OPTIMAL TAXATION WITH ENDOGENOUS DEFAULT UNDER INCOMPLETE MARKETS

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ABSTRACT. In a dynamic economy, I characterize the fiscal policy of the government when it levies distortionary taxes and issues defaultable bonds to finance its stochastic expenditure. Households predict the possibility of default, generating endogenous debt limits that hinder the government’s ability to smooth shocks using debt. Default is followed by temporary financial autarky. The government can only exit this state by paying a fraction of the defaulted debt. Since this payment may not occur immediately, in the meantime, households trade the defaulted debt in secondary markets; this device allows me to price the government debt before and during the default.

JEL: H3,H21,H63,D52,C60.

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1. INTRODUCTION

For many governments, debt and tax policies are conditioned by the possibility of default. For emerging economies, default is a recurrent event and is typically followed by a lengthy debt-restructuring process, in which the government and bond holders engage in renegotiations that conclude with the government paying a fraction of the defaulted debt.\(^1\)

\(^1\)See Pitchford and Wright (2008) and Benjamin and Wright (2009).
I find that emerging economies exhibit lower levels of indebtedness and higher volatility of government tax policy than do industrialized economies—where, contrary to emerging economies, default is not observed in my dataset—. Also, emerging economies, exhibit higher interest rate spreads, especially for high levels of domestic debt-to-output ratios, than industrialized economies. In fact, industrialized economies exhibit interest rate spreads that are low and roughly constant for different levels of domestic debt-to-output ratios. Moreover, in emerging economies, the highest interest rate spreads are observed after default and during the debt-restructuring period.

These empirical facts indicate that economies that are more prone to default display different government tax policy, as well as different prices of government debt, before default and during the debt-restructuring period. Therefore, the option to default, and the actual default event, will affect the utility of the economy’s residents: Indirectly, by affecting the tax policy and debt prices, but also directly, by not servicing the debt in the hands of the economy’s residents during the default event.

My main objective is to understand how the possibility of default and the actual default event affect tax policy, debt prices—before and during default—, and welfare of the economy. For this purpose, I analyze the dynamic taxation problem of a benevolent government in a closed economy under incomplete markets. The government chooses distortionary labor taxes, non-state-contingent debt, and whether to default, so as to maximize the representative household’s lifetime expected utility, and subject to the equilibrium restrictions imposed by the households’ optimal decisions, market clearing conditions and feasibility. Although the government cannot commit to pay the debt, it can commit itself to a path of taxes when the economy is not in default. If the government defaults, the economy enters temporary financial autarky and faces exogenous offers to pay a fraction of the defaulted debt that occur at an exogenous rate. The government has the option to accept the offer—and, thus,

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2To measure “indebtedness”, I am using government domestic debt-to-output ratios, where domestic debt is the debt issued under domestic law (see Panizza (2008)). I am using domestic and not total government debt because my model is a closed economy. As a proxy of tax policy, I am using government revenue-to-output ratio or inflation tax.

3Throughout this paper, I will also refer to the restructuring period as the default period.

4For Argentina’s default in 2001, almost 50 percent of the face value of debt to be restructured (about 53 percent of the total owed debt from 2001) is estimated to be in the hands of Argentinean residents; Local pension funds alone held almost 20 percent of the total defaulted debt (see Sturzenegger and Zettelmeyer (2006)).

5In this model, financial autarky is understood as the period during which the government is precluded from issuing new debt/savings.
exit financial autarky—or to stay in financial autarky until a new offer comes along. During temporary financial autarky, the defaulted debt still has positive value because it is going to be paid in the future with positive probability. Hence, households can trade the defaulted debt in a secondary market from which the government is excluded; the equilibrium price in this market is used to price the debt during a period of default.

In the model, the government has three policy instruments: (1) distortionary taxes, (2) government debt, and (3) default decisions that consist of: (a) whether to default on the outstanding debt and (b) whether to accept the offer to exit temporary financial autarky.

The government faces a trade-off between levying distortionary taxes to finance the stochastic process of expenditures and not defaulting, or issuing debt and thereby increasing the exposure to default risk. The option to default introduces some degree of state contingency on the payoff of the debt since the financial instrument available to the government becomes an option, rather than a non-state-contingent bond. This option, however, does not come free of charge: Households accurately predict the possibility of default, and the equilibrium incorporates it into the pricing of the bond; this originates a “Laffer curve” type of behavior for the debt income, thereby implying endogenous debt limits. In this sense, my model generates “debt intolerance” endogenously. This behavior hinders government’s ability to smooth shocks using debt, renders tax policy more volatile, and implies higher interest rate spreads. Hence, the possibility of default introduces a trade-off between the cost of the lack of commitment to repay the debt, reflected in the price of the debt, and the flexibility that comes from the option to default and partial payments, reflected in the pay-off of the debt.

In a benchmark case, with quasi-linear utility, i.i.d. process for the government expenditure, I characterize, analytically, the determinants of the optimal default decision and its effects on the optimal taxes, debt and allocations. For this purpose, I assume financial autarky forever after default. First, I show that default is more likely when the government’s expenditure or debt is higher. Second, I show how the law of motion of the optimal government policy is affected, on the one hand, by the benefit from having “more state-contingency” on the payoff of the bond, but, on the other hand, by the cost of having the option to default.

Finally, I calibrate a more complete model, with an auto-correlated process for the government expenditure and an exogenous process for the arrival of offers of partial payments.

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6 A term coined by Reinhart et al. (2003).
to exit financial autarky; the model is qualitatively consistent with the differences observed in the data between emerging and industrialized economies. In terms of welfare policy, the numerical simulations suggest a nonlinear relationship between welfare and the probability of receiving an offer of partial payments. In particular, increasing the probability of receiving offers for exiting autarky decreases welfare when this probability is low/medium to begin with, but increases it when the probability is high.

The paper is organized as follows. I first present the related literature. Section 2 presents some stylized facts. Section 3 introduces the model. Section 4 presents the recursive equilibrium, and section 5 presents the planner’s problem. Section 6 derives analytical results that characterize the government policy for a simple example. Section 7 contains some numerical exercises. Section 8 briefly concludes. All proofs are gathered in the appendices.

1.1. Related Literature. The paper builds on and contributes to two main strands in the literature: endogenous default and optimal taxation.

Regarding the first strand, I model the strategic default decision of the government as in Arellano (2008) and Aguiar and Gopinath (2006), which, in turn, are based on the seminal paper by Eaton and Gersovitz (1981). My model, however, differs from theirs in several ways. First, I consider distortionary taxation; Arellano (2008) and references therein implicitly assume lump-sum taxes. Second, my economy is closed—i.e., “creditors” are the representative household—; Arellano (2008) and references therein assume an open economy with foreign creditors. Note that under the closed-economy assumption, the default decision has a direct effect on the households’ wealth—and, thus, welfare—because the government does not honor the debt in the hands of the households. Third, in my model, the government must pay at least a positive fraction of the defaulted debt to exit financial autarky through a “debt-restructuring process”; in Arellano (2008) and references therein, the government is exempt from paying the totality of the defaulted debt upon exit of autarky. I model this “debt-restructuring process” exogenously, indexing it by two parameters, because I am interested in studying only the consequences of this process on the optimal fiscal policy and welfare. 7

Regarding the second strand, I base my work on Aiyagari et al. (2002), where, in a closed economy, the benevolent infinitely-lived government chooses distortionary labor taxes and

7See Benjamin and Wright (2009), Pitchford and Wright (2008) and Yue (2005) for ways of modeling the debt-restructuring process endogenously.
non-state-contingent risk-free debt, taking into account restrictions from the competitive equilibria, to maximize the households’ lifetime expected utility. My work relaxes this last assumption and, as a consequence, the option to default creates endogenous debt limits, reflected in the equilibrium prices.

Following the aforementioned literature, I assume that, although the government can commit itself to a tax policy outside temporary financial autarky, during this period, taxes are set mechanically so that tax revenues finance the government expenditure. This feature is related to Debortoli and Nunes (2008). Here the authors study the dynamics of debt in the Lucas and Stokey (1983) setting but with the caveat that at each time $t$, with some given probability, the government can lose its ability to commit to taxes; the authors refer to this as “loose commitment.” Thus, my model provides a mechanism that “rationalizes” this probability of “loosing commitment” by assuming that the government is not committed to paying debt and can default at any time. It is worth noting that, in their model, the budget constraint during the no-commitment stage remains essentially the same, whereas mine does not.

Finally, in recent independent papers, Doda (2007), Cuadra et al. (2009), study the procyclicality of fiscal policy in developing countries by solving an optimal fiscal-policy problem. Their work differs from mine in two main aspects. They assume, first, an open small economy (i.e., foreign lenders) and, second, no secondary markets.\(^8\)

2. Stylized Facts

In this section, I present stylized facts regarding the domestic government debt-to-output ratio and central government revenue-to-output ratio of several countries: Industrialized economies (IND, henceforth), emerging economies (EME, henceforth) and a subset of these: Latin American (LAC, henceforth).\(^9\)

As shown below, my theory predicts that endogenous borrowing limits are more active for a high level of indebtedness. That is, when the government debt is high (relative to output), the probability of default is higher, thus implying tighter borrowing limits, higher spreads

\(^8\) Aguiar et al. (2008) also allow for default in a small open economy with capital where households do not have access to neither financial markets nor capital and provide labor elastically. The authors’ main focus is on capital taxation and the debt “overhang” effect.

\(^9\) For the latter ratios, I used the data in Kaminsky et al. (2004), and for the first ratio, I used the data in Panizza (2008). See Appendix C for a detailed description of the data.
and higher volatility of taxes. But when this variable is low, default is an unlikely event, thereby implying slacker borrowing limits, lower spreads and lower volatility in the taxes. Hence, implications in the upper tail of the domestic debt-to-output ratio distribution can be different from those in the “central part” of it. Therefore, the mean and even the variance of the distribution are not too informative, as they are affected by the central part of the distribution; quantiles are better suited for recovering the information in the tails of the distribution.\textsuperscript{10}

Table 2.1. (A) Measure of default risk for EME and IND groups for different quantiles of debt-to-output ratio; (B) Std. Dev. of ctral. government revenue over GDP (%) for EME and IND groups for different quantiles of debt.

<table>
<thead>
<tr>
<th>Debt/GDP Quantile</th>
<th>(A)</th>
<th>(B)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>EME</td>
<td>IND</td>
</tr>
<tr>
<td>0.10</td>
<td>6.55</td>
<td>3.16</td>
</tr>
<tr>
<td>0.90</td>
<td>7.75</td>
<td>2.60</td>
</tr>
</tbody>
</table>

Table 2(A) compares a measure of default risk across different quantiles, between IND and EME matching them across the 10th and 90th debt quantiles.\textsuperscript{11} \textsuperscript{12} For both quantiles, the measure of default risk is higher for EME than for IND; however, for the former group, the measure increases as the debt quantile increases, whereas for the latter group, it decreases.

Table 2(B) compares the standard deviation of the central government revenue-to-output ratio. For the lowest quantile, both volatilities are very close to each other for both groups, but as we go to the higher quantile, the volatility for the EME group increases, but for the IND group decreases.\textsuperscript{13}

In addition, the amount of debt-to-output associated with the 10 percent quantile for IND and EME are, 19 percent and 10 percent, respectively; for the 90 percent quantiles, these amounts are 74 percent and 40 percent, respectively. That is, “high levels” of debt-to-output for EME are much lower than “high levels” of debt-to-output for IND.

Finally, it is worth noting that during the debt-restructuring period, the measure of default risk stays significantly higher than during “no default periods.” For instance, for Argentina,

\textsuperscript{10}I refer the reader to Koenker (2005) for a thorough treatment of quantiles and quantile-based econometric models.
\textsuperscript{11}I constructed the measure of default risk as the spread using the EMBI+ real index for countries for which it is available and using the 3-7 year real government bond yield for the rest, minus U.S. bond return.
\textsuperscript{12}I also studied the domestic debt net of foreign reserves; results are the same or are even enhanced.
\textsuperscript{13}I looked also at the inflation tax as a proxy for tax policy; results are qualitatively the same.
this measure was around 5 percent during 1997-2000 and 2005-2006 but around 60 percent
during 2001-2005; for Russia, it was around 4 percent during 2000-2006 but around 20
percent during 1998-1999; finally, for Ecuador, it was around 9 percent during 1997 and

3. The Model

3.1. The Setting. Let time be indexed as \( t = 0, 1, \ldots \) and let \((\Omega, \mathcal{F}, \Pr)\) denote the
underlying probability space. The government expenditure process \((g_t)_t\) is an exogenous stochas-
tic process such that \( g_t : \Omega \rightarrow \mathbb{G} \) with \( \mathbb{G} \) a compact and convex subset of \( \mathbb{R} \). Let
\( g^t \equiv (g_0, \ldots, g_t) \in \mathbb{G} \times \cdots \times \mathbb{G} \equiv \mathbb{G}^{t+1} \) be the history of government expenditures until time
t. Let \( \mathcal{G}^t \equiv \mathcal{F}(g^t) \) be the \( \sigma \)-algebra generated by \( g^t \). I assume further that the proba-
bility kernel of the \((g^t)_t\) is Markov—i.e., \( \pi_t(g_{t+1}|g^t) \) (the conditional probability of \( g_{t+1} \in \mathbb{G} \),
conditioned on \( g^t \in \mathbb{G}^{t+1} \)) equals \( \pi(g_{t+1}|g_t) \) (see Appendix A). Finally, let \( \pi_0(g_0) \) be the
unconditional probability of \( g_0 \); this probability can be degenerate at a point.

