Being Rational about Rational Expectations

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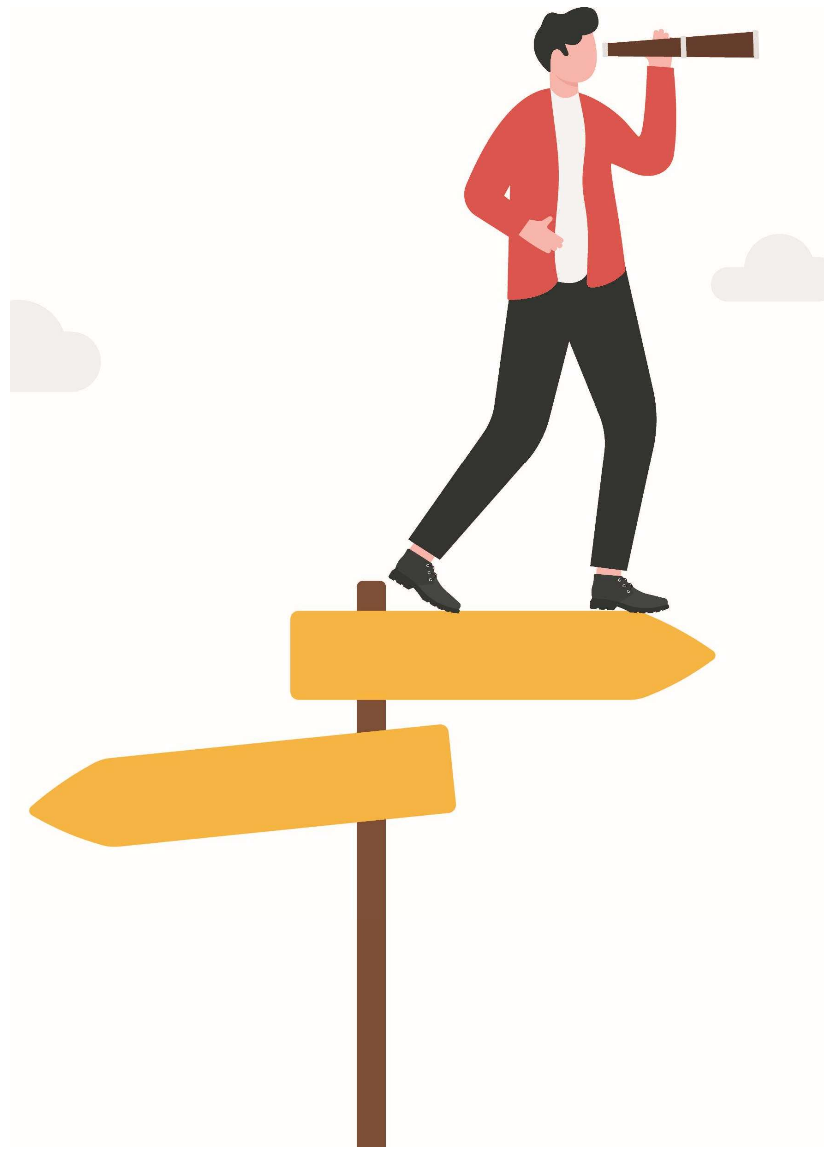
Joint AEA-AFA Lunch

*Based heavily on "Learning about the Long Run" with Leland Farmer & Jon Steinsson.

Rational Expectations

“Outcomes do not differ systematically (i.e., regularly or predictably) from what people expected them to be”

— Sargent, 2007



Rational Expectations

- Standard starting point for modeling in economics and finance
- Economics
  - Lucas critique
  - Ricardian equivalence
  - (Expectation-Augmented) Phillips curve

- Finance
  - Expectations hypothesis of the term structure
  - Uncovered interest rate parity
  - Equity premium puzzle

Tests of Rational Expectations

Benjamin Friedman 1979:

- Rational expectations should "yield predictions of future events which differ from the eventual outcomes only by errors which are themselves independent of the variables used to generate the predictions."

If not, rational agent should *use* this information to predict errors!

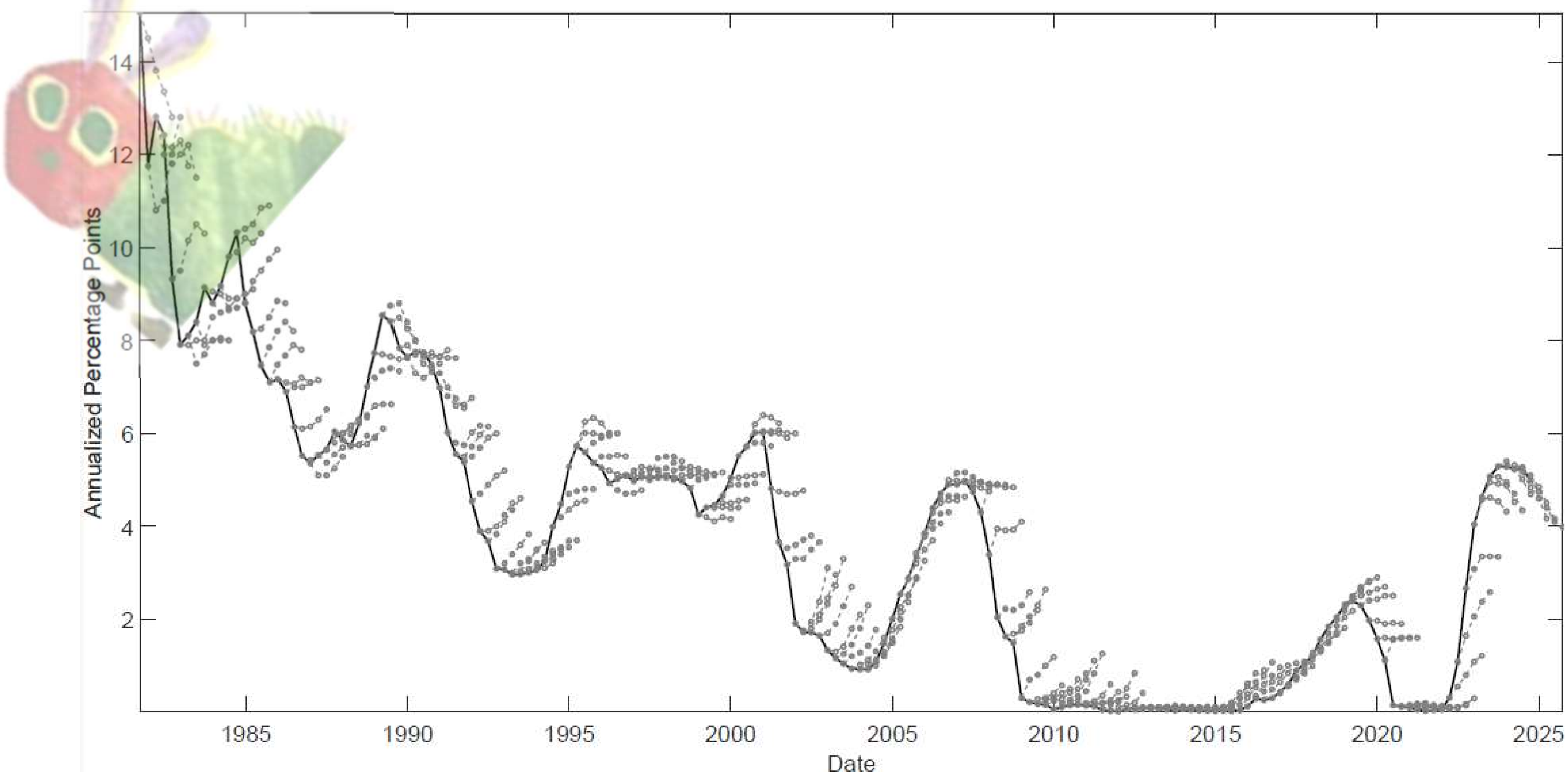
Classic Tests of Rational Expectations

1. Forecast Error Bias
2. Forecast Error Autocorrelation
3. Mincer-Zarnowitz Regression
4. Coibion-Gorodnichenko Regression

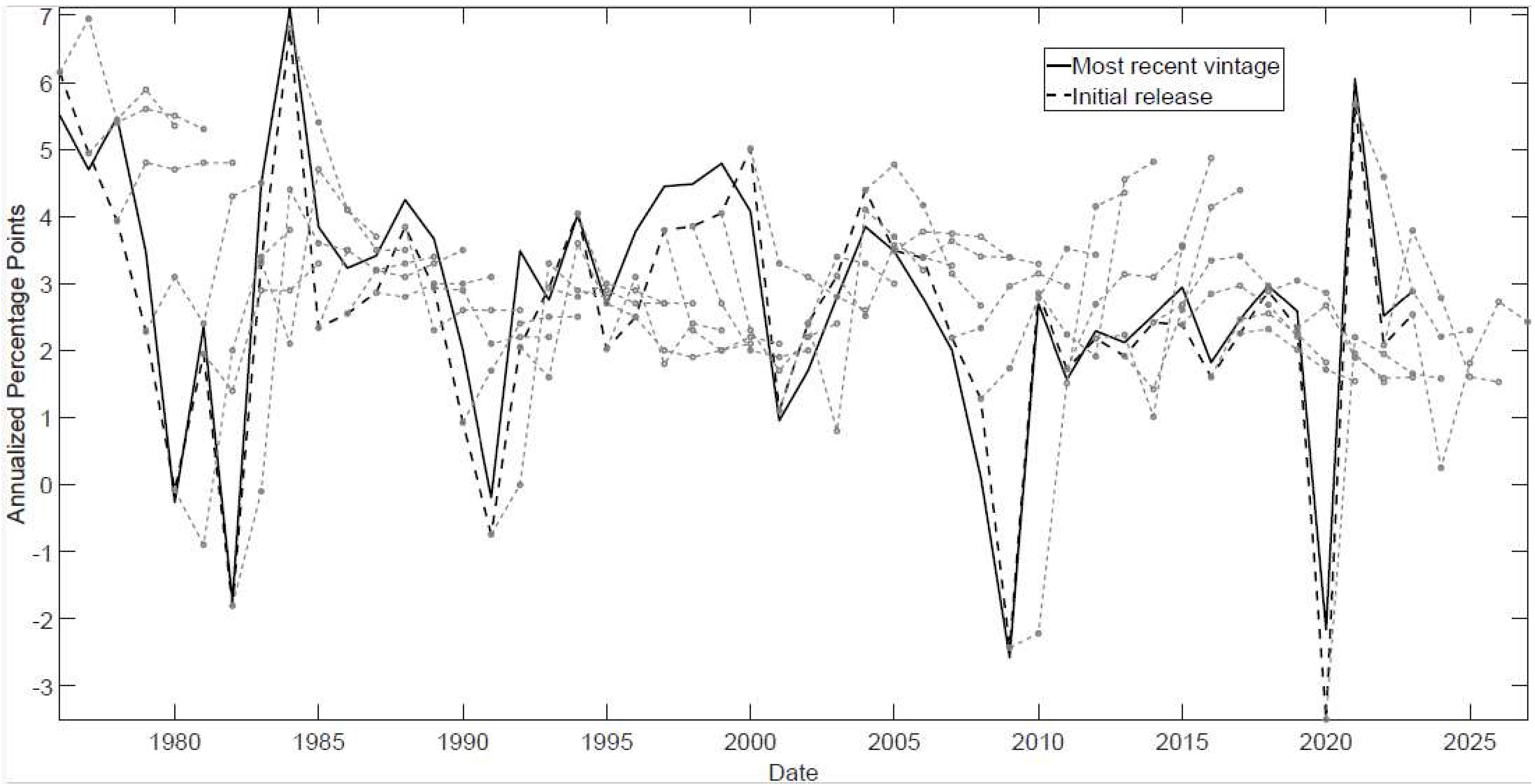
Expectations Hypothesis of Term Structure

