

What Do We Learn From Cross-Sectional Empirical Estimates in Macroeconomics?

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Abstract

Recent empirical work uses variation across cities or regions to identify the effects of economic shocks of interest to macroeconomists. The interpretation of such estimates is complicated by the fact that they reflect both partial equilibrium and local general equilibrium effects of the shocks. We present a simple method for recovering estimates of partial equilibrium effects from these cross-sectional empirical estimates. We apply our method to recent estimates of housing wealth effects based on city-level variation. For this case, we derive conditions under which the partial equilibrium effect of changes in house prices on consumption are equal to the city-level estimate divided by an estimate of the local fiscal multiplier.

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1 Introduction

A growing literature uses variation across cities or regions to identify the effects of economic shocks of interest to macroeconomists. What exactly these estimates identify is often complicated by the fact that city- or region-level outcomes reflect both the partial equilibrium effect of the shock and the local general equilibrium response to that shock. In this paper we propose an approach by which applied researchers can isolate the partial equilibrium effect of the shock. The partial equilibrium effect has a clear theoretical interpretation and can be easily calculated in standard one-region macro models for calibration purposes. Our method therefore allows researchers to avoid formulating and solving a multi-region macro model to be able to compare their empirical results to analogous concepts in a model.

We apply this method to the analysis of the housing wealth effect. The US housing boom and bust in the early 2000s focused the attention of economists on the implications of home prices changes for consumer spending. Prominent recent papers that estimate this housing wealth effect use regional data to relate outcomes such as spending, car registrations, and employment to changes in home prices (e.g. Mian et al., 2013; Mian and Sufi, 2014). The appropriate interpretation of these empirical results is not straightforward. House prices are endogenous at the level of a city and a shock that changes home prices surely alters consumption through other channels such as through the equilibrium response of wages and interest rates. In particular, suppose an increase in home prices triggers some additional spending in a city through a housing wealth effect. The extra spending will raise incomes locally, which will lead to more spending setting in motion a consumption multiplier. For these reasons, it is not immediately clear what we can learn from the change in city-level consumption in response to a change in house prices in the city.

We show how existing empirical estimates of the housing wealth effect on consumption can be mapped into the partial-equilibrium effect of house prices on consumption. We start from the point of view that some prices are determined nationally and some are determined locally. For example, financial markets are highly integrated at a national level while labor markets are quite local in nature. We lay out the conditions under which time fixed effects included in empirical regression specifications absorb the variation in national prices. Turning to the local general equilibrium effects, we show how the local fiscal multiplier can be used to gauge the effect of local general equilibrium multipliers on the response of consumption to changes in house prices. Under certain assumptions that we discuss in more detail below, we show that one can remove local gen-

eral equilibrium effects from a city-level estimate of the housing wealth effect simply by dividing that estimate by an estimate of the local fiscal multiplier. The end result is an estimate of the partial equilibrium housing wealth effect and marginal propensity to consume out of housing wealth (MPCH) that corresponds to a change in home prices holding fixed wages other non-housing prices.

In recent complementary work, we estimate the housing wealth effect based on city-level variation in house prices and using retail employment as a proxy for local consumption (Guren et al., 2018). We estimate an elasticity of retail employment with respect to house prices of 0.053, which implies an MPCH of 2.4 cents on the dollar. Nakamura and Steinsson (2014) estimate the local fiscal multiplier to be approximately 1.5. Dividing our housing wealth effect estimate by the Nakamura and Steinsson’s local fiscal multiplier estimate yields a partial equilibrium housing wealth effect estimate of $0.053/1.5 = 0.035$, which implies a partial equilibrium MPCH of 1.6 cents on the dollar.

The simple approach of dividing by the local fiscal multiplier makes sense if a change in house prices only directly stimulates the local economy through a housing wealth effect on consumption, which is then amplified by a consumption multiplier. An increase in house prices may however also stimulate the local economy by increasing construction activity. This is a separate channel from the standard consumption multiplier because the trigger is not consumption but residential investment. To assess the importance of this channel, we use the empirical research design that we developed in Guren et al. (2018) to estimate the response of construction employment to changes in house prices. We also show theoretically how to use such an estimate to strip out the effect of construction and get back to a partial equilibrium housing wealth effect on consumption. When we do this, we arrive at a partial-equilibrium housing wealth effect of 0.029 or an MPCH of 1.3 cents on the dollar.

We introduce our methods in the context of a static economy, but they extend to dynamic economies as well as we show in the context of our application. In a dynamic context, there is no single fiscal multiplier but an impulse response of output to a fiscal shock. For example, for the simple case the partial equilibrium effect of home prices on consumption is $F^{-1}E$ where E is a matrix where the (i, j) element gives the housing wealth effect of a price change at date j on consumption at date i and F is a matrix that gives the effect of a fiscal shock at date j on output at date i .

The paper is organized as follows. Section 2 lays out the general method. Section 3 applies the method to the housing wealth effect. Section 4 provides a fully structural, multi-region macro

model of the housing wealth, which we use to assess and motivate the identification assumptions used in Section 3. Section 5 shows how the method can be applied within the context of the full dynamic model. Section 6 conducts a Monte Carlo analysis of the fully structural model to evaluate the accuracy and robustness of the identification scheme used in Section 3.

2 The Method

We consider an economy composed of J regions that each produce and consume $\ell + L$ goods. The first ℓ goods are produced and consumed locally. The remaining L goods are traded in integrated markets. In total there are $J\ell + L$ goods. We normalize the price of the first good to 1. To fix ideas, the numeraire good could be final goods in the “home” region. Let p_j be the vector of local prices in region j and let P be the vector of national prices. Let x_j be a set of local shocks to region j and let X be a vector of aggregate shocks. Let the vector-valued functions $D(p_j, P, x_j, X)$ and $S(p_j, P, x_j, X)$ be the demand and supply curves for local goods in j . In the local equilibrium of region j we have $D(p_j, P, x_j, X) = S(p_j, P, x_j, X)$. If we assume linearity we can write

$$(D_p - S_p)p_j + (D_x - S_x)x_j + (D_P - S_P)P + (D_X - S_X)X = 0,$$

where D_p is the partial derivative of D with respect to p and so on. In this formulation the outcomes of the regions differ only because they experience different idiosyncratic shocks. In Section 4, we allow the economies to also differ in their housing supply curves.

We assume that the aggregate prices are insensitive to the idiosyncratic shocks. Using the implicit function theorem, the comparative statics of the effect of x_j on p_j are given by

$$\frac{dp_j}{dx_j} = -[D_p - S_p]^{-1}(D_x - S_x). \quad (1)$$

Let y_j be the equilibrium quantity of local goods produced in j so that

$$y_j = D(p_j, P, x_j, X) = S(p_j, P, x_j, X).$$

Differentiating, we have

$$\frac{dy_j}{dx_j} = D_p \frac{dp_j}{dx_j} + D_x. \quad (2)$$

Suppose we observe some elements of $\frac{dy_j}{dx_j}$ and some elements of $\frac{dp_j}{dx_j}$, and we wish to identify some of the elements of D_p , S_p , D_x , and/or S_x . Our approach is to impose some structural assumptions on these unknown matrices and then manipulate (1) and (2) to solve for the unknowns in terms of the observed responses.

Notice that the aggregate shocks and prices play no role in the analysis. Neither do the matrices D_P , S_P , D_X , or S_X . These aggregate disturbances shift quantities and prices uniformly across regions and are absorbed by regression constants or time fixed effects. This outcome relies on two assumptions: linearity and equal incidence of the aggregate shocks and prices (i.e. D_P , S_P , D_X , and S_X are common across regions).