At each time \( t \), the government can levy distortionary labor taxes, \( \tau^n_t \), and allocate one-
period, non-state-contingent bonds to the households to cover the expenses \( g_t \). I denote
\( B^G_t \in \mathbb{B} \) as the government bonds, where the set \( \mathbb{B} \) is a compact interval on \( \mathbb{R} \). A quantity
\( B^G_t > 0 \) means that the government has to pay the households \( B^G_t \) units of consumption
at time \( t \). The government, after observing the present government expenditure and the
outstanding debt to be paid this period, has the option to default on 100 percent of this
debt—i.e., the government has the option to refuse to pay the totality of the maturing debt.

As shown in figure D, if the government opts to exercise the option to default on 100 percent
the debt (node (A) in figure D), nature plays immediately, and with probability \( 1 - \lambda \), sends
the government to temporary financial autarky, where the government is precluded from
issuing bonds in that period. With probability \( \lambda \), the government enters a stage in which
nature draws a fraction \( 1 - \delta \) (with \( \delta \) distributed according to the probability function \( \pi_\delta(\delta) \))
of debt to be repaid, and the government has the option to accept or reject this offer. If the
government accepts, it pays the new amount (the outstanding debt times the fraction that
nature chose), and it is able to issue new bonds for the following period. If the government
rejects, it goes to temporary financial autarky (bottom branch in figure D).

The parameters \( (\lambda, \pi_\delta(\delta)) \) define the “debt-restructuring process.” These parameters cap-
ture the fact that debt-restructuring is time-consuming but, generally, at the end, a positive
fraction of the defaulted debt is honored (see Yue (2005), and Pitchford and Wright (2008) and Benjamin and Wright (2009) for two different ways of modeling this process as renegotiation between the government and the debt holders).

Finally, if the government is not in financial autarky—because it either chooses not to default, or it accepts the partial payment offer—then in the next period, it has the option to default, with new values of outstanding debt and government expenditure. If the government is in temporary financial autarky, then in the next period, it will face a new offer for partial payments with probability \( \lambda \).

**Remark 3.1.** I also consider an alternative option for the government to exit financial autarky. At the end of the period of financial autarky, with probability \( \alpha \), the government receives the option to leave autarky by paying 100 percent of the outstanding debt. The parameter \( \alpha \) summarizes transaction costs or other types of financial frictions that allow the government to exercise this option only occasionally. The bottom branch of figure D shows this alternative procedure.

Households are price takers and homogeneous; at each period \( t \), given their initial financial wealth \( z_t \), they decide how much to consume \( c_t \), how much to allocate to leisure \( l_t = 1 - n_t \) (which yields an after-tax labor income \( (1 - \tau^m_t)n_t \)), and how much to save \( b_{t+1}^G \) (if the economy is not in financial autarky) or how many shares, \( L_t \), of defaulted debt to trade (if the economy is in financial autarky).

Let \( d_t \in \mathbb{D} \cup \{1\} \equiv \{0\} \cup \Delta \cup \{1\} \) be a state variable that, at each time \( t \), indicates whether the government has paid 100 percent, a part or 0 percent of the debt. That is, \( d_t = 0 \) means that the government is not in default and has fully honored its outstanding debt; \( d_t = 1 \) means that the government has defaulted on the totality of the debt; finally, \( \Delta \equiv \{\delta_1, \ldots, \delta_\Delta\} \) (such that \( \delta_i < \delta_{i+1} \) and \( \delta_i \in (0,1) \) for all \( i \)) is a set of all possible fractions of debt on which the government could (partially) default. For instance, \( d_t \equiv \delta \) implies that

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\(^{14}\)The exogenous probabilities \( \pi_\delta \) and \( \lambda \) are set to be constant, but I can also allow for probabilities that depend on the state. For instance, I can have \( \pi_\delta \equiv \pi_\delta(b_t, d_t) \) denoting that possible partial payments depend on the credit history and level of debt. See Reinhart et al. (2003), Reinhart and Rogoff (2008) and Yue (2005) for an intuition behind this structure. Numerical simulations allowing for this structure are qualitatively the same as those shown in this chapter and are available upon request.
the government partially defaulted upon a fraction $\delta$ of the outstanding debt. I refer to $d_t$ as the “default indicator.” Finally, let $X \equiv G \times \{D \cup \{1\}\}$, $x_t \equiv (g_t, d_t) \in X$.\(^\text{15}\)

3.2. **The Household Problem.** The bellman equation of the household is

\[
V(z_t, \Theta_t) = \max_{c_t, n_t, z_{t+1}} \{U(c_t, 1 - n_t) + \beta E_t [V(z_{t+1}, \Theta_{t+1})]\}
\]

(2.1)

with $\Theta_t \equiv (x_t, B^G_t)$,

where $z_t$ is the initial financial wealth at the beginning of time $t$. The value function is also a function of the perceived law of motion of the households for the government expenditure, “default indicator” and debt: $\Theta_t \equiv (g_t, d_t, B^G_t)$. I summarize some standard conditions for $U(c_t, 1 - n_t)$ in Appendix A.

Due to the asymmetry between the financial assets described in section 3.1, I write the constraints for the household problem for the cases $d \in D$, and $d = 1$ separately.

3.2.1. **Household’s budget constraint for the case: $d_t \in D$.** For this case, the agents solve the problem in equation 2.1 subject to

\[
c_t + p^b_t b^{G}_{t+1} - (1 - \tau^n_t)n_t \leq z_t
\]

(2.3)

\[
z_{t+1}(d_{t+1}) = (1 - d_{t+1})b^{G}_{t+1}, \quad \forall d_{t+1} \in D,
\]

(2.4)

where $p^b_t$ is the price of the government bonds and $z_{t+1}(d)$ is defined as the financial wealth of household at the beginning of $t + 1$ when $d_{t+1} = d$. If $d_{t+1} = 1$

\[
z_{t+1}(1) = q_{t+1}b^{G}_{t+1},
\]

where $q_{t+1}$ is the secondary market price of defaulted government debt.

If $d_{t+1} = \delta$, then the household acknowledges that it receives only a part of its assets and if $d_{t+1} = 1$, the initial financial wealth of the household at $t + 1$ is whatever value the household can get out of its assets in the secondary market—i.e., $q_{t+1}b^{G}_{t+1}$.\(^\text{16}\)

\(^{15}\)The model could also allow for a “credit history”—i.e., $\tilde{d}_t \equiv (d_{t-K}, \ldots, d_t)$ where $\tilde{d}_t$ is the “credit history” of the last $K$ periods of the economy. For simplicity in this version I set $K = 0$.

\(^{16}\)The household also faces borrowing limits; but I assume that the exogenous borrowing limits for the household are always less stringent than those for the government, and, thus, in equilibrium the household problem is always an interior solution regarding its choice of assets.
3.2.2. Household’s budget constraint for the case: \( d_t = 1 \). Under this node, the government is in temporary financial autarky—i.e., does not honor the outstanding debt today and is also precluded from issuing new debt. Although the government is excluded from the financial markets, the households can trade the debt that the government owes them but is not honored today. Even though the households are homogeneous and, thus, no trade takes place in equilibrium, this secondary market yields an equilibrium price that reflects the fact that some fraction of the defaulted debt is going to be paid with positive probability at some point in the future. If the probability of the government repaying the debt in the future is naught, then the value of this secondary market’s asset is also naught; in this case, I can, without loss of generality, close this market—see, e.g., Arellano (2008).

I assume that households cannot issue debt. Thus, denoting \( L_t \) as the shares of defaulted debt the household can trade in the secondary markets, it follows that

\[
L_t \leq 1. 
\]

Therefore, the budget constraint is given by

\[
c_t + q_t L_t B_t^G - (1 - \tau_t^n)n_t \leq z_t, \tag{2.6}
\]

Note that, \( B_t^G \) and not \( b_t^G \) is in the budget constraint, because, under \( d_t = 1 \), the defaulted debt is exogenous for the household, and the only variable the household controls is the shares they trade.\(^{17}\)

At \( t + 1 \), the initial financial wealth of the household is given by

\[
\begin{align*}
z_{t+1}(d_{t+1}) &= (1 - d_{t+1})L_t B_t^G, \quad \forall d_{t+1} \in \mathbb{D}, \\
z_{t+1}(1) &= q_{t+1} L_t B_t^G.
\end{align*}
\]

3.3. The Government Problem. The government finances its stream of expenditure \((g_t)_t\) by levying time-varying taxes on labor, \( \tau_t^n \) and issuing government debt \( B_t^G \) in \( d_t \in \mathbb{D} \) such that they satisfy its budget constraint for \( d_t \in \mathbb{D} \)

\[
g_t + Z_t = \tau_t^n n_t + p_t b_t^G, \tag{2.7}
\]

\(^{17}\)The model could also encompass the case where, during financial autarky, the defaulted debt evolves according to a function \( \psi : \mathbb{B} \to \mathbb{B} \), i.e., \( B_{t+1}^G = \psi(B_t^G) \). See Yue (2005), where \( \psi(\cdot) = 1 + r \) with \( r \) being an exogenous risk-free rate.
and the budget constraint for $d_t = 1$,

$$g_t = \tau^n_t n_t,$$

where

$$Z_{t+1}(d_{t+1}) = (1 - d_{t+1})B^G_{t+1}, \quad \forall d_{t+1} \in \mathbb{D}$$

$$Z_{t+1}(1) = 0.$$  

Finally, as in Aiyagari et al. (2002), I assume that the government is subject to exogenous borrowing constraints,\(^{18}\)

$$M^G_0 \leq B^G_{t+1} \leq M^G_0, \quad \forall t. \quad (2.11)$$

4. The Recursive Competitive Equilibrium

This section defines the recursive equilibrium for this model. In order to achieve this goal, some intermediate definitions are needed. First, let $b_0 = B_0^G = b_0^G$ be the initial debt of this economy. Let $\tilde{\delta} \in \tilde{\Delta} \equiv \Delta \cup \{1\}$ be a random variable that denotes the lottery for receiving the offer for partial default (i.e., $\Pr(\tilde{\delta} = 1) = 1 - \lambda$) and also denotes the fraction of defaulted debt to be repaid (i.e., $\Pr(\tilde{\delta} = \delta_i) = \lambda \pi_\delta(\delta_i)$). Let $\tilde{\delta}^t$ be the history of this variable up to time $t$ and $\tilde{\Delta}^t \equiv \prod_{\tau=0}^t \tilde{\Delta}$; let $\Pi^t \equiv \mathbb{G} \times \tilde{\Delta}^t$ and let $\mathcal{I}^t$ denote the filtration until time $t$.

**Definition 4.1.** A government policy is a pair of sequences $(h_t, B^G_{t+1})_t$, such that for each $t$

$$h_t \equiv (g_t, \tau^n_t, d_t),$$

where $\tau^n_t : \{b_0\} \times \mathbb{I}^t \to [0, 1]$ is $\mathcal{I}^t$-measurable; $d_t : \{b_0\} \times \mathbb{I}^t \to \mathbb{D} \cup \{1\}$ is $\mathcal{I}^t$-measurable; and $B^G_{t+1} : \{b_0\} \times \mathbb{I}^t \to \mathbb{B} \subseteq \mathbb{R}$ is $\mathcal{I}^t$-measurable with $\mathbb{B}$ a compact interval in $\mathbb{R}$. And, finally, $\{b_0, (h_t, B^G_{t+1})_t\}$ satisfies the government budget constraint in equations 2.7-2.11 for each $t$.

Henceforth, let $\mathbb{H}_t \equiv \mathbb{G} \times [0, 1] \times (\mathbb{D} \cup \{1\})$ and $\mathbb{H}^t \equiv \prod_{\tau=0}^t \mathbb{H}_\tau$.

**Definition 4.2.** A feasible allocation is a sequence vector $(c_t, n_t, g_t)_t$ such that

$$c_t + g_t = \frac{n_t}{1 + \kappa}.$$  

\(^{18}\) The upper bound in this model is not important, because—as shown below—the option to default generates endogenous debt limits. The lower bound does not affect the results qualitatively, insofar as it is above the natural limit; otherwise, combined with lump-sum subsidies, the economy could build a “war chest” and finance all future expenditures with that; see Aiyagari et al. (2002).
with \( c_t : \{ b_0 \} \times \mathbb{H}^t \to \mathbb{R}_+ \) is \( \mathcal{I}^t \)-measurable; \( n_t : \{ b_0 \} \times \mathbb{H}^t \to [0, 1] \) is \( \mathcal{I}^t \)-measurable.

The government policy depends only on the exogenous history of random variables and the initial government debt; but, in the definition of feasible allocation, I define household consumption and labor as functions of the exogenous government policy. This asymmetry arises from the assumption that, in my model, households’ behavior is non-strategic.\(^{19}\) Finally, the parameter \( \kappa \) represents direct cost of defaulting (e.g., \( \kappa \geq 0 \)) if the government decides to default and zero otherwise. For simplicity, I take \( \kappa \equiv 0 \) and consider a different scheme only in the numerical simulations.\(^{20}\)

I now present the definition of recursive competitive equilibrium in this economy.

