

Economics 202A

Suggested Solutions to the New Keynesian problem set

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Spring 1999

1. As in all our models

$$p_i^* = p + \varphi y = \varphi m + (1 - \varphi) p \quad .$$

Assume that initially

$$y = p = m = 0 \Rightarrow p_i^* = 0 \quad .$$

Now at time 0 m jumps to m' .

a) A share of firms $s \in [0; 1]$ changes their prices. They change their prices to a new optimal level

$$p_i^* = \varphi m' + (1 - \varphi) p' \quad ,$$

where

$$p' = s(\varphi m' + (1 - \varphi) p') \quad ,$$

since the rest of the firms keep their prices equal to zero. This last equation can be solved for p' .

$$p' = \frac{s \varphi m'}{1 - s + \varphi s} \quad ,$$

and therefore

$$p_i^* = \varphi m' + (1 - \varphi) \frac{s \varphi m'}{1 - s + \varphi s} = \frac{\varphi m'}{1 - s + \varphi s} \quad .$$

Now, using our usual aggregate demand,

$$y' = m' - p' = m' - \frac{s \varphi m'}{1 - s + \varphi s} = m' \frac{1 - s}{1 - s + \varphi s} \quad .$$

b) Firm's incentive to adjust its price is the loss it would suffer from not adjusting the price $(p_i - p_i^*)^2$ minus the cost of adjusting the price Z . Call this incentive X . If it is positive, then firms will adjust the price, if it is negative, they will not. In this particular

case $p_i = 0$ and $p_i^* = \frac{\varphi m'}{1 - s + \varphi s}$. Therefore incentive is

$$X(s) = m^2 \left(\frac{\varphi}{1 - (1 - \varphi)s} \right)^2 - Z$$

Suppose first that $\phi < 1$, then the derivative

$$\frac{\partial X(s)}{\partial s} = 2m^2 \frac{\varphi^2(1 - \varphi)}{[1 - (1 - \varphi)s]^3}$$

is positive since the denominator is always positive ($\phi > 0$, $s < 1$) and in this case numerator is positive too. Then the incentive to adjust the price is increasing in number of firms that adjust their price.

c) If, on the other hand $\phi > 1$, then the incentive is decreasing in number of firms that adjust their price.

d) If all the firms adjust their prices, $s=1$. This will be the case if $X(1) \geq 0$ which is the case when

$$m^2 \left(\frac{\varphi}{1 - (1 - \varphi)} \right)^2 - Z \geq 0$$

which is equivalent to $m^2 \geq Z$.

If none of the firms adjust their prices then $s=0$. This will be the case if $X(0) \leq 0$ or

$$m^2 \varphi^2 - Z \leq 0 \Leftrightarrow m^2 \varphi^2 \leq Z$$

Therefore multiple equilibria are possible if and only if

$$m^2 \varphi^2 \leq Z \leq m^2$$

which necessarily requires that $\phi \leq 1$. This is intuitive because of our result in b). The more firms adjust the prices the greater is the incentive for the rest of the firms to adjust their prices, therefore if all the firms are symmetric, they either will all want to adjust their prices, or none of them will.

Note that for $\phi > 1$, there can be a situation when only some share $0 < s^* < 1$ will adjust their prices. For this situation to be an equilibrium, $X(s^*)$ must be equal to zero. Since the firms are identical, for them to behave differently, they have to be indifferent between adjusting and not adjusting their prices. One can solve for this s^* from the condition $X(s^*)=0$. Also, in this case, since incentive is decreasing in s , $s=0$ and $s=1$ can not be equilibrium situations at the same time, so we can not have multiple equilibria in this case.

2. Consider the Taylor model discussed in the textbook and in section 12. The only difference now is that $m_t = \varepsilon_t$. I will follow the steps in my section notes.

$$x_t = .5(p_t^* + E_t p_{t+1}^*) = .5(\varphi m_t + (1 - \varphi) p_t + \varphi E_t m_{t+1} + (1 - \varphi) E_t p_{t+1})$$

$$p_t = .5(x_t + x_{t-1})$$

Unlike in the book, because m is just a WN now, $E_t m_{t+1} = 0$. Therefore

$$x_t = .5(\varphi m_t + .5(1 - \varphi)(x_t + x_{t-1}) + .5(1 - \varphi)(x_t + E_t x_{t+1})) \quad ,$$

from which follows that

$$x_t = \frac{\varphi}{1 + \varphi} m_t + .5 \frac{1 - \varphi}{1 + \varphi} (x_{t-1} + E_t x_{t+1}) = \frac{1 - 2A}{2} m_t + A(x_{t-1} + E_t x_{t+1}) \quad .$$

As in the section we can use lag operators to solve this equation.

$$x_t = \frac{1 - 2A}{2} m_t + A(L + L^{-1})x_t \quad .$$

Now, as in the section, we also want to have $L^{-1} m_t$ term on the RHS. Remember

though that because m_t is now white noise, $E_t m_{t+1} = L^{-1} m_t = 0$ and therefore we can add as many of those terms on the RHS as we want. Therefore the above equation is equivalent to

$$x_t = \frac{1 - 2A}{2} (m_t + E_t m_{t+1}) + A(L + L^{-1})x_t = \frac{1 - 2A}{2} (1 + L^{-1})m_{t+1} + A(L + L^{-1})x_t \quad ,$$

which is exactly the same equation as we had in the random walk case.