The classic approach to simultaneous equation models uses instruments that each appear in only one of the equations. An instrument in this sense implies a particular structure for D_x and or S_x .

3 Application to the Housing Wealth Effect

We now present two examples to illustrate the method presented in section 2. We start with a simple example that involves strong assumptions to facilitate the exposition before turning to a richer example.

3.1 Simple Example

We start by considering a simple static model with three markets: a goods market, a housing market, and a labor market. As we discuss above, aggregate shocks and national prices do not affect the analysis. We, therefore, focus on the behavior of a single region taking aggregate prices as given.

The economy produces goods using labor. The production function is $Y = N$, where Y is goods produced and N is labor supply. There is a labor supply function $N(w, p)$ that depends on the wage, w , and potentially also the price of housing, p . Both prices are expressed in terms of goods. Household demand for goods is given by $C(w, p)$. In addition to this private consumption demand, goods are used for public consumption in amount g , where g is exogenous. The aggregate resource constraint is $Y = C(w, p) + g$. For housing, we specify an excess demand function $H(w, p, s)$, where s is a shock. Since we only use data on the price of housing and not on the quantity of housing that is traded, we do not need to specify housing supply and demand separately.

In this simple model there are no interest rates or taxes. We assume these variables are determined at the national level and do not differentially affect the regions so their consequences are absorbed by time fixed effects. We discuss these national prices in more detail in the context of the dynamic model presented in Section 4.

The equilibrium level of wages and house prices in this model is given by the solution to the following two equations:

$$\begin{aligned} C(w, p) + g &= N(w, p) \\ H(w, p, s) &= 0. \end{aligned}$$

Using the implicit function theorem as above, comparative statics for the price vector are

$$\frac{d(w, p)}{d(g, s)} = - \begin{pmatrix} C_w - N_w & C_p - N_p \\ H_w & H_p \end{pmatrix}^{-1} \begin{pmatrix} 1 & 0 \\ 0 & H_s \end{pmatrix},$$

which we can rearrange as

$$\frac{d(w, p)}{d(g, s)} = \underbrace{\left[N_w - C_w + (C_p - N_p) \frac{H_w}{H_p} \right]^{-1}}_{\equiv M} \begin{pmatrix} 1 & \frac{H_s}{H_p} (N_p - C_p) \\ -\frac{H_w}{H_p} & \frac{H_s}{H_p} (C_w - N_w) \end{pmatrix} \quad (3)$$

Using (2), we then have

$$\frac{dY}{d(g, s)} = \begin{pmatrix} C_w & C_p \end{pmatrix} \frac{d(w, p)}{d(g, s)} + \begin{pmatrix} 1 & 0 \end{pmatrix}. \quad (4)$$

Our goal is to estimate C_p , the partial equilibrium effect of home prices on consumption. Suppose we observe an instrumental variables (IV) estimate of the housing wealth effect based on regional variation. In our notation, this is $dY/dp = (dY/ds)/(dp/ds)$, where s is the shock (instrument) used to estimate dY/dp . The trouble is that our IV estimate is the total derivative of consumption with respect to house prices, not the partial derivative C_p .

We make two additional structural assumptions to simplify the model: 1) there are no (short-run) wealth effects on labor supply, $N_p = 0$; 2) house prices are independent of income, $H_w = 0$. The first of these assumptions we view as likely being a reasonable approximation to reality, while the second is less likely to be a good approximation. We will therefore relax the second of these

assumptions below. But for now, we make these two assumptions. This simplifies equation (3) to

$$\frac{d(w, p)}{d(g, s)} = \underbrace{[N_w - C_w]^{-1}}_{\equiv M} \begin{pmatrix} 1 & -\frac{H_s}{H_p} C_p \\ 0 & \frac{H_s}{H_p} (C_w - N_w) \end{pmatrix}$$

Combining this equation with equation (4) then yields

$$\begin{aligned} \frac{dY}{dg} &= 1 + MC_w \\ \frac{dp}{ds} &= M \frac{H_s}{H_p} (C_w - N_w) \\ \frac{dY}{ds} &= -MC_p \frac{H_s}{H_p} N_w, \end{aligned}$$

which imply

$$\frac{dY/ds}{dp/ds} = C_p \frac{N_w}{N_w - C_w} \tag{5}$$

$$\frac{dY}{dg} = \frac{N_w}{N_w - C_w}, \tag{6}$$

where the second line uses the definition of M . Finally, dividing the first of these two equations by the second yields

$$C_p = \frac{(dY/ds)/(dp/ds)}{dY/dg} = \frac{dY/dp}{dY/dg}. \tag{7}$$

This shows that given the two simplifying assumptions we made above, the partial equilibrium effect of house prices on consumption, C_p , is equal to the cross-region IV estimate of the housing wealth effect, dY/dp , divided by the local fiscal multiplier, dY/dg . Looking back at equations (5) and (6), we see that the the partial equilibrium effect differs from the cross-region IV estimate by a local general equilibrium multiplier ($N_w/(N_w - C_w)$) and that the local fiscal multiplier provides an estimate of this local general equilibrium multiplier. An estimate of the local fiscal multiplier can therefore be used to convert a cross-region IV estimate of the housing wealth effect into an estimate of the partial equilibrium effect of house prices on consumption. Intuitively, an increase in home prices of one unit spurs an extra C_p of spending, which then triggers local adjustments in wages with accompanying consumption effects. These same local adjustments would occur if the initial spending were due to a government spending shock.

In Guren et al. (2018) we estimate a marginal propensity to consume out of housing wealth (MPCH) of 2.4 cents on the dollar. This estimate corresponds to the total effect captured by

$(dY/ds)/(dp/ds)$. Nakamura and Steinsson (2014) estimate a local fiscal multiplier of about 1.5.¹ Equation (7) implies that the partial equilibrium MPCH is 1.6 cents on the dollar.

3.2 Richer Example

We now relax two assumptions from the previous example. We allow residential investment to respond to home price changes and we allow housing demand to respond to income (i.e. we do not restrict $H_w = 0$). In this case, the equilibrium level of wages and house prices is given by the solution to the following two equations:

$$\begin{aligned} C(w, p) + I(w, p) + g &= N(w, p) \\ H(w, p, s) &= 0. \end{aligned}$$

Relative to the previous example, we have now added demand for local goods coming from residential investment.

With this richer model, the conversion of the regional estimate of the housing wealth effect into the partial equilibrium effect of house prices on consumption will rely on us being able to observe several additional pieces of information. In particular, suppose we observe estimates of the response of house prices to government spending shocks, dp/dg , and the response of residential investment to a change in house prices, dI/dp , in addition to the estimates we assumed we had above.

As before, we assume that there is no (short-run) wealth effect on labor supply. In this case, we assume in addition that residential investment is independent of income, $I_w = 0$. (We could relax the $I_w = 0$ assumption if we observed dC/dg or dI/dg .) Under these assumptions, the partial-equilibrium effect of home prices on consumption is

$$C_p = \frac{E}{\frac{dY}{dg} (1 - EZ)} - \frac{dI}{dp} \tag{8}$$

where E is the housing wealth effect on total spending $E = \frac{dY/ds}{dp/ds} = \frac{(dC/ds)+(dI/ds)}{dp/ds}$ and $Z = \frac{dp/dg}{dY/dg}$ is the income effect on home prices. The derivation of equation 8 appears in Appendix A.