**Definition 4.3.** In this economy, a (recursive) competitive equilibrium is: an initial \( b_0 \); a set of value functions \( V(\cdot) \); a set of policy functions \( (c(\cdot), n(\cdot), b_{t+1}^G(\cdot), L(\cdot)) \); government policies; prices \( (p^b(\cdot), q(\cdot)) \); a perceived law of motion and actual law of motion for \( \Theta = (g, d, B^G) \); such that

a. Given the initial tuple, prices, government policies and perceived laws of motion, the policy functions and value functions solve the household’s problem.

b. Prices are such that the allocation is feasible and

\[
\begin{align*}
    b^G &= B^G \equiv b, \text{ for } d \in \mathbb{D}, \\
    L &= 1, \text{ for } d = 1.
\end{align*}
\]

c. Given a. and b., the actual and perceived laws of motion coincide.

Henceforth, I will continue to use sequence notation (indexing variable by \( t \)) for simplicity.

4.1. **Equilibrium Taxes and the Price of Government Debt.** I can obtain expressions for the equilibrium price of the government debt \( b_{t+1}^G \), the equilibrium price of one share of defaulted debt \( (L_t) \) traded in the secondary market, and for the labor taxes by first solving the household problem presented above and then substituting the equilibrium conditions in definition 4.2 and the market-clearing conditions in equation 2.13. I am going to impose the “correct” or actual law of motion for the \( \Theta_t \). In order to do this, I introduce two new objects \( D_t[|G] \equiv D(d_t, B^G_t)[|G] \subseteq \mathbb{G} \) and \( D_t[|\Delta] \equiv D(g_t, d_t, B^G_t)[|\Delta] \subseteq \Delta \). The first one is the set of

\(^{19}\)See Phelan and Stacchetti (2001) for a detailed discussion.

\(^{20}\)See Arellano (2008), Aguiar and Gopinath (2006), and Mendoza and Yue (2008) for a discussion about \( \kappa \).
government expenditures at time \( t \) such that the government does not pay the outstanding debt. The second object is the set of fractions of defaulted debt that the government does not accept—i.e., if \( \delta \in D_{t+1}[\Delta] \), the government rejects such an offer.

The expression for the taxes comes directly from the ratio of the first-order conditions for \( c_t \) and \( n_t \),

\[
1 - \tau^n_t = \frac{\nabla_t U(c_t, 1 - n_t)}{\nabla_c U(c_t, 1 - n_t)}.
\]

Let \( \Gamma_t \) be the Lagrange multiplier associated with equation 2.3. Then, from the first-order conditions of the household problem with respect to \( b^n_{t+1} \), it follows that

\[
\begin{align*}
0 = & p^b_t \Gamma_t + \beta E_t \left[ \{1 - \mathbb{I}\{D_{t+1}[\mathbb{G}]\}\} \nabla_z V(b_{t+1}, \Theta_{t+1}) \right. \\
& + \sum_{\delta \in \Delta} \left\{ \mathbb{I}\{D_{t+1}[\mathbb{G}]\} \lambda (1 - \delta)(1 - \mathbb{I}\{D_{t+1}[\Delta]\}) \pi_\delta(\delta) \right\} \nabla_z V((1 - \delta)b_{t+1}, \Theta_{t+1}) \\
& \left. + \left\{ \mathbb{I}\{D_{t+1}[\mathbb{G}]\} \left( (1 - \lambda) + \lambda \sum_{\delta \in \Delta} \mathbb{I}\{D_{t+1}[\Delta]\} \pi_\delta(\delta) \right) q_{t+1} \right\} \nabla_z V(q_{t+1}b_{t+1}, \Theta_{t+1}) \right] \\
\end{align*}
\]

where \( \mathbb{I}\{A\} \) is an indicator function that takes value one if the set \( A \) occurs.

By the envelope condition, it follows that

\[
\nabla_z V(z_t, \Theta_t) = -\Gamma_t.
\]

Let \( \mathcal{P} : \mathbb{G} \times \{\mathbb{D} \cup \{1\}\} \times \mathbb{B}^2 \to \mathbb{R}_+ \) such that \( \mathcal{P}_t \equiv \mathcal{P}(\Theta_t, b_{t+1}) \equiv p^b_t U_{c,t} \) where \( U_{c,t} \) denotes the marginal utility of consumption at time \( t \) when \( d_{t+1} = d \). From equations 2.16 - 2.17, the first-order condition with respect to \( c_t \) (which implies that \( U_{c,t} = \Gamma_t \)), the aggregate equilibrium conditions imply that

\[
\begin{align*}
\begin{align*}
0 = & p^b_t \Gamma_t + \beta E_t \left[ \{1 - \mathbb{I}\{D_{t+1}[\mathbb{G}]\}\} \frac{U_{c,t+1}(0)}{U_{c,t}} \right] \\
& + \beta E_t \left[ \sum_{\delta \in \Delta} \{ (1 - \delta)\mathbb{I}\{D_{t+1}[\mathbb{G}]\} \lambda (1 - \mathbb{I}\{D_{t+1}[\Delta]\}) \pi_\delta(\delta) \right] \frac{U_{c,t+1}(\delta)}{U_{c,t}} \\
& + \beta E_t \left\{ \mathbb{I}\{D_{t+1}[\mathbb{G}]\} \left( (1 - \lambda) + \lambda \sum_{\delta \in \Delta} \mathbb{I}\{D_{t+1}[\Delta]\} \pi_\delta(\delta) \right) q_{t+1} \right\} \frac{U_{c,t+1}(1)}{U_{c,t}}. \\
\end{align*}
\end{align*}
\]

A few noteworthy remarks are in order. First, each term in the equation above corresponds to a “branch” of the tree depicted in figure D. The first line represents the value of one unit of debt when the planner chooses to honor the entire debt. The second line represents the value of the debt if the planner decides not to pay the debt, but ends up in partial defaults.
The third line captures the value of the debt when the planner defaults on 100 percent of the debt, but the households can sell it in the secondary markets. Second, if $\lambda = \alpha = 0$ and $U_{c,t} = 1$, then the last two terms vanish and the price is analogous to the one obtained in Arellano (2008).

I now compute the expression for $q_t$. First, let $Q: \mathbb{G} \times \{\mathbb{D} \cup \{1\}\} \times \mathbb{B}^2 \to \mathbb{R}_+$ be such that $Q_t \equiv Q(\Theta_t, b_{t+1}) = q_t U_{c,t}$. The first-order condition and envelope conditions are basically the same as before; the difference lies in the law of motion for $d_{t+1}$. Following the same steps as before, but, replacing the “correct” law of motion for $d_{t+1}$, it follows that the secondary market price is

$$q_t \equiv \frac{Q_t}{U_{c,t}} = \beta E_t \left[ \sum_{\delta \in \Delta} \left( (1 - \delta) \lambda (1 - \mathbb{I}\{D_{t+1}[\Delta]\}) \pi_{\delta}(\delta) \right) \frac{U_{c,t+1}(\delta)}{U_{c,t}} \right]
+ \beta E_t \left[ \left\{ \left( 1 - \lambda \right) + \lambda \sum_{\delta \in \Delta} \mathbb{I}\{D_{t+1}[\Delta]\} \pi_{\delta}(\delta) \right) q_{t+1} \right] \frac{U_{c,t+1}(1)}{U_{c,t}}.$$  

If autarky is an absorbing state—i.e., $\lambda = \alpha = 0$—it follows that $q_t = \beta E_t \left[ q_{t+1} \frac{U_{c,t+1}(1)}{U_{c,t}} \right]$, which by substituting forward and standard transversality conditions, yields $q_t = 0$.

### 5. The Planner’s Problem

I define the planner’s problem as

**Definition 5.1.** Given an initial $b_0^G = B_0^G \equiv b_0$ and each period utility of $U(\cdot, \cdot)$, the planner chooses the (recursive) competitive equilibrium with the highest utility.

#### 5.1. Primal Approach.

As Kydland and Prescott (1980) pointed out, in order to write the planner’s problem recursively—when there is commitment on taxes—the addition of a new (co)state variable is needed in order to keep track of promises about future government actions. Hence, outside financial autarky, a new (co)state variable must convey this information. By inspecting the first-order conditions of the households, it is sufficient to set the (co)state variable, denoted as $\mu_t$, to be the marginal utility of consumption of the households at time $t$.  

---

Denote $U(b, g, \mu, d)$ as the value of the economy (i.e., the planner who is solving the primal approach) with financial wealth $b$, government expenditure $g$, a (co)state variable $\mu$ (which is defined below) and default indicator $d$ (i.e., no default, partial default or autarky).

If $d_t = 1$, then the government’s budget constraint is given by $g_t = \tau^n_t n_t$; from this equation, equation 2.15, the feasibility constraint for $d_t \in \mathbb{D}$ are as follows:

\begin{equation}
U_c(n_t - g_t, 1 - n_t) (n_t - g_t) - U_l(n_t - g_t, 1 - n_t) n_t = 0.
\end{equation}

Therefore, I can solve for $n_t$, which we denote it as $n^A_t \equiv n^A(g_t)$, and plug this solution into the household’s value function, thereby obtaining

\begin{equation}
U(b_t, g_t, 1) = U(n^A_t - g_t, 1 - n^A_t) + \max_{\mu_{t+1} \in \mathbb{M}_{t+1}} \{ E_t \left[ U^B(b_t, g_{t+1}, \mu_{t+1}) \right] \}
\end{equation}

where

$$
U^B(b, g, \mu) = \lambda \sum_{\delta \in \Delta} \max \left\{ U((1 - \delta)b, g, \mu, \delta), U(b, g, 1) \right\} \pi_\delta(\delta) + (1 - \lambda)U(b, g, 1),
$$

and $U^o(b, g, \mu) = \max \{ U(b, g, \mu, 0), U^B(b, g, \mu) \}$. The set $\mathbb{M}_{t+1} \subseteq \mathbb{R}_+$ is defined as $\{ \mu \in \mathbb{R}_+ : \mu = U_c(n^A_{t+1} - g_{t+1}) \text{ if } d_{t+1} = 1 \cap \mu = U_c(n^0_{t+1} - g_{t+1}) \text{ if } d_{t+1} \neq 1 \}$, where $n^0_{t+1}$ is defined below.

The function $U^B(b, g, \mu)$ is the value of the planner before nature plays and sends him to autarky with probability $1 - \lambda$ or to the offer of partial payment (node(B) in figure D) with expenditure $g$, outstanding debt $b$, and (co)state variable $\mu$. The function $U^o(b, g, \mu)$ is the value of the planner, which has the option to default (node (A) in figure D) with expenditure $g$, outstanding debt $b$, and (co)state variable $\mu$.

It now remains to construct $U(b_t, g_t, \mu_t, d_t)$, $d_t \in \mathbb{D}$. From the first-order conditions of the household with respect to consumption and labor (equation 2.15), the expression for the prices derived in section 4.1, the government budget constraint and feasibility constraint, the implementability condition at time $t$ is

\begin{equation}
U_{c,t}(n_t - g_t) - U_{c,t} b_t = U_{l,t} n_t - \mathcal{P}_t b_{t+1};
\end{equation}

note that under equilibrium, the beliefs embedded in $\mathcal{P}_t$ must be exactly those coming from the exogenous laws, $\pi_\delta$, $\lambda$, $\alpha$, and the endogenous government policies.
The value function $U(b_t, g_t, \mu_t, d_t)$ for $d_t \in \mathbb{D}$ is, thus, given by

$$U(b_t, g_t, \mu_t, d_t) = \max \{n_t, b_t + 1, \mu_t + 1\} \{U(n_t - g_t, 1 - n_t) + \beta E_t[U'(b_{t+1}, g_{t+1}, \mu_{t+1})]\},$$

subject to $\{n_t, b_{t+1}, \mu_{t+1}\} \in \{ (n_t, b_{t+1}, \mu_{t+1}) \in [0, 1] \times S : \mu_t = U_c(n_t - g_t, 1 - n_t) \text{ and eqn. (2.23) holds } \}$ and the exogenous debt limits 2.11. The set $S$ is defined as a fixed point, of the operator $S$:

$$S(Q) = \{(b_t, \mu_t) : \exists (b_{t+1}, \mu_{t+1}) \in Q \text{ such that eqn. (2.23) holds}\}$$

and has to be computed recursively.\(^{22}\)

The period $t = 0$ and the periods when the government returns from financial autarky have to be handled differently. I assume that, in these cases, the government starts without any binding promises; therefore, $\mu = U_c(n_0^0 - g_t)$ where $n_0^0$ is the solution to

$$\max \{n_t, b_t + 1, \mu_t + 1\} \{U(n_t - g_t, 1 - n_t) + \beta E_t[U'(b_{t+1}, g_{t+1}, \mu_{t+1})]\},$$

and, in particular, for the case $t = 0$, $(b_0, g_0, d_0)$ are given with $d_0 = 0$.

In the above equations, the government’s default decisions are constructed using the “max” operator. The intuition behind this construction stems from the assumption that the government is benevolent; it opts to pay the debt only inasmuch as it is in the best interest of the representative household.\(^{23}\) So, the sets $D_t[\mathbb{G}]$ and $D_t[\Delta]$, which characterize the default decisions, are constructed as follows:

$$D_t[\mathbb{G}] \equiv D(b_t, \mu_t)[\mathbb{G}] = \{g \in \mathbb{G} : U(b_t, g, \mu_t, 0) < U^B(b_t, g, \mu_t)\},$$

$$D_t[\Delta] \equiv D(g_t, b_t, \mu_t)[\Delta] = \{\delta \in \Delta : U((1 - \delta)b_t, g_t, \mu_t, \delta) < U(b_t, g_t, 1)\}.$$  

6. **Analytical Results**

In this section, I define a set of assumptions that constitutes the benchmark case; I characterize analytically the default sets, policy and pricing implications of the model, and implementable allocations.

