OPTIMAL TAXATION WITH ENDOGENOUS DEFAULT UNDER INCOMPLETE MARKETS

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Abstract. In a dynamic economy, I characterize the fiscal policy of the government when it levies distortionary taxes and issues defaultable bonds to finance its stochastic expenditure. Households predict the possibility of default, generating endogenous debt limits that hinder the government's ability to smooth shocks using debt. Default is followed by temporary financial autarky. The government can only exit this state by paying a fraction of the defaulted debt. Since this payment may not occur immediately, in the meantime, households trade the defaulted debt in secondary markets; this device allows me to price the government debt before and during the default.

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1. Introduction

For many governments, debt and tax policies are conditioned by the possibility of default. For emerging economies, default is a recurrent event and is typically followed by a lengthy debt-restructuring process, in which the government and bond holders engage in renegotiations that conclude with the government paying a fraction of the defaulted debt.[1]

I find that emerging economies exhibit lower levels of indebtedness and higher volatility of government tax revenue than do industrialized economies—where, contrary to emerging economies, default is not observed in my dataset—.[2] Also, emerging economies, exhibit higher interest

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[1]See Pitchford and Wright (2008) and Benjamin and Wright (2009).

[2]To measure "indebtedness", I am using government domestic debt-to-output ratios, where domestic debt is the debt issued under domestic law (see Panizza (2008)). I am using domestic and not total government debt

rate spreads, especially for high levels of domestic debt-to-output ratios, than industrialized economies. In fact, industrialized economies exhibit interest rate spreads that are low and roughly constant for different levels of domestic debt-to-output ratios. Moreover, in emerging economies, the highest interest rate spreads are observed after default and during the debt-restructuring period.[3] Finally, I find that in my dataset, higher spreads are associated with more volatile tax revenues.

These empirical facts indicate that economies that are more prone to default display different government tax policy, as well as different prices of government debt, before default and during the debt-restructuring period. Therefore, the option to default, and the actual default event, will affect the utility of the economy's residents: Indirectly, by affecting the tax policy and debt prices, but also directly, by not servicing the debt in the hands of the economy's residents during the default event.

My main objective is to understand how the possibility of default and the actual default event affect tax policy, debt prices—before default and during financial autarky—, and welfare of the economy. [4] For this purpose, I analyze the dynamic taxation problem of a benevolent government in a closed economy under incomplete markets which has access to distortionary labor taxes and non-state-contingent debt. I assume, however, that the government cannot commit to pay the debt, and in case the government defaults, the economy enters temporary financial autarky and faces exogenous offers to pay a fraction of the defaulted debt that occur at an exogenous rate.[5] The government has the option to accept the offer—and, thus, exit financial autarky—or to stay in financial autarky until a new offer comes along. During temporary financial autarky, the defaulted debt still has positive value because it is going to be paid in the future with positive probability. Hence, households can trade the defaulted debt in a secondary market from which the government is excluded; the equilibrium price in this market is used to price the debt during a period of default. Finally, I assume that the government commits itself to a path of taxes when the economy is not in financial autarky.

In the model, the government has three policy instruments: (1) distortionary taxes, (2) government debt, and (3) default decisions that consist of: (a) whether to default on the outstanding debt and (b) whether to accept the offer to exit temporary financial autarky.

The government faces a trade-off between levying distortionary taxes to finance the stochastic process of expenditures and not defaulting, or issuing debt and thereby increasing the exposure

because my model is a closed economy. As a proxy of tax policy, I am using government revenue-to-output ratio or inflation tax.

[3]Examples of this are Argentina 2001, Ecuador 1997, and Russia 1998.

[4]Throughout this paper, I will also refer to the restructuring period as the financial autarky.

[5]In this model, financial autarky is understood as the period during which the government is precluded from issuing new debt/savings.

to default risk. The option to default introduces some degree of state contingency on the payoff of the debt since the financial instrument available to the government becomes an option, rather than a non-state-contingent bond. This option, however, does not come free of charge: Households accurately predict the possibility of default, and the equilibrium incorporates it into the pricing of the bond; this originates a "Laffer curve" type of behavior for the debt income, thereby implying endogenous debt limits. In this sense, my model generates "debt intolerance" endogenously.[6]

The main insight of the paper is that this borrowing limits hinder the government's ability to smooth shocks using debt, thus rendering tax policy more volatile, and implying higher interest rate spreads. The possibility of default introduces a trade-off between the cost of the lack of commitment to repay the debt, reflected in the price of the debt, and the flexibility that comes from the option to default and partial payments, reflected in the pay-off of the debt.

In a benchmark case, with quasi-linear utility, and a Markov process for the government expenditure but allowing for offers of partial payments to exit financial autarky, I characterize, analytically, the determinants of the optimal default decision and its effects on the optimal taxes, debt and allocations. In particular, I first show that default is more likely when the government's expenditure or debt is higher, and that the government is more likely to accept any given offer to pay a fraction of the defaulted debt when the level of defaulted debt is lower. Second, by imposing additional restrictions, I show that prices — both outside and during financial autarky — are non-increasing on the level of debt, thus implying that spreads are non-decreasing. Third, I show that the law of motion of the optimal government tax policy departs from the standard martingale-type behavior found in Aiyagari et al. (2002); in particular, I show that the law of motion of the optimal government tax policy is affected, on the one hand, by the benefit from having "more state-contingency" on the payoff of the bond, but, on the other hand, by the cost of having the option to default. [7]

Finally, I calibrate a more complete model; the model is qualitatively consistent with the differences observed in the data between emerging and industrialized economies. In terms of welfare policy, the numerical simulations suggest a nonlinear relationship between welfare and the probability of receiving an offer of partial payments. In particular, increasing the probability of receiving offers for exiting autarky decreases welfare when this probability is low/medium to begin with, but increases it when the probability is high.

The paper is organized as follows. I first present the related literature. Section 2 presents some stylized facts. Section 3 introduces the model. Section 4 presents the competitive equilibrium, and section 5 presents the government's problem. Section 6 derives analytical results that

[6]A term coined by Reinhart et al. (2003).

[7]See also Farhi (2010) for an extension of Aiyagari et al. (2002) results to an economy with capital.

characterize the optimal government policies for a simple example. Section 7 contains some numerical exercises. Section 8 briefly concludes. All proofs are gathered in the appendices.

1.1. Related Literature

The paper builds on and contributes to two main strands in the literature: endogenous default and optimal taxation.

Regarding the first strand, I model the strategic default decision of the government as in Arellano (2008) and Aguiar and Gopinath (2006), which, in turn, are based on the seminal paper by Eaton and Gersovitz (1981). My model, however, differs from theirs in several ways. First, I consider distortionary taxation; Arellano (2008) and references therein implicitly assume lump-sum taxes. Second, in my model, the government must pay at least a positive fraction of the defaulted debt to exit financial autarky through a "debt-restructuring process"; in Arellano (2008) and references therein, the government is exempt from paying the totality of the defaulted debt upon exit of autarky. I model this "debt-restructuring process" exogenously, indexing it by two parameters, because I am interested in studying only the consequences of this process on the optimal fiscal policy and welfare. [8] Third, my economy is closed—i.e., "creditors" are the representative household—; Arellano (2008) and references therein assume an open economy with foreign creditors. This allows me to capture the direct impact of the default event in the residents of the economy. Empirical evidence seems to suggest that government default has a non negligible direct impact on domestic residents; either because a considerable portion of the foreign debt is in the hands of domestic residents, or because the government also defaults on domestic debt.[9] Ideally, a model should consider both type of lenders; and although outside the scope of this paper, this could be an interesting avenue for future research. [10]

Regarding the second strand, I base my work on Aiyagari et al. (2002), where, in a closed economy, the benevolent infinitely-lived government chooses distortionary labor taxes and non-state-contingent *risk-free* debt, taking into account restrictions from the competitive equilibria, to maximize the households' lifetime expected utility. My work relaxes this last assumption and, as a consequence, the option to default creates endogenous debt limits, reflected in the equilibrium prices.

[8]See Benjamin and Wright (2009), Pitchford and Wright (2008) and Yue (2010) for ways of modeling the deb-restructuring process endogenously.

[9]For Argentina's default in 2001, almost 50 percent of the face value of debt to be restructured (about 53 percent of the total owed debt from 2001) is estimated to be in the hands of Argentinean residents; Local pension funds alone held almost 20 percent of the total defaulted debt (see Sturzenegger and Zettelmeyer (2006)). See Reinhart and Rogoff (2008) for a discussion and stylized facts on domestic debt defaults.

[10]See Broner et al. (2010) for a paper studying this issue in a more stylized setting.

In their work, by imposing non-state-contingent debt, AMSS reconcile the behavior of optimal taxes and debt observed in the data with the theory developed in the seminal paper of Lucas and Stokey (1983), in which the government has access to state-contingent debt. These papers assume full commitment on taxes and risk-free debt. My work relaxes this last assumption and, as a consequence, the option to default creates endogenous debt limits, reflected in the equilibrium prices. It is worth to note that all these papers (and mine) take market incompleteness as exogenous, since the goal is study the implications of this assumption. Albeit outside the scope of this paper, it would be interesting to explore ways of endogenizing market incompleteness; the paper by Hopeynhan and Werning (2009) seems a promising avenue for this.

Following the aforementioned literature, I assume that, although the government can commit itself to a tax policy outside temporary financial autarky, during this period, taxes are set mechanically so that tax revenues finance the government expenditure. This feature is related to Debortoli and Nunes (2010). Here the authors study the dynamics of debt in the Lucas and Stokey (1983) setting but with the caveat that at each time t, with some given probability, the government can lose its ability to commit to taxes; the authors refer to this as "loose commitment." Thus, my model provides a mechanism that "rationalizes" this probability of "loosing commitment" by assuming that the government is not committed to paying debt and can default at any time. It is worth noting that, in their model, the budget constraint during the no-commitment stage remains essentially the same, whereas mine does not.

Finally, in recent independent papers, Doda (2007), Cuadra et al. (2010), study the procyclicality of fiscal policy in developing countries by solving an optimal fiscal-policy problem. Their work differs from mine in two main aspects. They assume, first, an open small economy (i.e., foreign lenders) and, second, no secondary markets.[11]

2. Stylized Facts

In this section, I present stylized facts regarding the domestic government debt-to-output ratio and central government revenue-to-output ratio of several countries: Industrialized economies (IND, henceforth), emerging economies (EME, henceforth) and a subset of these: Latin American (LAC, henceforth).[12]

In the dataset set, IND do not exhibit default events, whereas EME/LAC (LAC in particular) do exhibit several defaults. [13] Thus, I take the former group as a proxy for economies with

[11]Aguiar et al. (2008) also allow for default in a small open economy with capital where households do not have access to neither financial markets nor capital and provide labor inelastically. The authors' main focus is on capital taxation and the debt "overhang" effect.

[12]For the latter ratios, I used the data in Kaminsky et al. (2004), and for the first ratio, I used the data in Panizza (2008). See Appendix J for a detailed description of the data.

[13]For LAC, in my sample, four countries defaulted, and most notable, Argentina defaulted repeatedly.

access to risk-free debt and the latter group as a proxy for economies without commitment. It is worth to point out that I am not implying that IND economies are a type of economy that will never default; I am just using the fact that in *my dataset* IND economies do not show default events, to use them as a proxy for the type of economy modeled in AMSS (i.e., one with risk-free debt). There is still the question of what type characteristics of an economy will prompt it to behave like IND or EME/LAC economy. A possible explanation is that for IND default is more costly, due to a higher degree of financial integration. That is, default — and the posterior period of financial autarky — could have a larger impact on the financing of the firms, thus lowering the productivity of the economy. I delve more into this question, in the context of the model in section 7.

The main stylized facts that I found are, first, that EME/LAC economies have higher default risk than IND economies and that within the former group,the default risk is much higher for economies with high levels of debt-to-output ratio. Second, EME and LAC economies exhibit tighter debt ceilings than economies that do not default (in this dataset, represented by IND). Third, economies with higher default risk exhibit more volatile tax revenues than economies with low default risk, and this fact is particularly notable for the group of EME/LAC economies (where defaults are more pervasive).

As shown below, my theory predicts that endogenous borrowing limits are more active for a high level of indebtedness. That is, when the government debt is high (relative to output), the probability of default is higher, thus implying tighter borrowing limits, higher spreads and higher volatility of taxes. But when this variable is low, default is an unlikely event, thereby implying slacker borrowing limits, lower spreads and lower volatility in the taxes. Hence, implications in the upper tail of the domestic debt-to-output ratio distribution can be different from those in the "central part" of it. Therefore, the mean and even the variance of the distribution are not too informative, as they are affected by the central part of the distribution; quantiles are better suited for recovering the information in the tails of the distribution.[14]

Figure D.1 plots the percentiles of the domestic government debt-to-output ratio and of a measure of default risk for three groups: IND (black triangle), EME (blue square) and LAC (red circle).[15] The X-axis plots the time series averages of domestic government debt-to-output ratio, and the Y-axis plots the values of the measure of default risk.[16] For each group, the last point on the right correspond to the 95 percentile, the second to last to the 90 percentile

[14]I refer the reader to Koenker (2005) for a thorough treatment of quantiles and quantile-based econometric models.

[15]This type of graph is not the conventional QQplot as the axis have the value of the random variable which achieves a certain quantile and not the quantile itself. For my purposes, this representation is more convenient.

[16]I constructed the measure of default risk as the spread using the EMBI+ real index for countries for which it is available and using the 3-7 year real government bond yield for the rest, minus U.S. bond return. This is an imperfect measure of default risk for domestic debt since EMBI+ considers mainly foreign debt. However, it

and so on; these are comparable between groups as all of them represent a percentile of the corresponding distribution. EME and LAC have lower domestic debt-to-output ratio levels than IND, in fact the domestic debt-to-output ratio value that amounts for the 95 percentile for EME and LAC, only amounts for (approx.) 85 percentile for IND (which in both cases is only about 50 percent of debt-to-output ratio).[17] *Thus, economies that are prone to default (EME and LAC) exhibit tighter debt ceilings than economies that do not default (in this dataset, represented by IND).*

TABLE 2.1. (A) Measure of default risk for EME and IND groups for different levels of debt-to-output ratio; (B) Std. Dev. of ctral. government revenue over GDP (%) for EME and IND groups for different levels of default risk.

(A)			(B)		
Debt/GDP	EME	IND	DEF. RISK	EME	IND
$< 0.25\%$	5.4	2.0	$< 0.25\%$	0.9	1.4
$> 0.75\%$	10.7	2.9	$> 0.75\%$	2.5	1.7

Table 2(A) compares the measure of default risk between IND and EME matching them across low and high debt-to-output ratio levels. That is, for both groups (IND and EME) I select economies with debt-to-output ratio below the 25th percentile (these are economies with low debt-to-output) and for these economies I compute the average risk measure; I do the same for those economies with debt-to-output ratio above the 75th percentile (these are economies with high debt-to-output). For the case of low debt-to-output ratio, the EME group presents higher (approx. twice as high) default risk than the IND group; however, for high debt-to-output ratio economies, this difference is multiplied by a factor four. *Thus, economies that are prone to default (EME and LAC) exhibit higher default risk than economies that do not default (in this dataset, represented by IND), and, moreover, the default risk is much higher for economies in the former group that have high levels of debt-to-output ratio.*

Table 2(B) compares the standard deviation of the central government revenue-to-output ratio between IND and EME matching them across low and high default risk levels. It shows that for IND there is little variation of the volatility across low and high levels of default risk. For EME, however, there standard deviation of the central government revenue-to-output ratio is higher for economies with high default risk.[18] It is worth noting, that all the EME with high default risk levels defaulted at least once during our sample. *Thus, economies with higher default risk*

is still informative since domestic default are positively correlated with defaults on sovereign debt, at least for the period of 1950's onwards, see Fig. 10 in Reinhart and Rogoff (2008).

[17] I obtain this by projecting the 95 percentile point of the EME and LAC onto the X-axis and comparing with the 85 percentile point of IND.

[18] I looked also at the inflation tax as a proxy for tax policy; results are qualitatively the same.

exhibit more volatile tax revenues than economies with low default risk. This is particularly notable for the group of EME/LAC economies.

These stylized facts establish a link between (a) default risk/default events, (b) debt ceilings and (c) volatility of tax revenues. In particular, the evidence suggests that economies that show higher default risk, also exhibit lower debt ceilings and more volatile tax revenues. The theory below sheds a light upon the forces driving these facts. [19]

3. The Model

In this section I describe the stochastic structure of the model, the timing and policies of the government and present the households problem.

3.1. The setting

Let time be indexed as $t = 0, 1, ...$ Let (g_t, δ_t) be the exogenous government expenditure at time t and the fraction of the defaulted debt which is re-payed when exiting autarky, resp. These are the exogenous driving random variables of this economy. Let $\omega_t \equiv (g_t, \delta_t) \in \mathbb{G} \times \bar{\Delta}$, where $\mathbb{G} \subset \mathbb{R}$, $\bar{\Delta} \equiv \Delta \cup \{1\} \cup \{\bar{\delta}\}$ and $\Delta \subset [0, 1)$ are compact, and in order to avoid technical difficulties, I assume $|\mathbb{G}|$ and $|\Delta|$ are finite.[20] The set Δ models the offers — as fractions of outstanding debt — to repay the defaulted debt; $\{1\}$ represents the case where the government services the totality of its debt, and $\bar{\delta}$ is such that that the government rejects it in every possible state of the world, is designed to capture situations where the government does not receive an offer to repay.[21]

Finally, I denote histories as $\omega^t \equiv (\omega_0, \omega_1, ..., \omega_t) \in \Omega^t \equiv (\mathbb{G} \times \bar{\Delta})^t$ but I use $\omega \in \Omega$ to denote ω^∞.

3.2. The government policies and timing

Let $\mathbb{B} \subseteq \mathbb{R}$ be compact. Let B_{t+1} be the choice of debt at time t to be paid at time $t + 1$; τ_t is the labor tax; d_t is the default decision, it takes value 1 if the government decides to default and 0 otherwise; finally, let a_t is the decision of accepting an offer to repay the default debt, it takes value 1 if the offer is accepted and 0 otherwise.

[19] It is important to note that I am not arguing any type of causality; I am just illustrating co-movements. In fact, in the model below, all three features are endogenous outcomes of equilibrium.

[20] For a given set, $|S|$ is the cardinal of the set.

[21] An alternative way of modeling this situation is to work with $\bar{\Delta} \equiv \Delta \cup \{1\} \cup \{\emptyset\}$ where $\emptyset$ indicates no offer. Another alternative way is to add an additional random variable, $\iota \in \{0, 1\}$ that explicitly indicates if the government received an offer ($\iota = 1$) or not ($\iota = 0$) and let $\bar{\Delta} \equiv \Delta \cup \{1\}$.

The timing for the goverment is as follows. At each time t, the government can levy distortionary linear labor taxes, and allocate one-period, non-state-contingent bonds to the households to cover the expenses g_t. The government, after observing the present government expenditure and the outstanding debt to be paid this period, has the option to default on 100 percent of this debt—i.e., the government has the option to refuse to pay the totality of the maturing debt.

As shown in figure D.2, if the government opts to exercise the option to default (node (B) in figure D.2), nature plays immediately, and with some probability, sends the government to temporary financial autarky, where the government is precluded from issuing bonds in that period. If this does not occur, the government enters a stage in which nature draws a fraction δ of debt to be repaid, and the government has the option to accept or reject this offer. If the government accepts, it pays the new amount (the outstanding debt times the fraction that nature chose), and it is able to issue new bonds for the following period. If the government rejects, it goes to temporary financial outage (bottom branch in figure D.2).

Finally, if the government is not in financial autarky—because it either chooses not to default, or it accepts the partial payment offer—then in the next period, it has the option to default, with new values of outstanding debt and government expenditure. If the government is in temporary financial autarky, then in the next period, it will face a new offer for partial payments with probability λ.

The next assumption formalizes the probability model mentioned above.

ASSUMPTION 3.1. $\Pr(g_t \in G|\delta_t, \omega^{t-1}) \equiv \pi_{\mathbb{G}}(G|g_{t-1})$ *for any* $G \in \mathbb{G}$, $\Pr(\delta_t \in D|g_t, \omega^{t-1}) \equiv \Pr\{\delta_t \in D|d_t\}$ *for any* $D \in \bar{\Delta}$, *where:* [22]

$$\Pr\{\delta_t \in D|d_t\} = \begin{cases} 1\{1 \in D\} & if \ d_t = 0 \\ (1-\lambda)1\{\{\bar{\delta}\} \in D\} + \lambda\pi_\Delta(D) & if \ d_t = 1 \end{cases}$$

Essentially, this assumption imposes a Markov restriction on the probability and also additional restrictions across the variables. In particular, given g, $1-\lambda$ is the probability of $\delta = \bar{\delta}$ (i.e., not receiving an offer) and $\pi_\Delta(\cdot)$ is a probability over Δ. Finally, I use Π to denote the probability distribution over Ω generated by assumption 3.1, and $\Pi(\cdot|\omega^t)$ to denote the conditional probability over Ω, given ω^t.

The next definition formalizes the concept of government policy and the government budget constraint. In particular, it formally introduces the fact that debt is non-state contingent (i.e., B_{t+1} only depends on the history up to time t, ω^t).

[22]It is east to generalize this to a more general formulation such as λ and π_Δ depending on g.

Definition 3.1. *A government policy is a sequence* $(\sigma_t)_t$ *where* $\sigma_t \equiv (B_{t+1}, \tau_t, d_t, a_t) : \Omega \to \mathbb{B} \times [0,1] \times \{0,1\}^2$ *only depends on the history up to time* t, ω^t. [23]

Let $(p_t)_t$ be a stochastic process (p_t depends on ω^t) that denotes the price of one unit of government debt, at time t. I refer to this process as a price schedule.

Definition 3.2. *A government plan or policy* σ^∞ *is attainable (given a price schedule and initial debt* B_0*) iff for all* t

$$g_t + \phi_t \delta_t B_t = \kappa_t \tau_t n_t + \phi_t p_t B_{t+1}.$$

with $\phi_t \equiv (1-d_t) + d_t a_t$. [24]

I define $\kappa_t \equiv \kappa_t(\sigma^\infty)$ as the productivity process. For simplicity I restrict it to be non-random, and following the sovereign default literature I set it to $\kappa_t = 1$ if $(1-d_t) + d_t a_t = 1$ (i.e., either no default, or the country defaulted but accepted repayment offer) and $\kappa_t \equiv \kappa < 1$ otherwise, representing direct output costs of being in financial autarky. Also, observe that if the government defaults ($d_t = 1$) and rejects the offer of repayment ($a_t = 0$), its budget constraint boils down to $g_t = \tau_t n_t$, and if the government does default, $d_t = 1$ but accepts the offer to pay the defaulted debt, $a_t = 1$, then it has liabilities to be repaid for $\delta_t B_t$ and can issue new debt.

A few final remarks about the "debt-restructuring process" are in order. This process is defined by (λ, π_Δ). These parameters capture the fact that debt-restructuring is time-consuming but, generally, at the end, a positive fraction of the defaulted debt is honored. [25] [26] This debt-restructuring process intends to capture the fact that, after defaults (over domestic or international debt, or both), economies see their access to credit severely hindered. For, instance, this fact is well-documented for *sovereign* defaults; also, the data suggests that in many instances default on domestic debt and sovereign debt happen simultaneously. [27] Hence, the debt restructuring process intends to capture, up to some extend, this observed feature of the data.

[23]See appendix A for technical details.

[24]As defined, the government policy and prices depend on the particular history ω, so the equalities are understood to hold for all $\omega \in \Omega$.

[25]See Yue (2010), and Pitchford and Wright (2008) and Benjamin and Wright (2009) for two different ways of modeling this process as renegotiation between the government and the debt holders).

[26]I could also allow for, say, $\pi_\Delta(\cdot|g_t, B_t, d_t, d_{t-1}, ..., d_{t-K})$ some $K > 0$, denoting that possible partial payments depend on the credit history and level of debt. See Reinhart et al. (2003), Reinhart and Rogoff (2008) and Yue (2010) for an intuition behind this structure.

[27]For instance Argentina defaulted three times on its domestic debt between 1980 and 2001. Two of these defaults coincided with external defaults (1982 and 2001). Also, in Reinhart and Rogoff (2008) figure 10 shows the probability of external default versus the comparable statistic for domestic default either through inflation or explicit default, one can see that after 1950's there is a close co-movement.

3.3. The Household Problem

Households are price takers and homogeneous; they have time-separable preferences for consumption and labor processes. They also make debt/savings decisions by trading government bonds.

Given a government plan σ^{∞}, let $\varrho_t(\cdot;\sigma^{\infty}) : \Omega \to \mathbb{R}$ be the time t payoff of one unit of government debt, given that the government acts according to σ^{∞}; i.e.,

$$\varrho_t(\omega;\sigma^{\infty}) = (1 - d_t(\omega)) + d_t(\omega)\{a_t(\omega)\delta_t(\omega) + (1 - a_t(\omega))q_t(\omega)\}$$

where $q_t(\omega)$ is just notation for the price of selling one unit of government debt in the secondary market at time t; given that the government acts according to σ^{∞} and history ω.

A few remarks about ϱ_t are in order. Since the household takes government actions as given, from the point of view of the households the government debt is an asset with payoff that depends on the state of the economy. That is, if the government decides not to default ($d_t = 1$) then $\varrho_t = 1$; if the government decides to default ($d_t = 0$) but then accepts to repay a fraction δ_t, the household receives $\varrho_t = \delta_t$; finally, if the governments default and rejects the repayment option, the household can sell the unit of government debt in the secondary market and obtain $\varrho_t = q_t$. Observe that in cases where the government never repays a positive fraction (e.g., the model by Arellano (2008)), $\varrho_t = q_t = 0$. Finally, the dependence of ϱ on the state clearly illustrates, that default decisions add certain degree of state contingency to the government debt.

Definition 3.3. *A household allocation is a (c^{∞}, n^{∞}) such that $c_t : \Omega \to \mathbb{R}_+$ and $n_t : \Omega \to [0,1]$ depend only on the partial history up to time t, ω^t. A household debt plan is a b^{∞} such that for all t, $b_{t+1} : \Omega \to [\underline{b}, \overline{b}]$ depends only on the partial history up to time t, ω^t.* [28]

The household problem is given by: Given a (ω_0, b_0),

$$\sup_{(c^{\infty},n^{\infty},b^{\infty})\in\mathbb{B}(g_0,b_0;\sigma^{\infty})} E_{\Pi(\cdot|g_0)}\left[\sum_{t=0}^{\infty}\beta^t u(c_t(\omega), 1 - n_t(\omega))\right]$$

where $\mathbb{B}(g_0, b_0; \sigma^{\infty})$ is the set of household allocations and debt plans, such that for all t

$$c_t - (1 - \tau_t)\kappa_t n_t + p_t b_{t+1} = \varrho_t b_t,$$

and $b_{t+1} \leq b_t,\ if\ \phi_t = 0$. This restriction implies that, during financial autarky, when only secondary markets are open, the household cannot "print" debt. [29]

[28] I assume $b_{t+1} \in [\underline{b}, \overline{b}]$ and $[\underline{b}, \overline{b}] \supseteq \mathbb{B}$ so in equilibrium these restrictions will not be binding. See appendix A for more technical details regarding the debt and allocations.

[29] Observe that, by definition, allocations, prices and debt plans depend on ω, so all equalities and inequalities are understood to hold for all $\omega \in \Omega$; for instance $b_{t+1} \leq b_t$ should be understood as $b_{t+1}(\omega) \leq b_t(\omega)$ for all $\omega \in \Omega$ and so on.

4. Competitive Equilibrium with Government

I now define a competitive equilibrium, for a given government policy and derive the equilibrium taxes and prices.

