Economics 202A

Suggested Solutions to Old Keynesian problem set

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1. Start from IS and LM.

IS: $Y = E(Y, i - \pi^e, G, T)$

LM: $M/P = L(i, Y)$

Take full derivatives of both sides of both equations treating $\pi^\varepsilon$ as a constant[1]:

IS: $dY = E_Y dY + E_i di + E_G dG + E_T dT = E_Y dY + E_i di$

LM: $d(M/P) = L_i di + L_Y dY$

From the IS equation $dY = \dfrac{E_i di}{1 - E_Y}$, we can substitute it into LM equation

$d(M/P) = L_i di + L_Y \dfrac{E_i di}{1 - E_Y} = (L_i + \dfrac{L_Y E_i}{1 - E_Y}) di$, therefore

$$\boxed{\dfrac{di}{d(M/P)} = \dfrac{1 - E_Y}{L_i(1 - E_v) + L_v E} < 0}.$$

We also know that $\dfrac{dY}{d(M/P)} = \dfrac{E_i}{1 - E_Y} \dfrac{di}{d(M/P)} = \dfrac{E_i}{1 - E_Y} \dfrac{1 - E_Y}{L_i(1 - E_Y) + L_Y E_i}$, therefore

$$\boxed{\dfrac{dY}{d(M/P)} = \dfrac{E_i}{L_i(1 - E_Y) + L_Y E} > 0}.$$

We know that $L_i < 0$, $E_Y < 1$, $L_Y > 0$, $E_i < 0$.
Therefore we know that the denominator in
both equations is negative. Numerator is
positive in di and negative in dY equation.
Therefore, as M/P increases, Y increases and
i decreases. In terms of our diagram, LM
curve shifts to the right and IS curve does
not shift. This results in an increase in Y and
decrease in i.



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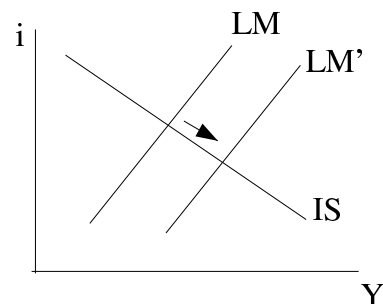
2. We are asked to analyze the effect of the "balanced budget". Take again the IS curve

$$Y = C(Y - T) + I(i - \pi^e) + G$$ and differentiate it now with respect to G holding i

and $\pi^\varepsilon$ as constants

$$dY/dG = C'(Y - T)(dY/dG - dT/dG) + 1 = C'(Y - T)(dY/dG - 1) + 1$$ , since

G=T. We can express now $$dY/dG = \frac{1 - C'(Y - T)}{1 - C'(Y - T)} = 1$$ . That is, the IS curve will

shift to the right by exactly as much as G and T change.
To answer the question about the AD curve, we should consider variable interest rate, but still fixed prices and expected inflation.

$$dY/dG = C'(Y - T)(dY/dG - 1) + I'(i - \pi^e)di/dG + 1$$ . We can also take a

derivative of LM equation with respect to G holding M and P constant.

LM: $$M/P = L(i,Y)$$ , $$0 = L_i di/dG + L_Y dY/dG$$ and therefore

$$di/dG = -\frac{L_Y}{L_i} dY/dG$$ . Substitute this back into our equation for dY/dG:

$$dY/dG = C'(Y - T)(dY/dG - 1) - I'(i - \pi^e)L_Y/L_i dY/dG + 1$$ , therefore

$$dY/dG = \frac{1 - C'(Y - T)}{1 - C'(Y - T) + I'(i - \pi^e)L_Y/L_i} < 1$$ , since I' is negative and $L_Y/L_i$ is also

negative. Therefore the AD curve shifts by less then IS curve. Part of the change in Y is offset by the change in the interest rate.

3. Now T=T(Y). Consider again general form of the expenditure function so that

IS: $$Y = E(Y,i - \pi^e,G,T)$$ . Differentiate both sides with respect to i to find a slope of

IS curve di/dY. $$dY/di = E_Y dY/di + E_i + E_T T'(Y)dY/di$$ , therefore

$$dY/di = \frac{E_i}{1 - E_Y - E_T T'(Y)}$$ and the slope of the IS curve is

$$\boxed{\frac{di}{dY} = \frac{1 - E_Y - E_T T'(Y)}{E_i}}$$ , which means that IS is steeper, since $E_T<0$, T'>0.

We now have to determine how T'(Y) affects the size of the shift of AD curve as a result of the change in G. Take derivatives of IS and LM with respect to G and combine them

to get $$dY/dG = \frac{E_G}{1 - E_Y + E_i L_Y/L_i - E_T T'(Y)} > 0$$ , since $E_T$ is negative. Now, if T'(Y)

increases, the denominator will increase and thus the shift of the AD curve will be smaller. The more responsive are the taxes to the change in output, the more they offset the effect of fiscal policy.

4.This was covered in section. In brief, the LM curve is horizontal and thus the AD curve is vertical. If AS is also vertical at a higher level of Y, then prices are falling and there is never an equilibrium.