- Fama-Bliss; Campbell-Shiller regressions

Interest Rate Forecasts



Real GDP Growth Forecasts



Forecast Error Bias

Regression equation:

$$e_{t+h|t} = \alpha + u_{t+h}$$

where $e_{t+h|t}$ is the h period ahead forecast error $(y_{t+h} - F_t y_{t+h})$

Forecast Error Bias

Regression equation:

$$e_{t+h|t} = \alpha + u_{t+h}$$

where $e_{t+h|t}$ is the h period ahead forecast error ($y_{t+h} - F_t y_{t+h}$)

	$\hat{\alpha},\ H_0 : \alpha = 0$ 81Q3-19Q4 for SPF, 76-19 for CBO				
	Forecast Horizon				
	1	2	3	4	5
T-Bill	-0.18***	-0.34***	-0.52***	-0.70***	–
	(0.05)	(0.09)	(0.14)	(0.19)	
GDP Growth	0.27	-0.27	-0.54	-0.62	-0.52
	(0.25)	(0.35)	(0.50)	(0.53)	(0.49)

Forecast Error Autocorrelations

Regression equation:

$$e_{t+h|t} = \alpha + \beta e_{t|t-h} + u_{t+h}$$

where $e_{t+h|t}$ is the h period ahead forecast error.

Forecast Error Autocorrelations

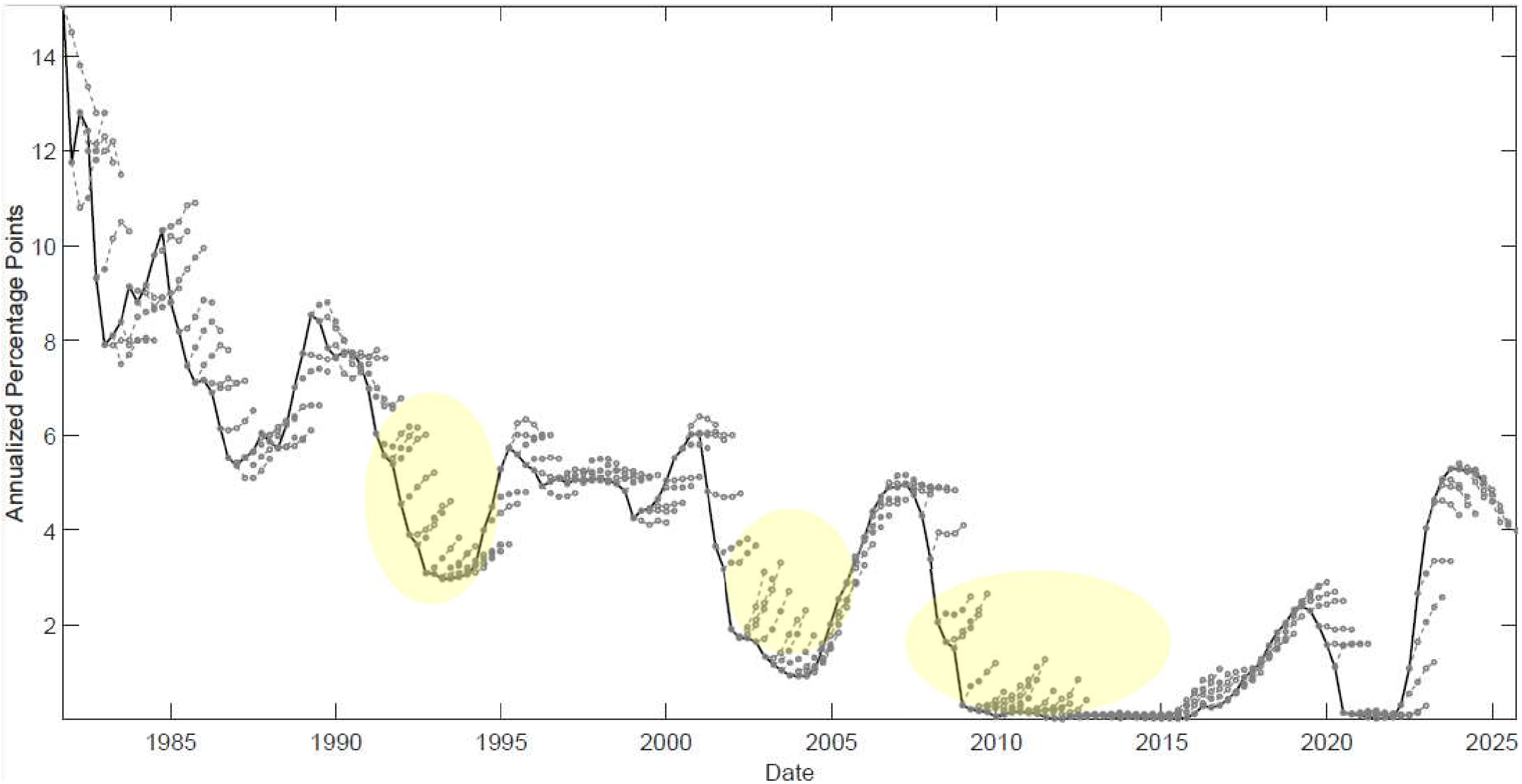
Regression equation:

$$e_{t+h|t} = \alpha + \beta e_{t|t-h} + u_{t+h}$$

where $e_{t+h|t}$ is the h period ahead forecast error.

	$\hat{\beta},\ H_0 : \beta = 0$ 81Q3-19Q4 for SPF, 76-19 for CBO				
	Forecast Horizon				
	1	2	3	4	5
T-Bill	0.30*	0.27**	0.24*	0.13	–
	(0.14)	(0.12)	(0.12)	(0.13)	
GDP Growth	0.22	0.16	0.11	0.08	0.08
	(0.12)	(0.14)	(0.13)	(0.18)	(0.10)

Interest Rate Forecasts



Mincer-Zarnowitz Regressions

Regression equation:

$$y_{t+h} = \alpha + \beta F_t y_{t+h} + u_{t+h}$$

where y_{t+h} is truth at $t+h$ and $F_t y_{t+h}$ is h period ahead forecast

Mincer-Zarnowitz Regressions

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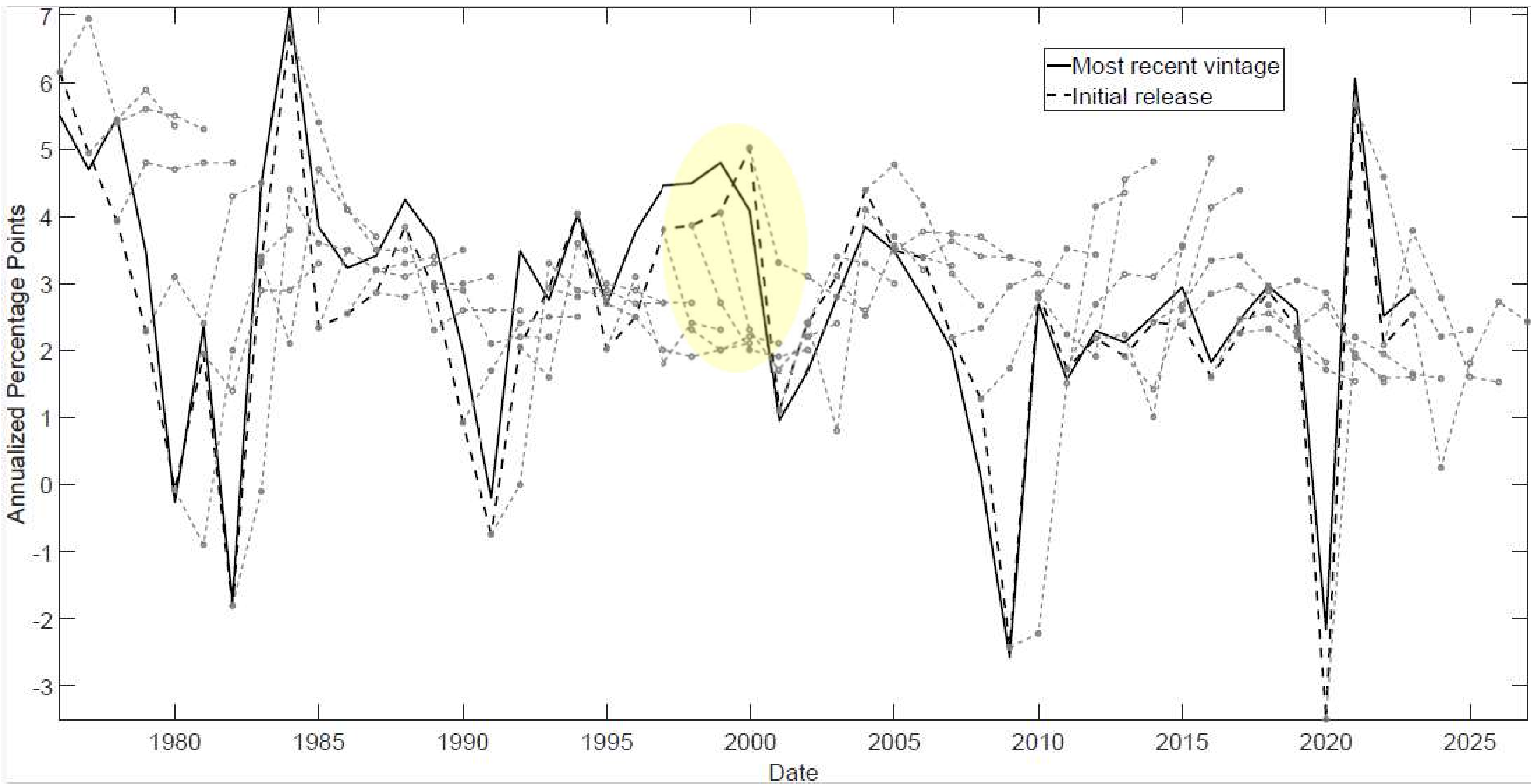
	$\hat{\beta}$, $H_0 : \beta = 1$ 81Q3-19Q4 for SPF, 76-19 for CBO				
	Forecast Horizon				
	1	2	3	4	5
T-Bill	0.97* (0.02)	0.94** (0.02)	0.90** (0.04)	0.86** (0.05)	–
GDP Growth	0.94 (0.10)	0.60 (0.38)	0.03** (0.27)	-0.42*** (0.18)	-0.43*** (0.29)