In equation (8), there are two considerations in addition to the fiscal multiplier intuition from the previous example. When house prices are affected by income, $H_w \neq 0$, a house price shock differs

¹Nakamura and Steinsson find larger multipliers in regional data than in state data. As the analysis of the housing wealth effect is undertaken at the city (CBSA) level, it may be appropriate to use a fiscal multiplier somewhat below 1.5.

from the government spending shock because part of the response to the government spending shock comes through home prices. In computing the housing wealth effect we compute the change in spending relative to the observed total change in home prices. It is not appropriate to apply the full local fiscal multiplier to the initial spending, but rather we only want to apply the part of the local fiscal multiplier that operates through wages. Second, some part of the initial spending response to home prices comes from residential investment. We subtract this component to isolate the part coming from consumption.

We can now use these results to generate a new estimate of the partial equilibrium effect of house prices on consumption by plugging empirical estimates into equation (8). For this purpose, it is to rewrite equation (8) in terms of elasticities:

$$\frac{p}{C}C_p = \frac{\frac{p}{C}E}{\frac{dY}{dg} \left(1 - \frac{C}{Y} \frac{p}{C} E \frac{Y}{p} Z\right)} - \frac{I}{C} \frac{p}{I} \frac{dI}{dp}. \quad (9)$$

We have estimated the elasticity of construction employment to home prices to be 0.309. This estimate is based on an analogous specification to the full-sample housing wealth effect estimate we present in Column 1 of Table 1 of Guren et al. (2018) with construction and real estate employment as the outcome variable. We use this as our estimate of the elasticity of residential investment to home prices. Lamont and Stein (1999) provide estimates of the short-run income elasticity of house prices, which imply it is less than 0.8 and more likely near 0.3. We use 0.3 as our estimate, but our conclusions are little changed by using 0.8. Our estimate of the housing wealth effect as an elasticity is 0.053. Between 1975 and 2017, the average ratio of residential investment to personal consumption was 0.068. We use this for I/C . Over the same period, the C/Y ratio averaged 0.649. We will again use the Nakamura and Steinsson (2014) estimate of the local fiscal multiplier of 1.5. We can rewrite $(p/C)E$ as

$$\frac{p}{C}E = \frac{p}{C} \frac{dC/ds}{dp/ds} + \frac{I}{C} \frac{p}{I} \frac{dI/ds}{dp/ds} = 0.053 + 0.068 \times 0.309 = 0.074.$$

Plugging these numbers into equation (9) we get

$$\frac{p}{C}C_p = \frac{0.074}{1.5(1 - 0.649 \times 0.074 \times 0.3)} - 0.068 \times 0.309 = 0.029. \quad (10)$$

In terms of an MPCH, we divide by a ratio of housing wealth to consumption of 2.17 to arrive at

1.3 cents on the dollar.²

4 Fully Structural Model

We now present a fully-microfounded, dynamic, macro model of multiple regions. After presenting the model we show how the method can be applied in the context of the full model. In subsequent sections we will use numeric solutions of this model to assess the accuracy of the formulae we derived using the simple examples we presented previously.

4.1 Model assumptions

Demographics There are two regions, “home” and “foreign.” The population of the entire economy is normalized to one with a share n in the home region. Within each region there are two representative households, which we call “patient” and “impatient.” Let $\omega(x)$ be the share of households that are $x \in \{\text{patient, impatient}\}$ in each region.

Preferences $\sum_{t=0} \beta(x)^t u(C_t(x), L_t(x), Q_{t+1}(x); \Omega_t)$, where the arguments are consumption, labor supply, units of housing Q_{t+1} , and Ω_t is an aggregate housing demand shock. x indexes a household’s patience.

Commodities and technology There is a final good assembled out of intermediate inputs that is used locally for consumption, residential investment, and government purchases. The production of the final good satisfies

$$\mathcal{Y}_{H,t} = \left[\phi_H^\eta Z_{H,t}^{\frac{\eta-1}{\eta}} + \phi_F^\eta Z_{F,t}^{\frac{\eta-1}{\eta}} \right]^{\frac{\eta}{\eta-1}}$$

$$\mathcal{Y}_{F,t} = \left[\phi_H^\eta (Z_{F,t}^*)^{\frac{\eta-1}{\eta}} + \phi_F^\eta (Z_{H,t}^*)^{\frac{\eta-1}{\eta}} \right]^{\frac{\eta}{\eta-1}},$$

where $Z_{H,t}$ and $Z_{F,t}$ are the home and foreign inputs to the home production of final goods. These inputs are “composite” goods produced in the home and foreign regions and described shortly. Normalize so that $\phi_H + \phi_F = 1$. The bundle shows home bias if $\phi_H > n$. The cost-minimization

²We take the value of owner-occupied housing from the flow of funds and divide it by total PCE less PCE on housing services and utilities and then take the average over 1975 to 2017 to arrive at a ratio of 2.17.

problem implies

$$\begin{aligned}
Z_{H,t} &= \phi_H \mathcal{Y}_{H,t} \left(\frac{P_{H,t}}{\mathcal{P}_{H,t}} \right)^{-\eta} && \text{Home demand for home composite good} \\
Z_{F,t} &= \phi_F \mathcal{Y}_{H,t} \left(\frac{P_{F,t}}{\mathcal{P}_{H,t}} \right)^{-\eta} && \text{Home demand for foreign composite good} \\
Z_{H,t}^* &= \phi_F \mathcal{Y}_{F,t} \left(\frac{P_{H,t}}{\mathcal{P}_{F,t}} \right)^{-\eta} && \text{Foreign demand for home composite good} \\
Z_{F,t}^* &= \phi_H \mathcal{Y}_{F,t} \left(\frac{P_{F,t}}{\mathcal{P}_{F,t}} \right)^{-\eta} && \text{Foreign demand for foreign composite good}
\end{aligned}$$

where $\mathcal{P}_{r,t}$ is the price of the final good in region $r \in \{H, F\}$. The two composite goods are produced in amounts $Y_{H,t}$ and $Y_{F,t}$ and are themselves aggregates of intermediate inputs

$$\begin{aligned}
Y_{H,t} &= \left(\int_0^1 y_{H,t}(z)^{\frac{\theta-1}{\theta}} dz \right)^{\frac{\theta}{\theta-1}} \\
Y_{F,t} &= \left(\int_0^1 y_{F,t}(z)^{\frac{\theta-1}{\theta}} dz \right)^{\frac{\theta}{\theta-1}}.
\end{aligned}$$

The usual cost-minimization problem results in price indices

$$\begin{aligned}
P_{H,t} &= \left(\int_0^1 p_{H,t}(z)^{1-\theta} dz \right)^{\frac{1}{1-\theta}} \\
P_{F,t} &= \left(\int_0^1 p_{F,t}(z)^{1-\theta} dz \right)^{\frac{1}{1-\theta}} \\
\mathcal{P}_{H,t} &= \left(\phi_H P_{H,t}^{1-\eta} + \phi_F P_{F,t}^{1-\eta} \right)^{\frac{1}{1-\eta}} \\
\mathcal{P}_{F,t} &= \left(\phi_H P_{F,t}^{1-\eta} + \phi_F P_{H,t}^{1-\eta} \right)^{\frac{1}{1-\eta}}
\end{aligned}$$

Each intermediate good is produced linearly out of labor $y_{H,t}(z) = L_{H,t}(z)$.

Housing supply The supply of housing satisfies

$$\begin{aligned}
Q_{H,t} &= (1 - \delta)Q_{H,t-1} + I_{H,t}^{\alpha_H} M_{H,t}^{1-\alpha_H} \\
Q_{F,t} &= (1 - \delta)Q_{F,t-1} + I_{H,t}^{\alpha_F} M_{F,t}^{1-\alpha_F}.
\end{aligned}$$

where $I_{H,t}$ is resources (in goods) devoted to residential investment in the home region and $M_{H,t}$ is units of construction permits sold by the federal government. The region-specific parameter α

allows the construction technology to differ across regions.

