Economics 202A
Suggested Solutions to RBC Problem Set

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1 From Romer, 4.4. CRRA utility from leisure

(a) First we solve a one–period problem. Use budget constraint to substitute for c_t in the utility function. Since there is only one period, we can drop the time subscript.

$$\max_l u = \ln(wl) + \frac{b(1-l)^{1-\gamma}}{1-\gamma}$$

F.O.C.

$$\frac{1}{l} = b(1-l)^{-\gamma},$$

which implicitly determines labor and we see that it does not depend on real wage. We know that $c = wl$; if we divide both sides of the equation for l by w, we will get

$$\frac{1}{c} = \frac{b}{w^{1-\gamma}(w-c)^{\gamma}},$$

which implicitly determines consumption as a function of real wage.

(b) Now we solve the two-period case. From the budget constraint

$$c_{2,t+1} = (w_t l_{1,t} - c_t)(1 + r_{t+1}) + w_{t+1} l_{2,t+1}.$$

Substitute it into the utility function:

$$\begin{aligned}
\max_{c_1,l_1,l_2} U &= \ln c_{1,t} + \frac{b(1-l_{1,t})^{1-\gamma}}{1-\gamma} + \\
&+ \mathrm{e}^{-\rho} \ln((w_t l_{1,t} - c_{1,t})(1+r_{t+1}) + w_{t+1} l_{2,t+1}) + \\
&+ \mathrm{e}^{-\rho} \frac{b(1-l_{2,t+1})^{1-\gamma}}{1-\gamma}
\end{aligned}$$

F.O.C.'s

$$\begin{aligned}
\frac{c_{2,t+1}}{c_{1,t}} &= \mathrm{e}^{-\rho}(1+r_{t+1}), \\
b(1-l_{1,t})^{-\gamma} &= \mathrm{e}^{-\rho} \frac{w_t(1+r_{t+1})}{c_{2,t+1}}, \\
b(1-l_{2,t+1})^{-\gamma} &= \frac{w_{t+1}}{c_{2,t+1}}.
\end{aligned}$$

Consumption path can be found from the first of the FOC's and the budget constraint. To find the path of labor we can divide the third condition by the second one to find

$$\frac{1-l_{1,t}}{1-l_{2,t+1}} = \left[\frac{1}{\mathrm{e}^{-\rho}(1+r_{t+1})} \frac{w_{t+1}}{w_t} \right]^{1/\gamma}.$$

If relative wage in period $t+1$ rises (w_{t+1}/w_t rises), then relative demand for leisure in period t rises. If the interest rate r rises then the relative demand for leisure in period t falls. It is also straightforward from the last equation that the responsiveness of the relative demand for leisure to the changes of relative wage and interest rate is higher the higher is $1/\gamma$. Remember from the beginning of the semester that $1/\gamma$ is the intertemporal elasticity of substitution. The higher is the elasticity of substitution, the larger is the response of the relative demand for leisure to the change in relative wage or interest rate.

2 From Romer 4.8. RBC with additive technology shocks

This model is similar to the one we considered in class, except for the fact that there is no labor and the technology shock is additive rather then multiplicative. We denote this technology shock as e and assume it follows random walk.

(a) Interest rate is equal to the marginal product of capital minus the rate of depreciation. MPK in this case is equal to A and the equation describing the dynamics of capital stock implies that there is no depreciation. Therefore the interest rate is equal to A.

(b) Assume $\rho = A$. Using the intuitive method we can write Euler equation.

$$\begin{aligned} MU(C_t) &= \mathrm{E}_t\{MU(C_{t+1})\} \\ 1 - 2\theta C_t &= 1 - 2\theta \mathrm{E}_t\{C_{t+1}\} \\ C_t &= \mathrm{E}_t(C_{t+1}). \end{aligned}$$

(c) We are now trying to find a value of consumption at each period of time that would satisfy Euler equation and all other equations of our model.

A little bird told us that we should first try the linear functional form

$$C_t = \alpha + \beta K_t + \gamma e_t.$$

Euler equation must hold. So we will find α, β and γ using method of undetermined coefficients equating LHS and RHS of Euler equation.

$$\begin{aligned} \text{LHS} &= \alpha + \beta K_t + \gamma e_t \\ \text{RHS} &= \mathrm{E}_t\{\alpha + \beta K_{t+1} + \gamma e_{t+1}\}. \end{aligned}$$

From the equation of motion of capital

$$K_{t+1} = K_t + AK_t + e_t - \alpha - \beta K_t - \gamma e_t,$$

we also know that $\mathrm{E}_t\{e_{t+1}\} = \phi e_t$ because e follows random walk. Note also that nothing else is random on the RHS after we substitute for K_{t+1}. We can substitute these results to get

$$\begin{aligned}\mathrm{RHS} &= \alpha(1-\beta) + \beta(1+A-\beta)K_t + \beta(1-\gamma)e_t + \gamma\phi e_t \\ &= \alpha(1-\beta) + \beta(1+A-\beta)K_t + [\beta(1-\gamma) + \gamma\phi]e_t.\end{aligned}$$

