Economics 202A
Suggested Solutions to the Mock Exam

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1 RBC. Intertemporal substitution and labor supply

The solution to this question can be found on pages 154-156 of the textbook. Here are the main steps. I think this question did not make it into real exam because it is too easy if you do it half-way and too messy otherwise. And also because it is straight from a textbook.

We have the consumption side of the RBC model

$$\begin{aligned} u_t &= \ln c_t + b\ln(1-l_t), \\ U &= \mathrm{E}_t\{\sum_{i=0}^{n-1}\beta^i u_{t+i}\}. \end{aligned}$$

And the intertemporal budget constraint will be

$$\begin{aligned} c_t &= l_t w_t + (1+r)s_{t-1} \\ s_0 &= 0 \\ s_{n-1} &= 0. \end{aligned}$$

(a) $n = 1$.

$$\max_{c,l} u = \ln c + b\ln(1-l)$$

s.t.

$$c = lw$$

We can substitute for c in the objective function to get

$$\max_{l} u = \ln(lw) + b\ln(1-l)$$

For which FOC is

$$\frac{1}{l} = b\frac{1}{1-l} \quad \Longrightarrow \quad l = \frac{1}{1+b}.$$

We can see that the desired labor supply does not depend on the real wage, because of the logarithmic utility. If utility is logarithmic, then income and substitution effect from the change in wage exactly cancel out and thus the total effect of the change in wage on the labor supply is zero.

b) Two periods. We now need to maximize the two-period utility function subject to the two-period budget constraint.

$$
\begin{aligned}
\max_{c_0,c_1,l_0,l_1} U &= \ln(c_0) + \beta \ln(c_1) + \mathrm{E}_0[b \ln(1-l_0) + \beta b \ln(1-l_1)] \\
c_0 + \frac{c_1}{1+r} &= l_0 w_0 + \frac{l_1 w_1}{1+r} \\
L &= \mathrm{E}_0[\ln(c_0) + \beta \ln(c_1) + b \ln(1-l_0) + \beta b \ln(1-l_1) + \lambda(l_0 w_0 + \frac{l_1 w_1}{1+r} - c_0 - \frac{c_1}{1+r})] \\
\frac{\partial L}{\partial l_0} &= -\frac{b}{1-l_0} + \lambda w_0 = 0 \\
\frac{\partial L}{\partial l_1} &= \mathrm{E}_0[-\frac{\beta b}{1-l_1} + \lambda \frac{w_1}{1+r}] = 0.
\end{aligned}
$$

If we divide one of the first order conditions by another and assume that (since we were not told explicitly what the uncertainty is) that only the interest rate is uncertain, and also assume certainty equivalence in labor decision (because it does not matter for our purposes), we will get that

$$\frac{1-l_1}{1-l_0} = \frac{\beta w_0 \mathrm{E}_0(1+r)}{w_1}.$$

This last equation allows us to conclude that if relative wage in the second period goes up, relative labor supply in the second period will go up. We can not draw conclusions about the absolute value of the labor supply in the first and second period without a messy derivation of the labor supply for each of the periods explicitly (this will require the use of two other FOC's and the budget constraint).

2 IS-LM. Inflation, Aggregate Demand and Related Topics

I think this question did not make it because it is kind of long and part a) is messy.

a) Substitute everything (including i derived from equation (4)) into equation (1) and solve it for Y. You will end up with quadratic equation

$$(1-c)hPY^2 - (Z+G)hPY - \alpha M = 0$$

where

$$Z = C_0 - cT + I_0 - \frac{\alpha}{h}L_0 + \alpha\pi^e.$$

This equation has only one positive solution

$$Y^* = \frac{Z + G + \sqrt{(Z+G)^2 + 4(1-c)hP\alpha M}}{2(1-c)hP}$$

and we can use it to find i from the LM equation

$$i^* = \frac{L_0}{h} - \frac{M}{hPY^*}.$$

It is important to realize that we do not need to derive these equations in order to answer further questions in this problem, because we can use implicit differentiation, which is much less messy then the explicit one in this case.

b) To find $\mathrm{d}Y/\mathrm{d}G$ (government spending multiplier, we can implicitly differentiate our quadratic equation above to get

$$2(1-c)hPY\mathrm{d}Y - hPY dG - (Z+G)hPdY = 0$$

from which

$$\frac{dY}{dG} = \frac{hPY}{2(1-c)hPY - (Z+G)hP} = \frac{1}{(1-c) + \frac{\alpha M}{hPY}}.$$

c) To derive the IS curve we take i as given and solve for Y

$$Y = \frac{C_0 - cT + I_0 - \alpha(i - \pi^e) + G}{1-c}.$$

The slope of the IS curve is $\mathrm{d}i/\mathrm{d}Y$. We can calculate explicitly

$$\frac{\mathrm{d}Y}{\mathrm{d}i} = -\frac{\alpha}{1-c},$$

and thus

$$\frac{\mathrm{d}i}{\mathrm{d}Y} = -\frac{1-c}{\alpha}.$$

d) If i is exogenous, the LM equation only insures the equilibrium on the money market (M adjusts given Y and i). Then the value of a multiplier is simply the derivative of Y with respect to G along the IS equation derived above.