Definition 4.1. *Given a $s_0 \equiv (g_0, B_0 = b_0, \phi_{-1})$, a competitive equilibrium with government is a government policy, σ^∞, a household allocation, (c^∞, n^∞), a household debt plan, b^∞, and a price schedule p^∞ such that:*

(1) *Given (g_0, b_0), the government policy and the price schedule, the household allocation and debt plan solve the household problem.*

(2) *Given B_0 and the price schedule, σ^∞ is attainable.*

(3) *For all t, $c_t + g_t = \kappa_t n_t$.*

(4) *For all t, $B_{t+1} = b_{t+1}$, and $B_{t+1} = B_t$ if $1 - d_t + d_t a_t = 0$.*

I use $CEG(s_0)$ to denote the set of all competitive equilibrium with government. Observe that, the market clearing for debt indicates that $B_{t+1} = b_{t+1}$. However, if the economy is in financial autarky — where the government cannot issue debt, and thus agents must only trade among themselves —, I impose $B_{t+1} = B_t$ which implies $b_t = b_{t+1}$, i.e., agents do not change their debt positions.

In appendix G, I characterize the set $CEG(s_0)$ exploiting the sufficiency of the first order conditions of the households and budget constraints.

4.1. Equilibrium Prices and Taxes

In this section I present the expressions for equilibrium taxes and prices of debt. The former quantity is standard (e.g. Aiyagari et al. (2002) and Lucas and Stokey (1983)); the latter quantity, however, incorporates the possibility of default of the government. The following assumption is standard and ensures that u is smooth enough to compute first order conditions.

Assumption 4.1. *(i) $u \in \mathbb{C}^2(\mathbb{R}_+ \times [0,1], \mathbb{R})$ with $u_c > 0$, $u_{cc} < 0$, $u_l > 0$ and $u_{ll} > 0$, and $\lim_{l \to 0} u_l(l) = \infty$.* [30]

[30] $\mathbb{C}^2(X, Y)$ is the space of twice continuously differentiable functions from X to Y. The assumption $u_{cc} < 0$ could be relaxed to include $u_{cc} = 0$ (see the section 6 below).

From the first order conditions of the optimization problem of the households, the following equations follow [31]

$$\frac{u_l(c_t, 1-n_t)}{u_c(c_t, 1-n_t)} = (1-\tau_t)\kappa_t(\sigma^\infty), \tag{4.1}$$

and

$$p_t = E_{\Pi(\cdot|\omega^t)}\left[\beta \frac{u_c(c_{t+1}, 1-n_{t+1})}{u_c(c_t, 1-n_t)} \varrho_{t+1}\right]. \tag{4.2}$$

From the definition of ϱ, and the restrictions on Π, equation 4.2 implies, for $d_t = 0$ or $a_t = 1$,

$$\begin{aligned} p_t =& \beta \int_{\mathbb{G}} \left(\frac{u_c(c_{t+1}, 1-n_{t+1})}{u_c(c_t, 1-n_t)} (1-d_{t+1}) \right) \pi_{\mathbb{G}}(dg_{t+1}|g_t) \\ &+ \beta \int_{\mathbb{G}} \lambda d_{t+1} \int_{\Delta} \frac{u_c(c_{t+1}, 1-n_{t+1})}{u_c(c_t, 1-n_t)} a_{t+1}\delta_{t+1}\pi_\Delta(d\delta_{t+1})\pi_{\mathbb{G}}(dg_{t+1}|g_t) \\ &+ \beta \int_{\mathbb{G}} \frac{u_c(c_{t+1}, 1-n_{t+1})}{u_c(c_t, 1-n_t)} d_{t+1} \left\{ \lambda \int_{\Delta} (1-a_{t+1})\pi_\Delta(d\delta_{t+1}) + (1-\lambda) \right\} q_{t+1}\pi_{\mathbb{G}}(dg_{t+1}|g_t), \end{aligned} \tag{4.3}$$

where q_t denotes the price p_t for $d_t = 1$ and $a_t = 0$, and is given by

$$\begin{aligned} q_t =& \beta \int_{\mathbb{G}} \lambda \int_{\Delta} \frac{u_c(c_{t+1}, 1-n_{t+1})}{u_c(c_t, 1-n_t)} a_{t+1}\delta_{t+1}\pi_\Delta(d\delta_{t+1})\pi_{\mathbb{G}}(dg_{t+1}|g_t) \\ &+ \beta \int_{\mathbb{G}} \frac{u_c(c_{t+1}, 1-n_{t+1})}{u_c(c_t, 1-n_t)} \left\{ \lambda \int_{\Delta} (1-a_{t+1})\pi_\Delta(d\delta_{t+1}) + (1-\lambda) \right\} q_{t+1}\pi_{\mathbb{G}}(dg_{t+1}|g_t). \end{aligned} \tag{4.4}$$

Each term in the equation 4.3 corresponds to a "branch" of the tree depicted in figure D.2. The first line represents the value of one unit of debt when the government chooses to honor the entire debt. The second line represents the value of the debt if the government decides not to pay the debt, but ends up in partial default. The third line captures the value of the debt when the government defaults on 100 percent of the debt, but the households can sell it in the secondary markets.

If $\lambda = 0$ and $u_c = 1$, then the last two terms vanish and the price is analogous to the one obtained in Arellano (2008). Also observe that, if $\lambda = 0$, it follows that $q_t = \int_{\mathbb{G}} \left(\frac{u_c(c_{t+1}, 1-n_{t+1})}{u_c(c_t, 1-n_t)} \right) q_{t+1}\pi_{\mathbb{G}}(dg_{t+1}|g_t)$, which by substituting forward and standard transversality conditions, yields $q_t = 0$.

The novelty of these pricing equations with respect to the standard sovereign default model, e.g., Arellano (2008) and Aguiar and Gopinath (2006) is the presence of secondary market prices, q_t. [32] By imposing positive repayment (with some probability), the model is able to deliver a price of defaulted debt during the financial autarky period. In sections 6 and 7, I shed some light on the pricing implications of this model and how its relates with the data. Delving

[31] As before, I omit dependence on ω to ease the notational burden. For the more detailed expression and the complete derivations, please see appendix B.

[32] See also Chatterjee and Eyingungor (2012) for the equilibrium prices in the presence of long term debt.

more on the pricing implications of equations 4.3 - 4.4, albeit outside the scope of this paper, seems like a promising avenue for future research.

5. The Government Problem

The government maximizes the welfare of the representative household by choosing the policies. The government, however, cannot commit to repaying the debt, but while having access to financial markets, commits to future tax promises. That is, as long as the government has access to financial markets, it honors past promises of taxes; when the government defaults and enters financial autarky ought to choose taxes to balance the budget by assumptions. Once the government exits financial autarky, it starts anew, without any outstanding tax promises.

It is worth to point out that, in the case where there is repayment for any state of the economy, the fiscal authority is the Ramsey problem studied by Aiyagari et al. (2002). [33] Due to space constraints I gathered the details and further discussions in the Appendix H. There, I also provide a succinct expression for the Bellman equation that defines the value function described in section 5.2 and the associated optimal policy functions, I also discuss and characterize the relevant state space which is an endogenous object; see Kydland and Prescott (1980) and Chang (1998). [34] Finally, in this appendix H I also establish the link between this solution and the "sequential solution" at $t = 0$.

Below, I first described the so-called implementability conditions for the government and then present the recursive formulation of the government's problem.

5.1. The Implementability Constraints

Recall that $\phi_t \equiv (1 - d_t) + d_t a_t$. By using the first order conditions 4.1 and 4.2, to replace taxes and prices in the government budget constraint, $\kappa_t \tau_t n_t - g_t + \phi_t\{p_t B_{t+1} - \delta_t B_t\} \geq 0$ (and $B_{t+1} = B_t$ if $\phi_t = 0$), I obtain

$$\left(\kappa_t - \frac{u_l(\kappa_t n_t - g_t, 1 - n_t)}{u_c(\kappa_t n_t - g_t, 1 - n_t)}\right) n_t - g_t + \phi_t\{p_t B_{t+1} - \delta_t B_t\} \geq 0.$$

Letting, $\mu_t \equiv u_c(\kappa_t n_t - g_t, 1 - n_t)$, then the display above can be cast as

$$Z_{\phi_t}(\mu_t, n_t, g_t) + \phi_t\{\mathcal{P}_t B_{t+1} - \delta_t \mu_t B_t\} \geq 0 \quad (5.5)$$

where $p_t = \mathcal{P}_t/\mu_t$ is given by the expression in equation 4.2 and $Z_{\phi_t}(\mu_t, n_t, g_t) \equiv (\kappa_t \mu_t - u_l(\kappa_t n_t - g_t, 1 - n_t))\, n$ $\mu_t g_t$.

[33] Also, in the case the government had access to lump-sum taxes, it will set distortionary taxes to zero and thus this model would be akin to that of Arellano (2008).

[34] I also relegate to this appendix an alternative timing which simplifies the state space; please see section H.5.

The variable μ_t should be viewed as the marginal utility of consumption at time t, that was promised at time $t-1$. The intuition behind this variable is that that CEG can be characterized by a sequence of equations (given by first order conditions and budget constraints; see appendix G for details) and each one connects only periods of "today" and "tomorrow". Moreover, from the perspective of any period, a CEG can be seen as the current policies and allocations, together with a "promise" of policies for next period; this is being captured by μ. See Kydland and Prescott (1980) and Chang (1998) for a more thorough discussion in similar settings.

In principle, μ_t should be specified for each $\omega_t \equiv (g_t, \delta_t)$, I thus use $\vec{\mu}_t$ to denote a function from $\mathbb{G} \times \bar{\Delta}$ to $\mathbb{R}_+$. Thus, $\vec{\mu}_t(g_t, \delta_t)$ denotes the function $\vec{\mu}_t$ evaluated at (g_t, δ_t). Also, observe that, from equation 4.2, equilibrium prices at time t are only a function of $(g_t, \vec{\mu}_{t+1})$ and of the default and debt-repayment strategies, i.e., $\mathcal{P}_t = \mathcal{P}(g_t, \vec{\mu}_{t+1}, d_{t+1}, a_{t+1})$ (henceforth, unless needed I leave the dependence on the default and debt-repayment strategies implicit, to ease the notational burden).

Thus, equation 5.5 imposes the following set of constraints: Given any (g, B, μ, ϕ),

$$\bar{\Gamma}(g, B, \mu, \phi) \equiv \{(n, B', \vec{\mu}') \in [0,1] \times \mathbb{B} \times \mathbb{M} \ : \mu = u_c(\kappa_\phi n - g, 1-n),$$

$$Z_\phi(\mu, n, g) + \phi \{\mathcal{P}(g, \vec{\mu}')B' - B\mu\} \geq 0, \ \textit{and if } \phi = 0, \ B' = B\}, \tag{5.6}$$

where $\kappa_\phi \equiv \kappa(1-\phi) + \phi$. It is clear by the last equality, that μ imposes restrictions on the choice of n. In fact, if $u_c(\kappa n - g, 1-n)$ is monotonic as a function of n (e.g., if u is separable in leisure and consumption and increasing in the latter) then there exists only one possible n given (μ, g). The set $\mathbb{M}$ summarizes the a-priori restrictions on $\vec{\mu}$ and it is given by $\mathbb{M} \equiv \cup_{g \in \mathbb{G}} \mathbb{M}(g)$ where $\mathbb{M}(g) \equiv \{m : \exists n \in [0,1], \ s.t. \ m(g) = u_c(n - g, 1-n)\}$.

5.2. The Government Optimization Problem

The government problem is divided in two parts: an "initial problem" (i.e., the problem at time 0) and the "continuation problem" at (i.e., at time $t \geq 1$). I start with the latter which can be cast recursively. For any $(g, B, \vec{\mu}, \phi)$, let $\mathbf{V}^*(g, B, \vec{\mu}, \phi)$ be the value of having the option to default (if $\phi = 1$) or having the option to repay the default debt (if $\phi = 0$), given g, an outstanding level of debt B and a profile $\vec{\mu}$. Also, $\mathbf{d}^*$ and $\mathbf{a}^*$ denote the optimal policy functions for default and for repayment of a fraction of the defaulted debt, resp. For expositional purposes, I separate the study of this problem into two cases: the case where the economy is in financial access ($\phi = 1$) and the one where the economy is in financial autarky ($\phi = 0$).

<u>Financial Access Case:</u> In this case (where $\phi = 1$), the government has the option to default on the debt. Thus, for any $(g, B, \vec{\mu}, 1) \in \mathbf{R}^*$,

$$\mathbf{V}^*(g, B, \vec{\mu}, 1) = \max\{\mathcal{V}_1^*(g, B, \vec{\mu}(g,1)), \mathbb{V}^*(g, B, \vec{\mu})\} \tag{5.7}$$

where

$$\mathbb{V}^*(g,B,\vec{\mu}) \equiv \lambda \int_{\Delta} \max\left\{\mathcal{V}_1^*(g,\delta B,\vec{\mu}(g,\delta)),\mathcal{V}_0^*(g,B)\right\}\pi_{\Delta}(d\delta) + (1-\lambda)\mathcal{V}_0^*(g,B). \tag{5.8}$$

The set $\mathbf{R}^*$ is the set of states, $(g,B,\vec{\mu},\phi)$ for which there exists a CEG that takes these as initial states; see the Appendix H for a detailed definition and characterization. The value $\mathcal{V}_1^*(g,\delta B,\mu)$ is the value of repaying a fraction δ of the outstanding debt, B, given (g,μ), and the value $\mathcal{V}_0^*(g,B)$ is the value of defaulting on the outstanding debt. The function $\mathbb{V}^*$ is the value function of the government that defaulted ($d=1$) and is awaiting the lottery to receive an offer of repayment (and has the option to reject it).

The "max" in equation 5.7 stems from the fact that the default authority optimally compares the value of not defaulting and paying the totality of the outstanding debt (i.e., $\mathcal{V}_1^*(g,B,\vec{\mu}(g,1))$) or defaulting, and waiting for an offer of repayment (i.e., $\mathbb{V}^*(g,B,\vec{\mu})$). The max in equation 5.8 arises from the fact that the default authority optimally compares the value of accepting the offer of repayment (given a fraction δ at hand) and the value of rejecting.

Therefore, the optimal policy functions are: [35] For any $(g,B,\vec{\mu},\phi)$,

$$\mathbf{d}^*(g,B,\vec{\mu},\phi) = 1\left\{\mathcal{V}_1^*(g,B,\vec{\mu}(g,1)) < \mathbb{V}^*(g,B,\vec{\mu})\right\}\ if\ \phi=1$$

(if $\phi=0$, the government is "forced" to choose $d=1$), and, for any (g,δ,B,μ)

$$\mathbf{a}^*(g,\delta,B,\mu) = 1\left\{\mathcal{V}_1^*(g,\delta B,\mu) \geq \mathcal{V}_0(g,B)\right\}\ if\ \delta\neq\bar{\delta}$$

(if $\delta=\bar{\delta}$ —which occurs with probability $1-\lambda$— the government is "forced" to choose $a=0$).

Finally, the function $\mathcal{V}^*$ is given by:

$$\mathcal{V}_\phi^*(g,B,\mu) = \max_{(n,B',\vec{\mu}')\in\bar{\Gamma}(g,B,\mu,\phi)}\left\{u(\kappa_\phi n - g, 1-n) + \beta\int_{\mathbb{G}}\mathbf{V}^*(g',B',\vec{\mu}',\phi)\pi_{\mathbb{G}}(dg'|g)\right\} \tag{5.9}$$

for any (g,B,μ,ϕ).

Observe that, when optimally choosing $(n,B',\vec{\mu}')$, the government takes as given that the (future) default authority acts according to $(\mathbf{d}^*,\mathbf{a}^*)$. This implies that equilibrium prices depend on B'; this is a feature of sovereign default models, see Arellano (2008).

Financial Autarky Case: In this case ($\phi=0$), the government is in financial autarky. It cannot issue new debt (or equivalently, the new debt ought to coincide with the outstanding defaulted debt). Additionally, since the economy is already in default, the government does not have to decide whether to default or not (or, equivalently, it is forced to choose $d=1$).

[35] As defined, I am imposing that if indifferent the government chooses not to default. This is just a normalization that is standard in the literature; it is easy to see that $\mathbf{d}^*$ could be defined as a correspondence, taking any value in $[0,1]$ if $\mathcal{V}_1^*(g,B,\vec{\mu}(g,1)) = \mathbb{V}^*(g,B,\vec{\mu})$.

In this case, $\mathbf{V}^*$, for any $(g, B, \vec{\mu}, 0) \in \mathbf{R}^*$, is given by,

$$\begin{aligned}\mathbf{V}^*(g, B, \vec{\mu}, 0) =& \mathbb{V}^*(g, B, \vec{\mu}) \\ =& \lambda \int_\Delta \max\left\{\mathcal{V}_1^*(g, \delta B, \vec{\mu}(g, \delta)), \mathcal{V}_0^*(g, B)\right\} \pi_\Delta(d\delta) + (1-\lambda)\mathcal{V}_0^*(g, B).\end{aligned} \tag{5.10}$$

Observe that in this case (of $\phi = 0$), $\bar{\Gamma}(g, B, \mu, \phi) = \{(n, B', \vec{\mu}) : B' = B,\ Z_\phi(\mu, n, g) \leq 0 \text{ and } \mu = u_c(\kappa n - g, 1-n)\}$. Therefore, n ought to be such that the budget is balanced (i.e., $Z(u_c(\kappa n - g, 1-n), n, g) = 0$); denote such n as $n_A(g)$ for any g. Moreover, μ does not impose any restrictions on $(B', \vec{\mu}')$ because, first, the government cannot issue new debt while in financial autarky and ought to keep track of the defaulted debt (i.e., $B' = B$); second, since in financial autarky there is no issuance of new debt, the government does not have any "outstanding past promising" of consumption. This implies that

$$\mathcal{V}_0^*(g, B) = u(\kappa n_A(g) - g, 1 - n_A(g)) + \beta \max_{\vec{\mu}' \in \mathbb{M}} \int_{\mathbb{G}} \mathbf{V}^*(g', B, \vec{\mu}', 0)\pi_{\mathbb{G}}(dg'|g).$$

That is, the government receives the payoff of running a balance budget, $u(\kappa n_A(g) - g, 1 - n_A(g))$, and next period will have the option to repay the outstanding debt, without any past tax promises, hence $\mathbf{V}^*(g, B, \vec{\mu}', 0)$ is being maximize over all (feasible) $\vec{\mu}'$ (the expression for $\mathbf{V}^*$ is below, in equation 5.10). Observe that the RHS of the equation does not depend on $\vec{\mu}$ or δ, thus, for case $\phi = 0$, $\mathcal{V}^*$ does not depend on $\vec{\mu}$.

To conclude, I present the problem of the government at time $t = 0$. The crucial difference is that the government starts the period without any outstanding past consumption promises and both (g_0, B_0, ϕ_{-1}) are given as parameters. [36] That is, $\mathbf{V}_o^*(g_0, B_0, \phi_{-1}) \equiv \max_{\vec{\mu} \in \mathbb{M}} \mathbf{V}^*(g_0, B_0, \vec{\mu}, \phi_{-1})$.

Below I present some particular cases where the value function gets simplified.

5.2.1. *Example I: The case of* $\lambda = 0$*.* Under this assumption, equation 5.7 boils down to

$$\mathbf{V}^*(g, B, \mu, 1) = \max\{\mathcal{V}_1^*(g, B, \mu), \mathcal{V}_0^*(g)\}.$$

Observe that there is no need to have the whole function $\vec{\mu}$ as part of the state, only $\vec{\mu}(g, 1)$ — which without loss of generality I denoted as μ —. Also, there is no need to keep B as part of the state during financial autarky, since defaulted debt is never repaid. Hence, $\mathcal{V}_0^*$ is given by

$$\mathcal{V}_0^*(g) = u(\kappa n_A(g) - g, 1 - n_A(g)) + \beta \int_{\mathbb{G}} \mathcal{V}_0^*(g')\pi_{\mathbb{G}}(dg'|g).$$

[36]There exists the restriction that (g_0, B_0, ϕ_{-1}) and the solution of $\vec{\mu}$ ought to be such that, taking these quantities as starting values, a CEG exists. That is, there is a continuation sequence that satisfy the restrictions for a CEG. See the Appendix H for details.

And

$$\mathcal{V}_1^*(g,B,\mu) = \max_{(n,B',\vec{\mu}')\in\bar{\Gamma}(g,B,\mu,1)} \left\{ u(n-g,1-n) + \beta \int_{\mathbb{G}} \mathbf{V}^*(g',B',\vec{\mu}'(g'),1)\pi_{\mathbb{G}}(dg'|g) \right\},$$

5.2.2. *Example II: The case of* $u_c = 1$ *and* π_Δ *degenerate at* 0. Under this assumption, μ can be dropped as a state variable (since $u_c = 1$ and thus it does not affect the pricing equation). Now, equation 5.7 boils down to

$$\mathbf{V}^*(g,B,1) = \max\{\mathcal{V}_1^*(g,B), \lambda\max\{\mathcal{V}_1^*(g,0),\mathcal{V}_0^*(g)\} + (1-\lambda)\mathcal{V}_0^*(g)\},$$

and

$$\mathcal{V}_0^*(g) = u(\kappa n_A(g) - g, 1-n_A(g)) + \beta\int_{\mathbb{G}} (\lambda\max\{\mathcal{V}_1^*(g',0),\mathcal{V}_0^*(g')\} + (1-\lambda)\mathcal{V}_0^*(g'))\,\pi_{\mathbb{G}}(dg'|g).$$

As in the previous example, there is no need to keep B as part of the state during financial autarky, since defaulted debt is never repaid. Finally,

$$\mathcal{V}_1^*(g,B) = \max_{(n,B')\in\bar{\Gamma}(g,B)} \left\{ u(n-g,1-n) + \beta\int_{\mathbb{G}} \mathbf{V}^*(g',B',1)\pi_{\mathbb{G}}(dg'|g) \right\},$$

where $\bar{\Gamma}(g,B) \equiv \bar{\Gamma}(g,B,1)$.

This is example is closely related to the models by Arellano (2008) and Aguiar and Gopinath (2006) (without distortionary taxes). [37]

5.2.3. *Example III: The case of* $\lambda = 0$ *and no default.* Consider an economy where there is no default (this is imposed ad-hoc), then the value function boils down to

$$\mathbf{V}^*(g,B,\mu) = \max_{(n,B',\vec{\mu}')\in\bar{\Gamma}(g,B)} \left\{ u(n-g,1-n) + \beta\int_{\mathbb{G}} \mathbf{V}^*(g',B',\vec{\mu}'(g'))\pi_{\mathbb{G}}(dg'|g) \right\},$$

and now $\mathbf{d}^*$ is set to never default, also note that $\mathbf{V}^*$ does not depend on ϕ, since trivially $\phi = 1$ always. This is precisely the type of model studied in Aiyagari et al. (2002).

6. Analytical Results

In this section I present some analytical results for a benchmark model that is characterized by quasi-linear per-period utility and stochastically ordered process for g. [38] The proofs for the results are gathered in appendix C.

[37] There is a slight difference in the timing; these models define $\mathbf{V}^*(g,B,1) = \max\{\mathcal{V}_1^*(g,B),\mathcal{V}_0^*(g)\}$. My model could be adapted to replicate this timing.

[38] By stochastically ordered, I mean that the transition probability of g satisfies that for any $g_1 \leq g_2$, $\pi_{\mathbb{G}}(\cdot|g_1) \leq_{FOSD} \pi_{\mathbb{G}}(.|g_2)$; where, for two probability measures P and Q, $P \leq_{FOSD} Q$ means that the corresponding cdf, $F_P(X \leq t) \geq F_Q(Y \leq t)$ for any t, where F_P (F_Q) is the cdf associated to the probability measure P (Q).

Assumption 6.1. *$u(c,n) = c + H(1-n)$ where $H \in \mathbb{C}^2((0,1),\mathbb{R})$ with $H'(0) = \infty$, $H'(l) > 0$, $H'(1) < \kappa$, $2H''(l) < H'''(l)(1-l)$*

This assumption imposes that the per-period utility of the households is quasi-linear and it is analogous to, say, assumption in p. 10 in AMSS. As noted above, under this assumption, μ can be dropped as a state variable. This implies that $\mathbf{V}^*$ is only a function of (g, B, ϕ) and the same holds true for the optimal policy functions. It is also clear from the previous section that $\mathbf{d}^*(g, B, \phi)$ is only non-trivial if $\phi = 1$; thus, henceforth, I omit the dependence of ϕ.

Moreover, to further simplify the technical details, I assume, unless stated otherwise, that $\mathbb{B}$ has only finitely many points each. [39]

6.1. Characterization of optimal default decisions

The next theorem characterizes the optimal decisions of default and acceptance offer to repay the defaulted debt as "threshold decisions"; it is analogous to the one in Arellano (2008), but extended to this setting. Recall that, $\mathbf{d}^*(g, B)$ and $\mathbf{a}^*(g, \delta, B)$ are the optimal decision of default and acceptance offer respectively, given the state (g, δ, B).

Theorem 6.1. *Suppose assumption 6.1 holds and suppose $\kappa = 1$ and $H'' < 0$. Then, there exists $\bar{\lambda}$ such that for all $\lambda \in (0, \bar{\lambda})$, the following holds:*

(1) *There exists a $\bar{\delta} : \mathbb{G} \times \mathbb{B} \to \Delta$ such that $\mathbf{a}^*(g, \delta, B) = 1\{\delta \leq \bar{\delta}(g, B)\}$ and $\bar{\delta}(g, B)$ non-increasing as a function of B.* [40]
(2) *If, in addition, for any $g_1 \leq g_2$, $\pi_{\mathbb{G}}(\cdot|g_1) \leq_{FOSD} \pi_{\mathbb{G}}(.|g_2)$, there exists a $\bar{g} : \mathbb{B} \to \mathbb{G}$ such that $\mathbf{d}^*(g, B) = 1\{g \geq \bar{g}(B)\}$ and $\bar{g}$ non-increasing.*

This result shows that default is more likely to occur for high levels of debt, but so are rejections of offers to exit financial autarky. The latter result implies that the average fraction of repaid debt, conditional on the offer being accepted, is decreasing with the level of debt.[41] It also follows that other things equal, higher debt levels are, on average, associated with longer financial autarky periods.Thus, these two results imply a positive co-movement between the (observed) average haircut and the average length of financial autarky. [42]

[39]This assumption is made for simplicity. It can be relaxed to allow for general compact subsets, but some of the arguments in the proofs will have to be change slightly. Also, the fact that $\mathbb{B} \equiv \{B_1, ..., B_{|\mathbb{B}|}\}$ is only imposed for the government; the households can still choose from convex sets; only in equilibrium I impose $\{B_1, ..., B_{|\mathbb{B}|}\}$.

[40]It turns out, that the first part of the statement holds for any λ.

[41]According to the theorem, this quantity equals $E_{\pi_{\mathbb{G}}}[\int_{\delta \in \Delta} \delta 1\{\delta \leq \bar{\delta}(g, B)\}\pi_{\Delta}(d\delta)]$.

[42]This last fact seems to be consistent with the data; see Benjamin and Wright (2009) Fact 3 in the paper. It is important to note, however, that I derived the implications by looking at *exogenous* variation of the debt

Finally, the result holds for $\lambda \in (0,\bar{\lambda})$, because for $0<\lambda\leq\bar{\lambda}$, i.e., λ's that are small enough, the variation of the continuation value of financial autarky with respect to the outstanding defaulted debt can be controlled and is dominated by the variation of the per-period payoff. This allows me to show that the value functions satisfy a single crossing condition that is the key to show the results.