We know that the solution can be written as

$$x_t = \lambda x_{t-1} + \frac{\lambda}{A} \frac{1 - 2A}{2} (m_t + (1 + \lambda)(E_t m_{t+1} + \lambda E_t m_{t+2} + \dots)) \quad ,$$

where λ is the solution to the equation $A + A\lambda^2 = \lambda$. However, since m is WN, $E_t m_{t+k} = 0$ for all k , therefore our solution can be rewritten as

$$x_t = \lambda x_{t-1} + \frac{\lambda}{A} \frac{1 - 2A}{2} m_t = \lambda x_{t-1} + \frac{(1 - \lambda)^2}{2} m_t \quad .$$

Lagging this equation one we can write that

$$x_{t-1} = \lambda x_{t-2} + \frac{(1 - \lambda)^2}{2} m_{t-1} \quad .$$

Therefore, using the fact that $m_t = \epsilon_t$,

$$p_t = .5\lambda(x_{t-1} + x_{t-2}) + \frac{(1 - \lambda)^2}{4}(m_t + m_{t-1}) = .5\lambda p_{t-1} + \frac{(1 - \lambda)^2}{4}(\epsilon_t + \epsilon_{t-1})$$

and

$$y_t = m_t - p_t = m_t - .5\lambda(x_{t-1} + x_{t-2}) + \frac{(1 - \lambda)^2}{4}(m_t + m_{t-1}) \quad ,$$

which after some manipulations can be simplified to

$$y_t = \lambda y_{t-1} + \frac{1}{4} \left[(\lambda^2 - 2\lambda + 5)\epsilon_t - (\lambda^2 - 6\lambda + 1)\epsilon_{t-1} \right] \quad .$$

As we can see, the AR(1) result is not following from the assumption that m follows RW. Even with serially uncorrelated monetary shocks we get the AR(1) process for the output.

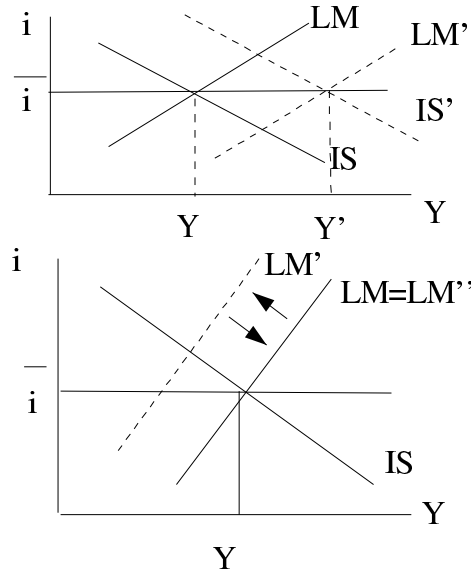
3. This is a monetary policy question, chapter 9.

$$y = c - ai + \epsilon_{IS} \quad , \quad m - p = hy - ki + \epsilon_{LM} \quad ,$$

where shocks are i.i.d. mean zero with variances σ_{IS}^2 and σ_{LM}^2 respectively.

a) When policy makers can control i , we assume that i is exogenous and is fixed at some level \bar{i} . Then $\text{Var}(i)=0$. And $\boxed{\text{Var}(y) = \sigma_{IS}^2}$. The intuition is as follows.

Consider the IS–LM framework with fixed interest rate. Then if IS curve shifts, Central Bank will have to adjust money supply so that the interest rate stays the same. For example, the rightward shift in the IS curve will cause the increase in output larger than with flexible interest rate, because LM curve will have to shift to the right (increase in money supply) to keep i fixed. On the other hand, if LM curve shifts because of the change in money demand, Central Bank will have to adjust money supply accordingly and bring the LM curve back to keep interest rate fixed. Therefore the shifts in the LM curve will have no effect on output, which explains why the shock to LM curve does not enter the equation for $\text{Var}(y)$.



b) When policy makers can control the money stock, we assume that they fix money stock at some level \bar{m} and thus $\text{Var}(m)=0$. We can then substitute LM equation for i into IS equation, because now i is endogenous.

$$y = c - \frac{a}{k} [hy + \epsilon_{LM} - m + p] + \epsilon_{IS} \quad ,$$

from which

$$y = \frac{k}{k + ah} (c + \epsilon_{IS}) - \frac{a}{k + ah} (\epsilon_{LM} - m + p) \quad .$$

Therefore, taking into account that p is fixed,

$$\boxed{\text{Var}(y) = \left[\frac{k}{k + ah} \right]^2 \sigma_{IS}^2 + \left[\frac{a}{k + ah} \right]^2 \sigma_{LM}^2} \quad .$$

c) If the variance of the LM curve becomes high, it is more likely that a good stabilization policy would be based on the control of interest rates, because then monetary policy is adjusting to the money demand shocks and the variability of LM curve does not affect the variance of the output. Basically, by controlling interest rate, Central Bank always counteracts the effects of money demand shocks.