Matlab Code for Simple Menu Cost Models

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September 27, 2007

This short note contains documentation for a package of matlab files that solves for an equilibrium of three simple versions of menu cost models of the type we study in Nakamura and Steinsson (2006a, 2006b). The package is divided into three folders. One of these folders contains files that solve a partial equilibrium model of the type studied in Nakamura and Steinsson (2006a) as well as an analogous Calvo model. The second and third folders contain files that solve a one sector version and a multi-sector version, respectively, of the general equilibrium menu cost model we analyze in Nakamura and Steinsson (2006b) as well as one sector and multi-sector versions of the CalvoPlus model analyzed in this same paper. The package uses one file in the Matlab Statistics Toolbox (normcdf.m). In order to run, the statistics toolbox must therefore be installed.

This package is provided for academic use. Users of this package (or modified version of it) should cite Nakamura and Steinsson (2006a, 2006b).

1 Partial Equilibrium Model

The "Partial Equilibrium" folder contains nine files. Two of these files are the main scripts (PartialEqBasic, PartialEqCalvo), while the other files are functions called by these scripts or by other functions. PartialEqBasic implements the solution of the partial equilibrium menu cost model described in Nakamura and Steinsson (2006a). PartialEqCalvo implements the solution of an analogous partial equilibrium Calvo model. These files solve for an equilibrium and then simulate the economy and report a number of statistics such as the frequency and size of price changes and the hazard function of price changes. PartialEqBasic should take only a few seconds to run on a standard PC desktop.

In PartialEqBasic, we solve for the optimal pricing policy of a firm that faces menu costs and takes the evolution of all aggregate variables as given. The firm's problem is dynamic because the menu cost leads to infrequent price adjustment. The solution method we use is value function

iteration on a discrete grid.

The basic structure of PartialEqBasic is that after defining parameters, the grids for the two state variables and transition matrices for the exogenous variables, the program calls the function VFIterationBasic. This function performs the value function iteration step. It uses the Bellman equation for the firm's problem to iterate on the expected value of the value function $E_t V(p_t(z)/P_{t+1}, A_{t+1}(z))$ until it converges. It then calculates the value function and the policy function (rV, and F, respectively) and returns these. PartialEqBasic then calls DataSimBasic, which simulates NSim periods of data from the model. The rest of the program uses this data to calculate various statistics such as the frequency of price change and the Kaplan-Meier hazard function of price change.

The structure of the PartialEqCalvo program is analogous.

2 One Sector General Equilibrium Model

The "One Sector GE" folder contains 23 files. Three of these files are scripts to be run by the user (ge1Basic.m, ge1CalvoPlus.m and SimboG1S.m). The other files are functions called by these scripts or by other functions. ge1Basic and ge1CalvoPlus solve for an equilibrium in a one sector general equilibrium menu cost model and CalvoPlus model, respectively, and store all the relevant information (parameter values, transition matrices, policy function, etc.) in a structure called "GE1Model". SimboG1S reads GE1Model, simulates NSim periods of data from the model and calculated the variance of output from this simulation. ge1Basic should take between 2 and 15 minutes to run on a standard PC desktop depending on the size of the state space and the quality of the initial guess for Γ.

In ge1Basic, we solve not only the firm's optimal pricing problem given the evolution of the aggregates but also for the equilibrium evolution of the aggregate variables given firm behavior. We are therefore looking for a fixed point that simultaneously solves two problems: 1) Firms are optimizing given the evolution of the aggregate variables, 2) The evolution of the aggregate variables is given by the appropriate aggregation of the actions of all the firm's in the economy.

The state space of the firm's problem is infinite dimensional since the evolution of the price level and other aggregate variables depend on the entire joint distribution of all firms' prices and productivity levels. Following Krusell and Smith (1998), we make the problem tractable by assuming that the firms perceive the evolution of the price level as being a function of a small number of moments of this distribution. Specifically, we assume that firms perceive that

$$\frac{P_t}{P_{t-1}} = \Gamma\left(\frac{S_t}{P_{t-1}}\right). \tag{1}$$

We use the following iterative procedure to solve for the equilibrium: 1) We specify a finite grid of points for the state variables, $A_t(z)$, $p_{t-1}(z)/P_t$ and S_t/P_t. 2) We propose a function $\Gamma(S_t/P_{t-1})$ on the grid. 3) Given the proposed Γ, we solve for the firm's policy function F by value function iteration on the grid. 4) We check whether Γ and F are consistent. If so, we stop and use Γ and F to calculate other features of the equilibrium. If not, we update Γ and go back to step 3.

The script ge1basic.m first defines parameters, the grids for the three state variables, transition matrices for the exogenous variables and an initial guess for the function Γ.[1] The initial guess for the function Γ (G in the program) is a discretization of a linear function. The user chooses the slope and the level of this function. ge1basic.m then passes all this information to a function EqSolverGE1Basic. This function calculates the equilibrium and returns the equilibrium values for: Γ (G), the value function (rV), the policy function (F) and the stationary distribution (Q). ge1Basic then calls the function RigidityStats, which returns a number of summary statistics for the equilibrium such as the frequency of price change. Finally, ge1Basic saves both the "inputs" and the "outputs" in a structure named GE1Model.

The function EqSolverGE1Basic consists of a while-loop that iterates on G until is converges. Each iteration involves three steps. First, VFIterationGEBasic solves the firm's problem for the current G and returns the value function (rV) and the policy function (F). Next, StaDistGEBasic calculates the stationary distribution (Q) over the state space using the current G and F. Finally, the function UpdateG uses Q, F and G to calculate a new G. If the new G is sufficiently similar to the old G, the algorithm exits the while-loop. Otherwise it does another iteration.

The structure of ge1CalvoPlus, which implements a one sector version of the CalvoPlus model described in Nakamura and Steinsson (2006b), is analogous to the structure of ge1Basic.

3 Multi-Sector General Equilibrium Model

The "Multisector GE"folder also contains 23 files, three of which are scripts to be run by the user (geNBasic.m, geNCalvoPlus.m and VarFromQ.m). The structure of genNBasic and geNCalvoPlus is analogous to the structure of ge1Basic and ge1CalvoPlus. VarFromQ inputs "GENModel" and uses the stationary distribution Q to calculate the variance of output.

[1] One of the parameters that is user specified is called "radshift". This parameter determines the level of the grid for real aggregate output. If radshift = 0, the middle of the grid for real aggregate output is equal to the steady state value of real aggregate output in a model with flexible prices and no uncertainty. This parameter will need to be adjusted by the user when the size of the menu cost and the variance of the idiosyncratic productivity shock are adjusted since these parameters affect the average level of output.

References

Krusell, P., and A. Smith (1998): "Income and Wealth Heterogeneity in the Macroeconomy," *Journal of Political Economics*, 106(5), 867–896.

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