6.2. Implications for equilibrium Prices and Taxes

I now study the implications of the above results on prices and taxes. I take the assumptions needed for theorem 6.1 as given throughout the section.

Observe that

$$\begin{aligned} p(g_t,B_{t+1}) =&\beta\int_{\mathbb{G}}1\{g'\leq\bar{g}(B_{t+1})\}\pi_{\mathbb{G}}(dg'|g_t)+\beta\lambda\int_{\mathbb{G}}1\{g'>\bar{g}(B_{t+1})\}D(g',B_{t+1})\pi_{\mathbb{G}}(dg'|g_t)\\ &+\beta\int_{\mathbb{G}}1\{g'>\bar{g}(B_{t+1})\}(\lambda\alpha(g',B_{t+1})+(1-\lambda))\}q(g',B_{t+1})\pi_{\mathbb{G}}(dg'|g_t), \end{aligned} \tag{6.11}$$

where $\alpha(g,B)\equiv\int_{\Delta}1\{\delta>\bar{\delta}(g,B)\}\pi_{\Delta}(d\delta)$, $D(g,B)\equiv\int_{\Delta}1\{\delta\leq\bar{\delta}(g,B)\}\delta\pi_{\Delta}(d\delta)$, and

$$q(g_t,B_t)=\beta\lambda\int_{\mathbb{G}}D(g',B_t)\pi_{\mathbb{G}}(dg'|g_t)+\beta\int_{\mathbb{G}}(\lambda\alpha(g',B_t)+(1-\lambda))\}q(g',B_t)\pi_{\mathbb{G}}(dg'|g_t). \tag{6.12}$$

Compare this with the case that there is no defaulted-debt repayment, $p^{o}(g_t,B_{t+1})\equiv\beta\int_{\mathbb{G}}1\{g'\leq\bar{g}(B_{t+1})\}\pi_{\mathbb{G}}(dg'|g_t)$. From analogous calculations to those in theorem 6.1, it follows that $p^{o}(g,B)$ is non-increasing as a function of B, given raise to endogenous debt limits; see Arellano (2008). By inspection of equation 6.11, it is easy to see that, other things equal, this result is attenuated by the presence of (potential) defaulted debt payments and secondary markets. Although, for a general π_{Δ} and $\pi_{\mathbb{G}}$ is hard to further characterize p and q, the next theorem shows that q and p are both non-increasing on the level of debt, once I impose additional restrictions on π_{Δ} and $\pi_{\mathbb{G}}$.

Theorem 6.2. *Suppose assumptions of theorem 6.1 hold. Suppose further that $\pi_{\Delta}=\mathbf{1}_{\delta_0}(\delta)$ and $g\sim iid\pi_{\mathbb{G}}$. Then: (1)*

$$q(g,B)=q(B)=\frac{\beta\lambda\delta_0 A(B)}{1-\beta+\beta\lambda A(B)},\ \textit{where}\ A(B)=\pi_{\mathbb{G}}(\{g\colon\delta_0\leq\bar{\delta}(g,B)\}),$$

and is non-increasing as a function of B. (2) $p(g,B)$ is non-increasing as a function of B.

This theorem shows that high levels of debt are associated with higher return on debt, both, before and during financial autarky. Moreover, it also shows that prices are increasing on the fraction of repayment (δ_0) and on the probability of receiving an offer λ. This result and those in theorem 6.1 imply a positive relationship between the return, the (observed) average recovery

level; in the data this quantity is endogenous and, in particular varies with g. This endogeneity issue should be taken into account if one would like to perform a more thorough test of the aforementioned implications.

rate and the length of financial autarky. Whether these results hold once I relax the strong assumptions over π_Δ and $\pi_{\mathbb{G}}$, is unknown to me. In the numerical simulation, however, I allow for more general formulations, and find that prices are still non-increasing in B (see figure D.3).

In the next theorem, I show that under some additional assumptions there exists a finite level of debt such that $p(g,B)B$ is maximal as a function of B, and thus an endogenous debt limit exists.

THEOREM 6.3. *Suppose assumptions of theorem 6.2 hold, $\mathbb{B} = [\underline{B}, \overline{B}]$ such that (i) $\underline{B} \leq 0$, (ii) $p(\overline{B}) = q(\overline{B}) = 0$, and (iii) $p(g,B)B$ is continuous as a function of B. Then there exists a $B^* \in (\underline{B}, \overline{B})$ such that the optimal level of debt belongs to $(\underline{B}, B^*)$.*

A few remarks about this theorem are in order. First, assumptions (i)-(ii) are designed to control $p(g,B)B$ at the boundary. In particular, assumption (i) ensures that the derivative is positive in $\underline{B}$ and (ii) ensures that the revenue from debt is naught at $\overline{B}$. Since the payoff obtained from tax revenues is bounded, one could always choose $\bar{B}$ to be so high that the government cannot levy enough taxes to pay it. Second, this result in conjunction with theorem 6.2, shed some light on the evidence regarding debt-to-ouput and default risk measures presented in section 2.

In order to analyze the ex-ante effect of default risk on taxes, I consider the case $\lambda = 0$ (i.e., autarky is an absorbing state) to simplify the analysis. By theorem 6.1, the default decision is a threshold decision, so, for each history $\omega \in \Omega$ I can define $T(\omega) = \inf\{t : g_t \geq \bar{g}(B_t(\omega))\}$ (it could be infinity) such that for all $t \leq T(\omega)$ the economy is not in financial autarky. For such t, the implementability constraint is given by (omitting the dependence on ω)

$$\begin{aligned} B_t u_c(n_t - g_t, 1 - n_t) + g_t u_c(n_t - g_t, 1 - n_t) \leq & \left(u_c(n_t - g_t, 1 - n_t) - u_l(n_t - g_t, 1 - n_t)\right) n_t \\ & + p_t u_c(n_t - g_t, 1 - n_t) B_{t+1}. \end{aligned}$$

Let $\nu_t(\omega)$ be the Lagrange multiplier associated to this restriction in the optimization problem of the government, given (t, ω). Observe that if $\nu_t = 0$, from the first display it is easy to see that $\tau_t = 0$.

From the FONC of the government it follows (see appendix C.1 for the derivation) [43]

$$u_c(n_t - g_t, 1 - n_t) - u_l(n_t - g_t, 1 - n_t) - \nu_t \frac{dA(n_t, g_t, B_t)}{dn_t} = 0,$$

[43] This derivation assumes that $\mathbb{B}$ is at least a convex set, so as to make sense of differentiation. It, also, assumes differentiability of $\bar{g}$.

where $A(n,g,B) \equiv (u_c(n-g,1-n) - u_l(n-g,1-n))\, n - (g+B)u_c(n-g,1-n)$. And

$$\nu_t \left(1 + \frac{dp_t}{dB_{t+1}} \frac{B_{t+1}}{p_t}\right) = \int_{\mathbb{G}} \nu_{t+1} \frac{1\{g' \leq \bar{g}(B_{t+1})\}}{\int_{\mathbb{G}} 1\{g' \leq \bar{g}(B_{t+1})\}\pi_{\mathbb{G}}(dg'|g_t)} \pi_{\mathbb{G}}(dg'|g_t). \tag{6.13}$$

The Lagrange multiplier associated with the implementability condition is constant in Lucas and Stokey (1983) and, thus, trivially a martingale. In Aiyagari et al. (2002) the Lagrange multiplier associated with the implementability condition is a martingale with respect to the probability measure $\pi_{\mathbb{G}}$.[44] Equation 6.13 implies that the law of motion of the Lagrange multiplier differs in two important aspects. First, the expectation is computed under the default-adjusted measure; this stems from the fact that the option to default adds "some" degree of state-contingency to the payoff of the government debt; this effect lowers the marginal cost of the debt. Second, the aforementioned expectation is multiplied by $\left(1 + \frac{dp_t(B_{t+1})}{dB_{t+1}} \frac{B_{t+1}}{p_t(B_{t+1})}\right)$, which can be interpreted as the "markup" that the government has to pay for having this option to default; this effect increases the marginal cost of the debt.

The next lemma studies further what happens to taxes and production on the eve of default. Namely, it establishes that, for states (g,B) for which the government chooses to default, labor (and thus production) in financial autarky is higher than what it would have been under financial access, and also, if H is concave (convex), taxes in financial autarky are lower (higher) than what it would have been under financial access.

Let $\mathbf{n}^*_C(g,\delta,B)$ be the optimal choice of labor under access to financial markets, given state (g,δ,B). Let $\boldsymbol{\tau}^*_A(g)$ and $\mathbf{n}^*_A(g)$ be the optimal choice of tax and labor under financial autarky, given state g. [45]

LEMMA 6.1. *Suppose assumption 6.1 holds, suppose $\kappa = 1$ and $H'' \neq 0$. Suppose that $\mathcal{D} \equiv \{(g,B)\colon \mathbf{d}^*(g,B) = 1 \text{ and } \mathbf{n}^*_C(g,1,B) < n_1(1)\} \neq \{\emptyset\}$. Then there exists a $\bar{\lambda}$ such that for all $0 < \lambda \leq \bar{\lambda}$:*

(1) $\mathbf{n}^*_C(g,1,B) < \mathbf{n}^*_A(g)$, for all $(g,B) \in \mathcal{D}$.

(2) If $H'' < (>) 0$ then $\boldsymbol{\tau}^*_A(g,B) < (>) \boldsymbol{\tau}^*_C(g,1,B)$, for all $(g,B) \in \mathcal{D}$.

The condition that $\mathbf{n}^*_C(g,1,B) < n_1(1)$ is an interiority assumption, and it implies that $\boldsymbol{\tau}^*_C(g,1,B) < 1$ for all $(g,B) \in \mathcal{D}$ and is needed to ensure that the per-period payoff is decreasing. [46] This result stems from the fact that, if default is chosen, it must be true that

[44]The martingale property is also preserved if capital is added to the economy; see Farhi (2010). This property, however, changes if I allow for ad-hoc borrowing limits (see Aiyagari et al. (2002))

[45]Observe that $\boldsymbol{\tau}^*_A$ and $\mathbf{n}^*_A$ do note depend on B, because the latter is obtained from balancing the budget, and the former is a function of the latter.

[46]$n_1(1)$ is defined in lemma C.1 and is the value of n such that makes tax revenue equal to zero.

the per-period payoff under financial autarky is high enough to compensate the government for exercising the option value of deferring default for one period.

7. Numerical Results

Throughout this section, I run a battery of numerical exercises in order to assess the performance of the model. I compare my findings with an economy in which the option to default is not present—precisely the model considered in Aiyagari et al. (2002). I denote the variables associated with this model with a (sub)superscript "AMSS"; variables associated to my economy are denoted with a (sub)superscript "ED" (short for Economy with Default).

In the dataset IND economies are proxies of the AMSS model and EME/LAC are proxies of my model. As discussed before, IND do not exhibit default events in the dataset. Thus, I take IND as a proxy for economies modeled in AMSS. There is the question of what characteristics of the economy will prompt it to behave like AMSS- or ED-type economies. One possible explanation is that by factors extraneous to the model, such as political instability, ED presents lower discount factor from the goverment and thus are more likely to default. Another possible explanation is that for AMSS-type/IND economy, default is more costly because these economies are financially more integrated, and a default — and the posterior period of financial autarky — could have a larger impact on financing of the firms, thus lowering the productivity (in the model represented by a lower κ). The next lemma shows that, for a simplified version of the economy, higher κ imply lower likelihood of default (the proof is in appendix E). [47]

Lemma 7.1. *Suppose the assumptions 6.1 - 3.1 hold, $H'' < 0$ and $\lambda = 0$. Let $\kappa < \kappa'$ and let $\mathcal{D}_\kappa \equiv \{(g, B) : \mathbf{d}^*(g, B) = 1\}$, then $\mathcal{D}_\kappa \subseteq \mathcal{D}_{\kappa'}$.*

For all the simulations the utility function is given by $u(c, 1-n) = c + C_1\frac{(1-n)^{1-\sigma}}{1-\sigma}$. The process for $(g_t)_t$ follows a 5 state discrete state space Markov chain "induced" by $g_{t+1} = \mu_g(1-\rho_1) + \rho_1 g_t + \sigma_g\sqrt{1-\rho_1^2}\varepsilon_{t+1},\ \varepsilon_{t+1} \sim N(0,1)$. By "induced", I mean that the transition probability follows from applying Tauchen's (Tauchen (1986)) results to the AR(1) process.

I calibrate the parameters of the model as follows. I choose $\beta = 0.9$, $\sigma = 3$, $\kappa = 1$ and $C_1 = 0.01$. This choice is taken from AMSS; the authors choose $C_1 = 1$ but work with 100 as the unit of time — to be split between leisure and labor — whereas I work with 1. The state space is given by $\mathbb{B} = [0, 0.5]$ and $\mathbb{G} = [0, 0.1]$.[48] For the benchmark parametrization (column (I)) I choose $\rho_1 = 0$, $\mu_g = 0.05$, $\sigma_g = 0.045$, $\lambda = 0.25$, and $\Delta = \{0.4, 0.6, 0.8\}$ where the probability π_Δ puts uniform probability over the interval. I choose β, λ and Δ to match: a default frequency

[47] This finding is still present in the numerical simulations that allow for more general parametrization.

[48] In this parametrization $|\mathbb{B}| = 50$.

between 3-6 percent, a recovery rate of (approx.) 50 percent and a autarky spell between 10-15 periods. [49]

I perform 1000 MC iterations, each consisting of sample paths of 1000 observations for which only the last 250 observations were considered in order to eliminate the effect of the initial values. [50]

Table 7.2. In the table, E and std denote the mean and standard deviation across time, respectively. All quantities are averaged across MC simulations. In parenthesis the 5% and 95% percentile of the MC sample. All results (except Default spell) are in percentages.

Whole sample	(I)		(II)	(III)	(IV)	(V)
	AMSS	ED	ED	ED	ED	ED
$E(b_t/n_t)$	6.1	3.4	3.5	5.8	3	0.4
	(5.1,7)	(3,3.8)	(3,4)	(5,6.8)	(2.8,3.5)	(0.3,0.5)
$E(\tau_t)$	7.0	6.5	6.5	6.7	6.5	6.4
	(6.5,7.6)	(6,7)	(6,7)	(6.2,7.2)	(6,6.7)	(5,7.8)
$Std(\tau_t)$	2.5	4.2	4.1	3.5	4.4	4.2
	(2.2,2.8)	(4,4.4)	(3.8,4.4)	(3.3,3.7)	(4.1,4.6)	(3.8,4.6)
$E(Spread)$		18	14	10	3.2	13
		(15,21)	(11,17)	(8.8,11.3)	(2.5,3.7)	(9,15)
E(Def. Spell)		9.3	10.5	2.7	17	38.4
E(Rec. Rate)		49.5	60	55	54	56
Pr(Def)		6.4	5.5	18.7	4.8	3.5
		(4.6,8.6)	(4.4,7.5)	(16,21.5)	(2.8,6.7)	(0.8,6.3)

Table 7.2 reports the results for the whole sample and tables 7.3 report the results for the "financial autarky" sample and "financial access" sample, resp. I constructed these latter sub-samples by separating, for each MC iterations, the periods in which the ED economy was in autarky from those in which the economy was not.

Column (I) (in all the tables) reports the result for the aforementioned parametrization which I use as benchmark. The average debt-to-output ratio (row 1) for the whole sample is around 6 percent for the AMSS economy; in the ED economy, however, it is around 3.5 percent because of the presence of the endogenous borrowing limits arising from the possibility of default. This

[49]The default recovery rate is taken from Yue (2010), where 35 percent is the recovery rate for Argentina and 65 percent is the recovery rate for Ecuador. See Pitchford and Wright (2008) for more details for the default spell.

[50]To solve the model I use standard techniques to iterate over value functions and policy functions and an "outer" loop that iterates on prices until convergence.

TABLE 7.3. In the table, E and std denote the mean and standard deviation across time, respectively. All quantities are averaged across MC simulations. In parenthesis the 5% and 95% percentile of the MC sample. All results are in percentages.

	Financial Autarky						Financial Access					
	(I)		(II)	(III)	(IV)	(V)	(I)		(II)	(III)	(IV)	(V)
	AMSS	ED	ED	ED	ED	ED	AMSS	ED	ED	ED	ED	
$E(b_t/n_t)$	7.2	4.4	5.2	8.1	3.5	0.4	3.6	1.4	1.5	3.7	1	0.2
	(6.3,8.1)	(4.3,4.6)	(5.2,5.2)	(7,9.2)	(2.9,4)	(0.3,0.5)	(2.6,4.7)	(1.2,1.6)	(0.9,1.4)	(3.1,4.1)	(0.8,1.2)	(0.1,0.2)
$E(\tau_t)$	7.8	7.0	7.0	7.4	6.8	6.6	5.7	5.6	5.8	6.0	5.5	3.3
	(7.2,8.3)	(6.4,7.6)	(6.5,7.7)	(6.8,8)	(6.3,7.3)	(5.3,7.9)	(4.9,6.2)	(5,6.3)	(5.2,6.3)	(5.5,6.5)	(4.9,6.1)	(2.2,4.6)
$Std(\tau_t)$	2.5	4.4	4.4	3.6	4.5	4.2	2.4	3.6	3.6	3.3	3.9	3.3
	(2.2,2.9)	(4.1,4.7)	(4.1,4.7)	(3.3,3.9)	(4.2,4.7)	(3.8,4.6)	(2.0,2.7)	(3.3,3.9)	(3.3,3.8)	(3,3.6)	(3.5,4.2)	(2.8,5.6)
$E(Spread)$		27	24	15	4.2	140		2.2	1.0	4.2	0.2	0.04

level is low compared to what is observed in the data: a ratio of approximately 23 percent for Argentina (1990-2005).[51] For the financial autarky sub-sample, the average debt-to-output ratio is actually the *defaulted* debt-to-output ratio, this provides additional evidence of endogenous borrowing limits being "active" in higher levels of debt and is consistent with the stylized facts presented in section 2.

The average tax rate (row 2) is slightly higher in the AMSS than in the ED economy, across all three samples. The volatility of the tax rate (row 3), however, is higher in the ED economy, especially in the financial autarky sub-sample, where is almost twice as high. A noteworthy remark is that taxes are more volatile in the ED economy, even during the financial access sub-sample, this is due to the presence of endogenous borrowing limits. This fact shows that the model is able to generate the corresponding stylized fact presented in section 2. Finally, when the ED economy is in autarky, the government is precluded from issuing debt, rendering taxes more volatile than in the other sub-samples.

In order to study the pricing implications, I compute the spread as $\frac{r_{t+1}}{p_t} - \frac{1}{\beta}$ where $r_{t+1} = 1$ if not in default and $r_{t+1} = \delta_{t+1}$ if an offer (of δ_{t+1}) was accepted and $r_{t+1} = q_{t+1}$ otherwise. The spread of the model around 18 percent, and during financial autarky is highest, around 27 percent. This feature — higher spreads during financial autarky — is (at least qualitatively) consistent with what we observe in the data (see the discussion in section 7.1); the model however, generates higher spreads than those observe for the whole sample.

Finally, the welfare in AMSS is 6.2 whereas in the ED economy is 2.1.[52]

[51]For the default period (2001-2005), this ratio was (approx.) 45 percent.

[52]Welfare is computed as the expectation of the value function with respect to the long run distribution for each model.

Column (II) (in all the tables) reports the case where $\underline{\delta} = \overline{\delta} = 0.6$. This parametrization has the same (unconditional) mean for Δ as the one in column (I), but no variance. Thus, it allows the model to shed light on the importance of the range of available offers to repay the debt and to quantify the selection bias coming from the fact that repayments of defaulted debt are *chosen* by the government.

Observe that the recovery rate in (I) is lower than the one in (II), and the expected duration spell is higher than in (I). These outcomes illustrate the fact that the government only accepts low offers of δ, thereby biasing downwardly the sample.

Column (III) (in all the tables) reports the case where $\lambda = 1$. A direct implication of this parametrization is that average default spell is reduced (relative to the model (I)) and the probability of default is higher. Also, the spread is lower for the financial autarky sub-sample; this outcomes illustrates the fact that *once* in default, the government re-pays (at least a fraction) of the defaulted debt in a shorter period of time. Finally, the welfare for this configuration is 3.2.

Column (IV) (in all the tables) reports the case where $\lambda = 0.15$.[53] The average default spell increases (relative to both (I) and (III)); this directly follows from a lower λ. Consequently, financial autarky becomes more costly and this implies lower frequency of default. For this case, the welfare is 1.4.

It is interesting to note that, at least for the current parametrization, *conditional on having default in equilibrium*, debt-restructuring process that yield a higher frequencies of offers (a higher λ), are preferred.

Column (V) (in all the tables) considers the case where $\rho_1 = 0.9$ and $\mu_g = \frac{0.05}{1-\rho_1}$ (the rest of the parameters are as in case (I)). For this case, the average default spell is around 38 periods (around 4 times as long as the one for (I)) and the default frequency drops to 3 percent. These facts reflect that default tends to occur when government expenditure is high, whereas acceptance of offers of repayment occur when the government expenditure is low. When the government expenditure is highly persistent, once in financial autarky, the government expenditure remains high thus prompting the government to reject more offers of repayment (relative to the case where expenditure is low). This effect implies that the default spell is longer and also that autarky is more costly, thus delivering less defaults in equilibrium. Interestingly, the average recovery rate is around 55 percent thus presenting a lower downward bias than the one in (I). This stems from the fact that, for this parametrization, the government rejects *any* offer when government expenditure high and accept almost all offers when government expenditure is low. Thus, since the realization of δ and g are independent, the sample of *accepted*

[53]For values of λ lower than 0.15 default was not present in the MC sample.

offers presents a low selection bias. Also, observe that the spread for the financial autarky subsample is an order of magnitude higher than the previous one, this reflects the long autarky spell. Welfare for this case is around 0.5.

7.1. Impulse Response Functions

In this subsection, I draw a particular path for g_t given by

$$g_t = \begin{cases} 0 & if\ t < T \\ 0.075 & if\ t \in [T, T+4] \\ 0 & if\ t \geq T+5 \end{cases} \tag{7.14}$$

This choice is completely arbitrary, chosen to showcase all the features of the model.

Figure D.4 presents the results. The dotted line in all the panels is the path d_t. The economy enters default during the third period of high government expenditure and stays in financial autarky for two periods. The upper-left panel shows the debt for both economies (ED solid and AMSS dashed); the endogenous borrowing limits present in the ED economy render lower levels of debt during "bad times." During autarky, since we keep track of the defaulted debt, we have a plateau; then, the economy leaves default by paying part of the outstanding debt.

The lower-left panel shows the tax path for both economies (ED solid and AMSS dashed); for the ED economy, the path is more volatile and for the AMSS taxes return more slowly to zero since they ought to finance the high levels of debt. The lower-right panel depicts consumption for both economies (ED solid and AMSS dashed). Finally, the upper-right panel shows the spread. The spread increases before the default event (during the periods of high expenditure and debt before default) and is maximal during financial autarky.

This last result is consistent with the data that shows that during the debt-restructuring period, the measure of default risk stays significantly higher than during "no default periods." For instance, for Argentina, this measure was around 5 percent during 1997-2000 and 2005-2006 but around 60 percent during 2001-2005; for Russia, it was around 4 percent during 2000-2006 but around 20 percent during 1998-1999; finally, for Ecuador, it was around 9 percent during 1997 and 2001-2006 and around 21 percent during 1998-2000.

In brief, the aforementioned figures show a summary of the dynamics generated by this model: endogenous debt limits, higher volatility of taxes, and higher spreads due to default risk, especially, during a default period.

8. Conclusion

I study a government's problem, in a closed economy, that consists of choosing distortionary taxes with only non-state-contingent government debt, but allowing for partial defaults on the debt.

First, I provide an explanation for the lower debt-to-output ratios and more volatile tax policies observed in emerging economies, vis-*à*-vis industrialized economies. This stems from the fact that the holders of government debt forecast the possibility of default, imposing endogenous debt limits. These limits restrict the ability of the government to smooth shocks using debt, thus rendering taxes more volatile.

Second, I propose a device to price the debt during temporary financial autarky. Numerical simulations show that the spread during the default period is higher than for the rest of the sample; this characteristic is consistent with data for defaulters—e.g., Argentina, Ecuador and Russia.

Third, and last, the numerical simulations suggest that increasing the probability of receiving offers for exiting autarky decreases welfare when this probability is low/medium to begin with, but increases it when the probability is high.

Although this model does a good job of explaining qualitatively the facts observed in the data, it does not do very well in matching the data quantitatively. A line of future research should delve further into the production side of this economy and its driving shocks, and also on developing the pricing implications.[54]

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[54]See Aguiar and Gopinath (2006) and Mendoza and Yue (2012), and Pouzo and Presno (2012) exploring some of these issues.

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Appendix A. Notation and Stochastic Structure of the Model

Throughout the appendix, for a generic mapping f from a set S to T, I use $s \mapsto f(s)$ or $f : S \to T$ to denote it. For the case that a mapping depends on many variables, the notation $s_1 \mapsto f(s_1, s_2)$ is used to denote the function f *only* as a function of s_1, keeping s_2 fixed.

For a generic X, let $X^\infty : \Omega \times \{0, 1, 2, ...\} \to \mathbb{R}$ be an stochastic process; $X_t(\omega) \equiv X(\omega, t)$ is the value of the stochastic process at time t and given ω; $X_t(\cdot, \omega^{t-1})$ is understood as a function from $\mathbb{G} \times \bar{\Delta}$ to $\mathbb{R}$ and is the function implied by the stochastic process, at time t, given the *past* history ω^{t-1}; X_t^s is an stochastic process given by the values $\omega \mapsto (X_t(\omega), ..., X_s(\omega))$; similarly, $X_t^s(\cdot, \omega^t)$ is an stochastic process given by the values $\tilde{\omega} \in \{\Omega : \tilde{\omega}^t = \omega^t\} \mapsto (X_t(\tilde{\omega}), ..., X_s(\tilde{\omega}))$.

As noted throughout the text, for stochastic processes, X^∞ and Y^∞, the equality $X_t = Y_t$ is understood as $X_t(\omega) = Y_t(\omega)$ for all $\omega \in \Omega$.

I denote $\mathcal{O} \equiv \mathcal{G} \times \mathcal{D}$ denote the product σ-algebra, where $\mathcal{G}$ and $\mathcal{D}$ are the σ-algebra attached to $\mathbb{G}$ and $\bar{\Delta}$ resp. I use $\mathcal{O}^t$ to denote the σ-algebra generated by ω^t, for all $t \in \{0, 1, ...\}$.

With this notation in place, definition 3.1 implies that a government policy $(\sigma_t)_t$ is such that $B_{t+1} : \Omega \to \mathbb{B}$ is $\mathcal{O}^t$-measurable; $\tau_t : \Omega \to [0, 1]$ is $\mathcal{O}^t$-measurable; $d_t : \Omega \to \{0, 1\}$ is $\mathcal{G} \times \mathcal{O}^{t-1}$-measurable; $a_t : \Omega \to \{0, 1\}$ is $\mathcal{O}^t$-measurable. Observe that the measurability restriction on d_t represents the fact that the government decides to default or not, before observing the (eventual) fraction of debt to be repaid and whether it receives such an offer or not, in case of default.

The price schedule is defined to be $\mathcal{O}^t$-measurable. At times, I will be explicit about the dependence of prices on σ^∞ and use $p_t(\omega; \sigma^\infty)$ to denote the price at time t, history ω and given policy σ^∞.

Similarly, definition 3.3 implies that $c_t : \Omega \to \mathbb{R}_+$ and $n_t : \Omega \to [0, 1]$ are $\mathcal{O}^t$-measurable and $b_{t+1} : \Omega \to [\underline{b}, \overline{b}]$ is $\mathcal{O}^t$-measurable.