Markets The two regions share the same money, which serves as the numeraire. Composite intermediate goods markets are competitive and completely integrated across regions and each region faces the price $P_{H,t}$ for the home good and $P_{F,t}$ for the foreign good. Intermediate variety firms face Calvo price-setting frictions with probability of adjusting their price of $1 - \chi$. The labor markets are local to each region and competitive with nominal wages $W_{H,t}$ and $W_{F,t}$. Households trade a nominal bond at nominal interest rate i_t . Units of housing trade at price $J_{r,t}$ in region $r \in \{H, F\}$.

Households face an LTV constraint

$$-B_{t+1} \leq s \frac{J_{r,t+1} Q_{t+1}}{1 + i_t}.$$

We assume this constraint is constantly binding on impatient households and never binding on patient households.

Intermediate goods firms produce profits, which are owned by the patient households in the region. We use $D_t(x)$ to denote the nominal profits received by household of type x .

Government There is a federal government that purchases goods in the two regions, sells construction permits in the two regions, and sets a common monetary policy. Let $G_{H,t}$ and $G_{F,t}$ be per capita spending in the two regions. These are exogenous and financed by national lump-sum taxes.

There is a monetary policy rule that sets the nominal interest rate

$$1 + i_t = \frac{1}{\beta(\text{patient})} + \varphi_\pi \bar{\pi}_t + \varphi_y \bar{y}_t,$$

where $\bar{\pi}_t = n\pi_{H,t} + (1 - n)\pi_{F,t}$ is the weighted average of the two inflation rates in the regions and \bar{y}_t is the weighted average of the log deviations of output from their steady state values.

The government sells construction permits according to the rules

$$\begin{aligned} M_{H,t} &= \bar{M}_H \left(\frac{J_{H,t}}{\mathcal{P}_{H,t}} \right)^{\gamma_H} \\ M_{F,t} &= \bar{M}_F \left(\frac{J_{F,t}}{\mathcal{P}_{F,t}} \right)^{\gamma_F}. \end{aligned}$$

The region-specific parameter γ reflects the fact that regions differ in the elasticity of supply of vacant land.

Decision problems A household solves (we omit the region subscripts because all prices in the household's problem are local)

$$\sum_{t=0} \beta(x)^t u(C_t(x), L_t(x), Q_{t+1}(x); \Omega_t),$$

subject to

$$\begin{aligned} \mathcal{P}_t C_t(x) + J_t Q_{t+1}(x) + B_{t+1}(x) &= W_t L_t(x) - T_t + (1 + i_{t-1}) B_t(x) + J_t Q_t(x) + D_t(x) \\ -B_{t+1}(x) &\leq s \frac{J_{t+1} Q_{t+1}(x)}{1 + i_t}. \end{aligned}$$

The Lagrangian is

$$\begin{aligned} \mathcal{L} &= \sum_{t=0} \beta(x)^t \{ u(C_t(x), L_t(x), Q_{t+1}(x); \Omega_t) \\ &\quad - \lambda_t(x) (\mathcal{P}_t C_t(x) + J_t Q_{t+1}(x) + B_{t+1}(x) - W_t L_t(x) + T_t - (1 + i_{t-1}) B_t(x) - J_t Q_t(x) - D_t(x)) \\ &\quad + \frac{\zeta_t(x)}{\mathcal{P}_t} \left[B_{t+1}(x) + \frac{J_{t+1} Q_{t+1}(x)}{1 + i_t} \right] \} \end{aligned}$$

The first-order conditions can be rearranged to

$$\begin{aligned} u_C &= \mathcal{P}_t \lambda_t \\ -u_L &= \frac{W_t}{\mathcal{P}_t} u_C \\ u_Q &= \frac{J_t}{\mathcal{P}_t} u_C - \beta \left[\frac{J_{t+1}}{\mathcal{P}_{t+1}} u'_C \right] - \zeta_t \left[\frac{\frac{J_{t+1}}{\mathcal{P}_{t+1}} \pi_{t+1}}{1 + i_t} \right] \\ u_C &= \beta \left[\frac{1 + i_t}{\pi_{t+1}} u'_C \right] + \zeta_t, \end{aligned}$$

where $\pi_{t+1} \equiv \mathcal{P}_{t+1}/\mathcal{P}_t$. For patient households we have $\zeta_t = 0$. For impatient households we have $\zeta_t > 0$ and the LTV constraint holds with equality.

The real estate developer solves

$$\max_I J_t I^\alpha M_t^{1-\alpha} - \mathcal{P}_t I,$$

with first order condition

$$\alpha J_t I^{\alpha-1} M_t^{1-\alpha} = \mathcal{P}_t.$$

The resources invested in housing are then

$$I_t = (\alpha J_t M_t^{1-\alpha} \mathcal{P}_t^{-1})^{\frac{1}{1-\alpha}}, \quad (11)$$

and the construction of new houses is

$$I^\alpha M_t^{1-\alpha} = \left(\alpha \frac{J_t}{\mathcal{P}_t} \right)^{\frac{\alpha}{1-\alpha}} M_t.$$

Substitute in the rule for sales of construction permits

$$I^\alpha M_t^{1-\alpha} = \left(\alpha \frac{J_t}{\mathcal{P}_t} \right)^{\frac{\alpha}{1-\alpha}} \bar{M} \left(\frac{J_t}{\mathcal{P}_t} \right)^\gamma.$$

It follows that the elasticity of new houses with respect to the price of housing is $\alpha/(1-\alpha) + \gamma$. This differs across regions because α and γ differ across regions. Finally, the resources invested in housing can be obtained by substituting the rule for permits into (11) to obtain:

$$I_t = \alpha^{\frac{1}{1-\alpha}} \bar{M} \left(\frac{J_t}{\mathcal{P}_t} \right)^{\gamma + \frac{1}{1-\alpha}}. \quad (12)$$

Notice that the housing supply elasticity is $\alpha/(1-\alpha) + \gamma$, but the elasticity of residential investment is $1/(1-\alpha) + \gamma$. Cities with more elastic housing supply (large α) will have residential investment respond more strongly to a given change in the price of housing. However, it is not clear which cities have more volatile residential investment because house prices will rise more in low elasticity cities.

The intermediate goods producer solves

$$\max_{P_0^*} \sum_{t=0}^{\infty} \chi^t \lambda_t [P_0^* y_{H,t}(z) - W_{H,t} L_{H,t}(z)],$$

where

$$y_{H,t}(z) = Y_{H,t} \left(\frac{P_0^*}{P_{H,t}} \right)^{-\theta}$$

$$L_{H,t}(z) = y_{H,t}(z).$$

Substituting the constraints into the objective yields

$$\max_{P_0^*} \sum_{t=0}^{\infty} \chi^t \lambda_t Y_{H,t} P_{H,t}^\theta (P_0^* - W_{H,t}) (P_0^*)^{-\theta}$$

with first-order conditions

$$\sum_{t=0}^{\infty} \chi^t \lambda_t Y_{H,t} P_{H,t}^\theta (\theta - 1) P_0^{-\theta} = \sum_{t=0}^{\infty} \chi^t \lambda_t Y_{H,t} P_{H,t}^\theta \theta W_{H,t} P_0^{-\theta-1}$$

$$P_0^* = \frac{\theta}{\theta - 1} \frac{\sum_{t=0}^{\infty} \chi^t \lambda_t Y_{H,t} P_{H,t}^\theta W_{H,t}}{\sum_{t=0}^{\infty} \chi^t \lambda_t Y_{H,t} P_{H,t}^\theta}.$$