I assume that offers of partial payments do not occur ($\lambda = 0$) and utility is quasilinear in consumption ($U_{c,t} \equiv 1$).\(^{24}\) Aiyagari et al. (2002) argue that by setting $U_{c,t} \equiv 1$, they are

---


\(^{23}\)This functional form is analogous to Eaton and Gersovitz (1981), Arellano (2008) and references therein.

\(^{24}\)See Appendix A.
imposing a competitive behavior on the planner as it is unable to control the (implied) prices; thereby drawing an analogy between this problem and the standard incomplete markets consumption-smoothing problem.\textsuperscript{25} In my model, the planner is still able to affect prices through the probability of default; thus, the analogy with the (competitive) representative agent in the consumption-smoothing problem no longer holds.

6.1. \textbf{Characterization of Default Sets.} The results obtained in this section show that the decision to default follows a debt-dependent threshold rule; these results are similar to the one obtained in Chatterjee et al. (2007) and Arellano (2008) without distortionary taxes.

\textbf{Proposition 6.1.} \textit{Under assumptions A.2-A.3(ii), if for a given }$b_t$, $D[G](b_t) \neq \emptyset$, \textit{then there does not exists a }$b_{t+1}: b_t - P(b_{t+1})b_{t+1} \leq 0$.

The proposition above implies that if default occurs (with positive probability), then it must be true that the government is unable to roll over the debt; otherwise, it would simply keep the option to default in this period and would default tomorrow on a higher debt; thus, default never occurs today.

The next theorem states that under additional assumptions, the decision of default is equivalent to a threshold rule—i.e., the government defaults if $g$ is above some $\overline{g}(b)$ given a level of debt $b$.

\textbf{Theorem 6.1.} \textit{Under assumptions A.2, A.3 and A.4, it follows that: if }$g_1 \in D_t[G]$, \textit{then for }$g_1 \leq g_2$, $g_2 \in D_t[G]$.

The next theorem establishes that default sets are increasing in the debt level, or given my previous theorem, that $\overline{g}(\cdot)$ is a decreasing function.

\textbf{Theorem 6.2.} \textit{Under assumptions A.2 and A.3, it follows that if }$b_1,t \leq b_2,t$, \textit{then }$D[G](b_1,t) \subseteq D[G](b_2,t)$.\textsuperscript{26}

6.2. \textbf{Implications on the optimal government policies and allocations.} By the results in sections 4.1 and 6.1, it follows that

$$P(b_{t+1}) = \beta E \left[ \mathbb{I}\{g \leq \overline{g}(b_{t+1})\} \right] = \beta \Pi(\overline{g}(b_{t+1}))$$

\textsuperscript{25}It is clear that the planner’s problem prices do not show up, but they are implicit in budget constraint. These “implied prices” are the ones I am referring to here.

\textsuperscript{26}Note that I do not impose $g_t \sim i.i.d.$ or any other restriction over $g_t$ other than the Markovian one.
where \( \Pi(G) \equiv \int_{g \leq G} \pi(g) dg \).

The debt value such that \( \nabla_b [\mathcal{P}(b^*)b^*] = 0 \) is given by \( b^* = -\frac{\Pi(\gamma(\cdot))}{\pi(\gamma(\cdot))\nabla_b \gamma(\cdot)} \). Defining \( b_* \equiv \arg \sup \{ \beta \in \mathbb{R} : \Pi(\gamma(b)) = 1 \} \) —i.e., the maximum debt level such that default never occurs—it follows that the region \([b_*, b^*]\), which can be empty, is the region where risky borrowing takes place.\(^{27}\)

I can now give a sharp characterization for the law of motion of the optimal taxes and debt. In order to achieve this, following Aiyagari et al. (2002), I characterize the law of motion of the Lagrange multiplier associated with equation 2.23. I denote this Lagrange multiplier as \( \gamma_t \). First, by the envelope condition, it follows that \( \gamma_t = -\nabla_b \mathcal{U}(b_t, g_t, 0) \) —i.e., \( \gamma_t \) is the marginal cost of debt in terms present-value utility—. Thus, by studying the law of motion of \( \gamma_t \), I can study the law of motion of the optimal debt by inverting the previous equation. Moreover, as the first-order condition with respect to \( n_t \) is given by \((1 - \mathcal{U}_{t,t})(1 + \gamma_t) = -\gamma_t U_{t,t} n_t \), the tax, \( \tau_t^* \), is also a nonlinear increasing function of \( \gamma_t \). Therefore, by studying the law of motion of \( \gamma_t \), I can also study the law of motion of the optimal taxes.

Under assumptions A.2-A.4, \( D_{t+1}[G] \) is the set of \( g \in G \) such that \( g > \overline{g}_{t+1} \); thus, assuming natural debt limits (i.e., interior solution for the debt), the first-order condition with respect to \( b_{t+1} \) is given by\(^{28}\)

\[
\gamma_t (\nabla_b [\mathcal{P}(b_{t+1})]b_{t+1} + \mathcal{P}(b_{t+1})) + \beta E \left[(1 - \mathbb{I}\{D_{t+1}[G]\}) \nabla_b [\mathcal{U}(b_{t+1}, g_{t+1}, 0)]\right]
= \beta E \left[\nabla_b \left[\mathbb{I}\{D_{t+1}[G]\}\right] \left(\mathcal{U}(g_{t+1}, 1) - \mathcal{U}(b_{t+1}, g_{t+1}, 0)\right)\right].
\]

The first expectation equals \(-E[(1 - \mathbb{I}\{D_{t+1}[G]\})\gamma_{t+1}]\) by the envelope condition. The derivative in the second expression is taken in the weak sense; the expression is \( \mathcal{U}(g_{t+1}, 1) - \mathcal{U}(b_{t+1}, g_{t+1}, 0) \) evaluated at \( g_{t+1} \in \partial D_{t+1}[G] \) (i.e., the boundary of \( D_{t+1}[G] \)), which consists of a singleton such that \( \mathcal{U}(g_{t+1}, 1) - \mathcal{U}(b_{t+1}, g_{t+1}, 0) = 0 \). Thus, I obtain

\[
\gamma_t (\nabla_b [\mathcal{P}(b_{t+1})]b_{t+1} + \mathcal{P}(b_{t+1})) = E[(1 - \mathbb{I}\{D_{t+1}[G]\})\gamma_{t+1}].
\]

\(^{27}\)See Arellano (2008) for sufficient conditions that ensure this region is not empty. In this section, I assume that \([b_*, b^*] \neq \emptyset\)

\(^{28}\)Differentiability of \( \mathcal{P}_t \) with respect to \( b_{t+1} \) follows from applying the implicit function theorem. I am assuming, though, that \( \mathcal{U}(b, g, 0) \) is differentiable. This is not necessary neither for my general analysis nor for computing the solution in the numerical analysis, but it provides better intuition for understanding the problem.
Finally, note that
\[ \nabla_b [\mathcal{P}(b_{t+1})] = \beta \nabla_b [E \{ g : g \leq \bar{g}_{t+1} \}] \equiv \nabla_b [\Pi(\bar{g}_{t+1})] = \beta \nabla_b [\Pi(\bar{g}_{t+1})], \]
where the last term is well defined by a direct application of the implicit function theorem, insofar as the value function is differentiable. Therefore,
\[ (2.26) \quad \gamma_t = \frac{\Pi(\bar{g}_{t+1})}{\pi(\bar{g}_{t+1}) \nabla_b [\Pi(\bar{g}_{t+1})]} \mathbb{E} [\gamma_{t+1}], \]
with \( \mathbb{E} \) being the expectation with respect to the default-adjusted measure 
\[ \frac{\mathbb{E} \{ g : g \leq \bar{g}_{t+1} \}}{\Pi(\bar{g}_{t+1})} \mathbb{E} (dg); \]
i.e., the possibility of default inserts a wedge that slants the probability measure \( \Pi(dg) \). Henceforth, I denote the first term on the right-hand side as \( \mathcal{M}(b_{t+1}) \equiv \frac{1}{1 - \zeta(b_{t+1})} \), with \( \zeta(b_{t+1}) \equiv -\nabla_b [\mathcal{P}(b_{t+1})] \frac{b_{t+1}}{\mathcal{P}(b_{t+1})}. \)

The Lagrange multiplier associated with the implementability condition is constant in Lucas and Stokey (1983) and, thus, trivially a martingale. In Aiyagari et al. (2002) the Lagrange multiplier associated with the implementability condition is a martingale with respect to the probability measure \( \pi. \) Equation 2.26 implies that the law of motion of the Lagrange multiplier differs in two important aspects. First, the expectation is computed under the default-adjusted measure; this stems from the fact that the option to default adds “some” degree of state-contingency to the payoff of the government debt; this effect lowers the marginal cost of the debt. Second, the aforementioned expectation is multiplied by \( \mathcal{M}(b_{t+1}) \), which can be interpreted as the “markup” that the planner has to pay for having this option to default; this effect increases the marginal cost of the debt.

The next theorem formalizes these observations and provides sufficient conditions for establishing the sign of these dichotomous effects.

**Theorem 6.3.** Under assumptions A.3 - A.4, if
\[ (2.27) \quad \nabla_g [\nabla_b [\mathcal{U}(b, g, 0)]] \leq 0, \]

---

29Proposition B.1 in Appendix B summarizes some properties of \( \mathcal{M}. \)
30The martingale property is also preserved if capital is added to the economy; see Farhi (2007). This property, however, changes if I allow for ad-hoc borrowing limits (see Aiyagari et al. (2002)). The proposition below shows how the results in Aiyagari et al. (2002)—with exogenous debt limits—, relate to the results in my model—with only the option to default—.
then:

\[
\gamma_t = E[\gamma_{t+1}] + \left\{ E[\gamma_{t+1}] \left( M(b_{t+1}) - 1 \right) \right\} - \left\{ M(b_{t+1}) \frac{Cov \{ I\{ g \geq \bar{g}(b_{t+1}) \}, \gamma_{t+1} \}}{\Pi(\bar{g}(b_{t+1}))} \right\}
\]

with \( E[\gamma_{t+1}] \left( M(b_{t+1}) - 1 \right) \geq 0 \) and \( \frac{1}{\Pi(\bar{g}(b_{t+1}))} Cov \{ I\{ g \geq \bar{g}(b_{t+1}) \}, \gamma_{t+1} \} \geq 0 \).

Proof. See the proposition B.2 in Appendix B. □

Equation 2.27 implies that the marginal cost of debt is increasing in \( g \)—i.e., \( \nabla_g \mathcal{U}(b, g, 0) \) is decreasing in \( g \)—. The first term in the curly brackets in equation 2.28 is positive. This term increases the marginal cost of debt at \( t \) (relative to its expectation at \( t+1 \)) and measures the “markup” the government has to pay due to the default risk. The second term in the curly brackets in equation 2.28 is also positive. This implies that \( Cov \{ I\{ g \geq \bar{g}(b_{t+1}) \}, \gamma_{t+1} \} \geq 0 \), which means that the marginal cost of debt and the event of default are positively correlated; thus, when the debt is more costly, the government tends to default more. Hence, this term reflects the benefit of having the option to default and, in fact, decreases the marginal cost of debt at \( t \) (relative to its expectation at \( t+1 \)).

Although, equation 2.27 are somewhat unsatisfactory because they impose ad-hoc restrictions on an endogenous object, they can easily be checked in numerical simulations and have a clear economic interpretation.

7. Numerical Simulations

Given the complexity of the model, it is difficult to characterize the planner’s policy analytically when I include options to exit autarky. Hence, in this section, I present a series of numerical simulations that account for these features.

Throughout this section, I compare my findings with an economy in which the option to default is not present—precisely the model considered in Aiyagari et al. (2002). I denote the variables associated with this model with a (sub)superscript “AMSS”; variables associated to my economy are denoted with a (sub)superscript “ED” (short for Economy with Default).

\[\text{Under more regularity conditions over the behavior of the curvature of } \mathcal{U}(b, g, 0) \text{ and } \Pi(\bar{g}(b)) \text{, the first term in the curly brackets of equation 2.28 is increasing in } b_{t+1}. \text{ Hence, this term can be seen as a “continuous” Lagrange multiplier of a debt limit; i.e., it is positive (with a negative sign in front), and increases continuously in } b_{t+1}; \text{ a Lagrange multiplier would be zero and then jump to positive values when the bound is active. The second term in the curly brackets is also increasing. Please see proposition B.2 in Appendix B for the formal results.}\]
The utility function is given by
\[ U(c, 1 - n) = \frac{c^{1-\sigma_c}}{1-\sigma_c} + C_1 \frac{(1 - n)^{1-\sigma}}{1 - \sigma}, \]
and \((g_t)_t\) follows a discrete state space Markov chain “induced” by the linear process
\[ g_{t+1} = \mu_g (1 - \rho_1) + \rho_1 g_t + \sigma_g \sqrt{1 - \rho_1^2} \varepsilon_{t+1}, \quad \varepsilon_{t+1} \sim N(0, 1). \]

I define \(g\) and \(\overline{g}\) as the lower and upper bound for \(g_t\); \(b\) and \(\overline{b}\) are defined as analogous for the debt.\(^{32}\)

7.1. The i.i.d. Shocks case. In this subsection, I study the case where \(\rho_1 = 0\)—i.e.,
\[ g_{t+1} = \mu_g + \sigma_g \varepsilon_{t+1}. \]

Table 2.2 presents the parameter values for the benchmark model. The goal of this section is, first, to verify the analytical results obtained in the previous sections and, second, to compare the debt policy functions for the ED and the AMSS economies.