Appendix B. Optimization Problem for the Households

The Lagrangian associated to the household problem is given by

$$\mathcal{L}(c^{\infty}, n^{\infty}, b^{\infty}, \lambda^{\infty}, \mu^{\infty}, \psi^{\infty}) \equiv$$

$$\sum_{t=0}^{\infty} \beta^t E_{\Pi(\cdot|\omega_0)} \left[\{u(c_t(\omega), 1 - n_t(\omega)) - \lambda_t(\omega)\{c_t(\omega) - (1 - \tau_t(\omega))\kappa_t(\sigma^{\infty})n_t(\omega) + p_t(\omega; \sigma^{\infty})b_{t+1}(\omega) - \varrho_t(\omega; \sigma^{\infty})b_t(\omega)\} \right.$$
$$\left. + \Psi_t(\omega)c_t(\omega) + \psi_{1t}(b_{t+1} - \underline{b}) + \psi_{2t}(\overline{b} - b_{t+1}) \} \right],$$

where λ_t is the Lagrange multiplier associated to the budge constraint and Ψ_t is the Lagrange multipliers associated to the restrictions that $c_t \geq 0$, and ψ_{it} $i = 1, 2$ is the Lagrange multiplier associated to the debt restrictions.

Assuming interiority of the solutions, the first order conditions (FONC) are given by:

$$\begin{aligned} c_t &: u_c(c_t(\omega), 1 - n_t(\omega)) - \lambda_t(\omega) = 0 \\ n_t &: \; - u_l(c_t(\omega), 1 - n_t(\omega)) - \lambda_t(\omega)(1 - \tau_t(\omega))\kappa_t(\omega; \sigma^{\infty}) = 0 \\ b_{t+1} &: p_t(\omega; \sigma^{\infty})\lambda_t(\omega) - E_{\Pi(\cdot|\omega^t)}[\beta\lambda_{t+1}(\omega)\varrho_{t+1}(\omega)] = 0. \end{aligned}$$

Then

$$\frac{u_l(c_t(\omega), 1 - n_t(\omega))}{u_c(c_t(\omega), 1 - n_t(\omega))} = (1 - \tau_t(\omega))\kappa_t(\omega; \sigma^{\infty}), \tag{B.15}$$

and

$$p_t(\omega; \sigma^{\infty}) = E_{\Pi(\cdot|\omega^t)} \left[\beta \frac{u_c(c_{t+1}(\omega), 1 - n_{t+1}(\omega))}{u_c(c_t(\omega), 1 - n_t(\omega))} \varrho_{t+1}(\omega) \right]. \tag{B.16}$$

From the definition of ϱ, equation B.16 implies, for $d_t = 0$ and $a_t = 1$,

$$\begin{aligned} p_t(\omega; \sigma^{\infty}) = & E_{\Pi(\cdot|\omega^t)} \left[\beta \frac{u_c(c_{t+1}(\omega), 1 - n_{t+1}(\omega))}{u_c(c_t(\omega), 1 - n_t(\omega))} (1 - d_{t+1}(\omega)) \right] \\ & + E_{\Pi(\cdot|\omega^t)} \left[\beta \frac{u_c(c_{t+1}(\omega), 1 - n_{t+1}(\omega))}{u_c(c_t(\omega), 1 - n_t(\omega))} d_{t+1}(\omega) a_{t+1}(\omega) \delta_{t+1} \right] \\ & + E_{\Pi(\cdot|\omega^t)} \left[\beta \frac{u_c(c_{t+1}(\omega), 1 - n_{t+1}(\omega))}{u_c(c_t(\omega), 1 - n_t(\omega))} d_{t+1}(\omega)(1 - a_{t+1}(\omega)) q_{t+1}(\omega; \sigma^{\infty}) \right]. \end{aligned}$$

For $d_t = 1$ and $a_t = 0$,

$$\begin{aligned} p_t(\omega; \sigma^{\infty}) = & E_{\Pi(\cdot|\omega^t)} \left[\beta \frac{u_c(c_{t+1}(\omega), 1 - n_{t+1}(\omega))}{u_c(c_t(\omega), 1 - n_t(\omega))} a_{t+1}(\omega) \delta_{t+1} \right] \\ & + E_{\Pi(\cdot|\omega^t)} \left[\beta \frac{u_c(c_{t+1}(\omega), 1 - n_{t+1}(\omega))}{u_c(c_t(\omega), 1 - n_t(\omega))} (1 - a_{t+1}(\omega)) q_{t+1}(\omega; \sigma^{\infty}) \right]. \end{aligned}$$

The next lemma shows that the FONC of the households are sufficient; see section F for the proof.

Lemma B.1. *Suppose assumption 4.1(i) holds. Then first order conditions, B.15 - B.16, are also sufficient.*

Appendix C. Proof of Lemmas in section 6

Throughout, I assume that $\mathbb{G} \equiv \{g_1, ..., g_{|\mathbb{G}|}\}$, $\mathbb{B} \equiv \{B_1, ..., B_{|\mathbb{B}|}\}$ and $\Delta \equiv \{\delta_1, ..., \delta_{|\Delta|}\}$ with $|\mathbb{G}|$, $|\mathbb{B}|$ and $|\Delta|$ all finite (for a generic set A, $|A|$ denotes the cardinality of a set).

Under assumption 6.1, it is easy to see that the revenue of the government, as a function of leisure and productivity, is $R(\kappa, n) = (\kappa - H'(1-n))n$. Moreover, the optimal level of labor belongs to $[n_2(\kappa), n_1(\kappa)]$ where $R(\kappa, n_1(\kappa)) = 0$ and $R'(\kappa, n_2(\kappa)) = 0$, and in this domain $R' < 0$; see lemma C.1(1) for the proof. Let, for any given κ, $R \mapsto n(\kappa, R)$ be the inverse mapping of $n \mapsto R(\kappa, n)$.

Let W be the per-period payoff, i.e.,

$$W(R) = \kappa n(\kappa, R) - g + H(1 - n(\kappa, R)) \asymp \kappa n(\kappa, R) + H(1 - n(\kappa, R)).$$

Lemma C.1(2) establishes that W is differentiable, uniformly bounded, non-increasing (decreasing over all R such that $(-\kappa + H'(1 - n(\kappa, R))) < 0)$ function.

The government budget constraint becomes: If $\mathbf{d}(g, B) = 0$, $g + \delta B - R \leq p(g, B'; \mathbf{e})B'$, where $\mathbf{e} \equiv (\mathbf{d}, \mathbf{a})$ and

$$\begin{aligned} p(g, B'; \mathbf{e}) =& \beta \int_{\mathbb{G}} (1 - \mathbf{d}(g', B'))\pi_{\mathbb{G}}(dg'|g) + \lambda \int_{\mathbb{G}} \mathbf{d}(g', B') \int_{\Delta} \mathbf{a}(g', \delta', B')\delta' \pi_{\Delta}(d\delta')\pi_{\mathbb{G}}(dg'|g) \\ &+ \int_{\mathbb{G}} \mathbf{d}(g', B') \left(\int_{\Delta} \{\lambda(1 - \mathbf{a}(g', \delta', B')) + (1 - \lambda)\}\pi_{\Delta}(d\delta') \right) q(g', B'; \mathbf{e})\pi_{\mathbb{G}}(dg'|g), \end{aligned}$$

with

$$q(g, B'; \mathbf{e}) = \beta \int_{\mathbb{G}} \left\{ \int_{\Delta} \{\lambda \mathbf{a}(g', \delta', B')\delta' + \{\lambda(1 - \mathbf{a}(g', \delta', B')) + 1 - \lambda\} q(g', B'; \mathbf{e})\} \pi_{\Delta}(d\delta') \right\} \pi_{\mathbb{G}}(dg'|g).$$

If $\mathbf{d}(g, B) = 1$, $g - R \leq 0$. Henceforth, I omit d, a from the prices.

Let

$$\begin{aligned} V(g, B) =& (1 - \mathbf{d}^*(g, B))V_C(g, B) \\ &+ \mathbf{d}^*(g, B) \left[\lambda \int_{\Delta} \{\mathbf{a}^*(g, \delta', B)V_C(g, \delta' B) + (1 - \mathbf{a}^*(g, \delta', B))V_A(g, B)\}\pi_{\Delta}(d\delta') + (1 - \lambda)V_A(g, B) \right], \end{aligned}$$

where [55]

$$V_C(g, \delta B) = W(g + \delta B - p(g, \mathbf{B}^*(g, \delta B); \mathbf{d}^*, \mathbf{a}^*)\mathbf{B}^*(g, \delta B)) + \beta \int_{\mathbb{G}} V(g', \mathbf{B}^*(g, \delta B))\pi_{\mathbb{G}}(dg'|g),$$

[55] Formally, W is defined only in the interval $[0, R(\kappa, n_2(\kappa))]$. Hence, it must be $g + B - p(g, B')B' \in [0, R(\kappa, n_2(\kappa))]$ for some $B' \in \mathbb{B}$. If $g + B - p(g, B')B' < 0$, it implies that the revenue from taxes is negative, i.e., the government is issuing subsidies. This can easily be handled by allowing lump-sum transfers. The case where $g + B - p(g, B')B' > R(\kappa, n_2(\kappa))$ is more problematic since it entails that there are not enough resources to cover the deficit. To ensure this does not happen: I can define the feasible set of debt choices to be $\{B' \in \mathbb{B} \colon g + B - p(g, B')B' \leq R(\kappa, n_2(\kappa))\}$, if for a given (g, B) the set is empty, I set the per-period payoff to a large negative value (but finite) and set $d(g, B) = 1$.

and

$$V_A(g,B) = W(g) + \beta \int_{\mathbb{G}} \left(\lambda \int_{\Delta} \{\mathbf{a}^*(g,\delta,B) V_C(g',\delta B) + (1-\mathbf{a}^*(g,\delta,B)) V_A(g',B)\} \pi_\Delta(d\delta) + (1-\lambda) V_A(g',B) \right) \times \pi_{\mathbb{G}}(dg'|g).$$

The policy functions are given

$$\mathbf{B}^*(g,B) = \arg\max_{B'} \left\{ W(g+B-p(g,B';\mathbf{e}^*)B') + \beta \int_{\mathbb{G}} V(g',B') \pi_{\mathbb{G}}(dg'|g) \right\},$$

$\mathbf{a}^*(g,\delta,B) = \arg\max\{V_C(g,\delta B), V_A(g,B)\}$ and $\mathbf{d}^*(g,B) = \arg\max \left\{ V_C(g,B), \bar{V}(g,B) \right\}$, where

$$\bar{V}(g,B) = \lambda \int_{\Delta} \{\mathbf{a}^*(g,\delta',B) V_C(g,\delta' B) + (1-\mathbf{a}^*(g,\delta',B)) V_A(g,B)\} \pi_\Delta(d\delta') + (1-\lambda) V_A(g,B).$$

Proof of Theorem 6.1. **Part (1).** By lemma C.4, $\delta \mapsto V_C(g,\delta B)$ is non-increasing, provided $B > 0$ (but this is the only case it matters since the government will never default on savings $B < 0$). On the other hand $V_A(g,B)$ is constant with respect to δ. Therefore if for some $\delta \in \Delta$, $\mathbf{a}^*(g,\delta,B) = 1$, then for all $\delta_1 \leq \delta$ the same must hold. Thus, there exists a $\bar{\delta} : \mathbb{G} \times \mathbb{B} \to [0,1]$ such that $1\{\delta \leq \bar{\delta}(g,B)\}$ iff $\mathbf{a}^*(g,\delta,B) = 1$.

To show that $B \mapsto \bar{\delta}(g,B)$ is non-increasing. It suffices to show that for all δ such that $\delta > \bar{\delta}(g,B_1)$ then $\delta > \bar{\delta}(g,B_2)$ for any $B_1 < B_2$. Since $\delta > \bar{\delta}(g,B_1)$, it follows that $V_C(g,\delta B_1) < V_A(g,B_1)$, $\forall (g,B_1,\delta) \colon \delta > \bar{\delta}(g,B_1)$.

Let $\epsilon(g,B_1,\delta) \equiv V_A(g,B_1) - V_C(g,\delta B_1) > 0$. Since g, B_1 and δ belong to discrete sets, there exists a $\epsilon > 0$ such that $\epsilon \leq \epsilon(g,B_1,\delta)$ $\forall (g,B_1,\delta) \colon \delta > \bar{\delta}(g,B_1)$.

Since $B \mapsto V_C(\bar{g},B)$ is non-increasing (see lemma C.4), it follows that $V_C(g,\delta B_2) \leq V_C(g,\delta B_1)$, $\forall (g,\delta) \in \mathbb{G} \times \Delta$. Therefore, $\forall (g,B_1,B_2,\delta) \colon \delta > \bar{\delta}(g,B_1)$,

$$\begin{aligned} V_C(g,\delta B_2) - V_A(g,B_2) \leq & V_C(g,\delta B_1) - V_A(g,B_2) \\ \leq & V_C(g,\delta B_1) - V_A(g,B_1) + \{V_A(g,B_1) - V_A(g,B_2)\}. \end{aligned}$$

Hence, if $\{V_A(g,B_1) - V_A(g,B_2)\} < \epsilon$, it follows that $V_C(g,B_2,\delta) - V_A(g,B_2) < 0$ and the desired result follows. To show that $\{V_A(g,B_1) - V_A(g,B_2)\} < \epsilon$, observe that

$$V_A(g,B_1) - V_A(g,B_2) = \beta \int_{\mathbb{G}} \{\bar{V}(g',B_1) - \bar{V}(g',B_2)\} \pi_{\mathbb{G}}(dg'|g).$$

And by lemma C.2, for any $\varepsilon > 0$, there exists a $\lambda(\varepsilon)$, such that

$$V_A(g,B_1) - V_A(g,B_1) < \varepsilon, \ \forall \lambda \in [0,\lambda(\varepsilon)].$$

Choosing $\varepsilon = \epsilon$ the result thus follows.

This implies that $V_A(g,B_2) < V_C(g,B_2,\delta)$, which means that δ is rejected, and by the first part of the argument, $\delta \geq \bar{\delta}(g,B_2)$.

Part (2). Following Arellano (2008) I show the result in two parts. Also, to simplify notation I omit $\mathbf{e}^*$ from p.

Step 1. I show that for any $B_1 < B_2$, $\mathbb{S}(B_1) \subseteq \mathbb{S}(B_2)$ where $\mathbb{S}(B) = \{g : \mathbf{d}^*(g,B) = 1\}$. If $\mathbb{S}(B_1) = \{\emptyset\}$ the proof is trivial, so I proceed with the case this does not hold and let $\bar{g} \in \mathbb{S}(B_1)$. If B_2 is not feasible, in the sense that there does not exist any B' such that $\bar{g} + B_2 - p(g,B')B' - R \leq 0$, then $\mathbb{S}(B_2) = \mathbb{G}$. And the result holds trivially, so I proceed with the case that B_2 is feasible, given $\bar{g}$.

It follows (since I assume that under indifference, the government chooses not to default) $V_C(\bar{g}, B_1) < \bar{V}(\bar{g}, B_1)$, and the same holds for any $(B,g) \in Graph(\mathbb{S})$. Since $B \mapsto V_C(\bar{g}, B)$ is no-increasing (see lemma C.4), it follows that

$$V_C(\bar{g}, B_2) \leq V_C(\bar{g}, B_1),\ \forall g \in \mathbb{G}\ and\ B_1 < B_2.$$

Therefore, for $(B_2, B_1, \bar{g}) \in \mathbb{B} \times Graph(\mathbb{S})$

$$\begin{aligned} V_C(\bar{g}, B_2) - \bar{V}(\bar{g}, B_2) \leq & V_C(\bar{g}, B_1) - \bar{V}(\bar{g}, B_2) \\ \leq & V_C(\bar{g}, B_1) - \bar{V}(\bar{g}, B_1) + \{\bar{V}(\bar{g}, B_1) - \bar{V}(\bar{g}, B_2)\}. \end{aligned}$$

I know that $V_C(\bar{g}, B_1) - \bar{V}(\bar{g}, B_1) \equiv -\epsilon(\bar{g}, B_1) < 0$. Thus, if $\bar{V}(\bar{g}, B_1) - \bar{V}(\bar{g}, B_2) < \epsilon(\bar{g}, B_1)$, then $V_C(\bar{g}, B_2, 1) < \bar{V}(\bar{g}, B_2)$ and the desired result follows.

Observe that $|\mathbb{B} \times Graph(\mathbb{S})| < \infty$, so there exists $\epsilon > 0$ such that $\epsilon \leq \epsilon(\bar{g}, B_1)$. By lemma C.2, there exists a $\lambda(\epsilon) > 0$ such that

$$|\bar{V}(g, B_1) - \bar{V}(g, B_2)| < \epsilon,\ \forall \lambda \in [0, \lambda(\epsilon)]\ and\ (g, B_1, B_2) \in \mathbb{G} \times \mathbb{B}^2.$$

Hence, $V_C(\bar{g}, B_2) - \bar{V}(\bar{g}, B_2) < 0$, thereby implying that $\bar{g} \in \mathbb{S}(B_2)$.

Step 2. I show that, for any B and any $g_1 < g_2$, if g_1 is such that $\mathbf{d}^*(g_1, B) = 1$, then $\mathbf{d}^*(g_2, B) = 1$.

Let n_i^C be the optimal choice of labor, when $g = g_i$; n_i^A and B_i^C are defined similarly. Let $BL(n,g) \equiv R(1, 1-n) - g$. Since $g_2 > g_1$, $BL(n_1^C, g_2) \leq BL(n_1^C, g_1) = B - p(g, B_1^C)B_1^C$ and $BL(n_2^C, g_1) \geq BL(n_2^C, g_2) = B - p(g, B_2^C)B_2^C$. Let $\tilde{n}$ be such that $BL(\tilde{n}, g_1) = BL(n_2^C, g_2)$. Since in the relevant domain $n \mapsto BL(n,g)$ is non-increasing, it follows that $\tilde{n} \geq n_2^C$; moreover, $(\tilde{n}, B_2^C)$ are feasible for (B, g_1). It is also true that $n_1^A < n_2^A$ since $g_1 < g_2$. Also, since in (g_1, B) the government defaults, it follows from the proof of lemma 6.1 that $B - p(g, B_2^C)B_2^C \geq 0$ — this follows because $(\tilde{n}, B_2^C)$ are feasible for (B, g_1) and it cannot roll over its debt — and thus $\tilde{n} \leq n_1^A$ since $n \mapsto BL(n,g)$ is non-increasing.

Therefore,

$$BL(n_2^C, g_2) = B - p(g, B_2^C)B_2^C = BL(\tilde{n}, g_1) \iff R(1, 1-n_2^C) - g_2 = R(1, 1-\tilde{n}) - g_1$$

iff $R(1, 1-n_2^C) - R(1, 1-n_2^A) = R(1, 1-\tilde{n}) - R(1, 1-n_1^A)$. Observe that $n_2^A > n_1^A$ implies $\tilde{n} > n_2^C$, and $\tilde{n} \leq n_1^A$ implies that $R(1, 1-n_2^C) - R(1, 1-n_2^A) \geq 0$, and thus $n_2^C \leq n_2^A$. There could be two possibilities (a) $n_2^C \leq n_2^A \leq \tilde{n} \leq n_1^A$ (with at least inequalities strict) or (b) $n_2^C < \tilde{n} \leq n_2^A < n_1^A$.

I now analyze (a). By our assumptions $n \mapsto BL(n,g)$ is non-increasing and strictly concave (since $R''(1,1-n) = 2H''(1-n) - H'''(1-n)n < 0$ by assumption 6.1). thus

$$\frac{R(1,1-n_2^C) - R(1,1-n_2^A)}{n_2^C - n_2^A} > \frac{R(1,1-\tilde{n}) - R(1,1-n_1^A)}{\tilde{n} - n_1^A}$$

which implies $\tilde{n} - n_1^A > n_2^C - n_2^A$. (b) is analogous and will not be repeated here.

Let $F(n,g) \equiv n - g + H(1-n)$, since $\pi_{\mathbb{G}}(.|g_1)$ is FOSD by $\pi_{\mathbb{G}}(.|g_2)$, $g \mapsto V_C(B,g)$ is no-increasing (see lemma C.4), and the fact that $(\tilde{n}, B_2^C)$ is feasible for (g_1, B) and $g_2 \in \mathbb{G}$,

$$F(\tilde{n}, g_1) - F(n_2^C, g_2) \leq V_C(B, g_1) - V_C(B, g_2).$$

Since $n \mapsto F(n,g) \equiv n - g + H(1-n)$ is concave it follows that $\frac{F(n_2^A,g_2)-F(n_2^C,g_2)}{n_2^A - n_2^C} > \frac{F(n_1^A,g_1)-F(\tilde{n},g_2)}{n_1^A - \tilde{n}}$, and since $\tilde{n} - n_1^A > n_2^C - n_2^A$, $F(n_2^A, g_2) - F(n_2^C, g_2) > F(n_1^A, g_1) - F(\tilde{n}, g_2)$. Therefore,

$$F(n_1^A, g_1) - F(n_2^A, g_2) < F(\tilde{n}, g_1) - F(n_2^C, g_2) \leq V_C(B, g_1) - V_C(B, g_2).$$

By the same arguments in the proof of lemma C.2, for any $\epsilon > 0$, there exists a $\bar{\lambda}$, such that $\forall \lambda \in [0, \bar{\lambda})$

$$\bar{V}(g_2, B) - \bar{V}(g_1, B) \leq \epsilon.$$

By choosing $\epsilon > 0$, smaller than $V_C(g_1,B) - V_C(g_2,B) - \{F(n_1^A, g_1) - F(n_2^A, g_2)\} > 0$ (since there are finitely many g_i and B, choose the minimal one), it follows that $\bar{V}(g_1,B) - \bar{V}(g_2,B) < V_C(B,g_1) - V_C(B,g_2)$, since $V_C(g_1,B) - \bar{V}(g_1,B) \leq 0$, this implies that $V_C(g_2,B) < \bar{V}(B,g_2)$, as desired.

Hence, step 2 establishes that $\mathbf{d}^*$ is of the threshold type, since it shows that, for any B, if $\mathbf{d}^*(g,B) = 1$, the same is true for any $g' > g$. That is $\{g : \mathbf{d}^*(g,B) = 1\}$ is of the form $\{g : g \geq \bar{g}(B)\}$. Step 1 shows that the $\bar{g}$ ought to be non-increasing. □

Proof of Theorem 6.2. **Part 1.** First, by equation 6.12 it is easy to see that when $g \sim iid\pi_{\mathbb{G}}$, $q(g,B) = q(B)$. Moreover,

$$q(B_t) = \beta\lambda \int_{\mathbb{G}} D(g', B_t)\pi_{\mathbb{G}}(dg') + \left(\beta \int_{\mathbb{G}} (\lambda\alpha(g', B_t) + (1-\lambda))\pi_{\mathbb{G}}(dg')\right) q(B_t),$$

and after simple algebra,

$$\begin{aligned} q(B_t) =& \frac{\beta\lambda \int_{\mathbb{G}} D(g', B_t)\pi_{\mathbb{G}}(dg')}{1 - \beta\left(\lambda \int_{\mathbb{G}} \alpha(g', B_t)\pi_{\mathbb{G}}(dg') + (1-\lambda)\right)} \\ =& \frac{\beta\lambda\delta_0\pi_{\mathbb{G}}(\{g\colon \delta_0 \leq \bar{\delta}(g, B_t)\})}{1 - \beta\left(1 - \lambda\pi_{\mathbb{G}}(\{g\colon \delta_0 \leq \bar{\delta}(g, B_t)\})\right)} \end{aligned}$$

where the last equality follows from the fact that, since $\pi_\Delta = \mathbf{1}_{\delta_0}(\delta)$, $D(g,B) \equiv \int_\Delta 1\{\delta \leq \bar{\delta}(g,B)\}\delta\pi_\Delta(d\delta) = \delta_0 1\{\delta_0 \leq \bar{\delta}(g,B)\}$ and $\int_\Delta 1\{\delta \leq \bar{\delta}(g,B)\}\pi_\Delta(d\delta) = 1\{\delta_0 \leq \bar{\delta}(g,B)\}$.

From theorem 6.1(1), $B \mapsto \bar{\delta}(g,B)$ is non-increasing. Therefore, $\{g\colon \delta_0 \leq \bar{\delta}(g,B_1)\} \supseteq \{g\colon \delta_0 \leq \bar{\delta}(g,B_2)\}$, for $B_1 \leq B_2$. Hence $B \mapsto \pi_{\mathbb{G}}(\{g\colon \delta_0 \leq \bar{\delta}(g,B)\})$ is non-increasing. It is easy to see that q is of the form $q(B) = \psi \circ \pi_{\mathbb{G}}(\{g\colon \delta_0 \leq \bar{\delta}(g,B)\})$ where $\psi(t) = \frac{\lambda\beta\delta_0 t}{1+\beta\lambda t - \beta}$ and ψ is increasing. So

$B \mapsto q(B)$ is non-increasing.

Part 2. From part 1, note that $q(B) \leq \delta_0$. Also, observe that under assumptions

$$\begin{aligned} p(g_t, B_{t+1}) = p(B_{t+1}) =& \beta \int_{\mathbb{G}} 1\{g < \bar{g}(B_{t+1})\}\pi_{\mathbb{G}}(dg) + \beta\lambda \int_{\mathbb{G}} 1\{g \geq \bar{g}(B_{t+1})\}D(g, B_{t+1})\pi_{\mathbb{G}}(dg) \\ &+ \beta \int_{\mathbb{G}} 1\{g \geq \bar{g}(B_{t+1})\}(\lambda\alpha(g, B_{t+1}) + (1-\lambda))\}\pi_{\mathbb{G}}(dg)q(B_{t+1}) \\ =& \beta \int_{\mathbb{G}} 1\{g < \bar{g}(B_{t+1})\}\pi_{\mathbb{G}}(dg) + \beta(1-\lambda) \int_{\mathbb{G}} 1\{g \geq \bar{g}(B_{t+1})\}\pi_{\mathbb{G}}(dg)q(B_{t+1}) \\ &+ \beta\lambda \int_{\mathbb{G}} 1\{g \geq \bar{g}(B_{t+1})\}F(g, B_{t+1})\pi_{\mathbb{G}}(dg) \end{aligned}$$

where $F(g, B) \equiv 1\{\delta_0 \leq \bar{\delta}(g, B)\}\delta_0 + (1 - 1\{\delta_0 \leq \bar{\delta}(g, B)\})q(B) = q(B) + 1\{\delta_0 \leq \bar{\delta}(g, B)\}[\delta_0 - q(B)]$. First note that $B \mapsto F(g, B)$ is non-increasing: Let $B_1 \leq B_2$, then

$$\begin{aligned} F(g, B_1) \geq& 1\{\delta_0 \leq \bar{\delta}(g, B_2)\}\delta_0 + (1 - 1\{\delta_0 \leq \bar{\delta}(g, B_2)\})q(B_1) \\ \geq& 1\{\delta_0 \leq \bar{\delta}(g, B_2)\}\delta_0 + (1 - 1\{\delta_0 \leq \bar{\delta}(g, B_2)\})q(B_2) = F(g, B_2), \end{aligned}$$

where the first line follows from the fact that $B \mapsto 1\{\delta_0 \leq \bar{\delta}(g, B)\}$ is non-increasing and $\delta_0 - q(B) \geq 0$; the second line follows from the fact that $B \mapsto q(B)$ is non-increasing. Second, note that $F(g, B) \in [0, \delta_0]$.