Real GDP Growth Forecasts



Coibion-Gorodnichenko Regressions

Regression equation:

$$e_{t+h|t} = \alpha + \beta(F_t y_{t+h} - F_{t-1} y_{t+h}) + u_{t+h}$$

where $e_{t+h|t} = y_{t+h} - F_t y_{t+h}$, and $F_t y_{t+h}$ is h period ahead forecast

Interpretation

Interest rates: Underreaction β>0

GDP Growth: Overreaction and longer horizons β <0

Term Structure Facts

- Failures of expectations hypothesis
  - Fama-Bliss / Campbell-Shiller Regressions
  - (Related to uncovered interest rate parity deviations)

(Also, vast literature on risk premia in bond markets- e.g., Wachter 06, Bansal-Shaliastovich 13, Vayanos-Vila 21)

- We will see how far we can go with learning-based explanations

Forecasting Changes in Short Rates

$$\frac{1}{n}\sum_{i=0}^{n-1} y_{t+i}^{(1)} - y_t^{(1)} = \alpha + \beta(y_t^{(n)} - y_t^{(1)}) + u_t$$

	$\hat{\beta},\ H_0: \beta = 1$						
	61Q3-19Q4						
	Long Horizon n						
	2	3	4	8	12	20	40
Future Short Rates	-0.01***	0.11***	0.18***	0.39**	0.57	0.74	0.71
	(0.23)	(0.23)	(0.23)	(0.23)	(0.26)	(0.23)	(0.20)

Forecasting Changes in Long Rates

$$y_{t+1}^{(n-1)} - y_t^{(n)} = \alpha + \beta \left(\frac{1}{n-1} \right) (y_t^{(n)} - y_t^{(1)}) + u_t$$

	$\hat{\beta},\ H_0 : \beta = 1$						
	61Q3-19Q4						
	Long Horizon *n*						
	2	3	4	8	12	20	40
Change in Long Rate	-1.02***	-0.91***	-1.03***	-1.29***	-1.61***	-2.04***	-2.75***
	(0.45)	(0.59)	(0.62)	(0.59)	(0.57)	(0.55)	(0.87)

Failures of Rational Expectations tests

Many other examples!

Froot (1989):

"If the attractiveness an economic hypothesis is measured by the number of papers which statistically *reject* it, the expectations theory of the term structure is a knockout."

What to Make of Rational Expectations Tests?

- Forecasters are irrational / inefficient
  - (Mincer-Zarnowitz 69, Friedman 80, Maddala 91, Croushore 98, Schuh 01)
- Behavioral bi... (e.g., Bordalo-Gennaioli-Ma-Shleifer 20)
- Informa...
  - Stic...
  - ...

Professional forecasters do not have sticky or noisy information regarding the Fed Funds rate....

Alternative: Not Knowing Model

Assumption that forecasters know the model very strong assumption

- Assumes crystal ball about distribution of states!

More realistic to assume that:

- Forecasters are learning about the model that generates the data

Parameter learning fundamentally changes dynamics

- Can lead standard rational expectations tests to fail
    - Friedman 79, Lewis 89, Barsky-DeLong 93, Timmermann 93, Lewellen-Schanken 02, Brav-Heaton 02, Gourinchas and Tornell 04, Cogley-Sargent 05, Collin-Dufresne-Johannes-Lochstoer 16, Johannes-Lochstoer-Mou 16, Singleton 21

Cieslak (2018):

"Ex post predictability of forecast errors does not imply that people make "obvious" mistakes that could be easily fixed in real time. Even when conducting a quasi-real-time estimation, an econometrician uses ex-post knowledge of a statistical relationship that would have been much harder to uncover in real time."

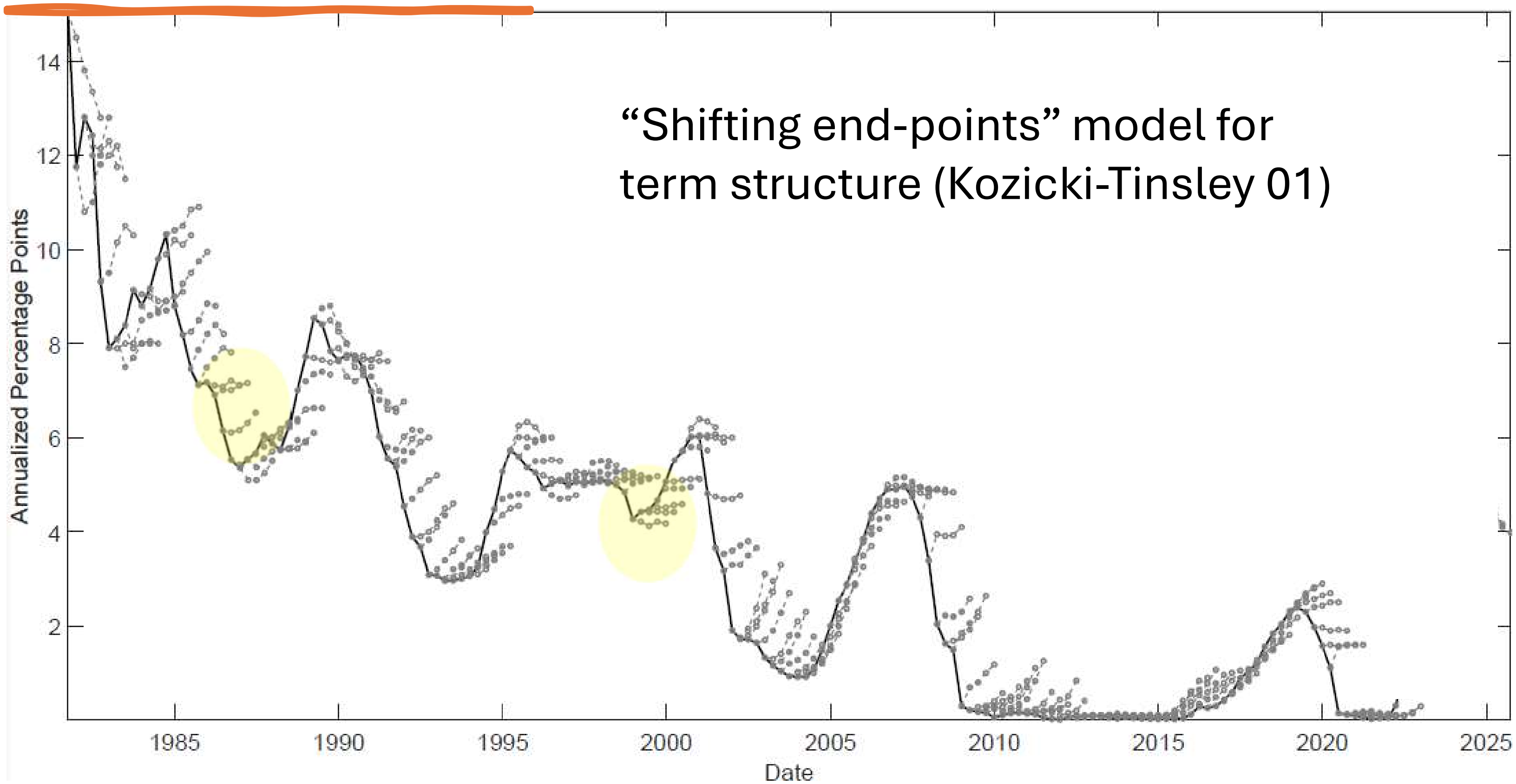
Andolfatto et al. (2008); Van Dijk et al., (2014); Cieslak & Povala (2015); Del Negro et al (2017); Johannsen & Mertens (2018); Crump, Eusepi & Moench (2018); Bauer and Rudebusch (2020); Hajdini and Kurmann (2022); Peso problems: Bekaert-Hodrick-Marshall (2001)

Using survey expectations to proxy for beliefs can help explain failures of rational expectations tests: Frankel and Froot (1987), Froot (1989), Piazzesi, Salomao and Schneider (2015), Nagel and Xu (2023)