$$\frac{\mathrm{d}Y}{\mathrm{d}G} = \frac{1}{1-c},$$

which is always greater than the multiplier we derived in b). Intuitively, when i is fixed, its change does not "crowd out" the effect of the fiscal expansion.

e) First, consider the case when M is exogenous. Differentiate implicitly our quadratic equation (with respect to Y and P) to get

$$\frac{\mathrm{d}Y}{\mathrm{d}P} = -\frac{\alpha M}{P^2((1-c)hY + \frac{\alpha M}{PY}},$$

which is clearly negative. Therefore the effect of the fall in price level is the increase in the output. On our IS-LM picture, this will correspond to the shift of the LM curve to the right.

Now, consider the case when i is exogenous. Then the price level changes do not have any effect on the output, because in order to keep the interest rate constant has to adjust money supply one-to-one to the changes in the price level.

f) See Old Keynesian problem set answers. Question #4. The LM curve is horizontal and the AD curve is vertical.

3 Monetary policy. Real money balances and inflation

This question seems a little long to make it into real final exam. Plus, part (c) of it is asking you something we did not cover in this course.

From the money demand equation we derive

$$-p_t - \gamma p_t = m_t + \alpha - \gamma \mathrm{E}_t p_{t+1}$$

which implies

$$p_t = \frac{m_t - \alpha + \gamma \mathrm{E}_t p_{t+1}}{1+\gamma}.$$

b) How can we get the solution that includes "bubbles" (non-zero limit of the price level)? The most straightforward way to do so is to forward the result of part (a) once, take the expectations as of time t and plug in the result in the original equation (result of part (a)). Then forward it once more, get the expression for $\mathrm{E}_t p_{t+2}$ and plug it in, etc. until you see the pattern. Since it is not hard to do, and a lot harder to type, I will give you the answer.

$$p_t = \lim T \to \infty \left(\frac{1}{1+\gamma}\right)^T \mathrm{E}_t p_{t+T} + \frac{1}{1+\gamma} \sum_{s=0}^{\infty} \left(\frac{\gamma}{1+\gamma}\right)^s (\mathrm{E}_t m_{t+s} - \alpha)$$

c) The first term in the result of (b) is the "bubble". We normally assume a "no-bubble" solution, because if this limit does not converge to zero, it means that prices are growing or falling infinitely and therefore rational agents will not hold any money, since they would expect price level to "explode". If we would rule out the "bubbles" from the beginning, we could use more efficient way of solving the equation - using lag operators. The solution we would get is

$$p_t = \frac{1}{1+\gamma}\sum_{s=0}^{\infty}\left(\frac{\gamma}{1+\gamma}\right)^s (\mathrm{E}_t m_{t+s} - \alpha).$$

We will use this no-bubble solution to answer the rest of the questions in this problem.

d) Suppose we expect that the path of money is the following:

$$\mathrm{E}_t m_{t+s} = m_t + gs.$$

Then we can substitute this into our solution for p_t to get[1]

$$p_t = \frac{1}{1+\gamma}[(m_t - \alpha)(1+\gamma) + gy(1+\gamma)] = m_t - \alpha + g\gamma.$$

e) Using our result in (d) we can derive

$$\frac{\partial p_t}{\partial g} = \gamma > 0.$$

Therefore the higher is the expected growth rate of money stock, the higher is the price level today. This is a general result of the rational expectations assumption.

4 NK. Fix- and Flexible-price firms

This question seems to be a little too messy to be in the final.

a) First, we substitute the equation for the price level into the expression of the rule according to which flexible-price firms set up their prices. Then we can solve this equation for p^f to get the result

$$p^f = p^r + \frac{\phi}{\phi + q - q\phi}(m - p^r).$$

[1] To derive the following equation you have to use two facts:

$$\sum_{s=0}^{\infty} x^s = \frac{1}{1-x},$$

and

$$\sum_{s=0}^{\infty} x^s s = \frac{x}{(1-x)^2}$$

b) Now we can plug our result into the expression we are given for p^r and solve for p^r. This is a messy part that miraculously reduces to the solution

$$p^r = \mathrm{E}m$$

c) To answer this question we should use our results in (a) and (b) to calculate the price level

$$p = \mathrm{E}m + \frac{\phi - \phi q}{\phi + q - \phi q}(m - \mathrm{E}m),$$

and the output

$$y = m - p = \frac{q}{\phi + q - \phi q}(m - \mathrm{E}m).$$

From the last equation we can see that expected variations in the moneys stock do not affect the output, they are offset by the adjustment of the price level, since even fix-price firms adjust their prices to the expected changes in the money stock.

d) Unexpected changes however, do affect the output level, because at least part of the firms can not adjust their prices and therefore the price level does not compensate completely for the change in the money stock.