Now consider $B_1 \leq B_2$,

$$\begin{aligned} p(B_1) \geq& \beta \int_{\mathbb{G}} 1\{g < \bar{g}(B_2)\}\pi_{\mathbb{G}}(dg) + \beta(1-\lambda) \int_{\mathbb{G}} 1\{g \geq \bar{g}(B_2)\}\pi_{\mathbb{G}}(dg)q(B_1) \\ &+ \beta\lambda \int_{\mathbb{G}} 1\{g \geq \bar{g}(B_2)\}F(g, B_1)\pi_{\mathbb{G}}(dg) \\ \geq& \beta \int_{\mathbb{G}} 1\{g < \bar{g}(B_2)\}\pi_{\mathbb{G}}(dg) + \beta(1-\lambda) \int_{\mathbb{G}} 1\{g \geq \bar{g}(B_2)\}\pi_{\mathbb{G}}(dg)q(B_2) \\ &+ \beta\lambda \int_{\mathbb{G}} 1\{g \geq \bar{g}(B_2)\}F(g, B_2)\pi_{\mathbb{G}}(dg) = p(g_t, B_2) \end{aligned}$$

where the first line follows from the fact that $B \mapsto 1\{g < \bar{g}(B)\}$ is non-increasing and $1 > (1-\lambda)q(B_1) + \lambda F(g, B_1)$; the second line follows from the fact that both q and $F(g, \cdot)$ are non-increasing. □

Proof of Theorem 6.3. First, assumption (i) implies that $\pi_{\mathbb{G}}(g \leq \bar{g}(\underline{B})) = 1$. Because either $\underline{B} < 0$ and the government never defaults or $\underline{B} = 0$ and (at most) the government is indifferent between defaulting or not, and by assumption it does not do so.

If $\underline{B} < 0$, then $\frac{dp(g,B)B}{dB}$ is clearly positive. If $\underline{B} = 0$, then

$$\begin{aligned}\frac{dp(g,0)}{dB} =& (\beta - \beta(1-\lambda)q(0))) \frac{d\pi_{\mathbb{G}}(g \leq \bar{g}(0))}{dB} + \beta\lambda \frac{d\int 1\{g > \bar{g}(B)\} F(g,B)\pi_{\mathbb{G}}(dg)}{dB} |_{B=0} \\ =& (\beta - \beta(1-\lambda)q(0)) \frac{d\pi_{\mathbb{G}}(g \leq \bar{g}(0))}{dB} + \beta\lambda \int 1\{g > \bar{g}(0)\} \frac{dF(g,B)}{dB} |_{B=0} \pi_{\mathbb{G}}(dg) \\ &+ \beta\lambda \int \frac{d1\{g > \bar{g}(B)\}}{dB} |_{B=0} F(g,0)\pi_{\mathbb{G}}(dg) \\ =& (\beta - \beta(1-\lambda)q(0)) \frac{d\pi_{\mathbb{G}}(g \leq \bar{g}(0))}{dB} + \beta\lambda \int \frac{d1\{g > \bar{g}(B)\}\pi_{\mathbb{G}}(dg)}{dB} |_{B=0} F(g,0),\end{aligned}$$

where the third line follows from the fact that $g > \bar{g}(\underline{B})$ for all g. Since $B \mapsto 1\{g > \bar{g}(B)\}$ is nondecreasing and $F > 0$, the second term is non-negative. By analogous argument, the same holds true for the first term. Therefore, $\frac{dp(g,0)}{dB} > 0$, thus implying $\frac{dp(g,B)B}{dB} |_{B=0} \geq p(0) > 0$.

Note that revenue from zero debt is zero. Therefore, taking an "infinitesimal" amount of positive debt increases the revenue from debt. On the other hand, by assumption (ii), $p(g, \bar{B}) = 0$. So, there exists an amount of debt in $(\underline{B}, \overline{B})$ that yields maximal $p(g,B)B$, I denote this value as B^* (does not depend on g, because p does not depend).

I claim that the government never chooses a level of debt above the value that achieves the highest point. To show this, suppose not. Take any $B' > B^*$. By continuity of $B \mapsto p(B)B$ and the fact that B^* is maximal, there exists a $B'' \in [0, B^*]$ such that $p(B'')B'' = p(B')B'$ and $V_C(g,B'') \geq V_C(g,B')$ (by lemma C.4, $B \mapsto V_C(g,B)$ is non-increasing); hence the government will never choose (optimally) B' over B''. □

Proof of Lemma 6.1. Since $\mathcal{D}$ is non-empty; there exists at least a pair $(g,B) \in \mathcal{D}$. By construction $V_C(g,1,B) < \bar{V}(g,B)$ (by assumption, if indifferent, the government does not default) where

$$\bar{V}(g,B) \equiv \lambda \int_{\Delta} \{\mathbf{a}^*(g,\delta',B)V_C(g,\delta' B) + (1-\mathbf{a}^*(g,\delta',B))V_A(g,B)\}\pi_{\Delta}(d\delta') + (1-\lambda)V_A(g,B).$$

By lemma C.1(2) W is decreasing, provided that $H'(1-\mathbf{n}_C^*(g,1,B)) < \kappa$. Since $H'' \neq 0$, then H' is monotonic, so the only point (in the relevant domain) for which $H'(1-n) = \kappa$ is $n = n_1(\kappa)$. Since by construction of $\mathcal{D}$, $1 - \mathbf{n}_C^*(g,1,B) \neq n_1(1)$, it follows that W is decreasing.

Step 1. I first show that for $(g,B) \in \mathcal{D}$, $B - p(g,\mathbf{B}^*(g,B);\mathbf{e}^*)\mathbf{B}^*(g,B) \geq 0$. Suppose not, that is, $B - p(g,\mathbf{B}^*(g,B);\mathbf{e}^*)\mathbf{B}^*(g,B) < 0$. Thus, for any $(g,B) \in \mathcal{D}$,

$$W(g + B - p(g,\mathbf{B}^*(g,B);\mathbf{e}^*)\mathbf{B}^*(g,B)) - W(g) \equiv \eta(g,B) > 0.$$

Since $\mathbb{G} \times \mathbb{B}$ are a finite collection of points, so is $\mathcal{D}$. Hence, it follows that there exists a $\eta > 0$ such that $\eta \leq \eta(g,B)$ for all $(g,B) \in \mathcal{D}$.

Also, note that for any $B' \neq B$

$$V(g,B') - \bar{V}(g,B) = V(g,B') - \bar{V}(g,B') + \{\bar{V}(g,B') - \bar{V}(g,B)\}.$$

By lemma C.2, for any $\epsilon > 0$, there exists a $\lambda(\epsilon)$ such that

$$V(g,B') - \bar{V}(g,B) \geq V(g,B') - \bar{V}(g,B') - \epsilon, \ \forall (B,B',g).$$

By choosing $\epsilon \equiv \beta^{-1}0.5\eta$ (and set $\bar{\lambda} \equiv \lambda(\beta^{-1}0.5\eta)$), it follows that, for all $(g,B) \in \mathcal{D}$.

$$\begin{aligned} V_C(g,B) - V_A(g,B) =& \eta(g,B) + \beta \int_{\mathbb{G}} \{V(g,\mathbf{B}^*(g,B)) - \bar{V}(g,B)\}\pi_{\mathbb{G}}(dg'|g) \\ \geq& \eta - \beta\epsilon + \beta \int_{\mathbb{G}} \{V(g,\mathbf{B}^*(g,B)) - \bar{V}(g,\mathbf{B}^*(g,B))\}\pi_{\mathbb{G}}(dg'|g) \\ \geq& 0.5\eta > 0 \end{aligned}$$

where the last line follows from our previous calculations and the fact that $V(g,\mathbf{B}^*(g,B)) - \bar{V}(g,\mathbf{B}^*(g,B)) \geq 0$ (because $V(g,B) = \max\{V_C(g,B), \bar{V}(g,B)\}$). This implies that $V_C(g,B) > V_A(g,B)$. Therefore, since $\lambda < 1$, it implies that $V(g,B) > \bar{V}(g,B)$. But this is a contradiction to the fact that $\mathbf{d}^*(g,B) = 1$.

Step 2. By step 1, it follows that $B \geq p(g,\mathbf{B}^*(g,B);\mathbf{e}^*)\mathbf{B}^*(g,B)$ for all $(g,B) \in \mathcal{D}$. This implies that the revenue under financial access ought to be strictly greater than the revenue under financial autarky, i.e.,

$$R(1,\mathbf{n}_C^*(g,1,B)) = g + B - p(g,\mathbf{B}^*(g,B);\mathbf{e}^*)\mathbf{B}^*(g,B) > g = R(\kappa,\mathbf{n}_A^*(g)).$$

Since $\kappa = 1$ and $R' < 0$ (see lemma C.1(1)) it follows that $\mathbf{n}_C^*(g,1,B) < \mathbf{n}_A^*(g)$. Since $H'' < 0$, the previous statement implies that $H'(1-\mathbf{n}_C^*(g,1,B)) < H'(1-\mathbf{n}_A^*(g))$ iff $1-\tau_C^*(g,B,1) < 1-\tau_A^*(g) \iff \tau_A^*(g) < \tau_C^*(g,B,1)$. □

C.1. Derivation of Equation 6.13

By our characterization of the default rule. In this setting, to default or not, boils down to choosing a T (contingent on ω) such that for all $t < T$ there is no default and for $t \geq T$ there is financial autarky. Thus, the optimal T, n^∞ and B^∞ must solve the following program: $\sup_T \sup_{(n^\infty,B^\infty)\in\Gamma(T)} \mathcal{L}(\omega_0;n^\infty,T)$, where:

$$\mathcal{L}(\omega_0;n^\infty,T) \equiv \int_\Omega \left(\sum_{t=0}^{T(\omega)-1} \beta^t u(n_t(\omega)-g_t(\omega),1-n_t(\omega)) + \sum_{t=T(\omega)}^{\infty} \beta^t u(n_t^A(\omega)-g_t(\omega),1-n_t^A(\omega)) \right) \Pi(d\omega|\omega_0).$$

Where $n_t^A(\omega)$ is such that $\left(\kappa_t(\omega) - \frac{u_l(\kappa_t(\omega)n_t(\omega)-g_t(\omega),1-n_t(\omega))}{u_c(\kappa_t(\omega)n_t(\omega)-g_t(\omega),1-n_t(\omega))}\right) n_t(\omega) = g_t(\omega)$ for all t and ω, and

$\Gamma(T) \equiv$

$\{(n^\infty,B^\infty)\colon \forall\omega\in\Omega,\ 0 \leq A(n_t(\omega),g_t(\omega),B_t(\omega)) + \mathcal{P}_t(\omega;B_{t+1})B_{t+1}(\omega)\ \forall t < T \textit{ and } n_t(\omega) = n_t^A(\omega)\ \forall t \geq T(\omega)\}$

where $A(n,g,B) = (u_c(n-g,1-g) - u_l(n-g,1-n))\,n - (g+B)u_c(n-g,1-g)$ and $\mathcal{P}_t(\omega;B_{t+1}) \equiv p(\omega;B_{t+1})u_c(n_t(\omega)-g_t,1-n_t(\omega))$.

By assumption, the solution of B_{t+1} is in the interior. Given a particular T, and for (t,ω) such that $t < T(\omega)$, the optimal choice for (n^∞,B^∞) ought to satisfy

$$u_c(n_t(\omega)-g_t(\omega),1-n_t(\omega)) - u_l(n_t(\omega)-g_t(\omega),1-n_t(\omega)) - \nu_t(\omega)\frac{dA(n_t(\omega),g_t(\omega),B_t(\omega))}{dn_t} = 0$$

and

$$-\nu_t(\omega)\left\{p_t(\omega;B_{t+1}) + \frac{p_t(\omega;B_{t+1})}{B_{t+1}}B_{t+1}\right\} + \beta\int_{\{\tilde{\omega}\in\Omega\colon\,\tilde{\omega}^t=\omega^t\}} \nu_{t+1}(\tilde{\omega})\Pi(d\tilde{\omega}|\omega^t) = 0.$$

Observe that $\int_{\{\tilde{\omega}\in\Omega\colon\,\tilde{\omega}^t=\omega^t\}} \nu_{t+1}(\tilde{\omega})\Pi(d\tilde{\omega}|\omega^t) = \int_{\mathbb{G}} \nu_{t+1}(\omega^t,g_{t+1})\pi_{\mathbb{G}}(dg_{t+1}|g_t)$.

Where $(t,\omega) \mapsto \nu_t(\omega)$ is the Lagrange multiplier of $0 \leq A(n_t(\omega), g_t(\omega)) + p_t(\omega; B_{t+1})B_{t+1}(\omega)$ and is non-negative. Also, note that if $\nu_t(\omega) = 0$, then

$$u_c(n_t(\omega) - g_t(\omega), 1 - n_t(\omega)) - u_l(n_t(\omega) - g_t(\omega), 1 - n_t(\omega)) = 0$$

which implies that $\tau_t(\omega) = 0$.

C.2. Supplementary Lemmas

The proofs for these lemmas are relegated to appendix I.

Lemma C.1. *Suppose assumption 6.1 holds. Then:*

(1) $R(\kappa, \cdot)\colon (0,1] \to \mathbb{R}$ *is such that the the optimal level of labor belongs to* $[n_2(\kappa), n_1(\kappa)]$ *where* $R(\kappa, n_1(\kappa)) = 0$ *and* $R'(\kappa, n_2(\kappa)) = 0$*, and in this domain* $R' < 0$*.*

(2) $W : [0, R(\kappa, n_2(\kappa))] \to \mathbb{R}$ *is differentiable, uniformly bounded, non-increasing (decreasing over all* R *such that* $(-\kappa + H'(1 - n(\kappa, R))) < 0$*) function. Where, for any* κ*,* $R \mapsto n(\kappa, R)$ *is the inverse of* $R(\kappa, \cdot)$ *(by part (1) exists in the relevant domain).*

Lemma C.2. *Suppose assumption 6.1 holds. For any* $\epsilon > 0$*, there exists a* $\lambda(\epsilon) > 0$ *such that, for all* B *and* $B + h$ *in* $\mathbb{B}$ *and* $g \in \mathbb{G}$*,* $\left|\bar{V}(g, B) - \bar{V}(g, B + h)\right| \leq \epsilon$*.*

Lemma C.3. *Suppose assumption 6.1 hold. Then* $V_C \in L^{\infty}(\mathbb{G} \times \mathbb{B})$ *and* $V_A \in L^{\infty}(\mathbb{G} \times \mathbb{B})$*.*

Lemma C.4. *Suppose assumption 6.1 hold. Then:*

(1) V_C *is non-increasing in* B *(and thus in* δ*, whenever* $B > 0$*).*

(2) V_A *is non-increasing in* B*.*

(3) *If* $\pi_{\mathbb{G}}(\cdot|g_1) <_{FOSD} \pi_{\mathbb{G}}(\cdot|g_2)$ *for* $g_1 < g_2$*,* V_A *and* V_C *are non-increasing as a function of* g*.*

Appendix D. Figures and Tables

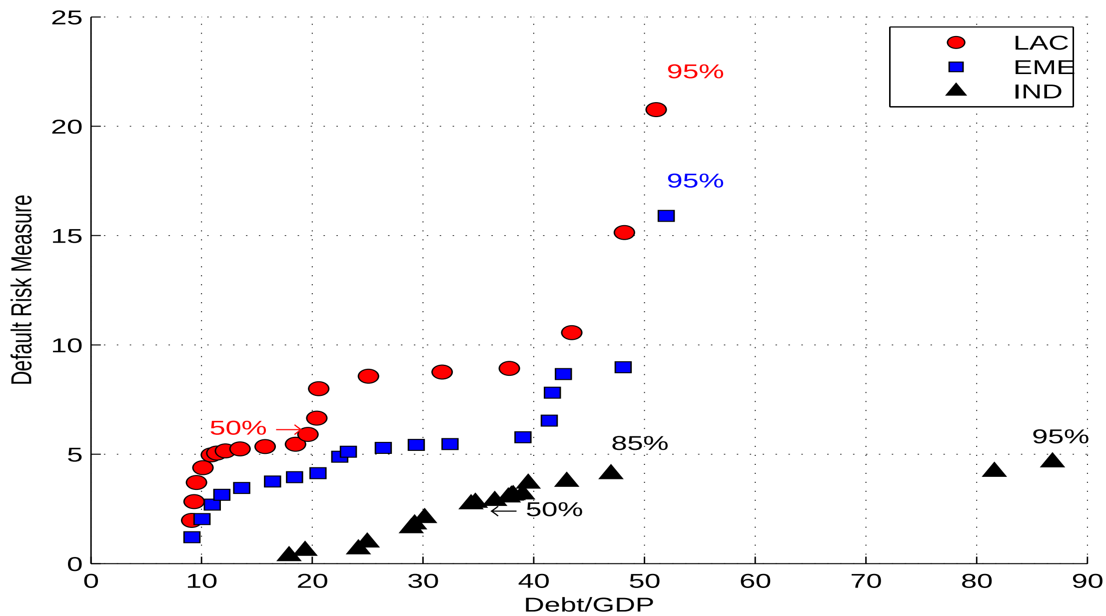

FIGURE D.1. The percentiles of the domestic government debt-to-output ratio and of a measure of default risk for three groups: IND (black triangle), EME (blue square) and LAC (red circle)

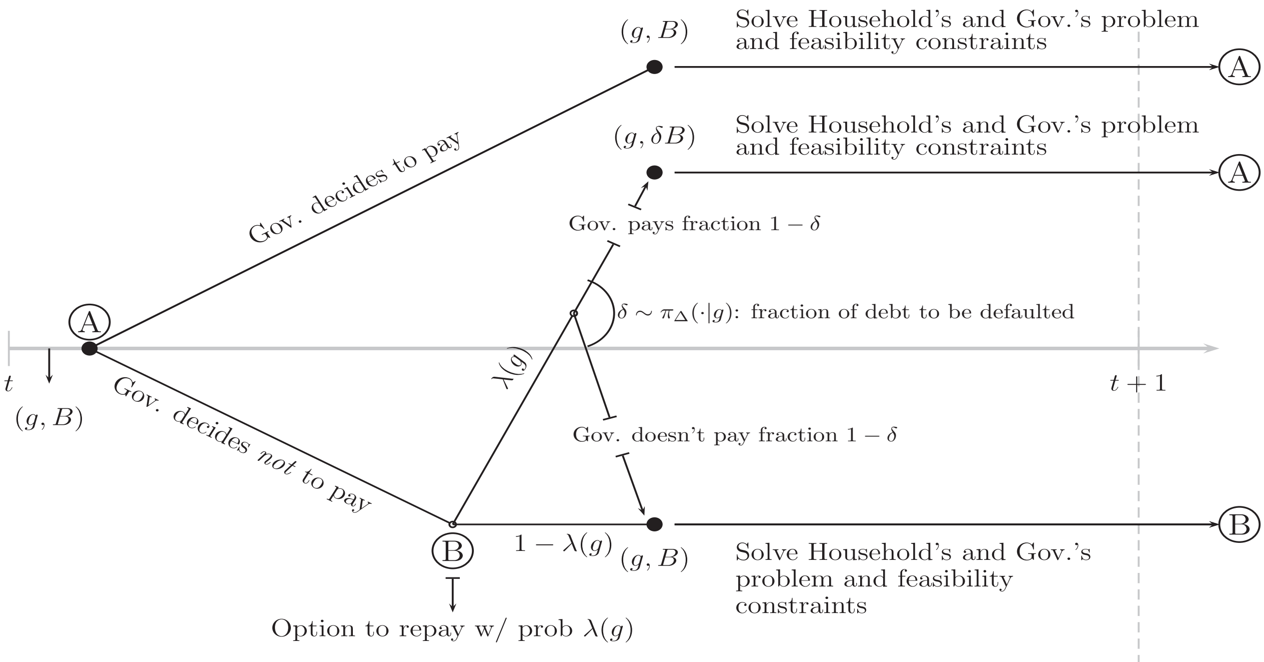

FIGURE D.2. Timing of the Model

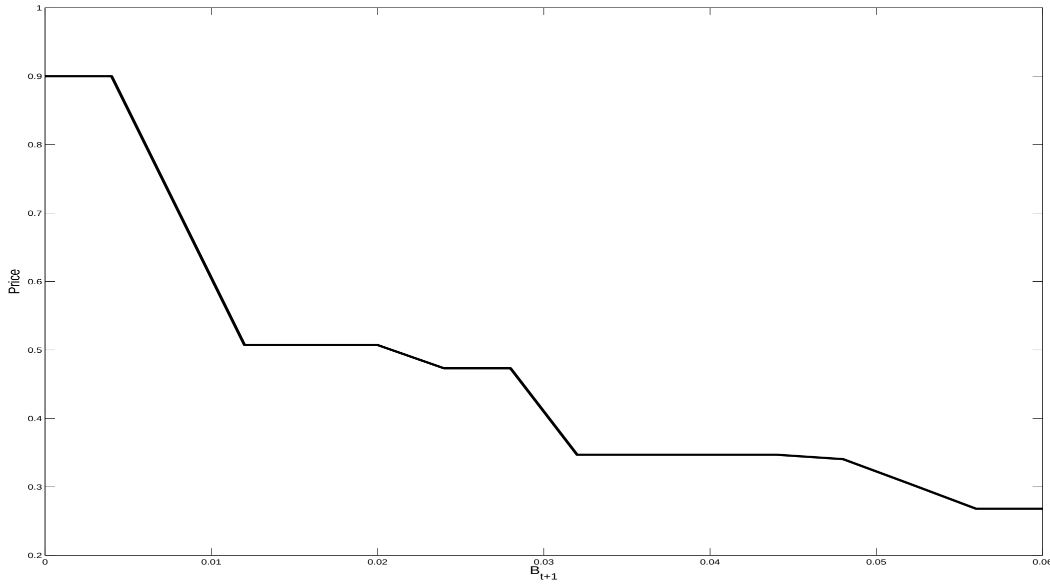

FIGURE D.3. Equilibrium price as a function of future debt.

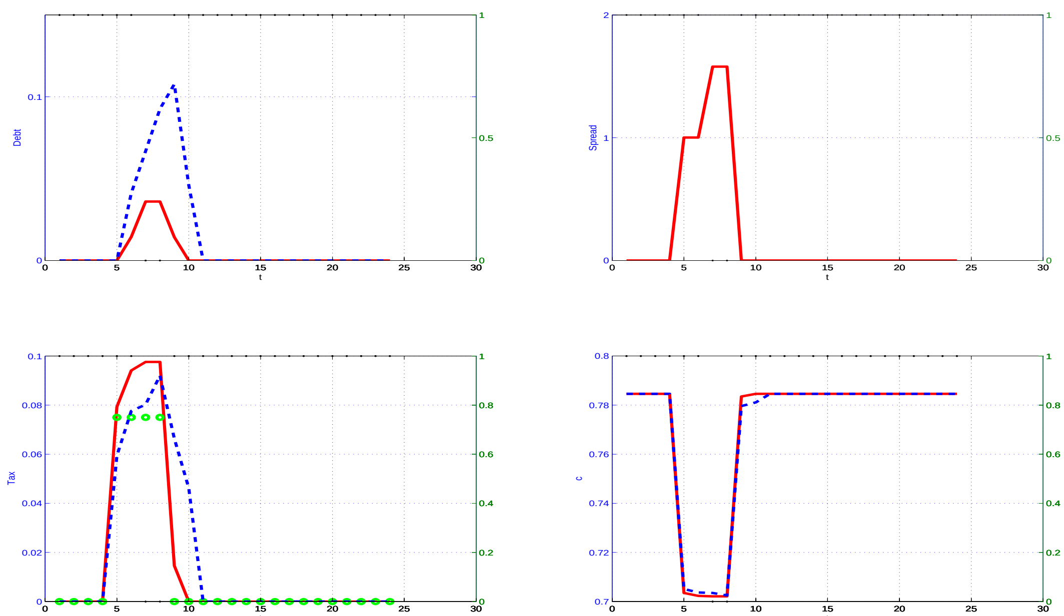

FIGURE D.4. Impulse Responses for a particular realization of $(g_t)_{t=0}^{25}$. In all the panels, solid red line belongs to the ED Economy and the dashed blue line belongs to the AMSS Economy.

Supplementary Online Material

Appendix E. Proof of Lemma 7.1

Proof of Lemma 7.1. Let $W_\kappa(g) \equiv \kappa n_A(g) - g + H(1 - n_A(g))$, where $n_A(g)$ is the solution for a balance primary budget, given g, and let $W(g, B, B')$ be the per-period payoff outside autarky. Let $\Delta W_\kappa(g, B, B') \equiv W(g, B, B') - W_\kappa(g)$.

It suffices to show that, for any state (g, B) which the government defaults under κ, it does under κ'.

From the definition of the Bellman equation (see appendix C), it follows

$$
\begin{aligned}
\tilde{V}_\kappa(g, B, \bar{B}) \equiv & V_C(g, B) - V_A(g, \bar{B}) \\
= & \Delta W_\kappa(g, B, \mathbf{B}^*(g, B)) \\
& + \beta \int_{\mathbb{G}} \max\{\tilde{V}_\kappa(g', \mathbf{B}^*(g, B), \mathbf{B}^*(g, B)) - \lambda \int_\Delta \max\{\tilde{V}_\kappa(g', \delta\mathbf{B}^*(g, B), \mathbf{B}^*(g, B)), 0\}\pi_\Delta(d\delta), 0\}\pi_{\mathbb{G}}(dg'|g) \\
& + \beta \int_{\mathbb{G}} \{\bar{V}(g', \mathbf{B}^*(g, B)) - \bar{V}(g', \bar{B})\}\pi_{\mathbb{G}}(dg'|g).
\end{aligned}
$$

Where $\bar{V}(g, B) \equiv \lambda \int_\Delta \max\{V_C(g, \delta B), V_A(g, B)\}\pi_\Delta(d\delta) + (1 - \lambda)V_A(g, B)$. Thus, if $\bar{B} = \mathbf{B}^*(g, B)$ the last term in the RHS vanishes and the above display defines a functional equation for which $\tilde{V}_\kappa(g, B, \mathbf{B}^*(g, B))$ is the fixed point. Formally, let $\mathbb{S}_\kappa$ be an operator from $L^\infty(\mathbb{G} \times \mathbb{B}^2)$ to itself such that, for any $F \in L^\infty(\mathbb{G} \times \mathbb{B})$,

$$
\begin{aligned}
\mathbb{S}_\kappa[F](g, B) \equiv & \Delta W_\kappa(g, B, \mathbf{B}^*(g, B)) \\
& + \beta \int_{\mathbb{G}} \max\{F(g', \mathbf{B}^*(g, B)) - J_\lambda(g', \mathbf{B}^*(g, B)), 0\}\pi_{\mathbb{G}}(dg'|g)
\end{aligned}
$$

where $J_\lambda(g', \mathbf{B}^*(g, B)) \equiv \lambda \int_\Delta \max\{F(g', \delta\mathbf{B}^*(g, B)), 0\}\pi_\Delta(d\delta)$. Observe that $\tilde{V}^*_\kappa \equiv \tilde{V}_\kappa(\cdot, \cdot, \mathbf{B}^*(\cdot, \cdot))$ is a fixed point of $\mathbb{S}_\kappa$.

Under our assumption that $\lambda = 0$, it follows that

$$
\mathbb{S}_\kappa[F](g, B) \equiv \Delta W_\kappa(g, B, \mathbf{B}^*(g, B)) + \beta \int_{\mathbb{G}} \max\{F(g', \mathbf{B}^*(g, B)), 0\}\pi_{\mathbb{G}}(dg'|g).
$$

Claim E.1 establishes that (i) $\mathbb{S}_\kappa$ is a contraction for all κ and that is monotonic, and that (ii) $\mathbb{S}_\kappa[F] > \mathbb{S}_{\kappa'}[F]$ for all $\kappa < \kappa'$.