Preference specification The preferences of the home households take the form

$$u = \frac{1}{1 - \sigma} \left[\left(C - \psi \frac{L^{1+\nu}}{1 + \nu} \right)^\kappa (Q - \Omega_t)^{1-\kappa} \right]^{1-\sigma}.$$

We then have

$$u_C = \left[\left(C - \psi \frac{L^{1+\nu}}{1 + \nu} \right)^\kappa (Q - \Omega_t)^{1-\kappa} \right]^{-\sigma} \kappa \left(C - \psi \frac{L^{1+\nu}}{1 + \nu} \right)^{\kappa-1} (Q - \Omega_t)^{1-\kappa}$$

$$u_L = - \left[\left(C - \psi \frac{L^{1+\nu}}{1 + \nu} \right)^\kappa (Q - \Omega_t)^{1-\kappa} \right]^{-\sigma} \kappa \left(C - \psi \frac{L^{1+\nu}}{1 + \nu} \right)^{\kappa-1} (Q - \Omega_t)^{1-\kappa} \psi L^\nu.$$

Using these derivatives, the labor supply curve is

$$\psi L_{H,t}(x)^\nu = \frac{W_{H,t}}{\mathcal{P}_{H,t}}. \quad (13)$$

Equilibrium definition Let lower-case denote real variables (i.e. normalized by \mathcal{P}_t). For $r \in \{H, F\}$ and $x \in \{\text{patient}, \text{impatient}\}$, the equilibrium variables are $j_{r,t}$, $w_{r,t}$, $\pi_{r,t}$, $\mathcal{Y}_{r,t}$, $Y_{r,t}$, $b_{r,t}(x)$, i_t , $Q_{r,t}(x)$, $Q_{r,t}$, $C_{r,t}(x)$, $L_{r,t}(x)$, $M_{r,t}$, $G_{r,t}$, t_t , $Z_{r,t}$, $Z_{r,t}^*$, $d_{r,t}(\text{patient})$, $I_{r,t}$, $E_t \equiv P_{H,t}/P_{F,t}$, where

$Y_{r,t}^*$ is foreign demand for the composite good produced in r . The equilibrium conditions are

$$Q_{r,t} = \sum_x \omega(x) Q_{r,t}(x) \quad \forall r \quad (14)$$

$$Y_{r,t} \int_0^1 \left(\frac{p_{H,t}(z)}{P_t} \right)^{-\theta} dz = \sum_x \omega(x) L_{r,t}(x) \quad \forall r \quad (15)$$

$$\psi L_{r,t}(x)^\nu = w_{r,t} \quad \forall r, x \quad (16)$$

$$\mathcal{Y}_{r,t} = \sum_x \omega(x) C_{r,t}(x) + G_{r,t} + I_{r,t} \quad \forall r \quad (17)$$

$$Y_{H,t} = n Z_{H,t} + (1-n) Z_{H,t}^* \quad (18)$$

$$Y_{F,t} = n Z_{F,t}^* + (1-n) Z_{F,t}^* \quad (19)$$

$$Z_{H,t} = \phi_H \mathcal{Y}_{H,t} \left(\phi_H + \phi_F E_t^{-(1-\eta)} \right)^{\frac{\eta}{1-\eta}} \quad (20)$$

$$Z_{F,t} = \phi_F \mathcal{Y}_{H,t} \left(\phi_H E_t^{1-\eta} + \phi_F \right)^{\frac{\eta}{1-\eta}} \quad (21)$$

$$Z_{H,t}^* = \phi_F \mathcal{Y}_{F,t} \left(\phi_H E_t^{-(1-\eta)} + \phi_F \right)^{\frac{\eta}{1-\eta}} \quad (22)$$

$$Z_{F,t}^* = \phi_H \mathcal{Y}_{F,t} \left(\phi_H + \phi_F E_t^{1-\eta} \right)^{\frac{\eta}{1-\eta}} . \quad (23)$$

Additionally there are two production functions for the final goods, two Phillips curves, four budget constraints (for the patient and impatient household in each region), the monetary policy rule, four housing first-order conditions, two Euler equations (for the patient agents), two LTV constraints (for the impatient agents), two exogenous G sequences, two housing permit supply rules, two dynamic equations for Q , one government budget, the definitions of dividend for each region, the first-order condition of the real estate developer in each region, and the evolution of the real exchange rate E_t

given by

$$\begin{aligned}
\mathcal{P}_{H,t} &= \left(\phi_H + \phi_F E_t^{-(1-\eta)} \right)^{\frac{1}{1-\eta}} P_{H,t} \\
\mathcal{P}_{F,t} &= \left(\phi_H + \phi_F E_t^{1-\eta} \right)^{\frac{1}{1-\eta}} P_{F,t} \\
\pi_{H,t} &= \frac{\left(\phi_H + \phi_F E_t^{-(1-\eta)} \right)^{\frac{1}{1-\eta}} P_{H,t}}{\left(\phi_H + \phi_F E_{t-1}^{-(1-\eta)} \right)^{\frac{1}{1-\eta}} P_{H,t-1}} \\
\pi_{F,t} &= \frac{\left(\phi_H + \phi_F E_t^{1-\eta} \right)^{\frac{1}{1-\eta}} P_{F,t}}{\left(\phi_H + \phi_F E_{t-1}^{1-\eta} \right)^{\frac{1}{1-\eta}} P_{F,t-1}} \\
\frac{\pi_{H,t}}{\pi_{F,t}} &= \frac{\left(\phi_H + \phi_F E_t^{-(1-\eta)} \right)^{\frac{1}{1-\eta}} \left(\phi_H + \phi_F E_{t-1}^{1-\eta} \right)^{\frac{1}{1-\eta}} E_t}{\left(\phi_H + \phi_F E_{t-1}^{-(1-\eta)} \right)^{\frac{1}{1-\eta}} \left(\phi_H + \phi_F E_t^{1-\eta} \right)^{\frac{1}{1-\eta}} E_{t-1}}.
\end{aligned}$$

5 Application in the Full Model

We now show how our method can be applied in the context of the full model. We begin with a preliminary step of showing that the equilibrium can be represented in terms of a consumption function and price functions.

5.1 Consumption and price functions

We consider perfect foresight transitions lasting \mathcal{T} dates. For simplicity we will assume houses are created out of permits alone ($\alpha \rightarrow 0$) and complete home bias ($\phi_F \rightarrow 0$), which implies $\mathcal{Y}_{r,t} = Y_{r,t}$. The key points established in this section are (i) for any t and region r , we can write the equilibrium solutions for $C_{r,t}$ and $Q_{r,t+1}$ as functions of $\{\mathcal{P}_{r,t}, J_{r,t}, W_{r,t}, i_t, D_{r,t}, \Omega_t\}_{t=1}^{\mathcal{T}}$ and (ii) we can write the equilibrium solutions for $W_{r,t}, \mathcal{P}_{r,t}, D_{r,t}$ as functions of $\{\mathcal{Y}_{r,t}\}_{t=1}^{\mathcal{T}}$.³

The first step is to establish that consumption and housing demand at date t along the transition can be written as a function of current, and future prices and preference shocks and an endogenous state that summarizes the asset holdings of the patient and impatient households. Let $\mathcal{A}_t \equiv (1 + i_{t-1})B_t + J_t Q_t$ be the wealth of an individual at date t . For an individual (patient or impatient)