5. Since we draw the IS–LM diagram so that there is nominal interest rate on the vertical axis, the change in inflationary expectations will affect IS curve, but not LM curve. If expected inflation falls, investments will fall too, because it will mean that for the same level of nominal interest rate, real interest rate is higher. Therefore the IS curve will shift to the left. As a result, both output and nominal interest rate will fall.
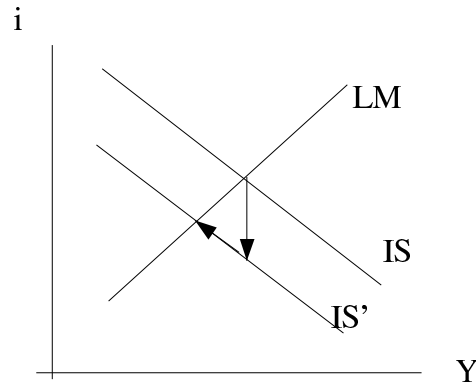


However, we can see that nominal interest rate falls by less then the fall in the expected inflation (IS curve shifted down by the amount of the change in expected inflation, but new equilibrium is to the left). This implies that real interest rate increases as a result in the fall in expected inflation.

6. Think of perfect capital mobility as a limiting case of imperfect capital mobility. With imperfect capital mobility this policy will shift the IS** curve to the right (open economy IS curve in output–interest rate space), because NX would increase as a result of the increase in the exchange rate. This would then increase interest rate and the output. However, with perfect capital mobility, the IS** curve is flat. Thus, its horizontal shift will have no effects. In terms of IS*–LM* diagram (in output–exchange rate space), the LM* curve will shift to the right because of the intervention and then back because of the sterilization. Therefore, sterilized intervention has no effects in case of perfect capital mobility.

7. The economy is described by the equations

$y = -\alpha(i - \pi), \alpha > 0$

$m - p = -ki, k > 0$

$\pi = \vartheta\, y, \vartheta > 0, \alpha\vartheta < 1$

$dp/dt = \pi$

Initially everything is equal to 0. Suddenly, m drops by $-\Delta m$ to a new level $-\Delta m$.

a) From the second equation, taking differences $-\Delta m - \Delta p = -k\,\Delta i$ therefore

$\Delta i = (\Delta m + \Delta p)/k$ . We know that price level can not jump, therefore $\Delta p = 0$. Therefore at time zero, after the fall in m, $\boxed{i = \Delta m/k}$ , which is positive.

b) Combining first and third equations, we get $y = -\alpha(i - \vartheta\, y)$ , therefore $y = \dfrac{-\alpha i}{1 - \alpha\vartheta}$ .

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We can now substitute our result from a) to get $y = \dfrac{-\alpha \Delta m}{k(1-\alpha\vartheta)}$ , which is negative.

So, after the fall in m, y falls and i rises.

c) From b) we can see that if θ is higher, then the drop in y is higher in absolute value. Intuitively, larger θ implies large response of inflation which in term further reduces equilibrium output (through fall in investment, because the fall in inflation means an increase in a real interest rate and this increase will be larger the larger the θ).

d) c) seems to contradict the conclusion of the AS–AD model that predicts smaller shocks to output if inflation adjusts faster. However, AS–AD only describes different steady states and does not describe initial shocks, therefore we should not be too confused by this contradiction.

e) Combining all the equations we can write

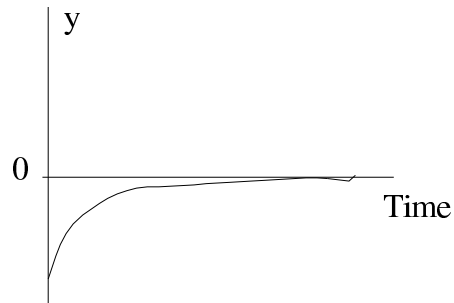output at any point of time as $y = \dfrac{\alpha(m-p)}{k(1-\alpha\vartheta)}$ ,

taking time derivative of both sides,

$\dot{y}(t) = \dfrac{-\alpha\,\dot{p}(t)}{k(1-\alpha\vartheta)} = \dfrac{-\alpha\pi}{k(1-\alpha\vartheta)} = \dfrac{-\alpha\vartheta\,y}{k(1-\alpha\vartheta)}$ ,

since m is constant after the change. Which is a simple differential equation in y. It has a solution

$y(t) = e^{[-\alpha\vartheta/(1-\alpha\vartheta)k]t}\, y(0) = -e^{[-\alpha\vartheta/(1-\alpha\vartheta)k]t}\, \dfrac{\alpha\Delta m}{k(1-\alpha\vartheta)}$ , which tells us that output is gradually

approaching 0, it's previous steady state.

f) Again, we know that no matter what happens to the economy, $y = \dfrac{\alpha(m-p)}{k(1-\alpha\vartheta)}$ .

Therefore $Var(y) = \left(\dfrac{\alpha}{k(1-\alpha\vartheta)}\right)^2 Var(m-p) = \left(\dfrac{\alpha}{k(1-\alpha\vartheta)}\right)^2 [Var(m) + Var(p) - 2Cov(m,p)]$ .

We know that in this model prices adjust slowly, in fact, no matter how much m jumps around, prices only change continuously, therefore we can assert that the variance of prices is much smaller then the variance of m and that covariance term can be ignored. This implies that higher θ will lead to the higher variance of output. This result is intuitive, because we know from c) that the higher the speed of price adjustment, the larger is initial impact on the output, therefore output will jump around more.

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