From these two properties, it follows that $\mathbb{S}^2_\kappa[F] = \mathbb{S}_\kappa[\mathbb{S}_\kappa[F]] \geq \mathbb{S}_{\kappa'}[\mathbb{S}_\kappa[F]] \geq \mathbb{S}_{\kappa'}[\mathbb{S}_{\kappa'}[F]] = \mathbb{S}^2_{\kappa'}[F]$, where the first inequality follows from (ii) and the second from (i, monotonicity). Hence, it follows that, for any t, $\tilde{V}^*_\kappa = \mathbb{S}^t_\kappa[\tilde{V}^*_\kappa] \geq \mathbb{S}^t_{\kappa'}[\tilde{V}^*_\kappa]$. Taking t to infinity, since $\mathbb{S}_{\kappa'}$ is a contraction, it must follows that $\mathbb{S}^t_{\kappa'}[\tilde{V}^*_\kappa] \to \tilde{V}^*_{\kappa'}$ and thus $\tilde{V}^*_\kappa \geq \tilde{V}^*_{\kappa'}$. Therefore, $V^\kappa_C(g, B) - V^\kappa_A(g, \mathbf{B}^*_\kappa(g, B)) \geq V^{\kappa'}_C(g, B) - V^{\kappa'}_A(g, \mathbf{B}^*_{\kappa'}(g, B))$ (V^κ_C and so on, is just notation for V_C, making the dependence on κ explicit). Also, since $\lambda = 0$ by assumption, it follows that $V^\kappa_A(g, B) = V^\kappa_A(g)$ so $V^\kappa_C(g, B) - V^\kappa_A(g) \geq V^{\kappa'}_C(g, B) - V^{\kappa'}_A(g)$.

Suppose now, that for κ, (g, B) is a state for which the government defaults. Then $V^\kappa_C(g, B) - V^\kappa_A(g) < 0$, which readily implies that $V^{\kappa'}_C(g, B) - V^{\kappa'}_A(g) < 0$. $\square$

The proof of the next claim is relegated to appendix I.

Claim E.1. *For any κ, the operator $\mathbb{S}_{\kappa}$ defined in the proof of lemma 7.1 is a contraction, monotonic and $\mathbb{S}_{\kappa}[F] > \mathbb{S}_{\kappa'}[F]$ for all $\kappa < \kappa'$.*

Appendix F. Proof of Lemma B.1

Proof of Lemma B.1. Under assumption 4.1(i) the objective function of the household optimization problem is strictly concave. The budget constraints and debt constraint form a convex set of constraints. Thus, if the transversality condition holds, the FONC are sufficient; this follows from a simple adaptation of the results in Stokey et al. (1989) Ch. 4.5.

In order to verify the transversality condition, it suffices to show that for any $\zeta_t(\omega)$ such that $b_t(\omega) + \zeta_t(\omega) \in \mathbb{B}$,

$$\lim_{T\to\infty} \beta^T E_{\Pi}[u_c(\kappa_T(\omega;\sigma^{\infty})n_T(\omega) - g_T(\omega), 1 - n_T(\omega))\varrho_T(\omega)\zeta_T(\omega)] = 0.$$

Since, by assumption, debt is constrained, this condition follows from Magill and Quinzii (1994) Thm. 5.2. □

Appendix G. Equilibrium Characterization

The next lemma characterizes the set $CEG(s_0)$ as a sequence of restrictions involving FONC and budget constraints.

Lemma G.1. *Suppose assumption 4.1(i) holds. The tuple $\{\sigma^{\infty}, c^{\infty}, n^{\infty}, b^{\infty}, p^{\infty}\} \in CEG(s_0)$ iff given a $B_0 = b_0$ and s_0, for all $\omega \in \Omega$, for all t,*

$$c_t(\omega) = \kappa_t(\omega;\sigma^{\infty})n_t(\omega) - g_t(\omega) \tag{G.17}$$

$$B_{t+1}(\omega) = b_{t+1}(\omega) \tag{G.18}$$

$$b_{t+1} = b_t \ if \ (1 - d_t(\omega) + d_t(\omega)a_t(\omega)) = 0; \tag{G.19}$$

$$\kappa_t(\omega;\sigma^{\infty})\tau_t(\omega) = \left(\kappa_t(\omega;\sigma^{\infty}) - \frac{u_l(\kappa_t(\omega;\sigma^{\infty})n_t(\omega) - g_t(\omega), 1 - n_t(\omega))}{u_c(\kappa_t(\omega;\sigma^{\infty})n_t(\omega) - g_t(\omega), 1 - n_t(\omega))}\right); \tag{G.20}$$

$$g_t(\omega) + \{(1 - d_t(\omega)) + d_t(\omega)a_t(\omega)\}\delta_t(\omega)B_t(\omega) - \{(1 - d_t(\omega)) + d_t(\omega)a_t(\omega)\}p_t(\omega;\sigma^{\infty})B_{t+1}(\omega) \tag{G.21}$$

$$= \left(\kappa_t(\omega;\sigma^{\infty}) - \frac{u_l(\kappa_t(\omega;\sigma^{\infty})n_t(\omega) - g_t(\omega), 1 - n_t(\omega))}{u_c(\kappa_t(\omega;\sigma^{\infty})n_t(\omega) - g_t(\omega), 1 - n_t(\omega))}\right) n_t(\omega). \tag{G.22}$$

Where

$$p_t(\omega;\sigma^{\infty}) = E_{\Pi(\cdot|\omega^t)}\left[\beta \frac{u_c n(\kappa_{t+1}(\omega;\sigma^{\infty})n_{t+1}(\omega) - g_{t+1}(\omega), 1 - n_{t+1}(\omega))}{u_c(\kappa_t(\omega;\sigma^{\infty})n_t(\omega) - g_t(\omega), 1 - n_t(\omega))}\varrho_{t+1}(\omega)\right], \ \forall t. \tag{G.23}$$

Proof of Lemma G.1. Take $\{\sigma^{\infty}, n^{\infty}, B^{\infty}\}$, and a price schedule $(p_t)_t$ that satisfy the equations. It is easy to see that feasibility and market clearing holds (assumptions 3 and 4). Also, by lemma B.1 optimality of the households is also satisfied.

To check feasibility of the government policy (assumption 3). Observe that, by equations G.17 - G.21 imply

$$\begin{aligned} & g_t(\omega) + \{(1-d_t(\omega)) + d_t(\omega)a_t(\omega)\}\delta_t(\omega)B_t(\omega) - \{(1-d_t(\omega)) + d_t(\omega)a_t(\omega)\}p_t(\omega;\sigma^{\infty})B_{t+1}(\omega) \\ =& \kappa_t(\omega;\sigma^{\infty})\tau_t(\omega)n_t(\omega). \end{aligned}$$

Finally, I check optimality of the households. I first check that the sequences satisfy the budget constraint. Observe that, by equations G.17 - G.21

$$\begin{aligned} & -c_t(\omega) + \kappa_t(\omega;\sigma^{\infty})n_t(\omega) + \{(1-d_t(\omega)) + d_t(\omega)a_t(\omega)\}\delta_t(\omega)B_t(\omega) - p_t(\omega;\sigma^{\infty})(1-d_t(\omega)+d_t(\omega)a_t(\omega))B_{t+1}(\omega) \\ =& \kappa_t(\omega;\sigma^{\infty})\tau_t(\omega)n_t(\omega). \end{aligned}$$

If $d_t(\omega) = 0$, then equation G.17 implies that $b_{t+1} = B_{t+1}$ for all t (and for b_0 I assume it is equal to B_0) and thus

$$-c_t(\omega) + \kappa_t(\omega;\sigma^{\infty})n_t(\omega) + b_t(\omega) - p_t(\omega;\sigma^{\infty})b_{t+1}(\omega) = \kappa_t(\omega;\sigma^{\infty})\tau_t(\omega)n_t(\omega).$$

This coincides with the budget constraint of the household. If $d_t(\omega) = 1$, but $a_t(\omega) = 1$, then equation G.17 implies that $b_{t+1} = B_{t+1}$ for all t, so

$$-c_t(\omega) + \kappa_t(\omega;\sigma^{\infty})n_t(\omega) + \delta_t(\omega)b_t(\omega) - p_t(\omega;\sigma^{\infty})b_{t+1}(\omega) = \kappa_t(\omega;\sigma^{\infty})\tau_t(\omega)n_t(\omega).$$

Finally, if $d_t(\omega) = 1$, but $a_t(\omega) = 0$, then equation G.17 implies that $b_t = b_{t+1} = 0$ for all t, so

$$-c_t(\omega) + \kappa_t(\omega;\sigma^{\infty})n_t(\omega) = \kappa_t(\omega;\sigma^{\infty})\tau_t(\omega)n_t(\omega).$$

This coincides with the budget constraint of the household and with the restriction that $b_{t+1} \leq b_t \ if \ (1-d_t(\omega)+d_t(\omega)a_t(\omega)) = 1$.

Take $\{\sigma^{\infty}, c^{\infty}, n^{\infty}, b^{\infty}, p^{\infty}\} \in CEG(s_0)$. Then it is easy to see that the associated $\{\sigma^{\infty}, n^{\infty}, B^{\infty}, p^{\infty}\}$ satisfy the equations. □

Let $\mu^{\infty} : \Omega \times \{0, 1, ...\} \to \mathbb{R}_+$ be such that μ_t is $\mathcal{O}^t$ - measurable and it is defined by

$$\mu_t(\omega) \equiv u_c(\kappa_t(\omega;\sigma^{\infty})n_t(\omega) - g_t(\omega), 1 - n_t(\omega)).$$

From the proof of lemma G.1, under assumption 4.1(i), it is easy to see that a competitive equilibrium with government can be expressed in terms of $(\mu^{\infty}, B^{\infty}, d^{\infty}, a^{\infty})$ such that they satisfy the following conditions: For all (ω, t),

$$Z_{\phi_t(\omega)}(\mu_t(\omega), g_t(\omega)) + \phi_t(\omega)\{\mathcal{P}_t(\omega)B_{t+1}(\omega) - \delta_t(\omega)B_t(\omega)\mu_t(\omega)\} \geq 0, \tag{G.24}$$

where $\phi_t(\omega) \equiv (1 - d_t(\omega)) + d_t(\omega)a_t(\omega)$, and

$$(\mu, g, \phi) \mapsto Z_{\phi}(\mu, g) \equiv (\mu - u_l(\kappa_{\phi}n(\mu, g, \kappa_{\phi}) - g, 1 - n(\mu, g, \kappa_{\phi})))n(\mu, g, \kappa_{\phi}) - \mu g. \tag{G.25}$$

Where $\kappa_\phi \equiv (1-\phi)\kappa + \phi$. That is, $Z_\phi(\mu, g)$ is the primary superavit, given (μ, g), and $n(\mu, g, \kappa)$ is the solution of $u_c(\kappa n - g, 1-n) = \mu$ (under assumption 4.1(i) there exists a unique one). And

$$If\ \phi_t(\omega) = 0,\ then\ B_{t+1}(\omega) = B_t(\omega) \quad \text{(G.26)}$$

and (ω_0, B_0) given.

The next set defines continuations of CEG.

Definition G.1. *For any $(g_0, B_0, \mu_0, \phi_{-1})$,*

(G.27)
$$\mathbf{S}(g_0, B_0, \mu_0, \phi_{-1}) \equiv \{(\mu^\infty, B^\infty, d^\infty, a^\infty) : \forall (t,\omega)\ satisfy\ equations\ G.24 - G.26\ and\ \mu_0(\omega) = \mu_0, \forall \omega\}.$$

The next lemma establishes that continuations of processes in CEG are still in CEG

Lemma G.2. *If $(\mu^\infty, B^\infty, d^\infty, a^\infty) \in \mathbf{S}(g_0, B_0, \mu_0, \phi_{-1})$, then for any ω^t,*

$$(\mu_t^\infty(\omega^t, \cdot), B_t^\infty(\omega^t, \cdot), d_t^\infty(\omega^t, \cdot), a_t^\infty(\omega^t, \cdot)) \in \mathbf{S}(g_t(\omega^t), B_t(\omega^t), \mu_t(\omega^t), \phi_{t-1}(\omega^{t-1})).$$

(Recall $\phi_t(\omega^t) \equiv (1 - d_t(\omega^t)) + d_t(\omega^t) a_t(\omega^t)$).

The notation for, say, $\mu_t^\infty(\omega^t, \cdot)$ was introduced at the beginning of this appendix and denotes continuation of stochastic processes given a history ω^t.

Proof of Lemma G.2. It readily follows from equations G.24 - G.26. □

The following definitions present sets related to $\mathbf{S}$ that are useful for the proofs below.

Definition G.2. *For any $e^\infty \in (\{0,1\}^2)^\infty$, let*

$$\mathbf{S}(g_0, B_0, \mu_0, \phi_{-1}; e^\infty) \equiv \mathbf{S}(g_0, B_0, \mu_0, \phi_{-1})\ but\ with\ (d^\infty, a^\infty) = e^\infty \quad \text{(G.28)}$$

Definition G.3. *For any $e^\infty \in (\{0,1\}^2)^\infty$, let*

$$\mathbf{T}_{e^\infty} \equiv \{(g, B, \mu, \phi) \colon \mathbf{S}(g, B, \mu, \phi; e^\infty) \neq \{\emptyset\}\}.$$

And, for any policy function **e**, [56]

$$\mathbf{T}_{\mathbf{e}} \equiv \mathbf{T}_{e^\infty}\ where\ e^\infty\ is\ generated\ by\ \mathbf{e}. \quad \text{(G.29)}$$

Definition G.4. *For any $e^\infty \in (\{0,1\}^2)^\infty$, let*

$$\mathbf{R}_{e^\infty} \equiv \{(g, B, \vec{\mu}, \phi) : (g, \delta B, \vec{\mu}(g,\delta), \phi) \in \mathbf{T}_{e^\infty} \neq \{\emptyset\},\ \forall \delta \in supp(\Pr(\cdot|g, d_0))\} \quad \text{(G.30)}$$

For any policy function **e** [57]

$$\mathbf{R}_{\mathbf{e}} \equiv \{(g, B, \vec{\mu}, \phi) : (g, \delta B, \vec{\mu}(g,\delta), \phi) \in \mathbf{T}_{\mathbf{e}} \neq \{\emptyset\},\ \forall \delta \in supp(\Pr(\cdot|g, \mathbf{d}^*))\}. \quad \text{(G.31)}$$

Finally, $\mathbf{R}^* \equiv \mathbf{R}_{\mathbf{e}^*}$.

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[56]See definitions H.1 and H.5 for the definition of policy function and the formal meaning of ”generated by”.

[57]For a probability measure Pr, *supp*(Pr) denotes the support of Pr, i.e., the smallest closed set containing all points with positive mass.

Appendix H. Appendix of Section 5

This Appendix is divided into several parts. First, I present the recursive formulation of the Government's problem; this is a more detailed version of the one presented in the text. I then present a sequential formulation of the government problem.

A convenient way of viewing the sequential problem is to divide the government into two parts: the "fiscal authority" and the "default authority". The latter should be viewed as a sequence of one period-lived authorities, thus reflecting the lack of commitment regarding the debt. Given the policy of (the sequence of) default authority, the fiscal authority chooses taxes and debt. The one period default authority chooses to default or not, simply by comparing value functions, taking as given the behavior of future authorities.

I then show the equivalence between these two approaches; this amounts to proving a version of the principle of optimality. Finally, I conclude with a section that characterizes the state space of this economy; this generalizes the results in Kydland and Prescott (1980) and Chang (1998). Throughout this section, let $\tilde{\omega} \equiv (g, \delta) \in \tilde{\Omega}$.

H.1. Recursive formulation of the government's problem

I now formally define a government policy and value functions.

Definition H.1. *A government policy function, $\boldsymbol{\sigma}$, is a tuple $\boldsymbol{\sigma} \equiv (\mathbf{B}, \vec{\boldsymbol{\mu}}, \mathbf{d}, \mathbf{a})$ where $\mathbf{B} : \mathbb{G} \times \bar{\Delta} \times \mathbb{B} \times \mathbb{R}_+ \times \{0,1\} \to \mathbb{B}$, $\vec{\boldsymbol{\mu}} : \mathbb{G} \times \bar{\Delta} \times \mathbb{B} \times \mathbb{R}_+ \times \{0,1\} \to \mathbb{M}$, $\mathbf{d} : \mathbb{G} \times \mathbb{B} \times \mathbb{M} \times \{0,1\} \to \{0,1\}$ and $\mathbf{a} : \mathbb{G} \times \bar{\Delta} \times \mathbb{B} \times \mathbb{R}_+ \to \{0,1\}$.*

Definition H.2. *A government value functions is a pair $(\mathbf{V}^*_o, \mathbf{V}^*)$ where $\mathbf{V}^* : \mathbb{G} \times \mathbb{B} \times \mathbb{M} \times \{0,1\} \to \mathbb{R}$ and $\mathbf{V}^*_o : \mathbb{G} \times \mathbb{B} \times \{0,1\} \to \mathbb{R}$, such that $\mathbf{V}^*$ is the value function of the benevolent government for the "continuation problem" and $\mathbf{V}^*_o$ is the value function of the benevolent government for the "initial problem".*

Henceforth, and for convenience, I use $\mathbf{e}^* \equiv (\mathbf{d}^*, \mathbf{a}^*)$ and $\mathbf{f}^* \equiv (\mathbf{B}^*, \vec{\boldsymbol{\mu}}^*)$. The following definitions are needed to define the recursive formulation of the government's problem.

(1) The mapping $(g, \mu, \phi) \mapsto F(\mu, g, \phi)$ is the per-period payoff of the government. That is,

   (a) If $\phi = 1$, $F(\mu, g, 1) \equiv u(n_1(\mu, g) - g, 1 - n_1(\mu, g))$, where $n_1(\mu, g)$ is the solution to $\mu = u_c(n - g, 1 - n)$. [58]

   (b) If $\phi = 0$, then $F(\mu, g, 0) \equiv F_A(g) \equiv u(\kappa n_0(\mu, g) - g, 1 - n_0(\mu, g))$, where $n_0(\mu, g) \equiv n_A(g)$ is chosen such that the budget constraint is balanced. [59]

[58] Observe that by this argument n is no longer a choice variable, since it is given by $n_1(\mu, g)$; this uses the fact that $n \mapsto u_c(n - g, 1 - n)$ is monotone increasing (assumption 6.1).

[59] $n_0(\mu, g)$ is constant with respect to μ.

(2) Let $\phi \mapsto \mathbb{D}(\phi)$ and $(d,\delta) \mapsto \mathbb{A}(d,\delta)$, be such that

$$\mathbb{D}(\phi) = \begin{cases} \{1\} & if\ \phi = 0 \\ \{0,1\} & if\ \phi = 1 \end{cases} \quad and\ \mathbb{A}(d,\delta) = \begin{cases} \{0\} & if\ \delta = \bar{\delta} \\ \{0,1\} & if\ \delta \neq \bar{\delta}\ and\ d = 1 \\ \{1\} & if\ d = 0 \end{cases}$$

That is, the correspondences reflect the restrictions on timing and default decisions I imposed in the text. [60]

(3) Given a policy function $\mathbf{e}$, let

$$(g,B,\mu,\phi) \mapsto \Gamma_{\mathbf{e}}(g,B,\mu,\phi) = \{(B',\vec{\mu}') : Z_{\phi}(g,\mu) + \phi\{\mathcal{P}(g,B',\vec{\mu}';\mathbf{e})B' - B\mu\} \geq 0\ and\ if\ \phi = 0,\ B' = B\}$$

where

$$Z_{\phi}(g,\mu) = \begin{cases} (\mu - u_l(n_1(\mu,g) - g, 1 - n_1(\mu,g)))\, n_1(\mu,g) - \mu g & if\ \phi = 1 \\ (\kappa\mu - u_l(\kappa n_0(g) - g, 1 - n_0(g)))\, n_0(g) - \mu g & if\ \phi = 0 \end{cases}$$

That is, Γ is a correspondence summarizing restriction from the CEG, i.e.,

(4) For any $(g,B',\vec{\mu}')$ and given $\mathbf{e}$,

$$\begin{aligned} \mathcal{P}(g,B',\vec{\mu}';\mathbf{e}) =& \beta \int_{\mathbb{G}} \vec{\mu}'(g',1)(1 - \mathbf{d}(g',B',\vec{\mu}',1))\pi_{\mathbb{G}}(dg'|g) \\ &+ \beta \int_{\mathbb{G}} \mathbf{d}(g',B',\vec{\mu}',1)\lambda \int_{\Delta} \vec{\mu}'(g',\delta')\mathbf{a}(g',\delta',B',\vec{\mu}'(g',\delta')))\pi_{\Delta}(d\delta')\pi_{\mathbb{G}}(dg'|g) \\ &+ \beta \int_{\mathbb{G}} \mathbf{d}(g',B',\vec{\mu}',1) \left\{1 - \lambda + \lambda \int_{\Delta} (1 - \mathbf{a}(g',\delta',B',\vec{\mu}'(g',\delta')))\pi_{\Delta}(d\delta')\right\} \mathcal{Q}(g',B',\vec{\mu}';\mathbf{e}) \\ &\times \pi_{\mathbb{G}}(dg'|g). \end{aligned}$$

Where

$$\begin{aligned} \mathcal{Q}(g,B,\vec{\mu}';\mathbf{e}) =& \beta \int_{\mathbb{G}} \lambda \int_{\Delta} \vec{\mu}'(g',\delta')\mathbf{a}(g',\delta',B,\vec{\mu}'(g',\delta')))\pi_{\Delta}(d\delta')\pi_{\mathbb{G}}(dg'|g) \\ &+ \beta \int_{\mathbb{G}} \left\{1 - \lambda + \lambda \int_{\Delta} (1 - \mathbf{a}(g',\delta',B,\vec{\mu}'(g',\delta')))\pi_{\Delta}(d\delta')\right\} \mathcal{Q}(g',B)\pi_{\mathbb{G}}(dg'|g). \end{aligned}$$

The next lemma characterizes $\mathbf{S}$ (see definition G.1) using Γ.

Lemma H.1. *Let $\sigma^{\infty} \equiv (f^{\infty}, e^{\infty})$ with $f^{\infty} = (B^{\infty},\mu^{\infty})$ and $e^{\infty} = (d^{\infty},a^{\infty})$ be a government policy. Given $(g_0,B_0,\mu_0,\phi_{-1})$, $f^{\infty} \in \mathbf{S}(g_0,B_0,\mu_0,\phi_{-1};e^{\infty})$ iff $\mu_0(\omega) = \mu_0$, and, for any (t,ω),*

$$(B_{t+1}(\omega), \mu_{t+1}(\omega^t,\cdot)) \in \Gamma_{e_{t+1}(\omega^t,\cdot)}(g_t(\omega), \delta_t(\omega)B_t(\omega), \mu_t(\omega), \phi_t(\omega)) \tag{H.32}$$

Proof. If equation H.32 is satisfied by all (t,ω), this implies equations G.24 and G.26, and $\mu_0(\omega) = \mu_0$ by assumption. □

[60] That is, if $\phi = 0$, the government can only decide whether to accept or not an offer of partial repayment, so (trivially) $\mathbb{D}(0) = \{1\}$; if $\phi = 1$ the government has the option to default or not, so $\mathbb{D}(1) = \{0,1\}$. Also, $\mathbb{A}(d,\bar{\delta}) \equiv \{0\}$, i.e., if $\delta = \bar{\delta}$ then no (acceptable) offer arrived so $a = 0$ (trivially), if $\delta \neq \bar{\delta}$ the government has the possibility to accept or reject, provided it choose $d = 1$.

The next definition defines a solution to the recursive formulation of the government problem. In order to ensure $\Gamma_{\mathbf{e}^*}(g,B,\mu,\phi) \neq \{\emptyset\}$, I only consider $(g_0,B_0,\vec{\mu}_0,\phi_{-1}) \in \mathbf{R}_{\mathbf{e}^*}$.

Definition H.3. *A solution to the recursive formulation of the government problem is a* $(\mathbf{V}_o^*,\mathbf{V}^*,\boldsymbol{\sigma}^*)$ *where* $\boldsymbol{\sigma}^*$ *is a government policy function, and* $(\mathbf{V}_o^*,\mathbf{V}^*)$ *are government value functions, such that: For any* $s \equiv (g,B,\vec{\mu},\phi) \in \mathbf{R}_{\mathbf{e}^*}$, *letting* $z \equiv (g,\delta,B,\vec{\mu}(g,\delta),\phi)$,

(1) *Given* $(\mathbf{V}_o^*,\mathbf{V}^*)$, $\boldsymbol{\sigma}^*$ *is optimal, i.e.,*

(a) *Given,* $\mathbf{e}^* = (\mathbf{d}^*,\mathbf{a}^*)$, *it follows*

$$\text{For } \phi = 1,\ (\mathbf{B}^*(z),\vec{\boldsymbol{\mu}}^*(z)) \in \arg\max_{(B',\vec{\mu}') \in \Gamma_{\mathbf{e}^*}(g,\delta B,\vec{\mu}(g,\delta),\phi^*)} \left\{ \int_{\mathbb{G}} \mathbf{V}^*(g',B',\vec{\mu}',1)\pi_{\mathbb{G}}(dg'|g) \right\}$$

$$\text{For } \phi = 0,\ \mathbf{B}^*(z) = B \text{ and } \vec{\boldsymbol{\mu}}^*(z) \in \arg\max_{\vec{\mu}' \in \mathbb{M}} \left\{ \int_{\mathbb{G}} \mathbf{V}^*(g',B,\vec{\mu}',0)\pi_{\mathbb{G}}(dg'|g) \right\}. \tag{H.33}$$

Where $\phi^* \equiv (1-\mathbf{d}^*(s)) + \mathbf{d}^*(s)\mathbf{a}^*(g,\delta,B,\vec{\mu}(g,\delta))$.

(b) *Given* $\mathbf{f}^* = (\mathbf{B}^*,\vec{\boldsymbol{\mu}}^*)$, *for any* $(g,\delta,B,\vec{\mu}(g,\delta))$,

$$\mathbf{a}^*(g,\delta,B,\vec{\mu}(g,\delta)) \in \arg\max_{a \in \mathbb{A}(\mathbf{d}^*(s),\delta)} \mathcal{V}^*(g,\delta B,\vec{\mu}(g,\delta),a);$$

and,

$$\mathbf{d}^*(s) \in \arg\max_{d \in \mathbb{D}(\phi)} \int_{\bar{\Delta}} \mathcal{V}^*(g,\delta B,\vec{\mu}(g,\delta),(1-d)+d\mathbf{a}^*(g,\delta,B,\vec{\mu}(g,\delta)))\Pr(d\delta|g,d).$$

Where

$$(g,B,\mu,\phi) \mapsto \mathcal{V}^*(g,B,\mu,\phi) = \max_{(B',\vec{\mu}') \in \Gamma_{\mathbf{e}^*}(g,B,\mu,\phi)} \left\{ F(g,\mu,\phi) + \beta \int_{\mathbb{G}} \mathbf{V}^*(g',B',\vec{\mu}',\phi)\pi_{\mathbb{G}}(dg'|g) \right\}.$$

(2) $\mathbf{V}_o^*(g_0,B_0,\phi_{-1}) = \sup_{\vec{\mu} \in \mathbb{M}} \mathbf{V}^*(g_0,B_0,\vec{\mu},\phi_{-1})$. *And,*

(H.34)

$$\mathbf{V}^*(s) = \int_{\bar{\Delta}} \left\{ F(g,\vec{\mu}(g,\delta),\boldsymbol{\phi}^*(g,\delta)) + \beta \int_{\mathbb{G}} \mathbf{V}^*(g',\mathbf{B}^*(z),\vec{\boldsymbol{\mu}}^*(z),\boldsymbol{\phi}^*(g,\delta))\pi_{\mathbb{G}}(dg'|g) \right\} \Pr(d\delta|g,\mathbf{d}^*(s)).$$

Remark H.1. *Some remarks are in order:*

(1) *Recall that the probability* $\Pr(\cdot|g,d)$ *was introduced in assumption 3.1 and is such that, if the government chooses to pay* ($d=0$), *then* $\delta = 1$; *and if* $d = 1$, *then* $\Pr(\delta|0)$ *is given by assumption 3.1.*

(2) *The value* $\mathcal{V}^*(g,B,\mu,\phi)$ *is the value from having decided to default and/or accept an offer of defaulted debt (implying a particular value for* ϕ*), given the exogenous part of the state is* g *and that the outstanding debt is* B *and the outstanding tax promise is* μ. *It is easy to see that* $\mathcal{V}^*(g,\delta B,\mu,\phi)$ *(in this appendix) equals* $\mathcal{V}_\phi^*(g,\delta B,\mu)$ *(in the text). In particular, it is easy to see that* $\mathcal{V}^*(g,B,\mu,0) = \mathcal{V}^*(g,B,0) \equiv \mathcal{V}_0^*(g,B)$.