Under this specification, the household has quasi-linear utility, and in order to facilitate the comparison with Aiyagari et al. (2002), I choose the parameters associated with the utility function and the discount factor equal to theirs.\(^{33}\) Since I am interested only in the dynamics when the government can default, the exogenous bounds on debt are such that the government is precluded from saving. Finally, under this parametrization, autarky is assumed to be an absorbing state \((\alpha = \lambda = 0)\).\(^{34}\)

### Table 2.2. Parameter values for the benchmark model

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
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<tr>
<td>(Discount Factor) (\beta)</td>
<td>0.97</td>
<td>(AR(1) coeff.) (\rho_1)</td>
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<tr>
<td>(Utility of leisure) (C_1)</td>
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<td>(Mean) (\mu_g)</td>
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<td>(Std. Dev.) (\sigma_g)</td>
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<tr>
<td>(Utility of consumption) (\sigma_c)</td>
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<td>(G) (g, \overline{g})</td>
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</tr>
<tr>
<td>(Prob. offer for partial payment) (\lambda)</td>
<td>0</td>
<td>(B) (\underline{b}, \overline{b})</td>
<td>(0, 0.3)</td>
</tr>
<tr>
<td>(Prob. of escaping autarky) (\alpha)</td>
<td>0</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

\(^{32}\)By “induced”, I mean that the transition probability follows from applying Tauchen’s (Tauchen (1986)) results to the AR(1) process.

\(^{33}\)In their paper, \(C_1 = 1\), but they assume that \(n \in [0, 100]\), as opposed to my case: \(n \in [0, 1]\). Hence their constant \(C_1\) should be re-scaled to \(100^{1-\sigma} = 0.01\).

\(^{34}\)I refer to Aiyagari et al. (2002) regarding the implications when ”natural” savings limits are imposed.
Figure D.3 presents the policy function $b(b, g)$ for the ED model (black dots), and for the Aiyagari et al. (2002) model (red dots); the default region is represented by yellow bars. Obviously, the debt policy function when the government is in default, is redundant. The first row shows the policy function as a function of the government expenditure for low-level of debt (first panel), mid-level debt (second panel) and high-level debt (third panel). The second row shows the policy function as a function of the current debt level for low-level government expenditure (first panel), mid-level government expenditure (second panel) and high-level government expenditure (third panel). This last row also presents the 45-degree line (solid) for better comparison.

Although in all cases, a higher level of current debt or of current government expenditure implies a higher level of debt tomorrow, in the ED economy, the option to default generates endogenous debt limits that produce lower levels of debt. In particular, for the case of high debt levels the government decides to default at almost any value of $g$ (first row, third panel).

Figure D.4 shows the equilibrium default set (bottom panel) and the price as a function of $(b, g)$ (top panel). The area in blue (the lighter area) are the pairs of $(g, b)$ for which the government opts to default. As predicted in the theoretical section, the set increases as the government expenditure increases and as the debt level increases and is convex across $g$ (i.e., for a given level of debt, I look at the projection over $g$ which is characterized by an interval).

7.2. Impulse Response Functions. Before looking at the Monte Carlo results, it is useful to look at the dynamics of one particular realization of $(g_t)_t$. I solve the model (see Table 2.3 for the parameter values, except for $\alpha = 0.30$, and $\lambda = 0$), and then I draw a particular path for $g_t$ given by

$$g_t = \begin{cases} \bar{g} & \text{if } t = T, T + 1 \\ \bar{g}/2 & \text{if } t = T + 2 \\ \bar{g} & \text{if } t = T + 3, T + 4 \end{cases}$$

(2.29)

This choice is completely arbitrary, chosen to showcase all the features of the model.

Figure D.2 presents the results. The dotted line in all the panels is the path $d_t$. For the first half of the sample, the economy is not in default; immediately after the second row.

35 Throughout this section when describing the figures the numbering of the panels increases from left to right and the numbering of the rows does it from top to bottom.
government shock (upper right panel), the economy enters autarky for two periods. The upper left panel depicts the spread, which should be taken as the envelope of $1/p_t^b - \beta^{-1}$ (solid) and $1/q_t - \beta^{-1}$ (dashed); the high level of $1/q_t - \beta^{-1}$ observed during financial autarky is qualitatively consistent with the spreads we see in the data (see the stylized facts section). The middle left panel shows the debt ($b_t$) for both economies; the endogenous borrowing limits present in the ED economy render lower levels of debt during “bad times.” During autarky, since we keep track of the defaulted debt, we have a plateau in $b_t^{ED}$; then, the economy leaves default by paying the outstanding debt, and, thus, $b_t^{ED}$ plummets to zero. The right middle panel shows the $\tau^n_t$ path for both economies; for the ED economy, the path is more volatile and has an additional “spike” to cover for the payment of defaulted debt. Finally, the last row presents the paths for $c_t$ and $l_t$, respectively.

7.3. Monte Carlo Simulations. In this subsection, I run a battery of Monte Carlo (MC) simulation exercises, allowing government expenditure to be auto-correlated, and also allowing for $\lambda$ to be nonzero. I, however, still maintain the linear utility on consumption assumption.\footnote{Experiments with $\sigma_c = 0.5$ present the same thresholds as in the quasi-linear case, but with lower probability of default; they are available upon request.} $^{37}$

I perform 1000 MC iterations, each consisting of sample paths of 1000 observations for which the first 900 observations were omitted in order to eliminate the effect of the initial values. Government expenditures are assumed to be AR(1) with moments $(\rho_1, \mu_g, \sigma_g)$, chosen to match the auto-correlation, mean and volatility of the general government expenditure-to-output ratio for Argentina during the period 1993-2003. These values are (approx.) 0.30, 0.20 and 0.05, respectively.

Table 2.3. Parameter values for the case with $\lambda = 0.08$.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>(Discount Factor) $\beta$</td>
<td>0.9875</td>
<td>(AR(1) coeff.) $\rho_1$</td>
<td>0.30</td>
</tr>
<tr>
<td>(Utility of leisure) $C_1$</td>
<td>0.01</td>
<td>(Mean) $\mu_g$</td>
<td>0.20</td>
</tr>
<tr>
<td>(Utility of leisure) $\sigma$</td>
<td>2</td>
<td>(Std. Dev.) $\sigma_g$</td>
<td>0.05</td>
</tr>
<tr>
<td>(Utility of consumption) $\sigma_c$</td>
<td>0</td>
<td>($\mathcal{G}$) $(g, \overline{g})$</td>
<td>(0, 0.325)</td>
</tr>
<tr>
<td>(Prob. offer for partial payment) $\lambda$</td>
<td>0.08</td>
<td>($\mathcal{B}$) $(b, \overline{b})$</td>
<td>(0, 0.3)</td>
</tr>
<tr>
<td>(Prob. of escaping autarky) $\alpha$</td>
<td>0.0</td>
<td>($\Delta$) $(\delta, \overline{\delta})$</td>
<td>${0, 0.5, 0.95}$</td>
</tr>
<tr>
<td>(Direct cost of default) $\kappa$</td>
<td>0.021</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

$^{36}$Numerical experiments for $\alpha > 0$ and $\lambda = 0$ are relegated to the supplementary material.
Table 2.3 presents the parametrization for the case in which offers for partial payments are allowed—i.e., $\lambda > 0$. I construct a grid of three elements for $(\bar{\delta}, \bar{\beta})$, with equal probability weights. I choose $\beta$, $\lambda$ and $\bar{\delta}$ to match: a probability of default with range $[3, 4]$ percent, “autarky spell” in a range of $[5, 15]$ periods, and a default recovery rate of 45 percent. The parameter $\kappa$ is the direct cost of default, which yields an output of $\frac{1}{1+\kappa} n_t$ the first period in which the government decides to default, and is chosen to yield a reduction in output of 1-4 percent with respect to the mean.

Averages across MC simulations of some statistics for the whole sample, the “no default” sample, and the “autarky or default” sample are presented in Table 2.4. I constructed these latter sub-samples by separating, for each MC iterations, the periods in which the ED economy was in autarky from those in which the economy was not.

Table 2.4. MC results for the case $\lambda = 0.08$. In the table, $E$ and $std$ denote the mean and standard deviation across time, respectively. All quantities are averaged across MC simulations.

<table>
<thead>
<tr>
<th>Sample</th>
<th>All</th>
<th>No Default</th>
<th>Default</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>AMSS ED</td>
<td>AMSS ED</td>
<td>AMSS ED</td>
</tr>
<tr>
<td>$E(b_t/n_t)$ (%)</td>
<td>12.65 1.870</td>
<td>12.43 1.196</td>
<td>22.70 17.19</td>
</tr>
<tr>
<td>$std(b_t/n_t)$ (%)</td>
<td>9.290 3.420</td>
<td>8.600 1.410</td>
<td>5.611 0.580</td>
</tr>
<tr>
<td>$E(\tau_t)$</td>
<td>0.218 0.218</td>
<td>0.219 0.218</td>
<td>0.218 0.235</td>
</tr>
<tr>
<td>$std(\tau_t)$</td>
<td>0.030 0.051</td>
<td>0.027 0.049</td>
<td>0.019 0.053</td>
</tr>
<tr>
<td>$E(\tau_t n_t)$</td>
<td>0.194 0.193</td>
<td>0.193 0.194</td>
<td>0.194 0.200</td>
</tr>
<tr>
<td>$std(\tau_t n_t)$</td>
<td>0.026 0.045</td>
<td>0.023 0.043</td>
<td>0.016 0.046</td>
</tr>
<tr>
<td>$E(spread)$ (%)</td>
<td>5.660 0.820</td>
<td>119.3</td>
<td></td>
</tr>
<tr>
<td>$E(\text{Default Spell})$</td>
<td>11.12</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$E(\text{Recovery Rate})$ (%)</td>
<td>46.50</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$Pr(\text{Default})$ (%)</td>
<td>4.120</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

The average debt-to-output ratio (row 1) for the whole sample is around 12 percent for the AMSS economy; in the ED economy, however, it is around 2 percent because of the presence of the endogenous borrowing limits arising from the possibility of default. This level is low compared to what is observed in the data: a ratio of approximately 23 percent for Argentina (1990-2005). For the default sub-sample, however, the average debt-to-output ratio in the

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38 The default recovery rate is taken from Yue (2005), where 30 percent is the recovery rate for Argentina and 60 percent is the recovery rate for Ecuador. So, I took the average of both.

39 Pitchford and Wright (2008) report that, on average, for their dataset, default began when output was (approx.) 1.5 percent below trend; Reinhart and Rogoff (2008) report a decline in output of (approx.) 4 percent (with respect to past levels) at the time of default.

40 For the default period (2001-2005), this ratio was (approx.) 45 percent.
ED economy is only 4 percent lower than in the AMSS economy. Since in this part of the sample, the average debt-to-output ratio is actually the defaulted debt-to-output ratio, this provides additional evidence of endogenous borrowing limits being “active” in higher levels of debt.

Although the average tax rate (row 3) is similar in both economies across all three samples, the volatility of the tax rate (row 4) is higher in the ED economy, especially in the default sub-sample. This is a consequence of endogenous borrowing limits, which implies that the debt is not as good an instrument to smooth shocks as it is in the AMSS economy. In particular, when the ED economy is in autarky, the planner is precluded from issuing debt, rendering taxes more volatile than in the other sub-samples.

I compute the spread as \(100\left(1/p_t - 1/\beta\right)\) for the “no default” sample, and as \(100\left(1/q_t - 1/\beta\right)\) for the “default” sample. The spread (row 8) is around 7 percent for the whole sample, and around 1 percent for the “no default” sample. This is below the 5 percent (approx.) registered for Argentina (1997 - 2000 and 2005-2006) using the EMBI+. During the “default” period, the spread in the model is around 120 percent; for Argentina during the default period (2001-2005), the spread using the EMBI+ was about 55 percent.

Figure D.5 presents the box-plots, which allow me to determine whether the MC average presents statistically significant differences; for all quantities except average taxes, this is the case.