The only way RHS and LHS can be equal for all values of K and e is when the coefficients in front of them and the constants are equal on both sides. Thus

$$\begin{aligned}\alpha &= \alpha(1-\beta), \\ \beta &= \beta(1+A-\beta), \\ \gamma &= \beta(1-\gamma) + \gamma\phi.\end{aligned}$$

Two sets of solutions are possible.

$$\alpha = 0, \quad \beta = A, \quad \gamma = \frac{A}{1+A-\phi};$$

and

$$\alpha = \text{anything}, \quad \beta = 0, \quad \gamma = 0.$$

The second one can be ignored since it does not make economic sense. Therefore our solutions for consumption and capital are

$$\begin{aligned}C_t &= AK_t + \frac{A}{1+A-\phi}e_t, \\ K_{t+1} &= K_t + \frac{1}{1+A-\phi}e_t.\end{aligned}$$

(d) Now we have to find impulse response functions resulting from the one-time shock to ε. Recall that AR(1) that e follows can be written as MA(∞)[1].

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[1]

$$\begin{aligned}e_t &= \phi e_{t-1} + \varepsilon_t = \phi L e_t + \varepsilon_t, \\ e_t &= \frac{1}{1-\phi L}\varepsilon_t = \sum_{i=0}^{\infty}\phi^i\varepsilon_{t-i}.\end{aligned}$$

Therefore we can rewrite our results in (c) as

$$\begin{aligned} C_t &= AK_t + \frac{A}{1+A-\phi}\sum_{i=0}^{\infty}\phi^i\varepsilon_{t-i}, \\ K_{t+1} &= K_t + \frac{1}{1+A-\phi}\sum_{i=0}^{\infty}\phi^i\varepsilon_{t-i}, \\ Y_t &= AK_t + \sum_{i=0}^{\infty}\phi^i\varepsilon_{t-i}. \end{aligned}$$

Suppose before time t both e and ε were equal to zero and that $\varepsilon_t = 1$ and afterwards all ε's are again zero. How do capital, output and consumption respond? In period t K_t is not affected since it is predetermined by the past. Y_t goes up by 1. C_t goes up by $\frac{A}{1+A-\phi}$. Starting $t+1$ K develops according the rule

$$K_{t+s} = K_{t+s-1} + \frac{1}{1+A-\phi}\phi^{s-1},$$

i.e. it is growing at a decreasing rate (assuming $\phi < 1$). Output follows the rule

$$Y_{t+s} = Y_{t+s-1} + \frac{\phi-1}{1+A-\phi}\phi^{s-1} + \phi^s.$$

Consumption follows the rule

$$C_{t+s} = C_{t+s-1} + \frac{A}{1+A-\phi}\phi^s.$$

See Figure 1 for the plot of these impulse response functions.

3 Advocacies and criticism of RBC model

(a) Advocacies

It is appealing to be able to use the same model in order to explain different economic phenomena.

Ramsey model has reasonable assumptions about consumers behavior and the production sector.

There is no market failures in the model, it is based on the assumptions of pure competition and therefore social optimum and market equilibrium give the same result, so that we can use different tools to solve the model.

Ramsey model is micro-based, therefore it allows for endogenous unemployment.

(b) Criticisms

Ramsey model assumes representative agent and therefore can not explain unemployment.

In Ramsey model consumption path is determined by individuals that have rational expectations and are trying to smooth consumption, therefore it is unlikely that we will be able to get consumption cycles in Ramsey model.

There is no market failures in Ramsey model and all markets are constantly in equilibrium, therefore the unemployment we get can only be voluntary, which is unrealistic.

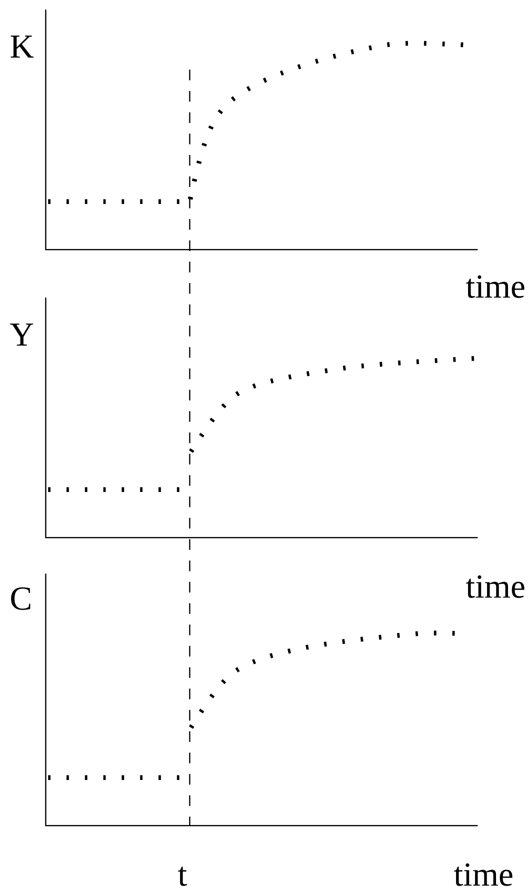

Figure 1: Impulse response functions