(3) *It is easy to see that* $Z_\phi(g,\mu)$ *in the appendix, equals* $Z_\phi(n,g,\mu)$ *in the text, imposing that* $n\ :\ \mu = u_c(n-g,1-n)$.

H.2. Sequential Formulation of the Government's Problem

I present the sequential formulation of the government's problem. For this, I need to introduce some notation.

(1) Let, for all t, $s_t \equiv (g_t, B_t, \vec{\mu}_t, \phi_{t-1})$ where $\vec{\mu}_t \in \mathbb{R}_+^{\tilde{\Omega}}$ where $\tilde{\Omega} \equiv \mathbb{G} \times \bar{\Delta}$.

(2) Let f^∞ denote a policy for the fiscal authority. That is, f^∞ is a stochastic process, such that $f_t(\omega) \equiv (B_t(\omega), \mu_t(\omega))$ for all t and ω.

(3) Let e^∞ denote a policy for the default authorities. That is, e^∞ is a stochastic process, such that $e_t(\omega) \equiv (d_t(\omega), a_t(\omega))$ for all t and ω.

(4) Let $\sigma^\infty \equiv (f^\infty, e^\infty)$ be a government policy.

(5) For any policy (f^∞, e^∞), and any (t, ω), let

$$V_t(\omega^{t-1}, g_t, d, a)(f^\infty, e^\infty)$$
$$\equiv \int_{\bar{\Delta}} \left\{ F(g_t, \mu_t(\omega^{t-1}, g_t, \delta), \phi) + \sum_{j=1}^{\infty} \beta^j \int_\Omega F(g_{t+j}, \mu_{t+j}(\omega), \phi_{t+j}(\omega)) \Pr(d\omega | g_t, \delta, e^\infty) \right\} \Pr(d\delta | g_t, d)$$

where $\phi \equiv 1 - d + da$ and $\phi_t(\omega) \equiv 1 - d_t(\omega) + d_t(\omega) a_t(\omega)$.

Remark H.2. *In order to avoid technical issues with measurability and interchanging limits with integration operations, I am working under the assumption $|\bar{\Delta}| < \infty$ and $|\mathbb{G}| < \infty$; see Stokey et al. (1989) Ch. 9 for a discussion about the consequence of relaxing this assumption.*

The next definition defines a solution to the sequential formulation of the government's problem.

Definition H.4. *Given g_0, B_0, ϕ_{-1}, a solution to the sequential formulation of the government's problem is a tuple $(V_o^*, V^*, \sigma^{*\infty})$ where $\sigma^{*\infty} = (f^{*\infty}, e^{*\infty})$ is a government plan and $V^* : \mathbb{G} \times \mathbb{B} \times \mathbb{M} \times \{0,1\} \to \mathbb{R}$ and $V_o^* : \mathbb{G} \times \mathbb{B} \times \{0,1\} \to \mathbb{R}$, such that*

(1) *Given $e^{*\infty}$ and V^*, $f^{*\infty}$ satisfies:*

For all $\delta \in supp(\Pr(\cdot | g_0, d_0^*))$ and $\mu_0(g_0, \delta)$ such that $(g_0, \delta B, \mu_0(g_0, \delta), \phi_{-1}) \in \mathbf{R}_{e^{*\infty}}$, [61]

$$(f_1^*, f_2^*, ...) \in \arg \max_{f^\infty \in \mathbf{S}(g_0, \delta B_0, \mu_0(g_0, \delta), \phi_0^*(g_0, \delta); e^{*\infty})} \sum_{j=1}^{\infty} \beta^j \int_\Omega F(g_j, \mu_j(\omega), \phi_j^*(\omega)) \Pr(d\omega | g_0, \delta, e^{*\infty}).$$

And

$$\mu_0^* \in \arg \max_{m \in \mathbb{M}} V^*(g_0, B_0, m, \phi_{-1}).$$

[61] The definition implicitly assumes the maximum in the RHS is achieved. One could define f^∞ as an approximate maximizer and thus avoid existence issues.

(2) *Given* $(e^{*\infty}, f^{*\infty})$, V^* *and* V_o^* *satisfy:*

$$V^*(g_0, \vec{\mu}_0, B_0, \phi_{-1})$$
$$= \int_{\bar{\Delta}} \left\{ F(g_0, \vec{\mu}_0(g_0,\delta), \phi_0^*(g_0,\delta)) + \sum_{j=1}^{\infty} \beta^j \int_{\Omega} F(g_j, \mu_j^*(\omega), \phi_j^*(\omega)) \Pr(d\omega | g_0, \delta, e^{*\infty}) \right\} \Pr(d\delta | g_0, d_0^*),$$

for any $\vec{\mu}_0 \in \mathbb{M}$, *and*

$$V_o^*(s_0) = V^*(g_0, \mu_0^*(\cdot), B_0, \phi_{-1})$$

(3) *Given* $f^{*\infty}$, $e^{*\infty}$ *satisfies: For all* (t,ω)*:*

(a) $d_t^*(\omega) = 0(1)$ *implies*

$$\int_{\Delta} \left\{ F(g_t, \vec{\mu}_t(\omega^{t-1}, g_t, \delta), a_t^*(\omega^{t-1}, g_t, \delta)) + \beta \int_{\mathbb{G}} V_t(\omega^t, g_{t+1}, 1, a_t^*(\omega^{t-1}, g_t, \delta))(f^{*\infty}, e^{*\infty}) \pi_{\mathbb{G}}(dg_{t+1}|g_t) \right\} \pi_{\Delta}(d\delta|g_t)$$
$$\leq (\geq) F(g_t, \vec{\mu}_t(\omega^{t-1}, g_t, 1), 1) + \beta \int_{\mathbb{G}} V_t(\omega^t, g_{t+1}, 0, 1)(f^{*\infty}, e^{*\infty}) \pi_{\mathbb{G}}(dg_{t+1}|g_t).$$

(b) $a_t^*(\omega) = 1(0)$ *implies* [62]

$$F_A(g_t) + \beta \int_{\mathbb{G}} V_t(\omega^t, g_{t+1}, 0, 0)(f^{*\infty}, e^{*\infty}) \pi_{\mathbb{G}}(dg_{t+1}|g_t)$$
$$\leq (\geq) F(g_t, \vec{\mu}_t(\omega^{t-1}, g_t, \delta_t), 1) + \beta \int_{\mathbb{G}} V_t(\omega^t, g_{t+1}, 0, 1)(f^{*\infty}, e^{*\infty}) \pi_{\mathbb{G}}(dg_{t+1}|g_t).$$

Remark H.3. *A few remarks are in order.*

(1) *Parts (1) and (2) of the previous definition states that the fiscal authority chooses plans* f^{∞}*, taking as given the actions of the default authorities, given by* $e^{*\infty}$*, in order to maximize the expected utility.*

(2) $V^*(s_0)$ *is the value, at time 0 and state* s_0*, of the fiscal authority that is chosen optimally fiscal policy* f^{∞} *and given that sequence of 1-period default authorities have a sequence of best responses given by* $e^{*\infty} \equiv (d^{*\infty}, a^{*\infty})$*.*

(3) *Part (3) of the previous definition states that, at time* t*, a default authority, chooses* d_t, a_t *optimally to maximize its payoffs, given that the fiscal authority is acting according to* $f^{*\infty}$ *and that future generations are also acting optimally and symmetrically. That is, I focus on the case where each default authority follows the same strategy.*

H.3. Equivalence between the sequential and recursive formulations

Before presenting the main theorems, I need the following definition that formalizes the notion of a policy function "generating" a plan.

Definition H.5. *Given* $(g_0, \delta_0, B_0, \vec{\mu}_0, \phi_{-1})$*, a policy function* $\boldsymbol{\sigma}$ *(see definition H.1) generates a government policy* $\sigma^{\infty} \equiv (B^{\infty}, \mu^{\infty}, d^{\infty}, a^{\infty})$ *iff*

[62] $F_A(g_t) \equiv u(n_A(g_t) - g_t, 1 - n_A(g_t))$ where $n_A(g)$ balance the primary budget, given g.

(1) $B_1(\omega_0) \equiv \mathbf{B}(g_0, \delta_0, B_0, \vec{\mu}_0(g_0,\delta_0), \phi_{-1})$, $\mu_1(\omega_0, \cdot) = \vec{\boldsymbol{\mu}}(g_0, \delta_0, B_0, \vec{\mu}_0(g_0,\delta_0), \phi_{-1})$, $d_0(\omega_0) = d_0(g_0) \equiv \mathbf{d}(g_0, B_0, \vec{\mu}_0, \phi_{-1})$ *and* $a_0(\omega_0) \equiv \mathbf{a}(g_0, \delta_0, B_0, \vec{\mu}_0(g_0,\delta_0))$.

(2) *For any* $t \geq 1$ *and* ω,

$$B_{t+1}(\omega) \equiv \mathbf{B}(z_t(\omega)) \text{ and } \mu_{t+1}(\omega^t, \cdot) = \vec{\boldsymbol{\mu}}(z_t(\omega))$$

where $z_t(\omega) \equiv (g_t, \delta_t, B_t(\omega), \mu_t(\omega), \phi_{t-1}(\omega))$. *And*

$$d_t(\omega) \equiv \mathbf{d}(g_t, B_t(\omega), \mu_t(\omega^{t-1}, \cdot), \phi_{t-1}(\omega)) \text{ and } a_t(\omega) \equiv \mathbf{a}(g_t, \delta_t, B_t(\omega), \mu_t(\omega))$$

where $\phi_t(\omega) \equiv 1 - d_t(\omega) + d_t(\omega)a_t(\omega)$.

Theorem H.1. *Given* g_0, B_0, ϕ_{-1}, *suppose the following transversality condition holds:*

$$\lim_{t\to\infty} \int \beta^t \mathbf{V}^*(g_t(\omega), B_t(\omega), \mu_t(\omega^{t-1}, \cdot), \phi^o_{t-1}(\omega)) \Pr(d\omega | g_0, \delta_0, e^{o\infty}) = 0, \tag{H.35}$$

for any government policy $\sigma^\infty \equiv (f^\infty, e^\infty)$ *such that* $f^\infty \in \mathbf{S}(g_0, \delta B_0, \mu_0(g_0,\delta), \phi_{-1}; e^{o\infty})$ *for any* $\delta \in$ *supp*$(\Pr(\cdot|g_0, d_0))$ *and* $e^{o\infty}$ *is generated by* $\boldsymbol{\sigma^*}$.

Then the solution to the recursive formulation, $(\mathbf{V}^*_o, \mathbf{V}^*, \boldsymbol{\sigma^*})$ *generates a solution to the sequential problem, in the sense that*

(1) $\boldsymbol{\sigma^*}$ *generates* $\sigma^{o\infty}$.

(2) $\sigma^{o\infty}$ *and* $(\mathbf{V}^*_o, \mathbf{V}^*)$ *satisfy the restrictions (1)-(3) in definition H.4.*

Theorem H.2. *Given* g_0, B_0, ϕ_{-1}, *a solution to the sequential formulation,* $(V^*_o, V^*, \sigma^{*\infty})$, *satisfies the following:*

$$V^*(s_0) = \int_{\bar{\Delta}} \left\{ F(g_0, \mu^*_0(g_0,\delta_0), \phi^*_0(g_0,\delta_0)) + \beta \int_{\mathbb{G}} V^*(g_1, B^*_1(g_0,\delta_0), \mu^*_1(g_0,\delta_0,\cdot), \phi^*_0(g_0,\delta_0)) \pi_{\mathbb{G}}(dg_1|g_0) \right\} \times \Pr(d\delta_0 | g_0, d^*_0(g_0)).$$

H.3.1. *Proofs of Theorems H.1 and H.2.*

Proof of Theorem H.1. By definition H.3, for any $s \equiv (g, B, \vec{\mu}, \phi)$, consider $(\mathbf{V}^*, \mathbf{f}^*, \mathbf{e}^*)$ that solve the recursive problem. That is,

$$\mathbf{V}^*(s) = \int_{\Delta} \left\{ F(g, \mu, \boldsymbol{\phi^*}(g,\delta)) + \beta \int_{\mathbb{G}} \mathbf{V}^*(g', \mathbf{B}^*(z), \vec{\boldsymbol{\mu}}^*(z), \boldsymbol{\phi^*}(g,\delta)) \pi_{\mathbb{G}}(dg'|g) \right\} \Pr(d\delta | g, \mathbf{d}^*(g, B, \vec{\mu}, \phi)), \tag{H.36}$$

where $z \equiv (g, \delta, B, \vec{\mu}(g,\delta), \phi)$ and $\boldsymbol{\phi^*}(g,\delta) = (1 - \mathbf{d}^*(g, B, \vec{\mu}, \phi)) + \mathbf{d}^*(g, B, \vec{\mu}, \phi)\mathbf{a}^*(g, \delta, B, \vec{\mu}(g,\delta))$.

In particular, take $s_0 \equiv (g_0, B_0, \mu_0, \phi_{-1})$, then, by equation H.36

$$\mathbf{V}^*(s_0) \geq \int_{\bar{\Delta}} \left\{ F(g_0, \mu_0(\omega_0), \boldsymbol{\phi^*}(\omega_0)) + \beta \int_{\mathbb{G}} \mathbf{V}^*(g_1, B', \vec{\mu}', \boldsymbol{\phi^*}(\omega_0)) \pi_{\mathbb{G}}(dg_1|g_0) \right\} \Pr(d\delta_0 | g_0, \mathbf{d}^*(s_0)), \tag{H.37}$$

for any $(B', \vec{\mu}') \in \Gamma_{\mathbf{e}^*}(g_0, \delta_0 B_0, \vec{\mu}_0(\omega_0), \phi^o_0(\omega_0))$, for any $\omega_0 = (g_0, \delta_0)$ such that $\delta_0 \in$ *supp*$(\Pr(\cdot | g_0, \mathbf{d}^*(s_0)))$. In particular, for the case $B' = B^o_1(\omega_0)$ and $\vec{\mu}' \equiv \mu^o_1(\omega_0, \cdot)$ where B^o_1 and μ^o_1 are part of the policy generated by $\boldsymbol{\sigma}^*$, equation H.37 holds with equality.

By iterating again, and using $\phi_0^o(\omega_0) = \boldsymbol{\phi}^*(\omega_0)$, and any $(B_1(\omega_0), \vec{\mu}_1(\omega_0, \cdot)) \in \Gamma_{\mathbf{e}^*}(g_0, \delta_0 B_0, \vec{\mu}_0(\omega_0), \phi_0^o(\omega_0))$

$$\begin{aligned}
\mathbf{V}^*(s_0) \geq & \int_{\bar{\Delta}} F(g_0, \mu_0(\omega_0), \phi_0^o(\omega_0)) \Pr(d\delta_0|g_0, \mathbf{d}^*(s_0)) \\
& + \beta \int_{\bar{\Delta}} \int_{\mathbb{G}} \left(\int_{\bar{\Delta}} \left\{ F(g_1, \mu_1(\omega^1), \boldsymbol{\phi}^*(\omega_1)) + \beta \int_{\mathbb{G}} \mathbf{V}^*(g_2, B', \vec{\mu}', \boldsymbol{\phi}^*(\omega_1)) \pi_{\mathbb{G}}(dg_2|g_1) \right\} \Pr(d\delta_1|g_1, \mathbf{d}^*(s_1)) \right) \\
& \times \pi_{\mathbb{G}}(dg_1|g_0) \Pr(d\delta_0|g_0, \mathbf{d}^*(s_0)) \\
= & \int_{\bar{\Delta}} \left\{ F(g_0, \mu_0(\omega_0), \phi_0^o(\omega_0)) + \beta \int_{\mathbb{G}\times\bar{\Delta}} F(g_1, \mu_1(\omega^1), \boldsymbol{\phi}^*(\omega_1)) \Pr(d\delta_1|g_1, \mathbf{d}^*(s_1)) \pi_{\mathbb{G}}(dg_1|g_0) \right\} \Pr(d\delta_0|g_0, \mathbf{d}^*(s_0)) \\
& + \int_{\bar{\Delta}} \left(\beta^2 \int_{(\mathbb{G})^2\times\bar{\Delta}} \mathbf{V}^*(g_2, B', \vec{\mu}', \boldsymbol{\phi}^*(\omega_1)) \pi_{\mathbb{G}}(dg_2|g_1) \Pr(d\delta_1|g_1, \mathbf{d}^*(s_1)) \pi_{\mathbb{G}}(dg_1|g_0) \right) \Pr(d\delta_0|g_0, \mathbf{d}^*(s_0)).
\end{aligned}$$

where $s_1 \equiv (g_1, B_1(\omega_0), \mu_1(\omega_0, \cdot), \phi_0^o(\omega_0))$. This display holds for any $(B', \vec{\mu}') \in \Gamma_{\mathbf{e}^*}(g_1, \delta_1 B_1(\omega_0), \mu_1(\omega^1), \boldsymbol{\phi}^*(\omega_1))$, for any $\omega_1 = (g_1, \delta_1)$ such that $\delta_1 \in supp(\Pr(\cdot|g_1, \mathbf{d}^*(s_1)))$.

Observe that by definition H.5, $d_1^o(\omega^1) = \mathbf{d}^*(s_1)$ and $a_1^o(\omega^1) = \mathbf{a}(g_1, \delta_1, B_1^o(\omega_0), \mu_1^o(\omega^1))$, thereby implying $\phi_1^o(\omega^1) = \boldsymbol{\phi}^*(\omega_1)$, thus

$$\begin{aligned}
\mathbf{V}^*(s_0) \geq & \int_{\bar{\Delta}} \left\{ F(g_0, \mu_0(\omega_0), \phi_0^o(\omega_0)) + \beta \int_{\mathbb{G}\times\bar{\Delta}} F(g_1, \mu_1(\omega^1), \phi_1^o(\omega^1)) \Pr(d\omega^1|g_0, \delta_0, d_1^o) \right\} \Pr(d\delta_0|g_0, d_0^o(g_0)) \\
& + \int_{\bar{\Delta}} \left(\beta^2 \int_{(\mathbb{G})^2\times\bar{\Delta}} \mathbf{V}^*(g_2, B', \vec{\mu}', \phi_1^o(\omega^1)) \pi_{\mathbb{G}}(dg_2|g_1) \Pr(d\omega^1|g_0, \delta_0, d_1^o) \right) \Pr(d\delta_0|g_0, d_0^o(g_0)).
\end{aligned}$$

Where $\Pr(d\omega^1|g_0, \delta_0, d_1^o) \equiv \Pr(d\delta_1|g_1, \mathbf{d}^*(s_1)) \pi_{\mathbb{G}}(dg_1|g_0)$.

Observe that if $f_1(\omega_0) = f_1^o(\omega_0)$ and $B' = B_2^o(\omega^1)$ and $\vec{\mu}' = \mu_2^o(\omega^1, \cdot)$, then the above display holds with equality.

I now proceed by choosing $B' = B_2(\omega^1)$ and $\vec{\mu}' = \mu_2(\omega^1, \cdot)$ such that

$$(B_2(\omega^1), \mu_2(\omega^1, \cdot)) \in \Gamma_{\mathbf{e}^*}(g_1, \delta_1 B_1(\omega_0), \mu_1(\omega^1), \boldsymbol{\phi}^*(\omega_1)).$$

By iterating n times, it follows

$$\begin{aligned}
\mathbf{V}^*(s_0) \geq & \int_{\bar{\Delta}} \left\{ F(g_0, \mu_0(\omega_0), \phi_0^o(\omega_0)) + \sum_{j=1}^{n} \beta^j \int_{\Omega} F(g_j, \mu_j(\omega^j), \phi_j^o(\omega^j)) \Pr(d\omega^j|g_0, \delta_0, d^{o\infty}) \right\} \Pr(d\delta_0|g_0, d_0^o(g_0)) \\
& + \beta^{n+1} \int_{\bar{\Delta}} \int_{\mathbb{G}\times\Omega^n} \mathbf{V}^*(g_{n+1}, B_{n+1}(\omega^n), \mu_{n+1}(\omega^n, \cdot), \phi_n^o(\omega^n)) \pi_{\mathbb{G}}(dg_{n+1}|g_n) \Pr(d\omega^n|g_0, \delta_0, d^{o\infty}) \\
& \times \Pr(d\delta_0|g_0, d_0^o(g_0)).
\end{aligned}$$

For any (B_1^n, μ_1^n) such that, for any $i = 1, ..., n$ and any ω, $(B_{i+1}(\omega^i), \mu_{i+1}(\omega^i, \cdot)) \in \Gamma_{\mathbf{e}^*}(g_i, \delta_i B_i(\omega^i), \mu_i(\omega^i), \phi_i^o(\omega^i))$

By considering $n \to \infty$ and invoking the transversality condition (equation H.35), it follows [63]

$$\mathbf{V}^*(s_0) \geq \int_{\bar{\Delta}} \left\{ F(g_0, \mu_0(g_0, \delta), \phi_0^o(g_0, \delta)) + \sum_{j=1}^{\infty} \beta^j \int_{\Omega} F(g_j, \mu_j(\omega^j), \phi_j^o(\omega^j)) \Pr(d\omega|g_0, \delta, d^{o\infty}) \right\} \Pr(d\delta|g_0, d_0^o(g_0)).$$

For any $f^o \equiv (B^o, \mu^o)$ such that, for any $i = 1, ...$ and any ω, $(B_{i+1}(\omega^i), \mu_{i+1}(\omega^i, \cdot)) \in \Gamma_{\mathbf{e}^*}(g_i, \delta_i B_i(\omega^i), \mu_i(\omega^i), \phi_i^o(\omega^i))$. This implies that $f^{o\infty} \in \mathbf{S}(g_0, \delta_0 B_0, \mu_0(g_0, \delta_0), \phi_{-1}; e^{o\infty})$ (see equation G.27 for definition of $\mathbf{S}$).

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[63] In this step, I am interchanging the limit with integration. Since $|\bar{\Delta}| < \infty$

Given $e^{o\infty}$ (which might not necessarily by the optimal solution for the sequential problem), calculations above show that

$$\begin{aligned}\mathbf{V}^{*}(s_0) \geq & \int_{\bar{\Delta}} F(g_0,\mu_0(g_0,\delta),\phi^{o}_0(g_0,\delta)) \Pr(d\delta|g_0,d^{o}_0(g_0)) \\ & + \int_{\bar{\Delta}} \left\{ \max_{f^{\infty}\in \mathbf{S}(g_0,\delta_0 B_0,\mu_0(g_0,\delta),\phi^{o}_0(g_0,\delta))} \sum_{j=1}^{\infty} \beta^{j} \int_{\Omega} F(g_j,\mu_j(\omega^{j}),\phi^{o}_j(\omega^{j})) \Pr(d\omega|g_0,\delta,d^{o\infty}) \right\} \Pr(d\delta|g_0,d^{o}_0(g_0)).\end{aligned}$$

Also, by the calculations above,

$$\mathbf{V}^{*}(s_0) = \int_{\bar{\Delta}} \left\{ F(g_0,\mu_0(g_0,\delta),\phi^{o}_0(g_0,\delta)) + \sum_{j=1}^{\infty} \beta^{j} \int_{\Omega} F(g_j,\mu^{o}_j(\omega^{j}),\phi^{o}_j(\omega^{j})) \Pr(d\omega|g_0,\delta,d^{o\infty}) \right\} \Pr(d\delta|g_0,d^{o}_0(g_0)).$$

Since $f^{o\infty} \in \mathbf{S}(g_0,\delta_0 B_0,\mu_0(g_0,\delta_0),\phi_{-1};e^{o\infty})$, it follows that $f^{o\infty}$ achieves the maximum, given $e^{o\infty}$. Thus, given $e^{o\infty}$, $f^{o\infty}$ satisfies part (1) in definition H.4. It also follows that, given $e^{o\infty}$ and $f^{o\infty}$, $\mathbf{V}^{*}(s_0) = V^{*}(s_0)$, so $\mathbf{V}^{*}$ (and $\mathbf{V}^{*}_o$) satisfy part (2) in definition H.4.

It remains to show that, given $f^{o\infty}$ and $\mathbf{V}^{*}$, $e^{o\infty}$ satisfies part (3) in definition H.4. Observe that, for any (d,a) and any (t,ω),

$$V_t(\omega^{t},g_t,d,a)(f^{o\infty},e^{o\infty}) = \mathbf{V}^{*}(g_t,B^{o}_t(\omega^{t}),\mu^{o}_t(\omega^{t},\cdot),(1-d)+da).$$

Hence it immediately follows from $\mathbf{e}^{*}$ and the construction of $e^{o\infty}$ that $e^{o\infty}$ satisfies part (3) in definition H.4. □

Proof of Theorem H.2. By definition of V^{*}, it follows that

$$\begin{aligned}V^{*}(s_0) = & \int_{\bar{\Delta}} \left\{ F(g_0,\mu_0(g_0,\delta),\phi^{*}_0(g_0,\delta)) + \beta \int_{\mathbb{G}} \left(\sum_{j=1}^{\infty} \beta^{j-1} F(g_j,\mu^{*}_j(\omega),\phi^{*}_j(\omega)) \Pr(d\omega|g_1,\delta,e^{*\infty}) \right) \pi_{\mathbb{G}}(dg_1|g_0) \right\} \\ & \times \Pr(d\delta|g_0,d^{*}_0(g_0)).\end{aligned}$$

Observe that

$$\begin{aligned} & \sum_{j=1}^{\infty} \beta^{j-1} F(g_j,\mu^{*}_j(\omega),\phi^{*}_j(\omega)) \Pr(d\omega|g_1,\delta,e^{*\infty}) \\ = & \int_{\bar{\Delta}} \left\{ F(g_1,\mu^{*}_1(\omega_0,g_1,\delta),\phi^{*}_1(\omega_0,g_1,\delta)) + \sum_{j=1}^{\infty} \beta^{j} F(g_{1+j},\mu^{*}_{1+j}(\omega),\phi^{*}_{1+j}(\omega)) \Pr(d\omega|g_1,\delta,e^{*\infty}) \right\} \Pr(d\delta|g_1,d^{*}_1(\omega_0,g_1)) \\ = & V^{*}(g_1,\mu^{*}_1(\omega_0,\cdot),B^{*}_1(\omega_0),\phi^{*}_0(\omega_0)).\end{aligned}$$

Thus,

$$V^{*}(s_0) = \int_{\bar{\Delta}} \left\{ F(g_0,\mu_0(\omega_0),\phi^{*}_0(\omega_0)) + \beta \int_{\mathbb{G}} V^{*}(g_1,\mu^{*}_1(\omega_0,\cdot),B^{*}_1(\omega_0),\phi^{*}_0(\omega_0)) \pi_{\mathbb{G}}(dg_1|g_0) \right\} \Pr(d\delta_0|g_0,d^{*}_0(g_0)).$$

Where $\omega_0 = (g_0,\delta_0)$. Hence, the display of the theorem follows.