**Welfare Comparison.** In order to assess the welfare implications of my model, let \(\Omega\) be the increment of labor income in the initial period—i.e., \(c_0^\Omega \equiv (1 + \Omega)n_0 - g\)—such that

\[
\int_{B \times G} U^{AMSS}(b, g) \Pi_{bg} (db, dg) = \int_{B \times G \times \Delta \cup \{1\}} U^\Omega(b, g, \delta) \Pi_{bg\delta} (db, dg, d\delta),
\]

where \(U^{AMSS}(b, g)\) is the present value expected utility in the AMSS economy and \(U^\Omega\) is the value function with \(c^\Omega\) instead of \(c\), for the initial period. Doing Taylor approximation of \(U\) and doing some algebra, it follows,

\[
\Omega \approx \frac{\int_{B \times G} U^{AMSS}(b, g) \Pi_{bg} (db, dg) - \int_{B \times G \times \Delta \cup \{1\}} U(b, g, \delta) \Pi_{bg\delta} (db, dg, d\delta)}{\int_{B \times G \times \Delta \cup \{1\}} U_c(n^*(b, g, \delta) - g) n^*(b, g, \delta) \Pi_{bg\delta} (db, dg, d\delta)},
\]
where \( n^* \) is the policy function. The probability measures \( \Pi_{bg\delta} \) and \( \Pi_{gb} \) are computed as the frequencies across 1000 Monte Carlo repetitions at time \( T = 1000 \) (so as to avoid any dependence on the initial values), for each economy.\(^{41}\)

I study the welfare implications for \( \lambda \in [0, 1] \); Figure D.6 presents the results. For \( \lambda = 0 \), default does not exist. As \( \lambda \) increases, the option value of defaulting increases, prompting the planner to default more; the planner, however, receives the repayment offer more often and, thus, the likelihood of repayment increases. For low values of \( \lambda \), the former effect dominates and the likelihood of being in autarky is high (dashed line in Figure D.6). For high values of \( \lambda \), the latter effect dominates and the likelihood of being in autarky is low (dashed line in Figure D.6). The solid line in this figure shows \( \Omega \) for \( \lambda \in [0, 1] \).

Finally, the magnitude of the difference is low: Less than one percent of the initial labor income. This is consistent with the findings in Aiyagari et al. (2002) when they compared the incomplete markets economy with the complete markets one.

8. Conclusion

I study a planner’s problem, in a closed economy, that consists of choosing distortionary taxes with only non-state-contingent government debt, but allowing for partial defaults on the debt.

First, I provide an explanation for the lower debt-to-output ratios and more volatile tax policies observed in emerging economies, vis-à-vis industrialized economies. This stems from the fact that the benevolent government not only chooses distortionary labor taxes and one-period non-state-contingent debt, but it also can default on its debt.

Second, I propose a device to price the debt during temporary financial autarky. Numerical simulations show that the spread during the default period is higher than for the rest of the sample; this characteristic is consistent with data for defaulters—e.g., Argentina, Ecuador and Russia.

Third, and last, the numerical simulations suggest that increasing the probability of receiving offers for exiting autarky decreases welfare when this probability is low/medium to begin with, but increases it when the probability is high.

41 Another measure is given by

\[
\mathcal{U}^{AMSS}(b, g) \Pi_{bg}(db, dg) - \mathcal{U}(b, g, \delta) \Pi_{bg\delta}(db, dg, d\delta),
\]

but as it gives qualitatively the same results, I do not report it.
Although this model does a good job of explaining qualitatively the facts observed in the data, it does not do very well in matching the data quantitatively. A line of future research should delve further into the production side of this economy and its driving shocks.\footnote{See Aguiar and Gopinath (2006) and Mendoza and Yue (2008).}

REFERENCES


28
Appendix A. Theoretical Assumptions

Recall that the underlying probability space is denoted as \((\Omega, \mathcal{F}, \Pr)\) and the expenditure process \((g_t)_t\) is such that \(g_t : \Omega \to \mathbb{B} \subseteq \mathbb{R}_+\). Let \(G = \{A \in \mathbb{B} : g^{-1}(A) \in \mathcal{F}\}\). In order to write the problem recursively, I assume that \(g_t\) is a Markov process. That is,

**Assumption A.1.** (Markov) \(\pi_t(G|g^t) = \pi(G|g_t), \forall G \in \mathcal{G}\).

**Assumption A.2.** (i) \(U : \mathbb{R}_+ \times [0,1] \to \mathbb{R}\) is twice continuously differentiable; (ii) \(U_c > 0, U_{cc} \leq 0, U_t \geq 0, U_{tt} \leq 0\) and \(\lim_{n \to 1} \nabla_t U = \infty\); (iii) there exists a \(n^* \in (0,1)\) such that \((1 - U_t(1-n^*)) = -U_{tt}(1-n^*)n^*\).

**Assumption A.3.** (i) \(\lambda = 0\); (ii) \(U_c \equiv 1\).

Part (i) states that offers of partial payments do not occur. Part (ii) implies that prices do not depend on marginal utilities. The next assumption is needed to control the option of default tomorrow and the “budget set” of the planner.

**Assumption A.4.** (i) \(g_t \sim i.i.d.;\) (ii) \(U_{tt}(1-n_t) - U_{ttl}(1-n_t)n_t \leq 0\).

Appendix B. Proofs

Throughout this section, I denote \(n^i_t, c^i_t, b^i_{t+1}\) with \(i \in \{C, A\}\) as the policy functions given \((b_t, g_t)\) for ”continuation” and ”autarky”, respectively.

**Proof of proposition 6.1.** First I need to show that \((1 - U_t(1-n_t))n_t\) is decreasing, for optimally chosen \(n_t\). Let \(\bar{n} = 1 - U_t(1-\bar{n})\). Under assumption A.2(i) the government optimal tax revenue is given by \(A(n) \equiv (1 - U_t(1-n_t))n_t\); which \(A(0) = 0\) and \(A(1) = -\infty\) (by assumption A.2(ii)). Moreover it follows that

\[
\nabla_n[A(n^*)] = 0 \iff (1 - U_t(1-n^*)) = -U_{tt}(1-n^*)n^*
\]

thereby implying \(A(n^*) = -U_{tt}(1-n^*)(n^*)^2 > 0\) by assumption A.2(ii)(iii). So by continuity \(A(n)\) has to decrease for a nontrivial interval included in \([n^*, 1]\). Since under assumption A.2 the utility: \(n_t - g_t + U(1-n_t)\) is increasing over \(n_t \in [n^*, \bar{n}]\), it is optimal for the government to choose \(n_t\) on the decreasing part of the tax revenue; otherwise the government achieves the same tax revenue but for a low level \(n_t\) yielding lower utility. Hence \((1 - U_t(1-n_t))n_t\) is decreasing, for optimally chosen \(n_t\).

\[43\text{For a generic function } f, f_x \text{ denotes the partial derivative of } f \text{ with respect to } x; f_{xx} \text{ denotes the second derivative, and so on.}\]
I show the result by contradiction. Assume that there exists a $\overline{\pi}_{t+1}$ such that $b_t - \mathcal{P}(\overline{b}_{t+1}) \leq 0$ then it follows from the implementability conditions and assumption A.3(ii) that $n_t = b_t - \mathcal{P}(\overline{b}_{t+1}) \leq 0$; hence denoting $n^C_t$ as the optimal choice under $d_t = 0$ and $n^C_t$ as the optimal choice under $d_t = 1$ it follows that $(1 - U_t(1 - n^A_t)) n^C_t \leq (1 - U_t(1 - n^A_t)) n^C_t$. Since $(1 - U_t(1 - n))$ is decreasing in $(n^*, 1)$, $n^C_t \geq n^A_t$, it follows that the immediate utility under $d_t = 0$ is higher than under autarky ($d_t = 1$). This result, plus the fact that the continuation utility is always higher under $d_t = 0$ (the planner has the option to go to autarky), yields that no default is always preferred to autarky; thus $D[\mathbb{G}](b_t) = \{\emptyset\}$, and I arrive to a contradiction. \hfill \Box

**Proof of theorem 6.1.** Define $BL(n_t; g_t) \equiv -U_t(1 - n_t) n_t + n_t - g_t$. Notice that $BL_n(n_t; g_t) = 1 - U_t(1 - n_t) + U_t(1 - n_t) n_t$. In the proof of proposition 6.1, I showed that $(1 - U_t(1 - n_t)) n_t < 0$ is decreasing for all $n \in (n^*, 1)$, hence $BL(\cdot; g)$ is also decreasing in $n \in (n^*, 1)$.

Moreover notice that, for $g_{1,t} \leq g_{2,t}$, $BL(n^C_{1,t}; g_{2,t}) \leq BL(n^C_{1,t}; g_{1,t}) = b_t - \mathcal{P}(b_{1,t+1}) b^C_{1,t+1}$ and $BL(n^C_{1,t}; g_{1,t}) \geq BL(n^C_{2,t}; g_{2,t}) = b_t - \mathcal{P}(b_{2,t+1}) b^C_{2,t+1}$ with $b^C_{1,t+1}$ and $n^C_t$ being the optimal policy function under $g_{1,t}$. Thus, there exists a $\overline{\pi}_{2,t}$ such that $BL(n^C_{2,t}; g_{1,t}) > BL(\overline{\pi}_{2,t}; g_{1,t}) = BL(n^C_{2,t}; g_{2,t})$. Since $n^C_t \geq n^*$ and $BL(\cdot; g_{1,t})$ is decreasing (in this part of the support), then $\overline{\pi}_{2,t} \geq n^C_t \geq n^*$. So, $(\overline{\pi}_{2,t}, \overline{\pi}_{2,t}, b^C_{2,t+1})$ is feasible under $g_{1,t}$. Given that at $g_{1,t}$, the government defaults, it must be true that the immediate utility under default exceeds the immediate utility of the bundle $(\overline{\pi}_{2,t}, \overline{\pi}_{2,t}, b^C_{2,t+1})$—i.e., $U(c^A_{1,t}, 1 - n^A_{1,t}) > U(\overline{\pi}_{2,t}, 1 - \overline{\pi}_{2,t})$. Moreover it follows that

\begin{equation}
(A.30) U(c^C_{1,t}, 1 - n^C_{1,t}) + \beta E \left[U^o(g, b^C_{1,t+1})\right] \geq U(\overline{\pi}_{2,t}, 1 - \overline{\pi}_{2,t}) + \beta E \left[U^o(g, b^C_{2,t+1})\right].
\end{equation}

To show that the government chooses to default under $g_{2,t}$, I need to verify that $U(c^A_{2,t}, 1 - n^A_{2,t}) + \beta E [U(g, 1)] \geq U(c^C_{2,t}, 1 - n^C_{2,t}) + \beta E [U^o(g, b^C_{2,t+1})]$.\footnote{Abusing notation I excluded $b$ from the value function of the planner under autarky as it is not needed anymore and I denote it as $U(\mathcal{P}(b_t), 1)$.} Invoking equations A.30, and that the government defaults under $g_{1,t}$; it suffices to show that

\begin{equation}
(A.31) \quad U(c^A_{1,t}, 1 - n^A_{1,t}) - U(c^A_{2,t}, 1 - n^A_{2,t}) < U(\overline{\pi}_{2,t}, 1 - \overline{\pi}_{2,t}) - U(c^C_{2,t}, 1 - n^C_{2,t}).
\end{equation}

Thus I need to show that the difference on the right is “less negative”, than the one on the left.

$BL(\cdot; g)$ is decreasing in $(n^*, 1)$ and $BL(n; \cdot)$ is decreasing in $\mathbb{G}$; thus, since $g_{1,t} \leq g_{2,t}$ it follows that $n^A_{1,t} \geq n^A_{2,t}$; recall that $n^C_{2,t} \leq \overline{\pi}_{2,t}$. By proposition 6.1, I know that $b_t - \mathcal{P}(b_{t+1}) b_{t+1} \geq$
and the optimal choices

I need to analyze two cases. First case, let \( n_{2,t}^C \leq n_{1,t}^A \leq \bar{n}_{2,t} \) and define \( V(n) = BL(n,g) + g \) then \( V(n_{2,t}^C) - V(n_{2,t}^A) = b_t - \mathcal{P}(b_{t+1}^C) ) ) - V(\bar{n}_{2,t}^A) - V(n_{1,t}^A) \). Since I am under the case \( n_{2,t}^C \leq n_{1,t}^A \leq \bar{n}_{2,t} \), the difference \( n_{2,t}^A - n_{2,t}^C \) is greater than \( n_{1,t}^A - \bar{n}_{2,t} \). \(\text{45}\)

By my assumptions \( U \) is increasing and concave, thus it follows that

\[
\frac{U(n_{2,t}^A - g_t, 1 - n_{2,t}^A) - U(n_{2,t}^C - g_t, 1 - n_{2,t}^C)}{n_{2,t}^A - n_{2,t}^C} \geq \frac{U(n_{1,t}^A - g_t, 1 - n_{1,t}^A) - U(n_{2,t}^A - g_t, 1 - \bar{n}_{2,t})}{n_{1,t}^A - \bar{n}_{2,t}},
\]

which implies that \( U(n_{2,t}^A - g_t, 1 - n_{2,t}^A) - U(n_{2,t}^C - g_t, 1 - n_{2,t}^C) \) is greater than \( U(n_{1,t}^A - g_t, 1 - n_{1,t}^A) - U(\bar{n}_{2,t} - g_t, 1 - \bar{n}_{2,t}) \) and doing some algebra it is easy to verify that this implies equation A.31.

In the case \( n_{2}^C \leq \bar{n}_{2,t} \leq n_{1,t}^A \), I can show the desired result using an analogous argument and is not be repeated here.

\(\square\)

**Proof of theorem 6.2.** Take \( g_t \in D[G](b_t) \) then it must be true that \( U(c_{1,t}^C, 1 - n_{1,t}^C) + \beta E \left[ U^o \left( b_{t+1}, g \right) \middle| g_t \right] < U(c_{1}, 1 - n_{1}) + \beta E \left[ U(g, 1) \middle| g_t \right] \) where \( n_{1,t}^i, c_{1,t} \), \( b_{i+1}^t \) with \( i \in \{ C, A \} \) denotes the policy functions given \( (b_t, g_t) \) for “continuation” and “autarky”, respectively.