³The analyses in Kaplan et al. (2018), Farhi and Werning (2017), and Auclert et al. (2018) also make use of an aggregate consumption function.

household, consumption at date t is the solution to

$$V_t(\mathcal{A}_t) = \max_{C_t, Q_{t+1}, L_t, B_{t+1}} u(C_t, L_t, Q_{t+1}; \Omega_t) + \beta V_{t+1}((1 + i_t)B_{t+1} + J_{t+1}Q_{t+1}),$$

subject to

$$\begin{aligned} \mathcal{P}_t C_t + J_t Q_{t+1} + B_{t+1} &= W_t L_t - T_t + \mathcal{A}_t + D_t \\ -B_{t+1} &\leq s \frac{J_{t+1} Q_{t+1}}{1 + i_t}. \end{aligned}$$

$V_{\mathcal{T}+1}$ is the steady state value function associated with steady state prices and preference shifter Ω , which are fixed. $C_{\mathcal{T}}$ then depends on $\mathcal{A}_{\mathcal{T}}$ and the prices at \mathcal{T} . Recursing backwards, C_t depends on \mathcal{A}_t and the prices at future dates. Similarly for the choices Q_{t+1} and B_{t+1} . For L_t we already know that labor supply only depends on the current real wage due to GHH preferences.

Given the initial, steady state value \mathcal{A}_0 , we have already established that \mathcal{A}_1 depends on the whole path $\{\mathcal{P}_t, J_t, W_t, i_t, D_t, \Omega_t\}_{t=1}^{\mathcal{T}}$. We then recurse forwards and say \mathcal{A}_{s+1} depends on current and future prices $\{\mathcal{P}_t, J_t, W_t, i_t, D_t, \Omega_t\}_{t=s}^{\mathcal{T}}$ and \mathcal{A}_s , which itself is a function of $\{\mathcal{P}_t, J_t, W_t, i_t, D_t, \Omega_t\}_{t=1}^{\mathcal{T}}$ so we can write \mathcal{A}_{s+1} as a function of the whole path of prices.

Finally, substitute these “solutions” for asset holdings into the solution for C_t to write C_t as function of the full path of prices. The same argument applies to Q_{t+1} and B_{t+1} .

Next we establish that there are functions that map past, current and future demand in region r , $\{\mathcal{Y}_{r,s}\}_{s=1}^{\mathcal{T}}$ to the current inflation rate, $\pi_{r,t}$, price level, $\mathcal{P}_{r,t}$, wage, $W_{r,t}$, and dividend, $D_{r,t}$. We use the linearized aggregate production function

$$\mathcal{Y}_{r,t} = \sum_x \omega(x) L_{r,t}(x).$$

From the labor supply curve (13) we see that patient and impatient types work the same hours so the production function can be written as

$$\mathcal{Y}_{r,t} = L_{r,t},$$

for the common labor supply $L_{r,t}$. Using the labor supply curve (13) this becomes

$$\mathcal{Y}_{r,t} = \left(\frac{W_{r,t}}{\psi \mathcal{P}_{r,t}} \right)^{1/\nu}, \quad (24)$$

and we can use this to solve for the real and nominal wages as a function of current demand and the current price level. The definition of the dividend gives $D_{r,t}$ in terms of $\mathcal{Y}_{r,t}$, $W_{r,t}$, and $L_{r,t}$. For the inflation rate and price level we work with the log-linearized Phillips curve

$$\pi_t = \beta\pi_{t+1} + \frac{(1-\beta\chi)(1-\chi)}{\chi} \left[\log\left(\frac{W_{r,t}}{\mathcal{P}_{r,t}}\right) - \log\left(\frac{\theta}{1-\theta}\right) \right].$$

Solving this forward and using (24) gives

$$\pi_t = \sum_{s=t}^{\mathcal{T}} \beta^{s-t} \frac{(1-\beta\chi)(1-\chi)}{\chi} \left[\log(\psi\mathcal{Y}_{r,t}^\nu) - \log\left(\frac{\theta}{1-\theta}\right) \right].$$

We can then accumulate the inflation rate to find the path of the price level

$$\mathcal{P}_t = \left(\prod_{s=1}^t \pi_s \right) \mathcal{P}_0.$$

5.2 Application

We consider a perfect foresight transition lasting \mathcal{T} dates. The previous section establishes that there are functions that map the paths for $\{\mathcal{P}_{r,t}, J_{r,t}, W_{r,t}, i_t, D_{r,t}, T_t, \Omega_t\}_{t=1}^{\mathcal{T}}$ to $C_{t,r}$ and $Q_{t,r}$ for each r and t . A first-order approximation of these functions gives

$$\begin{aligned} \hat{C}_r &= \mathbf{C}_J \hat{J}_r + \mathbf{C}_P \hat{P}_r + \mathbf{C}_i \hat{i} + \mathbf{C}_W \hat{W}_r + \mathbf{C}_D \hat{D}_r + \mathbf{C}_T \hat{T} + \mathbf{C}_\Omega \hat{\Omega} \\ \hat{Q}_r &= \mathbf{Q}_J \hat{J}_r + \mathbf{Q}_P \hat{P}_r + \mathbf{Q}_i \hat{i} + \mathbf{Q}_W \hat{W}_r + \mathbf{Q}_D \hat{D}_r + \mathbf{Q}_T \hat{T} + \mathbf{Q}_\Omega \hat{\Omega}, \end{aligned}$$

where \hat{C} is a \mathcal{T} vector of consumption deviations from steady state for the \mathcal{T} dates, \mathbf{C}_J is a $\mathcal{T} \times \mathcal{T}$ matrix where the (i, j) element gives the effect of \hat{J}_{t+j} on \hat{C}_{t+i} , and so on. We assume that the model is linearized around a symmetric steady state so the \mathbf{C} matrices are the same for the two regions. While the two cities differ in their housing supply elasticities, the consequences of these differences affect consumption through the paths of prices. Similarly, government purchases affect consumption through the paths of prices and taxes.

Next, the previous section established that there are functions of $\{\mathcal{Y}_{r,t}\}_{t=1}^{\mathcal{T}}$ that give $\mathcal{P}_{r,t}$, $W_{r,t}$,

and $D_{r,t}$ at each date t . The linearized version of these functions gives

$$\begin{aligned}\hat{\mathcal{P}}_r &= \mathbf{P}_y \hat{\mathcal{Y}}_r \\ \hat{W}_r &= \mathbf{W}_y \hat{\mathcal{Y}}_r \\ \hat{D}_r &= \mathbf{D}_y \hat{\mathcal{Y}}_r,\end{aligned}$$

where \mathbf{P} , \mathbf{W} , and \mathbf{D} , are $\mathcal{T} \times \mathcal{T}$ matrices.