Major Challenge

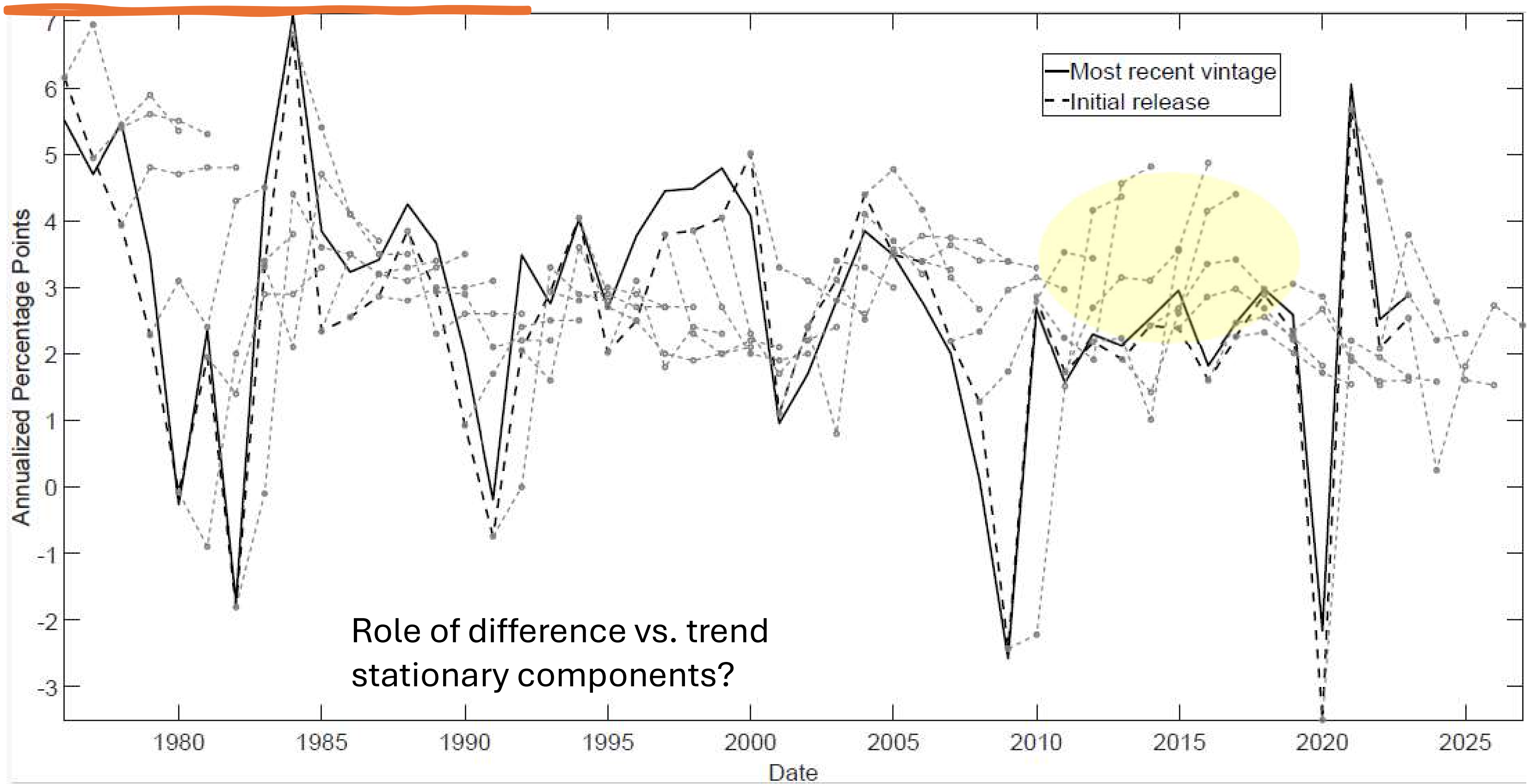
- Realistic models are hard to solve with parameter learning!
- Most earlier work on parameter learning used relatively simple models
  - In these models, Bayesian learning is fast
  - Can't explain persistent anomalies (i.e., over several decades)
- Informal discussion of parameter breaks that might sustain learning
- Not clear whether Bayesian (i.e. rational) learning can quantitatively explain forecasting anomalies over several decades

Unobserved Components Models: Interest Rates



"Shifting end-points" model for term structure (Kozicki-Tinsley 01)

GDP Growth: Trend vs. Difference Stationary



Slow Learning in More Complex Models

- Bayesian learning can be very slow in more realistic settings
  - e.g., Collin-Dufresne-Johannes-Lochstoer (2016)
- Such models can be very hard to learn!

- Different parameters can yield:
  - Similar fit to high frequency behavior
  - But very different implications for low frequency behavior
- "Anomalies" can potentially persist for decades

Interest Rates: Unobserved Components

3-month treasury bill yield: y_t

$$y_t = \mu_t + x_t$$

$$\mu_t = \mu_{t-1} + \sqrt{\gamma}\sigma\eta_t, \qquad \eta_t \sim N(0,1)$$

$$x_t = \rho x_{t-1} + \sqrt{(1-\gamma)}\sigma\omega_t, \qquad \omega_t \sim N(0,1)$$

Parameters:

- γ: variance share of permanent component
- ρ: persistence of transitory component
- σ: conditional volatility of short yield

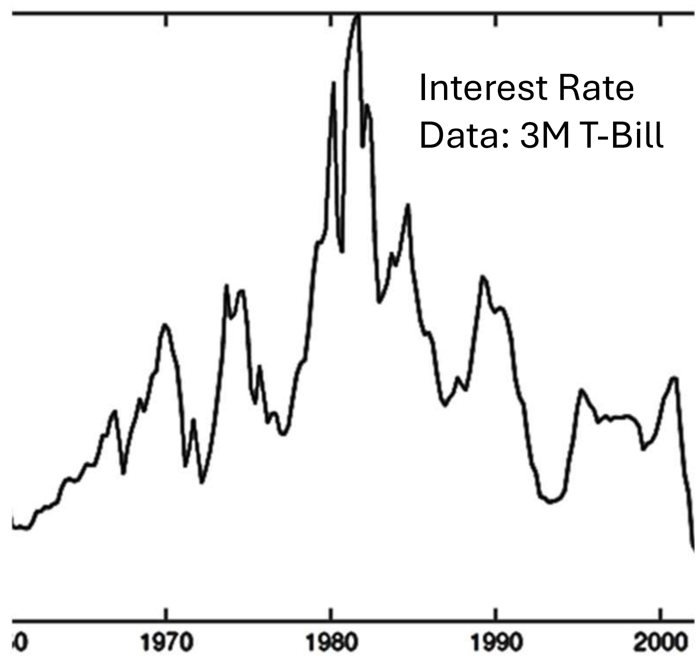
Farmer, Nakamura & Steinsson (2024)

Rational Bayesian Forecasters

- Endow them with unobserved components model

  \+ initial beliefs about parameters
- Feed in observations of short-term interest rate
- Have them learn about model parameters in real time
- Have them generate real-time forecasts
  - Start with initial beliefs in 1951Q2
  - Abstract from ZLB
  - Constant risk/term premium

- Assess whether resulting forecasts are "anomalous"

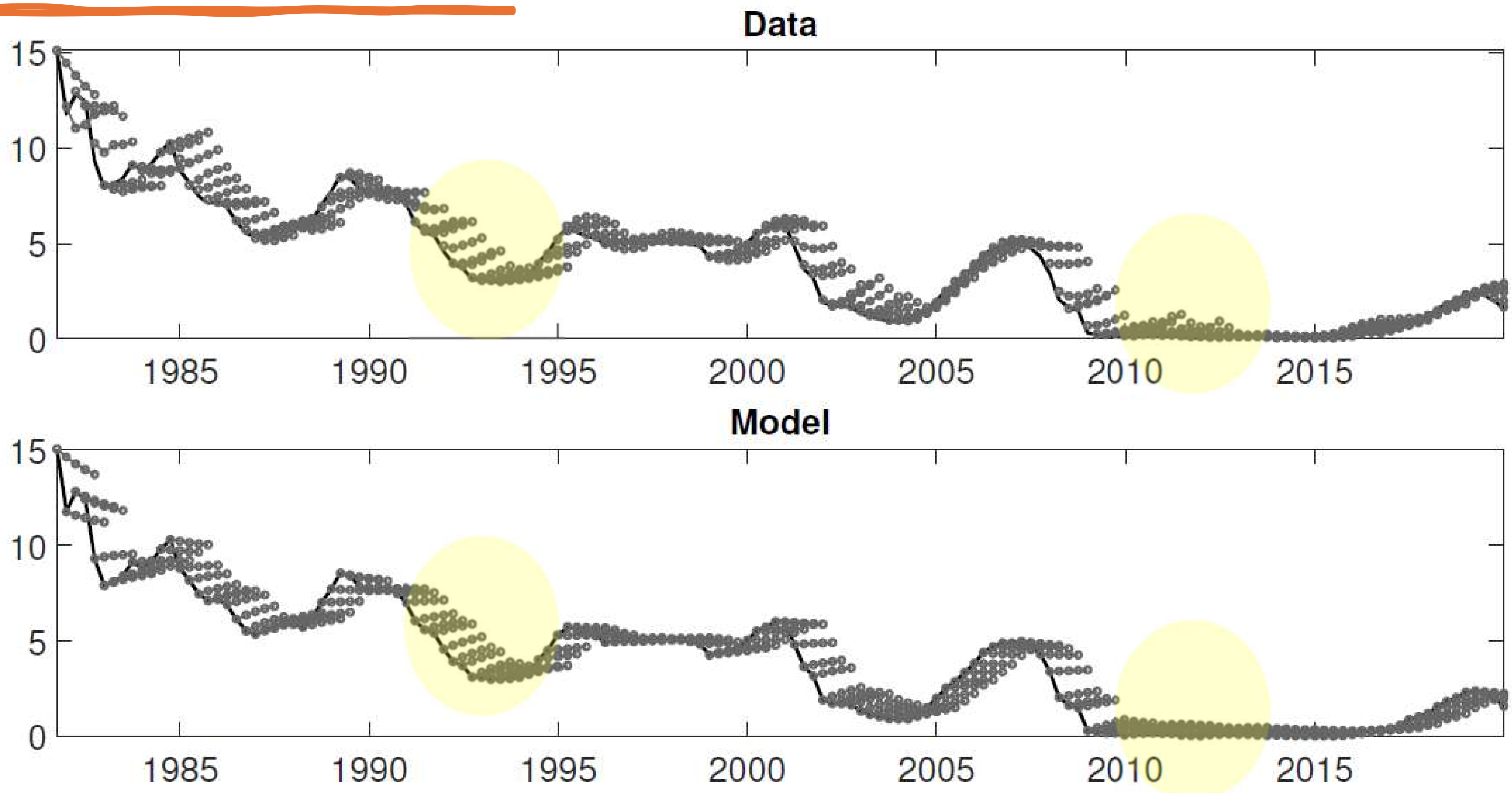


Initial Beliefs Will Matter

- Since learning will be slow, the initial beliefs of forecasters will matter
- Unlike in Rational Expectations we are adding another “free parameter”: Initial Beliefs
  - Search over the space of “hyperparameters” over initial beliefs for what fits anomaly regressions best (can also target data on beliefs)

- Important question:
  - Can we match anomalies while assuming “reasonable” initial beliefs for forecasters?