First note that the right hand side is constant as a function of \( b_t \). Second note that, as \( b_1 \leq b_2 \), then (using the notation in the proof of theorem 6.1)

\[
BL(n_{1,t}^C; g_t) + \mathcal{P}(b_{t+1})b_{t+1} = b_1 \leq b_2 = BL(n_{2,t}^C; g_t) + \mathcal{P}(b_{t+1})b_{t+1}, \forall b_{t+1} \in \mathbb{B}.
\]

Given that \( BL(n; g_t) \) is decreasing as a function of \( n \) in \( (n^*, 1) \) (see the proof of theorem 6.1) and the optimal choices \( n_{j,t}^i, i \in \{ A, C \} \) and \( j = 1, 2 \) belong to \( (n^*, 1) \), then it must follow that \( n_{1,t}^C \geq n_{2,t}^C \). Given that \( U(n - g_t, 1 - n) \) is increasing it must follow that

\[
U(n_{1,t}^C - g_t, 1 - n_{1,t}^C) + \beta E \left[ U^o \left( b_{t+1}, g \right) \middle| g_t \right] \geq U(n_{2,t}^C - g_t, 1 - n_{2,t}^C) + \beta E \left[ U^o \left( b_{t+1}, g \right) \middle| g_t \right] .
\]

Given that \( U(c_{1}, 1 - n_{1}) + \beta E \left[ U(g, 1) \middle| g_t \right] \) is constant as a function of the debt it must follow that \( U(c_{1}, 1 - n_{1}) + \beta E \left[ U(g, 1) \middle| g_t \right] > U(n_{2,t}^C - g_t, 1 - n_{2,t}^C) + \beta E \left[ U^o \left( b_{t+1}, g \right) \middle| g_t \right] \) and thus \( g_t \in D[G](b_2) \). \(\square\)

**Proposition B.1.** \( \mathcal{M}(b) \equiv \frac{1}{1-g(b)} \) is such that:

\(\text{45}\) For concave and decreasing function \( f(x) \) with \( a < b < c < d \), it follows \( \frac{f(c) - f(b)}{c - d} \geq \frac{f(c) - f(d)}{c - d} \). So, if \( f(a) - f(b) = f(c) - f(d) \) it must hold that \( b - a \geq d - c \).
(1) $\mathcal{M}(b) \geq 1$ for all $b \in [b_*, b^*]$ and $\mathcal{M}(0) = 1$.\footnote{Recall that $[b_*, b^*]$ is the region where risky borrowing takes place.}

(2) $\mathcal{M}(b)$ is increasing (decreasing) iff $\zeta(b)$ is increasing (decreasing).

(3) $\zeta(b)$ is increasing (decreasing) iff $\nabla_b \left[ \log \left( -\nabla_b \left[ \log (\overline{\pi}(b)) \right] \right) \right] + \frac{1}{b} \leq \left( \leq \right) 0$.\footnote{We denote $\nabla_b \left[ \cdot \right]$ as the derivative operator, i.e., $\nabla_x^k f = f_{x^k}$ for any $k \geq 1$.}

Proof of Proposition B.1. (1) It suffices to show that $\zeta(b) \in [0, 1]$. By definition $\zeta(b) = -\frac{\pi(\overline{\pi}(b)) \nabla_b (\overline{\pi}(b) b)}{\Pi(b)}$ and I already showed that $\nabla_b \left[ \log (\overline{\pi}(b)) \right]$ is decreasing, thus $\zeta(b) \geq 0$. Also note that $\zeta(b) \leq 1$ if and only if $0 \leq \Pi(b) + \pi(\overline{\pi}(b)) \nabla_b (\overline{\pi}(b) b)$ which has the same sign as the derivative of $\mathcal{P}(b) b$ with respect to $b \in [b_*, b^*]$. The latter expression is non-negative by optimality of the choice of $b$, otherwise the planner could perceive higher debt income by lowering the debt, and that contradicts optimality of the debt.

(2) It is easy to see that $\nabla_b \left[ \mathcal{M}(b) \right] = \frac{\nabla_b \zeta(b)}{(1-\zeta(b))^2}$ and thus the desired result follows.

(3) I show this for the increasing part, the decreasing is analogous. Note that

$$\nabla_b \left[ \zeta(b) \right] = -\frac{\nabla_b \nabla_b \Pi(b) b}{\Pi(b)} + \left( \frac{\nabla_b \Pi(b)}{\Pi(b)} \right)^2 b = -\frac{\nabla_b \Pi(b)}{\Pi(b)} b - \frac{\nabla_b \Pi(b)}{\Pi(b)} + \left( \frac{\nabla_b \Pi(b)}{\Pi(b)} \right)^2 b$$

$$= \frac{\nabla_b \Pi(b)}{\Pi(b)} \left[ \frac{-\nabla_b \Pi(b)}{\nabla_b \Pi(b)} b - 1 + \frac{\nabla_b \Pi(b)}{\Pi(b)} \right].$$

Given that $\frac{\nabla_b \Pi(b)}{\Pi(b)} \leq 0$, then $\nabla_b \left[ \zeta(b) \right] \leq 0$ iff

$$\frac{\nabla_b \Pi(b)}{\Pi(b)} \leq \frac{1}{b} \iff \nabla_b \left[ \log (\Pi(b)) \right] - \nabla_b \left[ \log \left( -\nabla_b \left[ \log (\Pi(b)) \right] \right) \right] \geq \frac{1}{b}$$

$$\iff \nabla_b \left[ \log \left( -\nabla_b \Pi(b) \right) \right] \leq -\frac{1}{b} \iff \nabla_b \left[ \log \left( -\nabla_b \left[ \log (\Pi(b)) \right] \right) \right] \leq -\frac{1}{b}.$$

It is easy to see that if $\Pi(b)$ satisfies the condition in the proposition then this inequality holds.

\[ \square \]

Proposition B.2. Under assumptions A.3 - A.4,

(1) if

\[ \nabla_g \left[ \nabla_b \left[ \mathcal{M}(b, g, 0) \right] \right] \geq 0, \]

then $\gamma_t > E[\gamma_{t+1}]$ a.s., conditional on no defaulting. Moreover if $\Pi(\overline{\pi}(b_t)) \geq \exp \{ -C/t^2 \}$

then $\gamma_t \to \gamma_\infty$ w.p.p.
(2) if

\(\nabla_g [\nabla_b [\mathcal{U}(b, g, 0)]] \leq 0,\)

then:

\[
\gamma_t = E[\gamma_{t+1}] + \{E[\gamma_{t+1}] (\mathcal{M}(b_{t+1}) - 1)\} - \left\{\frac{Cov(\mathbb{I}\{g \geq \overline{g}(b_{t+1})\}, \gamma_{t+1})}{\Pi(\overline{g}(b_{t+1}))(1 - \zeta(b_{t+1}))}\right\}
\]

with \(E[\gamma_{t+1}] (\mathcal{M}(b_{t+1}) - 1) \geq 0\) and \(\frac{1}{\Pi(\overline{g}(b_{t+1}))}Cov(\mathbb{I}\{g \geq \overline{g}(b_{t+1})\}, \gamma_{t+1}) \geq 0.\)

(a) If \(\nabla_b^2[\mathcal{U}(b, g, 0)] \leq 0\) and \(\nabla_b [\log (-\nabla_b [\log (\Pi(\overline{g}(b_{t+1})))])] + \frac{1}{b_{t+1}} \geq 0,\)

then: \(E[\gamma_{t+1}] (\mathcal{M}(b_{t+1}) - 1)\) is increasing in \(b_{t+1}.\)

(b) If, in addition, \(\nabla_b^3[\mathcal{U}(b, g, 0)] \leq 0,\) then: \(\frac{Cov(\mathbb{I}\{g \geq \overline{g}(b_{t+1})\}, \gamma_{t+1})}{\Pi(\overline{g}(b_{t+1}))(1 - \zeta(b_{t+1}))}\) is increasing in \(b_{t+1}.\)

**Proof of Proposition B.2.** (1) First note that under equation A.32 and the envelope condition \(\gamma\) is a decreasing function of \(g.\) Second, note that the default adjusted measure, \(\int_{g < g_t} \frac{\mathbb{I}\{g \geq \overline{g}(b_{t+1})\}}{\Pi(\overline{g}(b_{t+1}))} \pi(dg)\) is first order dominated by \(\int_{g \leq g_t} \pi(dg).\) Thus putting both results together it follows that \(E[\gamma_{t+1}] \geq E[\gamma_{t+1}].\)

The expression for \(\gamma_t\) can be written as \(E[\gamma_{t+1}|\mathcal{M}(b_{t+1})] - E[\gamma_{t+1}],\) where \(E\) is the expectation under the measure \(\frac{\mathbb{I}\{g \geq \overline{g}(b_{t+1})\}}{\Pi(\overline{g}(b_{t+1}))} \pi(g)dg.\) As \(\mathcal{M}(b_{t+1}) \geq 1\) it follows that is bounded from below by \(E[\gamma_{t+1}] - E[\gamma_{t+1}]\) which by my previous result I know it is positive; hence \(\gamma_t > E[\gamma_{t+1}].\)

This implies that \(\gamma_t\) converges almost surely to some limit, \(\gamma_\infty\) (see Billingsley (1995)) for all the histories \(g^\infty\) such that the planner does not default. Therefore in order to prove that \(\gamma_t \to \gamma_\infty\) w.p.p. it suffices to show that \(\text{Pr} (\text{No default}) \geq c > 0.\) It is easy to see that

\[
\text{Pr} (\text{No default until } t) = \text{Pr} (\text{No default at } t) \text{Pr} (\text{No default until } t - 1),
\]

and by my previous results \(\text{Pr} (\text{No default at } t) = \Pi(\overline{g}(b_t)).\) Therefore iterating, it follows

\[
\log(\text{Pr} (\text{No default})) = \sum_t - \log(1/\Pi(\overline{g}(b_t))).
\]

So a sufficient condition for \(\text{Pr} (\text{No default}) \geq c > 0\) is that \(- \log(1/\Pi(\overline{g}(b_t)))\) decays faster than \(C/t^2.\) The condition in the proposition ensures that this holds.
(2) Note that $\mathcal{M}(\cdot) \geq 0$, and
\[
\frac{1}{\Pi(\overline{g}(b_{t+1}))} Cov\left(\mathbb{I}\{g \geq \overline{g}(b_{t+1})\}; \gamma_{t+1}\right) = \frac{1}{\Pi(\overline{g}(b_{t+1}))} Cov\left(\mathbb{I}\{g \geq \overline{g}(b_{t+1})\} - 1, \gamma_{t+1}\right)
\]
\[= \frac{1}{\Pi(\overline{g}(b_{t+1}))} Cov\left(-\mathbb{I}\{g \leq \overline{g}(b_{t+1})\}, \gamma_{t+1}\right)
\]
\[= \frac{1}{\Pi(\overline{g}(b_{t+1}))} E\left[\left(\Pi(\overline{g}(b_{t+1})) - \mathbb{I}\{g \leq \overline{g}(b_{t+1})\}\right) \gamma_{t+1}\right]
\]
\[= \left(E[\gamma_{t+1}] - E[\gamma_{t+1}]\right).
\]
Where the first equality holds because the covariance of a random variable and a constant is zero, and the second equality follows from $1 = \mathbb{I}\{g \leq \overline{g}(b_{t+1})\} + \mathbb{I}\{g \geq \overline{g}(b_{t+1})\}$. The third equality is true because, for generic random variables $(X, Y)$, $Cov(X, Y) = E[(X - E[X])(Y - E[Y])] = E[(X - E[X])Y]$. By equation A.33 and the envelope condition $\gamma$ is an increasing function of $g$, thus the last term is positive. The term $E[\gamma_{t+1}](\mathcal{M}(b_{t+1}) - 1)$ is positive by the properties presented in proposition B.1.

(a) $\nabla_b[E[\gamma_{t+1}(\mathcal{M}(b_{t+1}) - 1)]]$ is given by $\nabla_b[\mathcal{M}(b_{t+1})]E[\gamma_{t+1}] + \nabla_b[E[\gamma_{t+1}]][\mathcal{M}(b_{t+1}) - 1]$. By proposition B.1(3) and my assumptions the first term is positive. Invoking the envelope conditions, the second terms is given by
\[
\left\{ \int \nabla_b[\gamma(b_{t+1}, g)]\Pi(\text{dg}) \right\} (\mathcal{M}(b_{t+1}) - 1).
\]
By assumption $\nabla_b[\gamma(b_{t+1}, g)] = -\nabla^2_b[\mathcal{U}(b_{t+1}, g, 0)] \geq 0$, and thus the first term is positive; implying that $\nabla_b[E[\gamma_{t+1}]] \geq 0$.