The aggregate resource constraint is

$$\hat{\mathcal{Y}}_r = \hat{C}_r + \hat{I}_r + \hat{G}_r.$$

Combining all these equations we have

$$\hat{C}_r = \mathbf{C}_J \hat{J}_r + \underbrace{(\mathbf{C}_P \mathbf{P}_y + \mathbf{C}_W \mathbf{W}_y + \mathbf{C}_D \mathbf{D}_y)}_{\equiv \mathbf{m}} (\hat{C}_r + \hat{I}_r + \hat{G}_r) + \mathbf{C}_i \hat{i} + \mathbf{C}_T \hat{T} + \mathbf{C}_\Omega \hat{\Omega} \quad (25)$$

$$(I - \mathbf{m}) \hat{C}_r = \mathbf{C}_J \hat{J}_r + \mathbf{m} \hat{I}_r + \mathbf{m} \hat{G}_r + \mathbf{C}_i \hat{i} + \mathbf{C}_T \hat{T} + \mathbf{C}_\Omega \hat{\Omega}$$

$$\hat{C}_r = \underbrace{(I - \mathbf{m})^{-1}}_{\equiv \mathbf{M}} \left[\mathbf{C}_J \hat{J}_r + \mathbf{m} \hat{I}_r + \mathbf{m} \hat{G}_r + \mathbf{C}_i \hat{i} + \mathbf{C}_T \hat{T} + \mathbf{C}_\Omega \hat{\Omega} \right]. \quad (26)$$

Finally, consider the effect on output

$$\begin{aligned}\hat{\mathcal{Y}}_r &= \hat{C}_r + \hat{I}_r + \hat{G}_r \\ &= \mathbf{M} \left[\mathbf{C}_J \hat{J}_r + \mathbf{m} \hat{I}_r + \mathbf{m} \hat{G}_r + \mathbf{C}_i \hat{i} + \mathbf{C}_T \hat{T} + \mathbf{C}_\Omega \hat{\Omega} \right] + \hat{I}_r + \hat{G}_r \\ &= \mathbf{M} \left[\mathbf{C}_J \hat{J}_r + \mathbf{C}_i \hat{i} + \mathbf{C}_T \hat{T} + \mathbf{C}_\Omega \hat{\Omega} \right] + [\mathbf{M} \mathbf{m} + I] (\hat{I}_r + \hat{G}_r) \\ &= \mathbf{M} \left[\mathbf{C}_J \hat{J}_r + \hat{I}_r + \hat{G}_r + \mathbf{C}_i \hat{i} + \mathbf{C}_T \hat{T} + \mathbf{C}_\Omega \hat{\Omega} \right],\end{aligned} \quad (27)$$

where the last equality follows from the fact that $\mathbf{M} \mathbf{m} + I = I + \mathbf{m} + \mathbf{m}^2 + \dots = (I - \mathbf{m})^{-1} = \mathbf{M}$. This makes clear that the consumption-income multiplier that applies to the partial equilibrium effects of the general equilibrium home price change is the same as the multiplier that applies to the government purchases shock.

From (12), we can see that we can write

$$\hat{I}_r = \mathbf{I}_{r,J} \hat{J}_r, \quad (28)$$

where, abusing notation, \hat{J}_r is the *relative* price of housing in terms of goods. Plug this in for \hat{I}_r in (27)

$$\hat{Y}_r = \mathbf{M} \left[(\mathbf{C}_J + \mathbf{I}_{r,J}) \hat{J}_r + \hat{G}_r + \mathbf{C}_i \hat{i} + \mathbf{C}_T \hat{T} + \mathbf{C}_\Omega \hat{\Omega} \right]. \quad (29)$$

This expression makes clear that the local general equilibrium demand effects resulting from home price changes are driven by both the consumption and residential investment responses.

What we need to add to the analysis is that \hat{J}_r is an equilibrium outcome. To a first-order approximation, market clearing for housing requires

$$\mathbf{Q}_J \hat{J}_r + \mathbf{Q}_i \hat{i} + \mathbf{Q}_W \hat{W}_r + \mathbf{Q}_D \hat{D}_r + \mathbf{Q}_T \hat{T} + \mathbf{Q}_\Omega \hat{\Omega} = \mathbf{Q}_{r,J}^S \hat{J}_r,$$

where $\mathbf{Q}_{r,J}^S$ determines the linearized supply curve. The argument that the supply curve can be written this way is as follows: at each date the government sells an amount of permits given by the real price of housing so the current supply of housing is discounted sum of the current and past real housing prices. Rearranging and substituting we have

$$(\mathbf{Q}_J - \mathbf{Q}_{r,J}^S) \hat{J}_r + \mathbf{Q}_W \mathbf{W}_y \hat{Y}_r + \mathbf{Q}_D \mathbf{D}_y \hat{Y}_r + \mathbf{Q}_i \hat{i} + \mathbf{Q}_T \hat{T} + \mathbf{Q}_\Omega \hat{\Omega} = 0$$

$$(\mathbf{Q}_{r,J}^S - \mathbf{Q}_J) \hat{J}_r = (\mathbf{Q}_W \mathbf{W}_y + \mathbf{Q}_D \mathbf{D}_y) \hat{Y}_r + \mathbf{Q}_i \hat{i} + \mathbf{Q}_T \hat{T} + \mathbf{Q}_\Omega \hat{\Omega}$$

$$\hat{J}_r = \mathbf{J}_{r,y} \hat{Y}_r + \underbrace{(\mathbf{Q}_{r,J}^S - \mathbf{Q}_J)^{-1} \mathbf{Q}_i}_{\equiv \mathbf{J}_{r,i}} \hat{i} + \underbrace{(\mathbf{Q}_{r,J}^S - \mathbf{Q}_J)^{-1} \mathbf{Q}_T}_{\equiv \mathbf{J}_{r,T}} \hat{T} + \underbrace{(\mathbf{Q}_{r,J}^S - \mathbf{Q}_J)^{-1} \mathbf{Q}_\Omega}_{\equiv \mathbf{J}_{r,\Omega}} \hat{\Omega}, \quad (30)$$

where

$$\mathbf{J}_{r,y} \equiv (\mathbf{Q}_{r,J}^S - \mathbf{Q}_J)^{-1} (\mathbf{Q}_W \mathbf{W}_y + \mathbf{Q}_D \mathbf{D}_y).$$

Now we substitute this expression for \hat{J}_r into (29) to obtain

$$\begin{aligned} \hat{Y}_r &= \mathbf{M} (\mathbf{C}_J + \mathbf{I}_{r,J}) \left[\mathbf{J}_{r,y} \hat{Y}_r + \mathbf{J}_{r,i} \hat{i} + \mathbf{J}_{r,T} \hat{T} + \mathbf{J}_{r,\Omega} \hat{\Omega} \right] + \mathbf{M} \left[\hat{G}_r + \mathbf{C}_i \hat{i} + \mathbf{C}_T \hat{T} + \mathbf{C}_\Omega \hat{\Omega} \right] \\ &= [I - \mathbf{M} (\mathbf{C}_J + \mathbf{I}_{r,J}) \mathbf{J}_{r,y}]^{-1} \mathbf{M} \\ &\quad \times \left\{ (\mathbf{C}_J + \mathbf{I}_{r,J}) \left[\mathbf{J}_{r,i} \hat{i} + \mathbf{J}_{r,T} \hat{T} + \mathbf{J}_{r,\Omega} \hat{\Omega} \right] + \left[\hat{G}_r + \mathbf{C}_i \hat{i} + \mathbf{C}_T \hat{T} + \mathbf{C}_\Omega \hat{\Omega} \right] \right\}. \end{aligned} \quad (31)$$

From (29), the housing wealth effect on output taking house prices as given is

$$E \equiv \mathbf{M}(\mathbf{C}_J + \mathbf{I}_{r,J}).$$

From (30), the response of home prices to income is

$$Z \equiv \mathbf{J}_{r,y}.$$

From (31), the fiscal multiplier is

$$\begin{aligned} F &\equiv [I - \mathbf{M}(\mathbf{C}_J + \mathbf{I}_{r,J}) \mathbf{J}_{r,y}]^{-1} \mathbf{M} \\ &= [I - EZ]^{-1} \mathbf{M}. \end{aligned}$$

For now, take these as the definitions of E , F , and Z and in the next subsection we relate them to typical empirical specifications. It then follows that

$$\mathbf{C}_J = [(1 - EZ)F]^{-1} E - \mathbf{I}_{r,J}.$$

This result is very similar to what we found in the static economy (see equation 8), however, here the components E , F , and so on are matrices rather scalars. To fix ideas, E is a $\mathcal{T} \times \mathcal{T}$ matrix in which the (i, j) element gives the response of output in period i to a change in home prices in period j . Similarly, the (i, j) element of F gives the response of output at period i to a change in government spending at date j . If these matrices have important off-diagonal elements, then the logic of our static examples is complicated by dynamic responses of the economy. On the other hand, if the matrices are close to diagonal, then the logic of the static economy goes through.