□

H.4. Continuation set of CGE

In this section I characterize the set $\mathbf{T_e}$ for any default policy function $\mathbf{e} \equiv (\boldsymbol{d}, \boldsymbol{a})$. [64] Namely, I show that $\mathbf{T_e}$ can be viewed as a fixed point of a certain operator. Recall that $\mathbf{T_e} \equiv \{(g, B, \mu, \phi) : \mathbf{S}(g, B, \mu, \phi; \mathbf{e}) \neq \{\emptyset\}\}$; see definitions G.2 and G.3.

For any correspondence $s \mapsto \Gamma$ and any set A in the image, let $\Gamma^{-}(A) \equiv \{s \ : \ \Gamma(s) \cap A \neq \{\emptyset\}\}$.

Theorem H.3. *Given* $\mathbf{e}$*,*

$$\mathbf{T_e} = \Gamma^{-}_{\mathbf{e}}(\Lambda_{\mathbf{e}}(\mathbf{T_e}))$$

where $\Lambda_{\mathbf{e}} : 2^{\mathbb{S}} \to 2^{\mathbb{B}\times\mathbb{M}}$ *such that* $\forall Q \subseteq \mathbb{S}$, $\Lambda_{\mathbf{e}}(Q) \equiv \{(B', \vec{\mu}') : \forall (g, \delta) \ s.t. \ (g, \delta B', \vec{\mu}'(g, \delta), \phi) \in Q\}$.

Before stating the proof a few remarks are in order. First, although the notation is more involved, $\mathbf{T_e}$ is analogous to the set Ω in Chang (1998). In his problem, the only state is the promised marginal utility of money. In my setup, due to having debt and a stochastic structure, the state becomes more involved. Second, the composite mapping $\Gamma^{-}_{\mathbf{e}} \circ \Lambda_{\mathbf{e}}$ acts as $\mathbb{B}$ in Chang (1998); in particular, $\lambda_{\mathbf{e}}$ is needed because $\Gamma_{\mathbf{e}}$ maps states onto $(B, \vec{\mu})$.

Proof of Theorem H.3. Observe that

$$\Lambda_{\mathbf{e}}(\mathbf{T_e}) \equiv \{(B', \vec{\mu}') : \forall (g, \delta) \ s.t. \ (g, \delta B', \vec{\mu}'(g, \delta), \phi) \in \mathbf{T_e}\}.$$

I first show that $\mathbf{T_e} \subseteq \Gamma^{-}_{\mathbf{e}}(\Lambda_{\mathbf{e}}(\mathbf{T_e}))$. If $\mathbf{T_e} = \{\emptyset\}$ then the proof is trivial; so I proceed to show the result for the case that this is not true. Take any $s \in \mathbf{T_e}$, then $\mathbf{S}(g, B, \mu, \phi; \mathbf{e}) \neq \{\emptyset\}$. Thus, there exists $f^{\infty} \in \mathbf{S}(g, B, \mu, \phi; \mathbf{e})$ such that

$$(B_1(g, \delta), \mu_1(g, \delta, \cdot)) \in \Gamma_{\mathbf{e}}(g, \delta B, \mu, \boldsymbol{\phi}(g, \delta))$$

for any $\delta \in supp(\Pr\{\cdot | g, \mathbf{d}(s)\})$. Moreover, by definition of $\mathbf{S}$, for any $\hat{\omega} \equiv (\hat{g}, \hat{\delta})$,

$$(\hat{g}, B_1(g, \delta), \mu_1(g, \delta, \hat{\omega}), \boldsymbol{\phi}(g, \delta)) \in \mathbf{T_e}.$$

Therefore, $(B_1(g, \delta), \mu_1(g, \delta, \cdot)) \in \Lambda_{\mathbf{e}}(\mathbf{T_e})$. Hence, the two displays indicate that

$$(B_1(g, \delta), \mu_1(g, \delta, \cdot)) \in \Gamma_{\mathbf{e}}(g, \delta B, \mu, \boldsymbol{\phi}(g, \delta)) \cap \Lambda_{\mathbf{e}}(\mathbf{T_e}),$$

thus implying that $s \in \Gamma^{-}_{\mathbf{e}}(\Lambda_{\mathbf{e}}(\mathbf{T_e}))$.

I now show that $\mathbf{T_e} \supseteq \Gamma^{-}_{\mathbf{e}}(\Lambda_{\mathbf{e}}(\mathbf{T_e}))$. Assuming the RHS set is non-empty (otherwise the proof is trivial), take $s \in \{s : \Gamma_{\mathbf{e}}(s) \cap \Lambda_{\mathbf{e}}(\mathbf{T_e}) \neq \{\emptyset\}\}$. This implies that there exists a $(B', \vec{\mu}')$ such that (i) $(B', \vec{\mu}') \in \Gamma(s)$ and (ii) $(g, \delta B', \vec{\mu}'(g, \delta), \phi) \in \mathbf{T_e}, \ \forall (g, \delta)$.

To show the desired result, it suffices to show that there exists $\hat{f}^{\infty} \equiv (\hat{B}^{\infty}, \hat{\mu}^{\infty})$ such that $\hat{f}^{\infty} \in \mathbf{S}(s; \mathbf{e})$. To show this, let $\hat{B}_1 = B'$ and $\hat{\mu}(\cdot) = \vec{\mu}')$ (with B' and $\vec{\mu}'$ in (i)) and regarding $(\hat{f}^{\infty}_t)_{t \geq 2}$, it follows from part (ii) that there exists a continuation sequence that belongs to $\mathbf{S}(g, \hat{B}_1, \hat{\mu}_1, \phi; \mathbf{e})$. □

[64] See definition H.1 for the definition of policy function.

H.5. Alternative Recursive Formulation

In this section I present an alternative timing for the recursive formulation. The advantage of this timing over the one in the text is that is not necessary to keep $\vec{\mu}$ as state variable, only μ.

The following lemma formally states this result.

Lemma H.2. *Let* $z \equiv (g, B, \mu, \phi) \in \mathbb{G} \times \mathbb{B} \times \mathbb{R}_+ \times \{0,1\}$ *and*

$$z \mapsto \mathbf{v}(z) = \max_{\vec{e}' \in G(\phi)} \max_{(B', \vec{\mu}') \in \Gamma_{\vec{e}'}(z)} \int_{\mathbb{G}\times\bar{\Delta}} \left\{ F(g', \vec{\mu}'(\tilde{\omega}'), \vec{\phi}'(\tilde{\omega}')) + \beta \mathbf{v}(g', \delta' B', \vec{\mu}'(\tilde{\omega}'), \vec{\phi}'(\tilde{\omega}')) \right\} \Pr(d\tilde{\omega}'|g, \vec{d}').$$

Where $\vec{\phi}'(\tilde{\omega}') \equiv 1 - \vec{d}'(\tilde{\omega}') + \vec{d}'(\tilde{\omega}')\vec{a}'(\tilde{\omega}')$, $\Pr(\tilde{\omega}'|g,d) \equiv \Pr(\delta'|g',d)\pi_{\mathbb{G}}(dg'|g)$ *and*

$$\phi \mapsto G(\phi) \equiv \{\vec{e} : \forall (g,\delta) \colon \vec{d}(g) \in \mathbb{D}(\phi) \text{ and } \vec{a}(g,\delta) \in \mathbb{A}(\vec{d}(g), \delta)\}.$$

Then

$$\mathbf{v}(z) = \max_{(B', \vec{\mu}') \in \Gamma_{\vec{e}'}(z)} \int_{\mathbb{G}} \mathbf{V}^*(g', B', \vec{\mu}', \phi)\pi_{\mathbb{G}}(dg'|g), \tag{H.38}$$

where $\vec{e}' \equiv \mathbf{e}^*$.

Before showing the result a few remarks are in order. First, as it was the case for $\mathbf{V}^*$, $\mathbf{v}$ is defined over an endogenous state space which is analogous to the on for $\mathbf{V}^*$. Second, $\mathbf{v}$ for $\phi = 0$ (financial autarky), is constant with respect to μ. This follows from the assumption that after financial autarky, the government starts anew in terms of past "tax promises". Third, an appealing feature of $\mathbf{v}$ is that it contains μ (and not $\vec{\mu}$) as part of the state. This is due to the fact that, as indicated by equation H.38, $\mathbf{v}$ is defined as the value function prior to making the decisions.

Proof of Lemma H.2. It suffices to show the following statements: (i) $\vec{e}' \equiv \mathbf{e}^*$ and the fact that $\vec{e}'$ is optimal for $\mathbf{v}$, then equation H.38 holds, and (ii) given equation H.38, the optimal $\vec{e}'$ is such that $\vec{e}' \equiv \mathbf{e}^*$.

Part (ii) follows by using equation H.38 to replace $\mathbf{v}$ by $\mathbf{V}^*$ in the RHS of the Bellman equation above and simple (but tedious) algebra.

For part (i). Observe that, under the fact that $\vec{e}' \equiv \mathbf{e}^*$ is optimal, it follows by definition of $\mathbf{v}$, for any $z \equiv (g, B, \mu, \phi)$,

$$\mathbf{v}(z) = \max_{(B', \vec{\mu}') \in \Gamma_{\vec{e}'}(z)} \int_{\mathbb{G}\times\bar{\Delta}} \{ F(g', \vec{\mu}'(\tilde{\omega}'), \boldsymbol{\phi}^*(\tilde{\omega}')) + \beta \mathbf{v}(g', \delta' B', \vec{\mu}'(\tilde{\omega}'), \boldsymbol{\phi}^*(\tilde{\omega}')) \} \Pr(d\tilde{\omega}'|g, \vec{d}')$$

where $\boldsymbol{\phi}^*$ is defined as ϕ but using $\mathbf{e}^*$. It is easy to see that the operator defining $\mathbf{v}$ in the display above is a contraction. Thus, if I show that $z = (g, B, \mu, \phi) \mapsto \max_{(B', \vec{\mu}') \in \Gamma_{\vec{e}'}(z)} \int_{\mathbb{G}} \mathbf{V}^*(g', B', \vec{\mu}', \phi)\pi_{\mathbb{G}}(dg'|g)$ is a fixed point for the operator, the result follows.

In order to do this, by definition of $\mathbf{V}^*$ it follows that

$$\begin{aligned}\mathbf{V}^*(g', B', \vec{\mu}', \phi) = &\int_{\bar{\Delta}} \{F(g', \vec{\mu}'(\tilde{\omega}'), \boldsymbol{\phi}^*(\tilde{\omega}')) \\ &+\beta \max_{(B'', \vec{\mu}'') \in \Gamma_{\mathbf{e}^*}(g', \delta' B', \vec{\mu}'(\tilde{\omega}'), \boldsymbol{\phi}^*(\tilde{\omega}'))} \int_{\mathbb{G}} \mathbf{V}^*(g', B', \vec{\mu}', \boldsymbol{\phi}^*(\tilde{\omega}')) \pi_{\mathbb{G}}(dg'|g) \Big\} \Pr(\delta'|g', \mathbf{d}^*) \\ \equiv &\int_{\bar{\Delta}} \{F(g', \vec{\mu}'(\tilde{\omega}'), \boldsymbol{\phi}^*(\tilde{\omega}')) + \beta v(g', \delta' B', \vec{\mu}'(\tilde{\omega}'), \boldsymbol{\phi}^*(\tilde{\omega}'))\} \Pr(\delta'|g', \mathbf{d}^*).\end{aligned}$$

Where the second line follows from defining $v(g', B', \vec{\mu}'(\tilde{\omega}'), \boldsymbol{\phi}^*(\tilde{\omega}'))$ (observe that, for now, it could be that $v \neq \mathbf{v}$) as $\max_{(B'', \vec{\mu}'') \in \Gamma_{\mathbf{e}^*}(g', \delta' B', \vec{\mu}'(\tilde{\omega}'), \boldsymbol{\phi}^*(\tilde{\omega}'))} \int_{\mathbb{G}} \mathbf{V}^*(g', B', \vec{\mu}', \boldsymbol{\phi}^*(\tilde{\omega}')) \pi_{\mathbb{G}}(dg'|g)$. Taking expectation with respect to $\pi_{\mathbb{G}}$ at both side of the display,

$$\int_{\mathbb{G}} \mathbf{V}^*(g', B', \vec{\mu}', \phi) \pi_{\mathbb{G}}(dg'|g) = \int_{\mathbb{G}} \int_{\bar{\Delta}} \{F(g', \vec{\mu}'(\tilde{\omega}'), \boldsymbol{\phi}^*(\tilde{\omega}')) + \beta v(g', \delta' B', \vec{\mu}'(\tilde{\omega}'), \boldsymbol{\phi}^*(\tilde{\omega}'))\} \Pr(\delta'|g', \mathbf{d}^*) \pi_{\mathbb{G}}(dg'|g).$$

By definition, $\Pr(\delta'|g', \mathbf{d}^*) \pi_{\mathbb{G}}(dg'|g) = \Pr(dg' d\delta'|g, \mathbf{d}^*) = \Pr(d\tilde{\omega}'|g, \mathbf{d}^*)$. This fact, and the fact that the previous display holds for all $(B', \vec{\mu}') \in \Gamma_{\vec{e}'}(z)$ (for suitably chosen z), it follows that

$$\begin{aligned}&\underbrace{\max_{(B', \vec{\mu}') \in \Gamma_{\vec{e}'}(z)} \int_{\mathbb{G}} \mathbf{V}^*(g', B', \vec{\mu}', \phi) \pi_{\mathbb{G}}(dg'|g)}_{v(z)} \\ = &\max_{(B', \vec{\mu}') \in \Gamma_{\vec{e}'}(z)} \int_{\mathbb{G} \times \bar{\Delta}} \{F(g', \vec{\mu}'(\tilde{\omega}'), \boldsymbol{\phi}^*(\tilde{\omega}')) + \beta v(g', \delta' B', \vec{\mu}'(\tilde{\omega}'), \boldsymbol{\phi}^*(\tilde{\omega}'))\} \Pr(d\tilde{\omega}'|g, \vec{d}').\end{aligned}$$

Thus, this shows that v is a fixed point of the same operator that defines $\mathbf{v}$. Since the fixed point is unique, it follows that $v = \mathbf{v}$. □

Appendix I. Proofs of Supplementary Results

Proof of Lemma C.1. (1) Note that $R(\kappa, 0) = 0$ and $R(\kappa, 1) = -\infty$. Also $\frac{dR(\kappa,n)}{dn} = (\kappa - H'(1-n)) + H''(1-n)n$ where $\frac{dR(\kappa,0)}{dn} = (\kappa - H'(1)) > 0$ and $\frac{d^2R(\kappa,n)}{dn^2} = 2H''(1-n) - H'''(1-n)n < 0$; this follows from assumption 6.1. Therefore, $R(\kappa, \cdot)$ is a concave function with $R(\kappa, 0) = 0$ and $R(\kappa, 1) = -\infty$.

Let $n_2(\kappa)$ be such that $R'(\kappa, n_2(\kappa)) = 0$; clearly $n_2(\kappa) < 1$. Since $\frac{dR(\kappa,0)}{dn} > 0$ and $R(\kappa, \cdot)$ is concave, it follows that $n_1(\kappa) > 0$. Also, since $\frac{dR(\kappa,0)}{dn} > 0$ and $R(\kappa, 0) = 0$, it follows that $R(\kappa, n) > 0$ for all $n \in [0, n_2(\kappa)]$.

Following AMSS, it is easy to show that the optimal level of labor belongs to $[n_2(\kappa), n_1(\kappa)]$ where $R(\kappa, n_1(\kappa)) = 0$, and in this domain $R' < 0$. I note that $R(\kappa, n_1(\kappa)) = 0$ iff $\kappa = H'(1 - n_2(\kappa))$ or $n_2(\kappa) = 0$; by optimality, the latter will never be an option.

Hence, I can define $r \mapsto n(\kappa, r) = R^{-1}(\kappa, r)$ for any $r \in [0, R(\kappa, n_2(\kappa))]$. Since R is decreasing and differentiable, it is easy to see that $r \mapsto n(\kappa, r)$ is too.

(2) Note that $\frac{dW(R)}{dR} = (\kappa - H'(1 - n(\kappa, R)))n'(\kappa, R)$. From (1), it follows that $n'(\kappa, \cdot) < 0$ and $(\kappa - H'(1 - n(\kappa, R))) \leq 0$ and with equality only if $\tau = 0$.

I argued that $[0, R(\kappa, n_2(\kappa))]$ is the image of $[n_2(\kappa), n_1(\kappa)]$ under R. Thus, W can be viewed as a mapping, $\tilde{W}$, from $[n_2(\kappa), n_1(\kappa)]$ to $\mathbb{R}$. Moreover, (I omit κ to ease the notational burden)

$$\tilde{W}(n_2) \asymp \kappa n_2 + H(1-n_2) \geq const. > -\infty$$

because $n_2 < 1$. Similarly, $\tilde{W}(n_1) \leq Const. < \infty$. Thus, $\tilde{W}$ (and hence W) is uniformly bounded. □

Proof of Lemma C.2. By definition of $\bar{V}(g, B)$, (which is valid under assumption 6.1),

$$\begin{aligned}
&\left|\bar{V}(g,B) - \bar{V}(g,B+h)\right| \\
\leq&\lambda \int_{\Delta} |\mathbf{a}^*(g,\delta',B)V_C(g,\delta' B) - \mathbf{a}^*(g,\delta',(B+h))V_C(g,\delta'(B+h))|\, \pi_\Delta(d\delta') \\
&+ \lambda \int_{\Delta} \{(1-\mathbf{a}^*(g,\delta',B))V_A(g,B) - (1-\mathbf{a}^*(g,\delta',(B+h)))V_A(g,B+h)\}\pi_\Delta(d\delta') \\
&+ (1-\lambda)|V_A(g,B) - V_A(g,B+h)|.
\end{aligned}$$

And

$$\max_{g \in \mathbb{G}} |V_A(g,B) - V_A(g,B+h)| \leq \beta \int_{\mathbb{G}} \left|\bar{V}(g',B) - \bar{V}(g',B+h)\right| \pi_{\mathbb{G}}(dg'|g).$$

By lemma C.3, $V_C \in L^\infty(\mathbb{G} \times \mathbb{B})$ and $V_A \in L^\infty(\mathbb{G} \times \mathbb{B})$; hence, from the previous equations it follows that $\left|\bar{V}(g,B) - \bar{V}(g,B+h)\right| \leq const.$ (where $const.$ is some positive constant)

$$\left|\bar{V}(g,B) - \bar{V}(g,B+h)\right| \leq Const. \times \left(\lambda + (1-\lambda)\beta \int_{\mathbb{G}} \left|\bar{V}(g',B) - \bar{V}(g',B+h)\right| \pi_{\mathbb{G}}(dg'|g)\right)$$

where $Const. > 0$ is related to the uniform bounds of V_C and V_A. In particular, it is easy to see that it ought to be true that

$$\left|\bar{V}(g,B) - \bar{V}(g,B+h)\right| \leq \lambda \frac{Const.}{1-(1-\lambda)\beta}.$$

The desired result follows with $\lambda(\epsilon)\colon \lambda(\epsilon)\frac{Const.}{1-(1-\lambda(\epsilon))\beta} = \epsilon$. □

Proof of Lemma C.3. By lemma C.1(2), W is uniformly bounded. Thus by standard arguments, e.g. Stokey et al. (1989) Ch. 4 and 9, $V \in L^\infty(\mathbb{G} \times \mathbb{B})$ and $V_C \in L^\infty(\mathbb{G} \times \mathbb{B})$ and $V_A \in L^\infty(\mathbb{G} \times \mathbb{B})$. □

Proof of Lemma C.4. Recall that

$$V_C(g, \delta B) = \max_{B'} \left\{ W(g + \delta B - p(g, B'; \mathbf{d}^*, \mathbf{a}^*)B') + \beta \int_{\mathbb{G}} V(g', B')\pi_{\mathbb{G}}(dg'|g) \right\}.$$

By lemma C.1(2), $R \mapsto W(R)$ is non-increasing and since $B > 0$ this immediately implies that $\delta \mapsto W(g + \delta B - p(g, B'; \mathbf{d}^*, \mathbf{a}^*)B')$ is non-increasing. The maximum does not affect this property, so $\delta \mapsto V_C(g, \delta B)$ is non-decreasing, with $B > 0$.

Regarding B, it follows by similar arguments that $B \mapsto W(g+\delta B-p(g, B'; \mathbf{d}^*, \mathbf{a}^*)B')$ is non-increasing. This implies that, if $B \mapsto V(g, B)$ is non-increasing, the Bellman operator preserves this property. By standard arguments (e.g. see Stokey et al. (1989) Ch. 9) it follows that $B \mapsto V_C(g, B)$ and $B \mapsto V_A(g, B)$ are non-increasing.

Regarding g, suppose $g \mapsto V(g,B)$ is non-increasing. Then under the assumption $\pi_{\mathbb{G}}(\cdot|g_1) \leq \pi_{\mathbb{G}}(\cdot|g_2)$ for $g_1 < g_2$., then

$$g \mapsto \int_{\mathbb{G}} V(g',B')\pi_{\mathbb{G}}(dg'|g)$$

is non-increasing too. In addition, $g \mapsto W(g+\delta B - p(g,B';\mathbf{d}^*,\mathbf{a}^*)B')$ and $g \mapsto W(g)$ are non-increasing. So, by standard arguments (e.g. see Stokey et al. (1989) Ch. 4 and 9) it follows that $g \mapsto V(g,B)$ is non-increasing. It thus follows trivially that V_A and V_C are non-increasing as a function of g. □

Proof of Claim E.1. The first part follows from applying Blackwell sufficient conditions. Both conditions, monotonicity and discounting are straightforward to verify. To show the second part, observe that one can cast $\mathbb{S}_\kappa$ as

$$\mathbb{S}_\kappa[F](g,B) = \max_{B'}\{\Delta W_\kappa(g,B,B') + \beta \int_{\mathbb{G}} \max\{F(g',B'),0\}\pi_{\mathbb{G}}(dg'|g)\}.$$

Observe that

$$\frac{dW_\kappa(g)}{d\kappa} = n_A(g;\kappa) + (\kappa - H'(1-n_A(g;\kappa)))\frac{dn_A(g;\kappa)}{d\kappa}$$

where, for any g, $\kappa n_A(g;\kappa) - H'(1-n_A(g;\kappa))n_A(g;\kappa) = g$. It follows that

$$\frac{dn_A(g;\kappa)}{d\kappa} = -\frac{n_A(g;\kappa)}{\kappa - H'(1-n_A(g;\kappa)) + H''(1-n_A(g;\kappa))n_A(g;\kappa)}.$$

Hence

$$\begin{aligned}\frac{dW_\kappa(g)}{d\kappa} =& n_A(g;\kappa)\left(1 - \frac{\kappa - H'(1-n_A(g;\kappa))}{\kappa - H'(1-n_A(g;\kappa)) + H''(1-n_A(g;\kappa))n_A(g;\kappa)}\right)\\ =& n_A(g;\kappa)\left(\frac{H''(1-n_A(g;\kappa))n_A(g;\kappa)}{\kappa - H'(1-n_A(g;\kappa)) + H''(1-n_A(g;\kappa))n_A(g;\kappa)}\right).\end{aligned}$$

Observe that $\kappa - H'(1-n_A(g;\kappa)) + H''(1-n_A(g;\kappa))n_A(g;\kappa) < 0$ by lemma C.1)(1) and $H'' < 0$ be assumption. Thus $\frac{dW_\kappa(g)}{d\kappa} > 0$.

This implies that, for $\kappa < \kappa'$,

$$\Delta W_\kappa(g,B,B') + \beta \int_{\mathbb{G}} \max\{F(g',B'),0\}\pi_{\mathbb{G}}(dg'|g) > \Delta W_{\kappa'}(g,B,B') + \beta \int_{\mathbb{G}} \max\{F(g',B'),0\}\pi_{\mathbb{G}}(dg'|g)$$

and the max operator does not change this. □

Appendix J. Description of the Data

In this section I describe how I constructed the figures presented in section 2.

The industrialized economies group consists of AUSTRALIA (1990-1999), AUSTRIA (1990-1999), BELGIUM (1990-2001), CANADA (1990-2003), DENMARK (1990-2003), FINLAND (1994-1998), FRANCE (1990-2003), GERMANY (1990-1998), GREECE (1990-2001), IRELAND (1995-2003), ITALY (1990-2003), JAPAN (1990-1993), NETHERLANDS (1990-2001), NEW ZEALAND (1990-2003), NORWAY (1990-2003), PORTUGAL (1990-2001), SPAIN (1990-2003), SWEDEN (1990-2003), SWITZERLAND (1990-2003), UNITED KINGDOM (1990-2003) and UNITED STATES (1990-2003).

The emerging economies group consists of ARGENTINA[1] (1998-2003), BOLIVIA[1] (2001-2003), BRAZIL[1] (1997-2003), CHILE[1] (1993-2003), COLOMBIA[1] (1999-2003), ECUADOR[1] (1998-2003), EL SALVADOR[1] (2000-2003), HONDURAS[1] (1990-2003), JAMAICA[1] (1990-2003), MEXICO[1] (1990-2003), PANAMA[1] (1997-2003), PERU[1] (1998-2003), VENEZUELA[1] (1997-2003), ALBANIA (1995-2003), BULGARIA (1991-2003), CYPRUS (1990-2003), CZECH REPUBLIC (1993-2003), HUNGARY (1991-2003), LATVIA (1990-2003), POLAND (1990-2003), RUSSIA (1993-2003), TURKEY (1998-2003), ALGERIA (1990-2003), CHINA (1997-2003), EGYPT (1993-2003), JORDAN (1990-2003), KOREA (1990-2003), MALAYSIA (1990-2003), MAURITIUS (1990-2003), MOROCCO (1997-2003), PAKISTAN (1990-2003), PHILIPPINES (1997-2003), SOUTH AFRICA (1990-2003), THAILAND (1999-2003) and TUNISIA (1994-2003). The LAC group is conformed by the countries with "[1]".

For section 2 I constructed the data as follows. First, for each country, I computed time average, or time standard deviations or any quantity of interest (in parenthesis is the number of observations use to construct these). Second, once I computed these averages, I group the countries in IND, EME and LAC. I do this procedure for (a) central government domestic debt (as % of output) ; (b) central government expenditure (as % of output) ; (c) central government revenue (as % of output) , and (d) Real Risk Measure. The data for (a) is taken from Panizza (2008) ; the data for (b-c) is taken from Kaminsky et al. (2004) ; finally the data for (d) is taken from www.globalfinancialdata.com.[65] [66] [67]

[65]For Greece and Portugal I use central government public debt because central government domestic debt was not available. For Sweden, Ecuador and Thailand I use general government expenditure because central government expenditure was not available. For Albania, Bulgaria, Cyprus, Czech Rep., Hungary, Latvia, Poland and Russia no measure of government expenditure was available and thus were excluded from the sample for the calculations of this variable. The same caveats apply to the central government revenue sample.

[66]I gratefully acknowledge that Kaminsky et al. (2004) and Panizza (2008) kindly shared the dataset used in their respective papers (see references).

[67]For Argentina, Brazil, Colombia, Ecuador, Egypt, Mexico, Morocco, Panama, Peru, Philippines, Poland, Russia, Turkey and Venezuela I used the real EMBI+ as a measure of real risk. For the rest of the countries I used government note yields of 1-5 years maturity, depending on availability.