(b) Defining $\Pi_{b_{t+1}}(\cdot)$ as the “default-adjusted” measure, $\frac{\mathbb{I}\{g \leq \overline{g}(b_{t+1})\}\Pi(\text{dg})}{\Pi(\overline{g}(b_{t+1}))}$, and $\pi_{b_{t+1}}(g) \equiv \frac{\mathbb{I}\{g \leq \overline{g}(b_{t+1})\}\pi(\text{dg})}{\Pi(\overline{g}(b_{t+1}))}$ as the pdf. It follows
\[
\nabla_b \left[ \int\int G \gamma(b_{t+1}, g)\Pi_{b_{t+1}}(\text{dg}) \right] = \int\int G \nabla_b[\gamma(b_{t+1}, g)]\Pi_{b_{t+1}}(\text{dg})
\]
\[+ \int G \gamma(b_{t+1}, g)\nabla_b[\pi_{b_{t+1}}(g)]dg.
\]
Since
\[
\nabla_b[\pi_{b_{t+1}}(g)] = \frac{\pi(\text{dg})}{\Pi(\overline{g}(b_{t+1}))} \left\{ \delta_{\{g(b_{t+1})\}}(g) - \frac{\mathbb{I}\{g \leq \overline{g}(b_{t+1})\}\pi(\overline{g}(b_{t+1}))}{\Pi(\overline{g}(b_{t+1}))} \right\} \nabla_b[\overline{g}(b_{t+1})]
\]
\[= \nabla_b[\overline{g}(b_{t+1})]
\]
where $\delta_{\{\bar{g}(b_{t+1})\}}(\cdot)$ is the Dirac delta function at $\bar{g}(b_{t+1})$. Hence
\[
\nabla_b \left[ \int_G \gamma(b_{t+1}, g) \Pi_{b_{t+1}}(dg) \right] = \int_G \nabla_b [\gamma(b_{t+1}, g)] \Pi_{b_{t+1}}(dg) + \gamma(b_{t+1}, \bar{g}(b_{t+1})) \int_G \pi(g)dg
\]
\[
\quad + \frac{\nabla_b [\bar{g}(b_{t+1})]}{\Pi(\bar{g}(b_{t+1}))} \int_G \gamma(b_{t+1}, g) \left\{ \delta_{\{\bar{g}(b_{t+1})\}}(g) \right\} \pi(g)dg
\]
\[
\quad - \frac{\pi(\bar{g}(b_{t+1})) \nabla_b [\bar{g}(b_{t+1})]}{\Pi(\bar{g}(b_{t+1}))} \int_G \gamma(b_{t+1}, g) \left\{ \frac{\Pi\{g: g \leq \bar{g}(b_{t+1})\}}{\Pi(\bar{g}(b_{t+1}))} \right\} \pi(g)dg
\]
\[
= \int_G \nabla_b [\gamma(b_{t+1}, g)] \Pi_{b_{t+1}}(dg)
\]
\[
\quad + \frac{\pi(\bar{g}(b_{t+1})) \nabla_b [\bar{g}(b_{t+1})]}{\Pi(\bar{g}(b_{t+1}))} \left( \gamma(b_{t+1}, \bar{g}(b_{t+1})) - \int_G \gamma(b_{t+1}, g)\pi_{b_{t+1}}(g)dg \right)
\]
\[
\equiv \int_G \nabla_b [\gamma(b_{t+1}, g)] \Pi_{b_{t+1}}(dg) + T_2.
\]
By assumption $\gamma(b_{t+1}, \cdot)$ is increasing and thus
\[
\int_G \gamma(b_{t+1}, g)\pi_{b_{t+1}}(g)dg \leq \gamma(b_{t+1}, \bar{g}(b_{t+1})) \int_{\{g: g \leq \bar{g}(b_{t+1})\}} \frac{\pi(g)dg}{\Pi(\bar{g}(b_{t+1}))} = \gamma(b_{t+1}, \bar{g}(b_{t+1})).
\]
Therefore $T_2 \leq 0$.

Putting these results together,
\[
\nabla_b \left[ E[\gamma_{t+1}] - \mathbb{E}[\gamma_{t+1}] \right] = \nabla_b \left[ \int_G \gamma(b_{t+1}, g) \Pi(dg) \right] - \nabla_b \left[ \int_G \gamma(b_{t+1}, g) \Pi_{b_{t+1}}(dg) \right]
\]
\[
= \int_G \nabla_b \gamma(b_{t+1}, g)(\pi(g) - \pi_{b_{t+1}}(g))dg - T_2.
\]
Since, $\nabla_b^2 [\gamma(b_{t+1}, g)] = -\nabla_b^2 [\mathcal{U}(b_{t+1}, g, 0)]$, and by assumption is positive, it follows that $\nabla_b [\gamma(b_{t+1}, g)]$ is an increasing function of $g$, which implies that the first term of the equation above is positive. I already showed that $T_2 \leq 0$.

Therefore $\nabla_b [E[\gamma_{t+1}] - \mathbb{E}[\gamma_{t+1}]]$ is positive. Hence, since,
\[
\mathcal{M}(b_{t+1}) \frac{\text{Cov}(\Pi\{g: g \geq \bar{g}(b_{t+1})\}, \gamma_{t+1})}{\Pi(\bar{g}(b_{t+1}))} = \mathcal{M}(b_{t+1})(E[\gamma_{t+1}] - \mathbb{E}[\gamma_{t+1}]).
\]
and $\nabla_b [\mathcal{M}(b)] \geq 0$ (see part 2(a)) then, it follows that $\mathcal{M}(b_{t+1}) \frac{\text{Cov}(\Pi\{g: g \geq \bar{g}(b_{t+1})\}, \gamma_{t+1})}{\Pi(\bar{g}(b_{t+1}))}$ is increasing.

\[\square\]

**APPENDIX C. DESCRIPTION OF THE DATA**

For section 2 I constructed the data as follows. First, for each country, I computed time average, or time standard deviations or any quantity of interest. Second, once I computed
these averages, I group the countries in IND, EME and LAC. I do this procedure for (a) central government domestic debt (as % of output); (b) central government expenditure (as % of output); (c) central government revenue (as % of output), and (d) Real Risk Measure.


**APPENDIX D. FIGURES**

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48For Greece and Portugal I use central government public debt because central government domestic debt was not available. For Sweden, Ecuador and Thailand I use general government expenditure because central government expenditure was not available. For Albania, Bulgaria, Cyprus, Czech Rep., Hungary, Latvia, Poland and Russia no measure of government expenditure was available and thus were excluded from the sample for the calculations of this variable. The same caveats apply to the central government revenue sample.

49I gratefully acknowledge that Kaminsky et al. (2004) and Panizza (2008) kindly shared the dataset used in their respective papers (see references).

50For Argentina, Brazil, Colombia, Ecuador, Egypt, Mexico, Morocco, Panama, Peru, Philippines, Poland, Russia, Turkey and Venezuela I used the real EMBI+ as a measure of real risk. For the rest of the countries I used government note yields of 1-5 years maturity, depending on availability.

51The LAC group is conformed by the countries with “\(^1\)”.
Fig. D.1. Timing of the Model. $g_t$: government expenditure; $b_t$: government equilibrium debt; $\lambda$: Prob. of receiving an offer; $\delta \in \Delta$: Fraction of defaulted government debt; $\alpha$: Prob. of having the option to leave autarky.
Figure D.2. Impulse Response functions.
Figure D.3. Policy function of debt for ED Economy (black dot), AMSS (red dot) and default region (yellow area).

Figure D.4. Top Panel: Price function $p^b(b, g)$ for ED Economy. Bottom Panel: Default set for ED Economy. I.i.d. Case and $\lambda = \alpha = 0$. 
Figure D.5. Box-plots of government policy for the whole sample.

Figure D.6. $\Omega$ (solid) and probability of autarky (dashed).
Table E.5. Parameter values for the case with $\alpha = 0.30$

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>(Discount Factor) $\beta$</td>
<td>0.9825</td>
<td>(AR(1) Coeff.) $\rho_1$</td>
<td>0.30</td>
</tr>
<tr>
<td>(Utility of leisure) $C_1$</td>
<td>0.01</td>
<td>(Mean) $\mu_g$</td>
<td>0.20</td>
</tr>
<tr>
<td>(Utility of leisure) $\sigma$</td>
<td>2</td>
<td>(Std. Dev.) $\sigma_g$</td>
<td>0.05</td>
</tr>
<tr>
<td>(Utility of consumption) $\sigma_c$</td>
<td>0</td>
<td>$\mathbb{E} (g, \bar{g})$</td>
<td>(0, 0.325)</td>
</tr>
<tr>
<td>(Prob. of offer for partial payment) $\lambda$</td>
<td>0</td>
<td>$\mathbb{E} (b, \bar{b})$</td>
<td>(0, 0.3)</td>
</tr>
<tr>
<td>(Prob. of escaping autarky) $\alpha$</td>
<td>0.30</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(Direct cost of default) $\kappa$</td>
<td>0.021</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table E.6. MC results for the case $\alpha = 0.30$. In the table $E$ and $std$ denote the mean and standard deviation across time, respectively. All quantities are averaged across MC simulations.

<table>
<thead>
<tr>
<th>Sample</th>
<th>All AMSS</th>
<th>ED</th>
<th>No Default AMSS</th>
<th>ED</th>
<th>Default AMSS</th>
<th>ED</th>
</tr>
</thead>
<tbody>
<tr>
<td>$E(b_t/n_t)$ (%)</td>
<td>12.64</td>
<td>5.820</td>
<td>12.41</td>
<td>5.520</td>
<td>29.70</td>
<td>18.20</td>
</tr>
<tr>
<td>std($b_t/n_t$) (%)</td>
<td>9.290</td>
<td>5.100</td>
<td>8.490</td>
<td>4.590</td>
<td>0.033</td>
<td>0.001</td>
</tr>
<tr>
<td>$E(\tau_t)$</td>
<td>0.219</td>
<td>0.218</td>
<td>0.219</td>
<td>0.218</td>
<td>0.264</td>
<td>0.247</td>
</tr>
<tr>
<td>std($\tau_t$)</td>
<td>0.030</td>
<td>0.040</td>
<td>0.027</td>
<td>0.038</td>
<td>0.023</td>
<td>0.049</td>
</tr>
<tr>
<td>$E(\tau_t n_t)$</td>
<td>0.194</td>
<td>0.193</td>
<td>0.194</td>
<td>0.193</td>
<td>0.233</td>
<td>0.218</td>
</tr>
<tr>
<td>std($\tau_t n_t$)</td>
<td>0.027</td>
<td>0.035</td>
<td>0.023</td>
<td>0.032</td>
<td>0.020</td>
<td>0.042</td>
</tr>
<tr>
<td>$E(spread)$ (%)</td>
<td>0.410</td>
<td>-</td>
<td>0.13</td>
<td>-</td>
<td>10.30</td>
<td>-</td>
</tr>
<tr>
<td>$E(Default Spell)$</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>5.694</td>
<td>-</td>
</tr>
<tr>
<td>$E(Recovery Rate)$ (%)</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>$Pr(Default)$ (%)</td>
<td>3.300</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>

The MC mean of average debt-output ratio is higher in AMSS than in my default economy (row 1). The ratio in the former economy is approximately 13 percent, whereas in my economy this ratio is about 6 percent. For the default sample, however, the average debt-to-output ratio in the ED economy is much higher, around 18 percent.

The flip side of the endogenous borrowing limits, a higher volatility in the taxes, can be seen in rows 3-4 (tax) and 5-6 (tax revenue) across all three sub-samples.
In the ED economy the average spread is defined as the spread associated to $p^b_t$ in times of “no default” and to $q_t$ in “default”; the value of this quantity (row 8) is 0.4%. For the default sample, however, the spread is around 11 percent; although still low, it is much higher than for the “no default” sample.

We observe that the results in the $(\alpha > 0, \lambda = 0)$ and $(\alpha = 0, \lambda > 0)$ experiments are qualitatively similar. Both cases have lower debt-to-output ratio, and more volatile taxes (or revenues). In both cases the defaulted debt-to-output ratio is higher than the ratio for “no default” samples, and the spread is also higher in this period. These similarities notwithstanding, there are some notable differences. First, debt-to-output ratio in the economy with $\alpha > 0$ and $\lambda = 0$ is higher on average than in the economy with $\alpha = 0$ and $\lambda > 0$. Second, the autarky spell is higher in the $(\alpha = 0, \lambda > 0)$ experiment. Finally, the spread for all three sub-samples is higher in the $(\alpha = 0, \lambda > 0)$ experiment.

The last fact is driven by two effects. First, the probability of default is slightly higher in the $(\alpha = 0, \lambda > 0)$; also this experiment exhibits higher ”outliers”. Second, the autarky spell is lower in the economy with $\alpha > 0$ and $\lambda = 0$. Given that $p^b$ conveys information regarding “distant” defaults and autarky spells through the price $q$; a lower autarky spell translates into a higher probability of getting paid at some point in the future.

**E.1. Welfare calculations for $\alpha \in [0, 1]$ and $\lambda = 0$.** For the case of $\alpha \in [0, 1]$ and $\lambda = 0$, if $\alpha$ increase the value option of defaulting also increases but the government always has to spend at least one period under autarky; this non-vanishing cost implies that even for $\alpha = 1$, the probability of autarky is not zero (dashed line in figure E.9). The solid line in figure E.9 show $\Omega$ for $\alpha \in [0, 1]$ and $\lambda = 0$; its behavior is similar to the one in the previous case.
Figure E.7. Box-plots of government policy for the whole sample.

Figure E.8. Box-plots of prob. of default (top-left), Avg. default spell (top-right) and spread of ”envelope” price (bottom-left) for the whole sample. MC experiment: AR(1) (with $\rho = 0.30$) Case and $\lambda = 0$ and $\alpha = 0.30$. 

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Figure E.9. \(\Omega\) (solid) and probability of autarky (dashed).