Can Match the Anomalies!



Anomalies in Model

1. Forecast Error Bias
2. Forecast Error Autocorrelation
3. Mincer-Zarnowitz Regression
4. Coibion-Gorodnichenko Regression

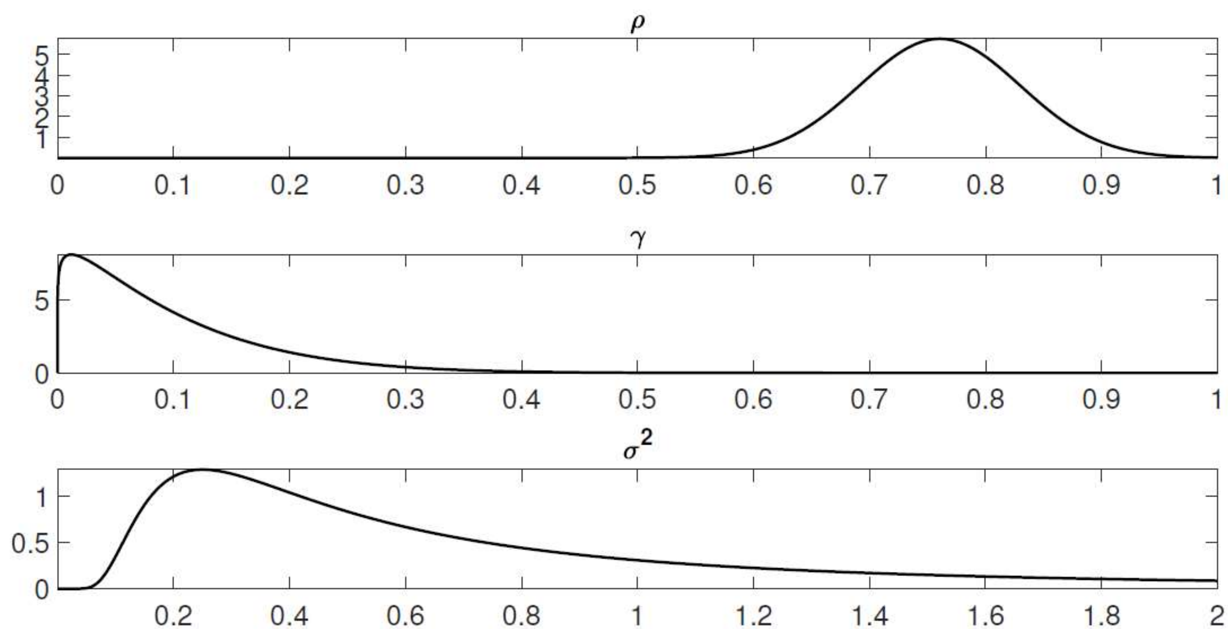
Also:

- Fama-Bliss / Campbell-Shiller regression results

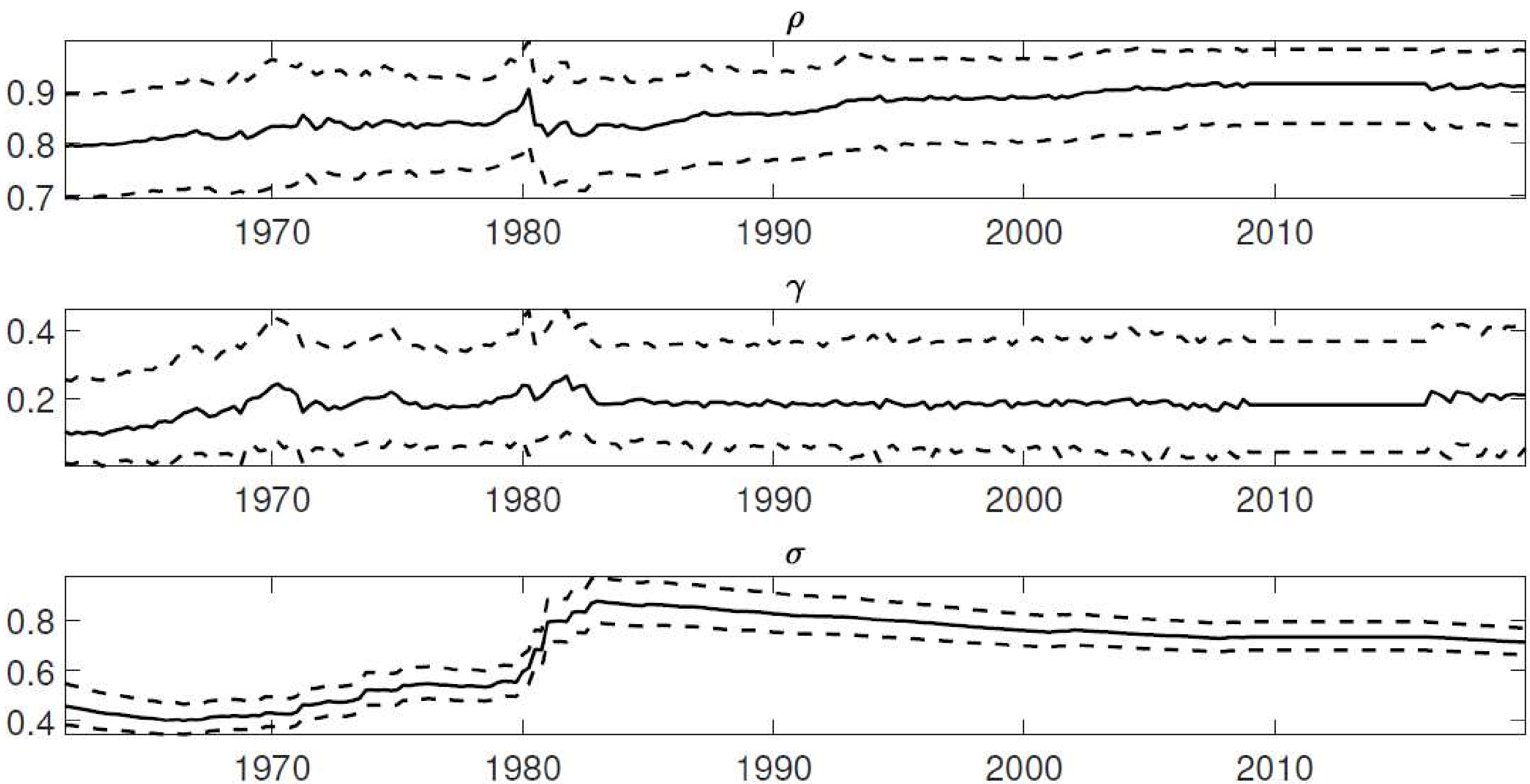
Model Simulations:
Forecasting Changes in Long Rates

	$\hat{\beta},\ H_0 : \beta = 1$						
	Long Horizon n						
	2	3	4	8	12	20	40
Data	-1.02***	-0.91***	-1.03***	-1.29***	-1.61***	-2.04***	-2.75***
	(0.45)	(0.59)	(0.62)	(0.59)	(0.57)	(0.55)	(0.87)
UC Model	-1.21***	-1.25***	-1.28***	-1.40***	-1.54***	-1.84***	-2.55**
	(0.63)	(0.64)	(0.65)	(0.70)	(0.76)	(0.88)	(1.52)

Reasonable Initial Beliefs?



Trending Parameter Estimates



Fama (2006)

There was little prior experience with a fiduciary currency when the right to exchange currency for gold was discontinued in 1971...It turns out, however, that the Federal Reserve...won...a long-odds game; they learned how to manage a fiduciary currency to bring about low inflation and interest rates. The result is a sequence of mostly negative permanent shocks to the spot rate.