5.3 Empirical specifications

At this stage, it is useful to consider what regression specifications are used in the cross-sectional analysis of housing wealth effects and fiscal multipliers. An estimate of the housing wealth effect might regress the change in consumption on the change in home prices and a constant or time fixed effect. We can rearrange equation (26) to show how it is related to this regression specification. To do so, we will assume that there are no dynamics so that the matrices are all diagonal. Substitute

(28) into (26), first difference, and rearrange to obtain

$$\Delta\hat{C}_r = \underbrace{\mathbf{M}(\mathbf{C}_J + \mathbf{m}\bar{\mathbf{I}}_{r,J})}_{\text{coef. of interest}} \Delta\hat{J}_r + \underbrace{\mathbf{M}[\mathbf{C}_i\Delta\hat{i} + \mathbf{C}_T\Delta\hat{T} + \mathbf{C}_\Omega\Delta\hat{\Omega}]}_{\text{constant or time fixed effect}} + \underbrace{\mathbf{Mm}\Delta\hat{G}_r + \mathbf{Mm}(\mathbf{I}_{r,J} - \bar{\mathbf{I}}_{r,J}) \Delta\hat{J}_r}_{\text{error}}.$$

The key thing to note here is that changes in aggregate variables (i , T , and Ω) are absorbed by the time fixed effect.

This equation shows a potential source of bias in the housing wealth effect regression that has not been analyzed before: to the extent that cities differ in their housing supply elasticities they will differ in the response of residential investment to home prices and cities with larger price changes will have smaller elasticities of residential investment. In our Monte Carlo analysis we can explore how severe this bias is.

For the local fiscal multiplier, one would regress the change in output on the change in government spending. House prices are typically not included in this regression. One can rearrange equation (31) as

$$\Delta\mathcal{Y}_r = \underbrace{F}_{\text{coef. of interest}} \Delta\hat{G}_r + \underbrace{F[\mathbf{C}_i\Delta\hat{i} + \mathbf{C}_T\Delta\hat{T} + \mathbf{C}_\Omega\Delta\hat{\Omega}]}_{\text{constant or time fixed effect}} + \bar{U}_r + \underbrace{U_r - \bar{U}_r}_{\text{error}}$$

where F is defined as above and

$$U_r \equiv F(\mathbf{C}_J + \mathbf{I}_{r,J}) [\mathbf{J}_{r,i}\Delta\hat{i} + \mathbf{J}_{r,T}\Delta\hat{T} + \mathbf{J}_{r,\Omega}\Delta\hat{\Omega}].$$

6 Monte Carlo analysis

[To be added.]

7 Conclusion

Cross-sectional empirical estimates have become part of the macroeconomist toolkit, but the appropriate interpretation of these estimates can be difficult as they often blend together partial-equilibrium responses with local general equilibrium effects. We have presented a method that allows the researcher to isolate the partial equilibrium effect, which we view as more useful as it is more easily compared to the predictions of standard one-region models.

We applied this method to compute the partial equilibrium housing wealth effect. The key step in the application is to use an estimate of the local fiscal multiplier to gauge the strength of the local general equilibrium effects. Accounting for local general equilibrium reduces the housing wealth effect almost by half.

A Proof for Section 3.2

We proceed as in the first example to arrive at

$$\frac{d(w, p)}{d(g, s)} = \underbrace{\left[N_w - C_w - I_w + (C_p + I_p - N_p) \frac{H_w}{H_p} \right]^{-1}}_{\equiv M} \begin{pmatrix} 1 & \frac{H_s}{H_p} (N_p - C_p - I_p) \\ -\frac{H_w}{H_p} & \frac{H_s}{H_p} (C_w + I_w - N_w) \end{pmatrix}$$

and

$$\frac{dY}{d(g, s)} = \begin{pmatrix} C_w + I_w & C_p + I_p \end{pmatrix} \frac{d(w, p)}{d(g, s)} + \begin{pmatrix} 1 & 0 \end{pmatrix}.$$

The observables are

$$\frac{dY}{dg} = 1 + M \left[C_w - (C_p + I_p) \frac{H_w}{H_p} \right] \quad (32)$$

$$\frac{dp}{ds} = M \frac{H_s}{H_p} (C_w - N_w) \quad (33)$$

$$\frac{dC}{ds} = -C_w M \frac{H_s}{H_p} (C_p + I_p) + C_p M \frac{H_s}{H_p} (C_w - N_w) \quad (34)$$

$$\frac{dp}{dg} = -M \frac{H_w}{H_p} \quad (35)$$

$$\frac{dI}{ds} = I_p M \frac{H_s}{H_p} (C_w - N_w) \quad (36)$$

and the definition of M gives

$$M (N_w - C_w) = 1 - M (C_p + I_p) \frac{H_w}{H_p}. \quad (37)$$

Use (35) in (32) to arrive at

$$\frac{dY}{dg} = 1 + M C_w + (C_p + I_p) \frac{dp}{dg}. \quad (38)$$

Using (33), (34), and (36) we have

$$\begin{aligned} E \equiv \frac{(dC/ds) + (dI/ds)}{dp/ds} &= \frac{-C_w M \frac{H_s}{H_p} (C_p + I_p) + C_p M \frac{H_s}{H_p} (C_w - N_w) + I_p M \frac{H_s}{H_p} (C_w - N_w)}{M \frac{H_s}{H_p} (C_w - N_w)} \\ &= \frac{C_w M (C_p + I_p)}{M (N_w - C_w)} + (C_p + I_p). \end{aligned}$$

Now use (37) and then (35)

$$\begin{aligned} E &= \frac{C_w M (C_p + I_p)}{1 - M (C_p + I_p) \frac{H_w}{H_p}} + (C_p + I_p) \\ &= \frac{C_w M (C_p + I_p)}{1 + (C_p + I_p) \frac{dp}{dg}} + (C_p + I_p). \end{aligned}$$

Now use (38) to substitute for $C_w M$

$$E = \frac{\left[\frac{dY}{dg} - 1 - (C_p + I_p) \frac{dp}{dg} \right] (C_p + I_p)}{1 + (C_p + I_p) \frac{dp}{dg}} + (C_p + I_p)$$

and rearrange

$$\begin{aligned} E &= (C_p + I_p) \frac{\frac{dY}{dg}}{1 + (C_p + I_p) \frac{dp}{dg}} \\ E + E (C_p + I_p) \frac{dp}{dg} &= (C_p + I_p) \frac{dY}{dg} \\ E &= (C_p + I_p) \left[\frac{dY}{dg} - E \frac{dp}{dg} \right] \\ \frac{E}{\frac{dY}{dg} \left[1 - E \frac{dp/dg}{dY/dg} \right]} &= C_p + I_p. \end{aligned}$$

I_p can be calculated from (33) and (36) so we have

$$C_p = \frac{E}{\frac{dY}{dg} \left[1 - E \frac{dp/dg}{dY/dg} \right]} - \frac{dI/ds}{dp/ds}$$

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