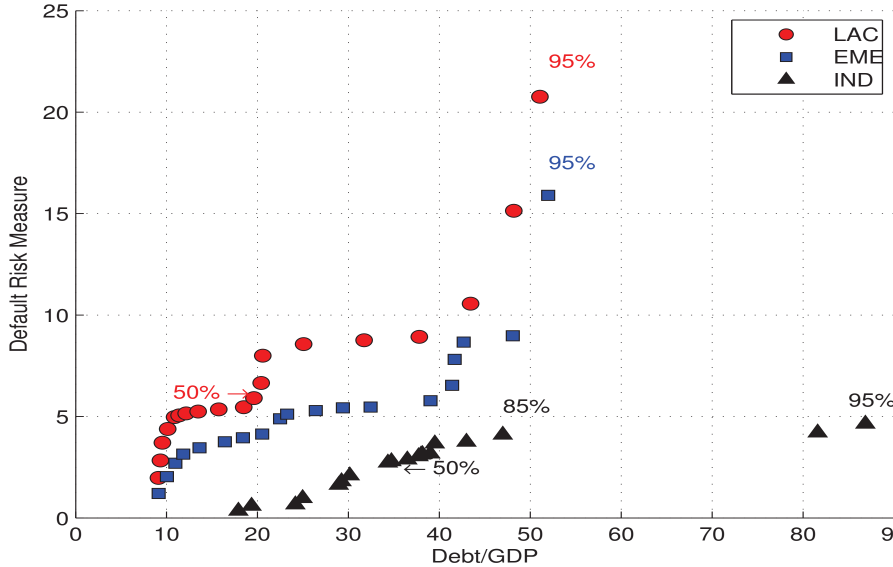

Figure F.7: The percentiles of the domestic government debt-to-output ratio and of a measure of default risk for three groups: IND (black triangle), EME (blue square) and LAC (red circle)

EME group presents higher (approximately twice as high) default risk than the IND group. For high debt-to-output ratio economies, however, this difference is quadrupled. *Thus, economies that are prone to default (EME and LAC) exhibit higher default risk than economies that do not default (in this dataset, represented by IND), and, moreover, the default risk is much higher for economies in the former group that have high levels of debt-to-output ratio.*

Table F(B) compares the standard deviation of the central government revenue-to-output ratio between IND and EME for low and high default risk levels. It indicates that for IND there is little variation of the volatility across low and high levels of default risk. For EME, however, the standard deviation of the central government revenue-to-output ratio is dramatically higher for economies with high default risk.[69]. It is worth pointing out that all the EME with high default risk levels defaulted at least once during our sample period. *Thus, economies with higher default risk exhibit more volatile tax revenues than economies with low default risk.* This is particularly notable for the group of EME/LAC economies.

These stylized facts establish a link between (a) default risk/default events, (b) debt ceilings and (c) volatility of tax revenues. In particular, the evidence suggests that economies that show higher default risk, also exhibit lower debt ceilings and more volatile tax revenues. The theory behind our model helps shed light upon the forces driving these facts.[70]

[69] We looked also at the inflation tax as a proxy for tax policy; results are qualitatively the same.

[70] It is important to note that we are not arguing any type of causality; we are just illustrating co-movements. In fact, in the model below all three features are endogenous outcomes of equilibrium.

G Description of the Data

In this section we describe how we constructed the figures presented in section F.

The industrialized economies group consists of AUSTRALIA (1990-1999), AUSTRIA (1990-1999), BELGIUM (1990-2001), CANADA (1990-2003), DENMARK (1990-2003), FINLAND (1994-1998), FRANCE (1990-2003), GERMANY (1990-1998), GREECE (1990-2001), IRELAND (1995-2003), ITALY (1990-2003), JAPAN (1990-1993), NETHERLANDS (1990-2001), NEW ZEALAND (1990-2003), NORWAY (1990-2003), PORTUGAL (1990-2001), SPAIN (1990-2003), SWEDEN (1990-2003), SWITZERLAND (1990-2003), UNITED KINGDOM (1990-2003) and UNITED STATES (1990-2003).

The emerging economies group consists of ARGENTINA[1] (1998-2003), BOLIVIA[1] (2001-2003), BRAZIL[1] (1997-2003), CHILE[1] (1993-2003), COLOMBIA[1] (1999-2003), ECUADOR[1] (1998-2003), EL SALVADOR[1] (2000-2003), HONDURAS[1] (1990-2003), JAMAICA[1] (1990-2003), MEXICO[1] (1990-2003), PANAMA[1] (1997-2003), PERU[1] (1998-2003), VENEZUELA[1] (1997-2003), ALBANIA (1995-2003), BULGARIA (1991-2003), CYPRUS (1990-2003), CZECH REPUBLIC (1993-2003), HUNGARY (1991-2003), LATVIA (1990-2003), POLAND (1990-2003), RUSSIA (1993-2003), TURKEY (1998-2003), ALGERIA (1990-2003), CHINA (1997-2003), EGYPT (1993-2003), JORDAN (1990-2003), KOREA (1990-2003), MALAYSIA (1990-2003), MAURITIUS (1990-2003), MOROCCO (1997-2003), PAKISTAN (1990-2003), PHILIPPINES (1997-2003), SOUTH AFRICA (1990-2003), THAILAND (1999-2003) and TUNISIA (1994-2003). The LAC group is conformed by the countries with "[1]".

For section F we constructed the data as follows. First, for each country, we computed time average, or time standard deviations or any quantity of interest (in parenthesis is the number of observations use to construct these). Second, once we computed these averages, we group the countries in IND, EME and LAC. We do this procedure for (a) central government domestic debt (as % of output) ; (b) central government expenditure (as % of output) ; (c) central government revenue (as % of output) , and (d) Real Risk Measure. The data for (a) is taken from Panizza (2008) ; the data for (b-c) is taken from Kaminsky et al. (2004) ; finally the data for (d) is taken from www.globalfinancialdata.com.

For Greece and Portugal we use central government public debt because central government domestic debt was not available. For Sweden, Ecuador and Thailand we use general government expenditure because central government expenditure was not available. For Albania, Bulgaria, Cyprus, Czech Rep., Hungary, Latvia, Poland and Russia no measure of government expenditure was available and thus were excluded from the sample for the calculations of this variable. The same caveats apply to the central government revenue sample. For Argentina, Brazil, Colombia, Ecuador, Egypt, Mexico, Morocco, Panama, Peru, Philippines, Poland, Russia, Turkey and Venezuela we used the real EMBI+ as a measure of real risk. For the rest of the countries we used government note yields of 1-5 years maturity, depending on availability.