UK Consol Rate (1727-2016)

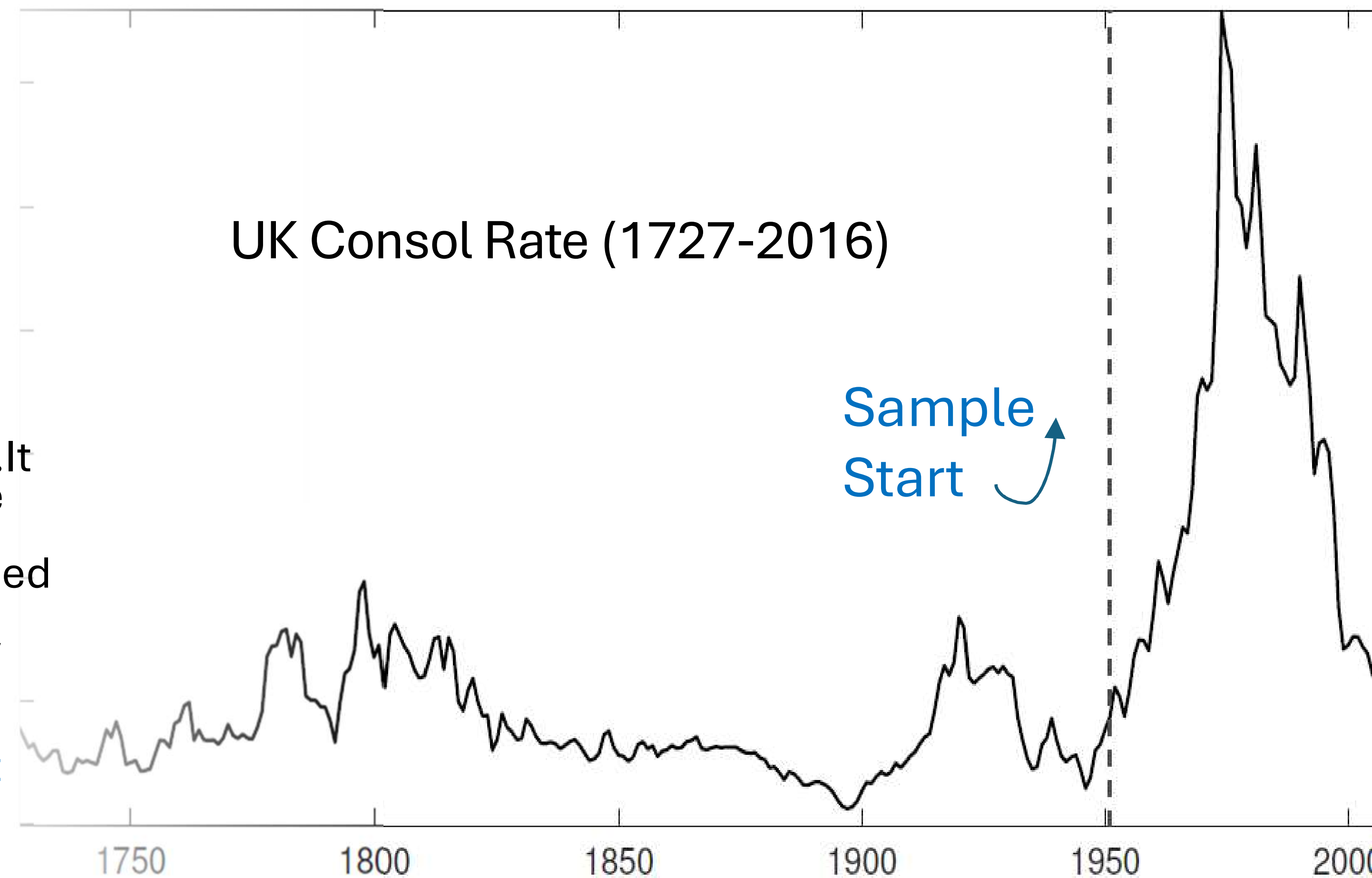
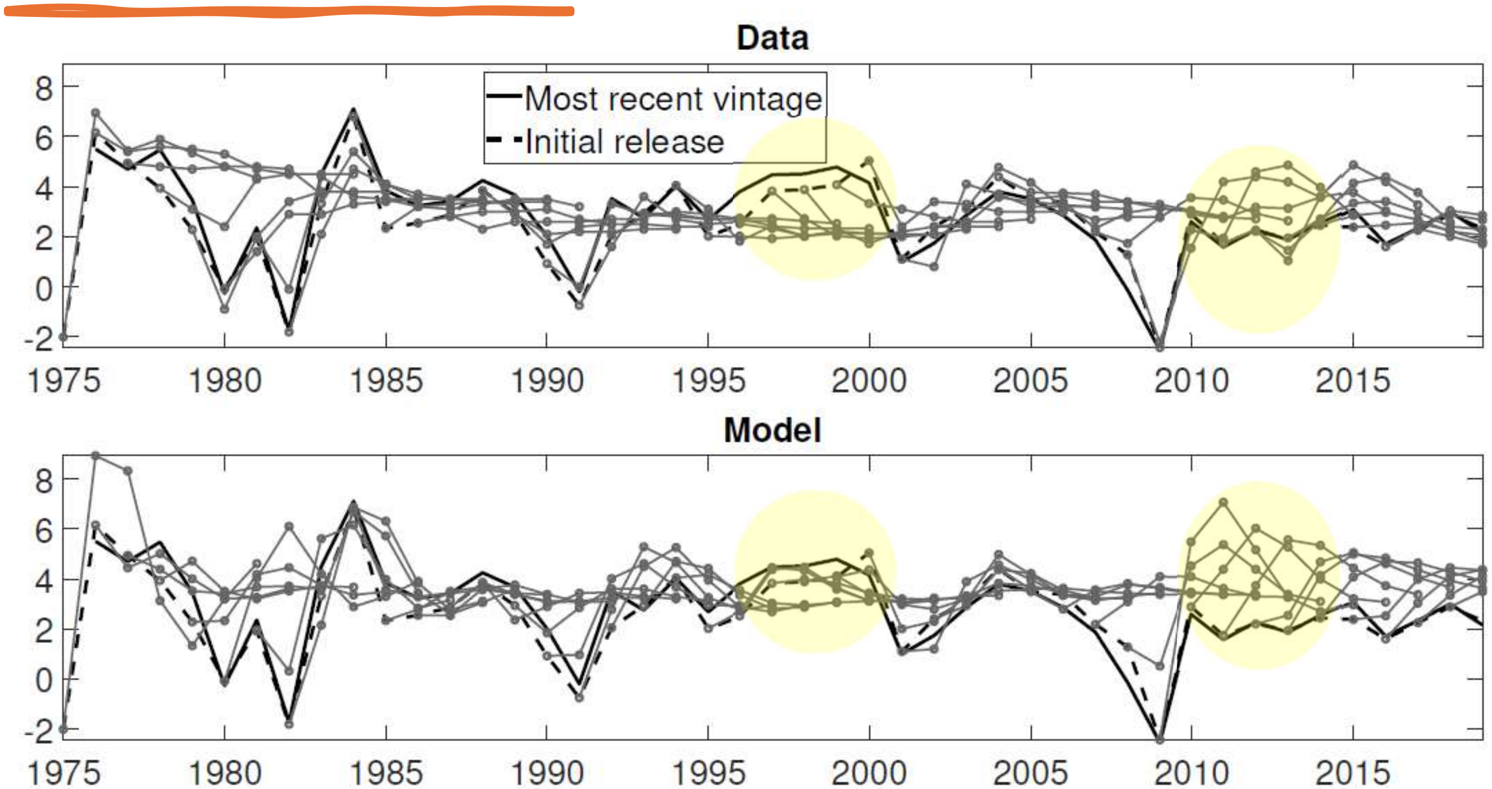


Figure 5: UK Console Rate

Growth Expectations: Data vs. Model



Why Does it Work? Monte Carlo

- Key ingredients:
  - Informative (but not dogmatic!) priors
  - Slow learning due to unobserved *persistent* component
  - Hard to distinguish random walk and highly persistent component!
- Monte Carlo
  - Simplified interest rate model

$$y_t = \mu_t + x_t$$

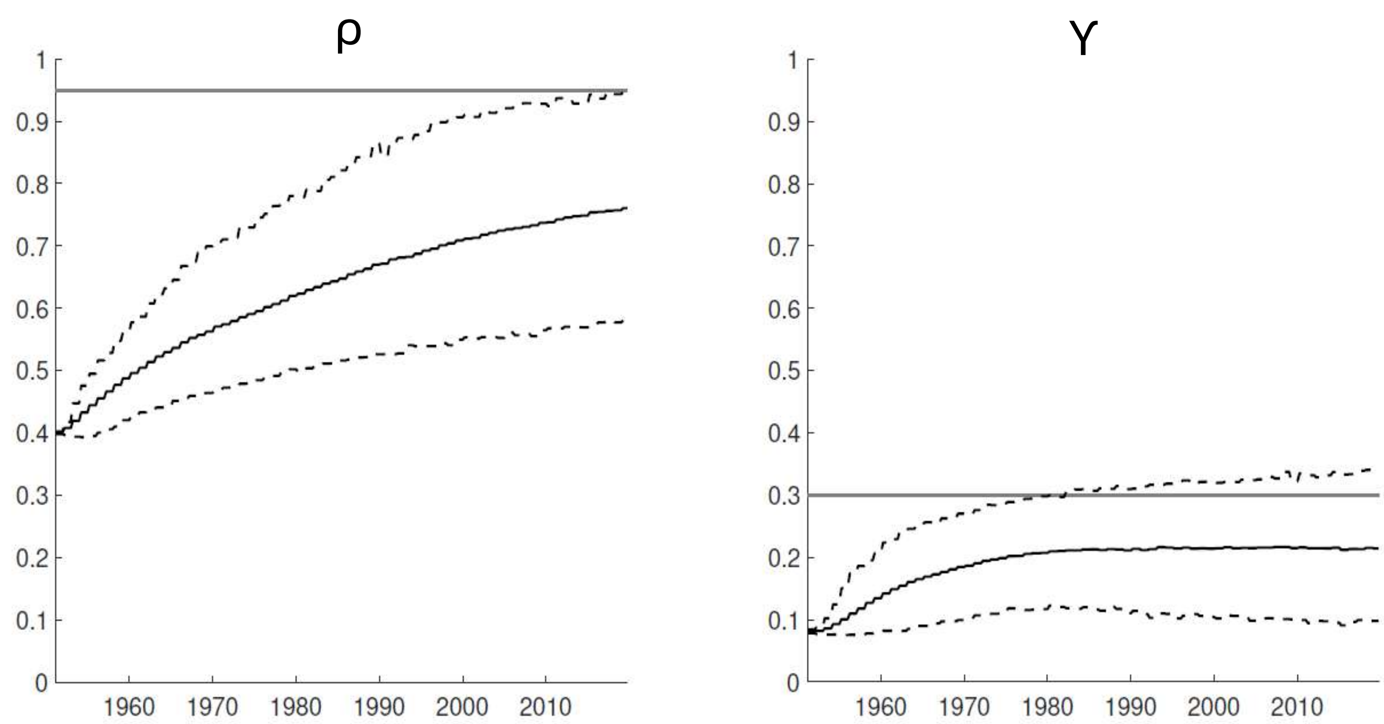
$$\mu_t = \mu_{t-1} + \sqrt{\gamma}\sigma\eta_t, \qquad \eta_t \sim N(0,1)$$

$$x_t = \rho x_{t-1} + \sqrt{(1-\gamma)}\sigma\omega_t, \qquad \omega_t \sim N(0,1)$$

When beliefs about ρ (persistence of x_t) are too low then expectation errors are positively autocorrelated!

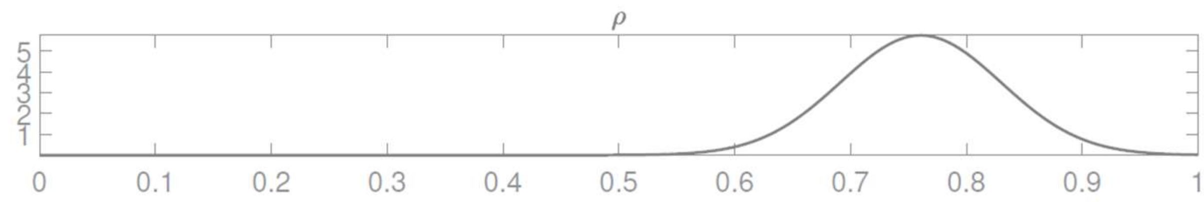
Very slow Learning!

- Unobserved components model
- Hard to distinguish random walk and persistent stationary componenents



Worry #1:
Are informative priors “rational”?

- I assumed “informative priors”

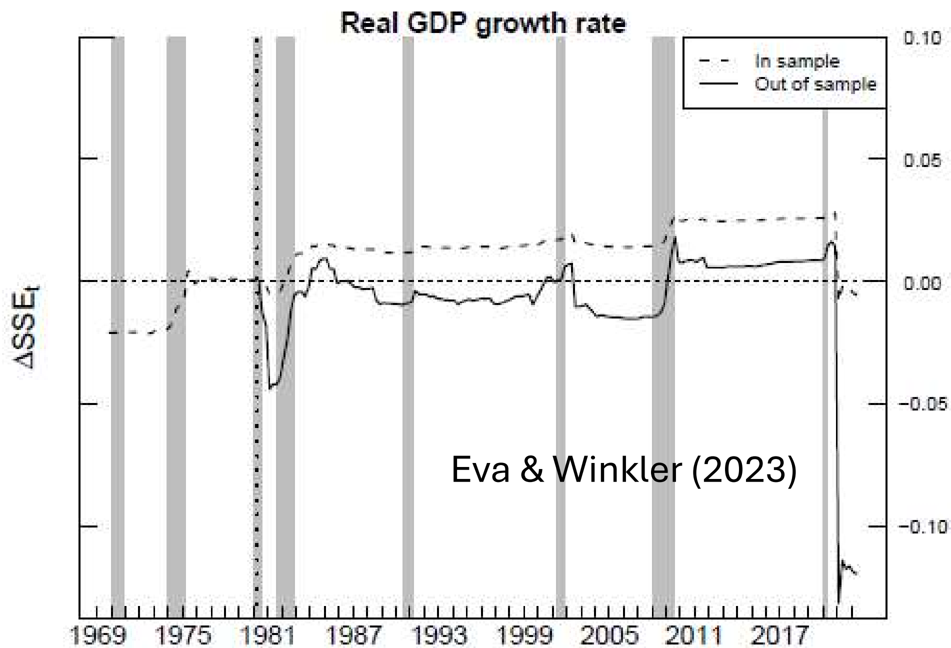


  - Alternative: flat (uniform) priors within finite range
  - Should a rational agent have flat priors?
- But…
  - From a forecasting perspective, flat priors are often *worse*
  - E.g. “Shrinkage” in James Stein estimator
  - Having a “view” can sometimes improve prediction even if your view is wrong

Worry #2:
Why Can’t Rational Agent “Use” Rational Expectations Tests to Forecast?

- Failures of rational expectations test suggest a “rational agent” could use predictable relationships to improve forecasting out of sample
- Is this really true?
- Often the answer is no!!

Using Anomaly Regressions to Forecast Often Hurts



- Eva and Winkler (2023)
  - Study many failures of ratio
  - Relationships often too uns en if they are really “there”
- Bianchi, Ludvigson & Ma (2022)
  - Coefficients on e.g. forecas is “shrunk to zero” in “optim
  - Including these variables *worsens* out-of-sample forecasting performance
  - Yet, excluding these variables leads to failures of rational expectations tests!

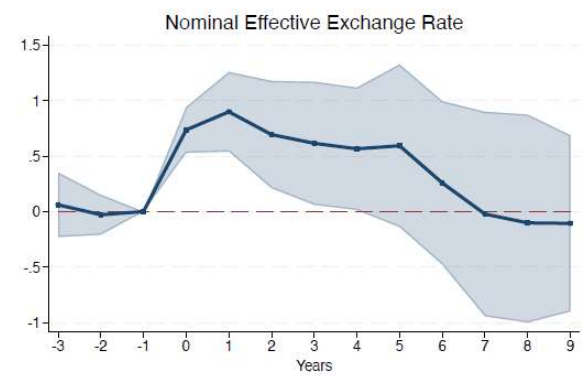
Intuition: Optimal forecasts

- In finite samples, optimal forecasts will often ignore complexities that are components of "true" model
- Such forecasts have biases
  - Could lead to failures of RE tests
  - Tradeoff between overfitting spurious relationships in finite samples and exploiting all relationships available

- Forecasting lore:
  - Simple random walk models often outperform unrestricted VAR's

Example:
“Are” Exchange Rates a Random Walk?

- Meese & Rogoff (1983) show random walk provides the best pseudo-out-of-sample forecasts for exchange rates
- Does that mean exchange rates *are* a random walk?
  - No!
  - Predictable deviations from random walks
  - But deviations often too unstable to improve forecasting



Machine Learning methods (Susan Athey)

Goal: estimate $\mu(x) = E[Y|X = x]$ and minimize MSE in a new dataset where only X is observed

- MSE: $\frac{1}{i}\sum_i \left(Y_i - \hat{\mu}(X_i)\right)^2$
- No matter how complex the model, the output, the prediction, is a single number
- Can hold out a test set and evaluate the performance of a model
- Ground truth is observed in a test set
- Only assumptions required: independent observations, and joint distribution of (Y,X) same in test set as in training set

Note: minimizing MSE entails bias-variance tradeoff, and always accept some bias

- Idea: if estimator too sensitive to current dataset, then procedure will be variable across datasets
- Models are very rich, and overfitting is a real concern, so approaches to control overfit necessary

Idea of ML algorithms

- Consider a family of models
- Use the data to select among the models or choose tuning parameters
- Common approach: cross-validation
  - Break data into 10 folds
  - Estimate on 9/10 of data, estimate MSE on last tenth, for each of a grid of tuning parameters
  - Choose the parameters that minimize MSE

ML works well because you can accurately evaluate performance without add'l assumptions

- Your robotic research assistant then tests many models to see what performs best

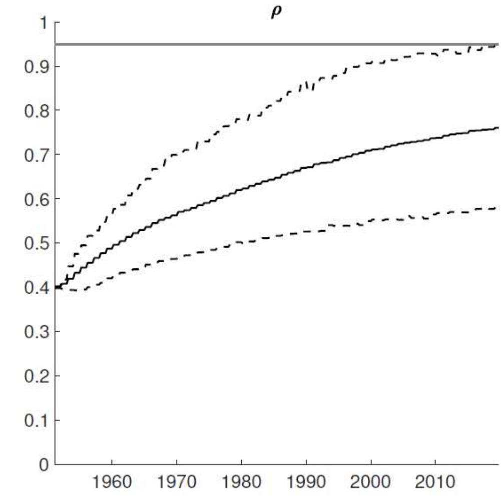
Machine Learning and Regularization

- Machine learning is all about forecasting
- Goal: maximize pseudo out of sample forecasting performance
- Central idea:
  - Machine learner will ignore lots of things that are part of the true model
  - Generates “regularization bias”

Bianchi, Ludvigson & Ma (2022):
“Machine learning null” more rational benchmark than rational expectations

Worry #3: What happens in the Very Long-Run?

- Won't we learn everything in the long-run even in these models?
- Yes, but...
  - Models are fundamentally limited in their scope



In the Very Long-Run we may be living on Mars

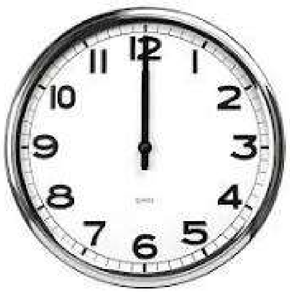
- That is not in the model!
  - Even “complicated” models abstract from many forms of structural change
- “True model” would allow scope of structural change to grow with sample size
- In this setting, we might “never” know the long-run parameters with certainty
  - Least-squares learning “gain” parameter often motivated (heuristically) by structural breaks

Worry #4: Can learning rationalize any beliefs?

- No!
- High frequency errors cannot be justified by rational learning
- If lunch is at 12:00 every day and you always show up at 12:05, this cannot be explained by rational learning

Can explain errors when "effective sample size" is small

- e.g., infrequent regime change



Rise of Learning Models

Some Recent Examples:

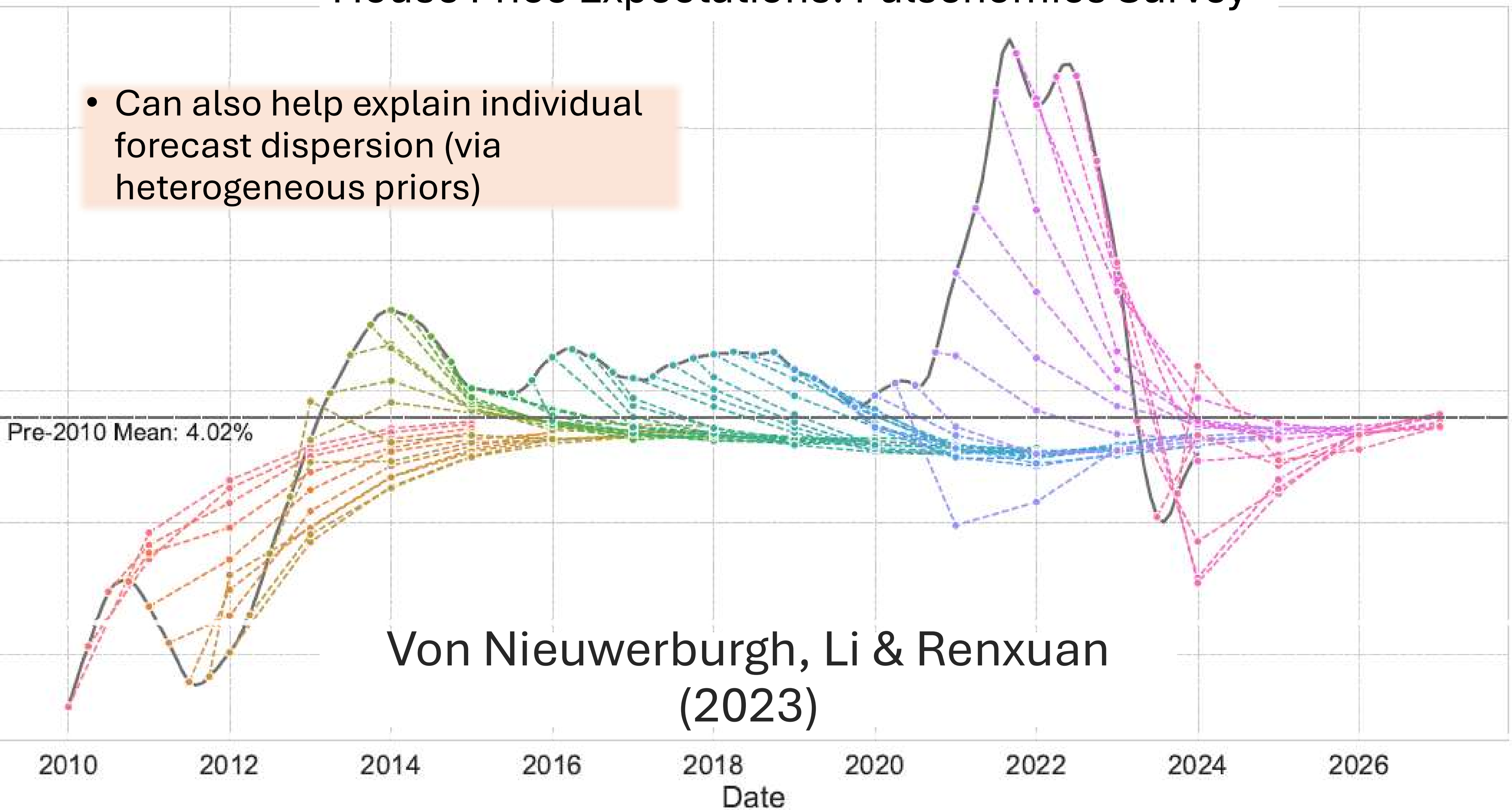
- Beliefs about long-run house price valuations (Van Nieuwerburgh, Li & Renxuan, 2024)
- Valuation of software firms (Gomez-Cram, 2024)
- Out-of-sample return forecastability (Nagel & Xu, 2023)
- Expectations about growth (Cogley & Sargent,2008; Kozlowski, Veldkamp & Venkateswaran, 2020)

<u>New Ammunition:</u>

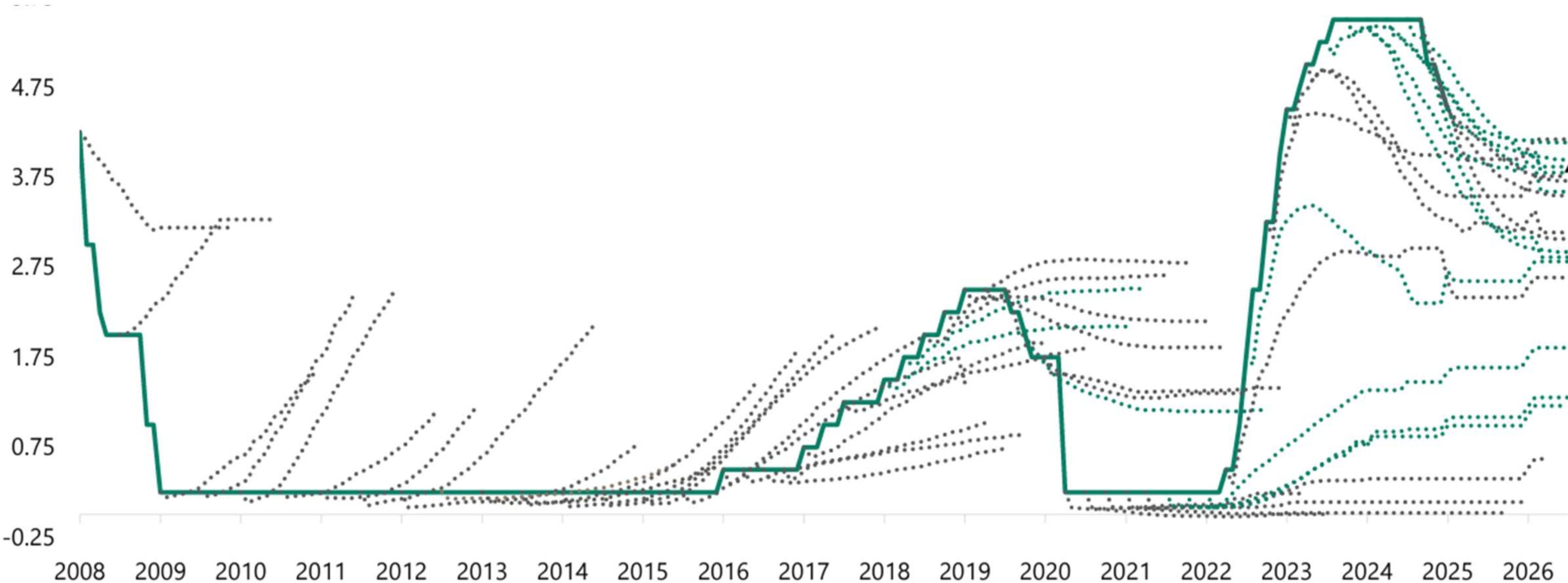
- Growing availability/comfort with survey data
- Bigger, better computers/ estimation algorithms

House Price Expectations: Pulsenomics Survey

- Can also help explain individual forecast dispersion (via heterogeneous priors)



Von Nieuwerburgh, Li & Renxuan (2023)

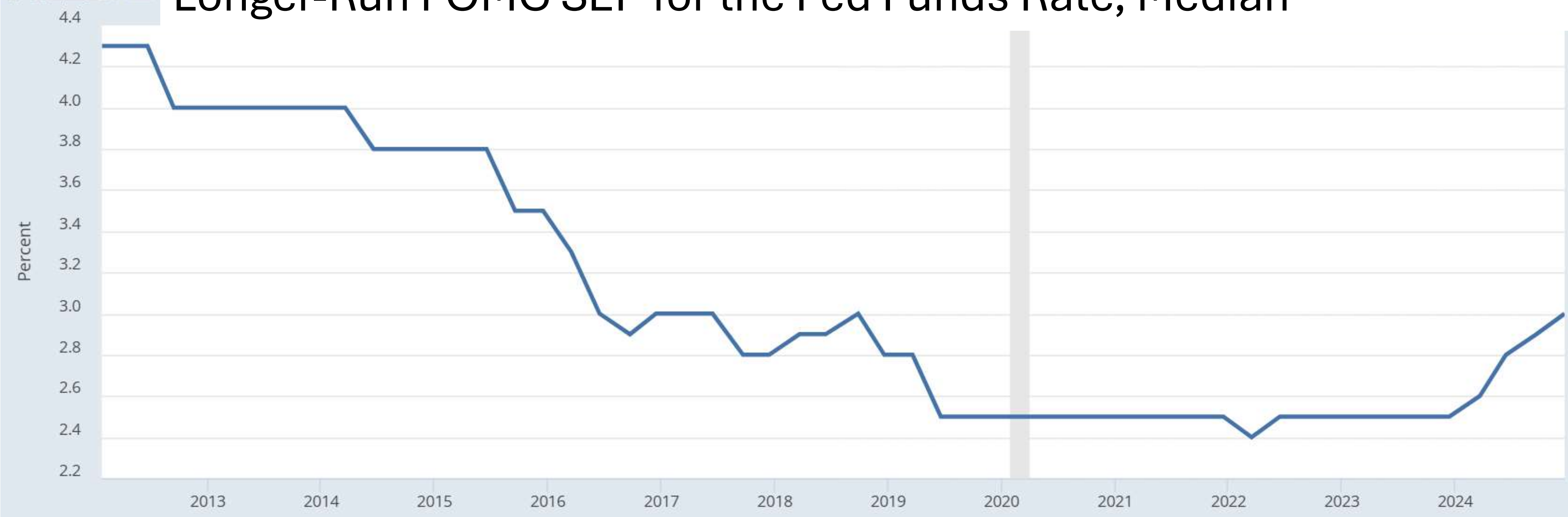


Hair Plot since 2008

- Fed Funds rate and financial market expectations (from futures)
- Sources: Federal Reserve Board, Bloomberg, Apollo Chief Economist

Fed Funds Rate since 2015

FRED Longer-Run FOMC SEP for the Fed Funds Rate, Median



Learning about the Long Run

- Why do model parameters seems persistently uncertain (even if you are trying to be rational)?
- Many macro/financial variables have low frequency dynamics
  - E.g., Interest rates, macro growth
  - World is hard to learn, even for a Bayesian!
- Learning models constitute more rational “null” than traditional rational expectations