Online Appendix for:
The Slope of the Phillips Curve: Evidence from U.S. States

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January 25, 2022

A Theoretical Appendix

A.1 Household Optimality Conditions

Households optimally trade off current consumption and current labor supply. This implies the following labor supply curve must hold in our model:

$$-\frac{u_n(C_{Ht}, N_{Ht})}{u_c(C_{Ht}, N_{Ht})} = \frac{W_{Ht}}{P_{Ht}},$$

where P_{Ht} denotes the lowest cost of purchasing a unit of the composite consumption good C_{Ht} and subscripts on the utility function denote partial derivatives. Using expressions for $u_n(C_{Ht}, N_{Ht})$ and $u_c(C_{Ht}, N_{Ht})$, we can rewrite the home labor supply curve as

$$\chi N_{Ht}^{\varphi^{-1}} = \frac{W_{Ht}}{P_{Ht}}. \tag{1}$$

Households optimally trade off current consumption and consumption in the next period. This implies the following consumption Euler equation must hold in our model:

$$\beta R_t^n E_t \left[\frac{u_c(C_{H,t+1}, N_{H,t+1})}{u_c(C_{Ht}, N_{Ht})} \frac{P_{Ht}}{P_{H,t+1}} \right] = 1. \tag{2}$$

where R_t^n is the gross nominal interest rate, which is common to both regions in the monetary union. Household optimization also implies a standard transversality condition must hold in model and it implies that the stochastic discount factor takes a standard form.

Households choose how much to purchase of the various goods in the economy to minimize the cost of attaining the level of consumption C_{Ht} they choose. This implies the following demand curves for home and foreign tradeable and non-tradeable goods:

$$C_{Ht}^N = \phi_N C_{Ht} \left(\frac{P_{Ht}^N}{P_{Ht}} \right)^{-\eta}, \tag{3}$$

$$C_{Ht}^{TH} = \phi_T \tau_{Ht}^H C_{Ht} \left(\frac{P_{Ht}^T}{P_{Ht}} \right)^{-\eta}, \quad \text{and} \quad C_{Ht}^{TF} = \phi_T \tau_{Ht}^F C_{Ht} \left(\frac{P_{Ft}^T}{P_{Ht}} \right)^{-\eta}. \tag{4}$$

Utility maximization, furthermore, implies the following demand curves for each of the varieties of goods produced in the economy:

$$C^N_{Ht}(z) = C^N_{Ht}\left(\frac{P^N_{Ht}(z)}{P^N_{Ht}}\right)^{-\theta} \quad C^{TH}_{Ht}(z) = C^{TH}_{Ht}\left(\frac{P^T_{Ht}(z)}{P^T_{Ht}}\right)^{-\theta} \quad C^{TF}_{Ht}(z) = C^{TF}_{Ht}\left(\frac{P^T_{Ft}(z)}{P^T_{Ft}}\right)^{-\theta} \tag{5}$$

The cost minimizing price indexes are given by

$$P^N_{Ht} = \left[\int_0^1 P^N_{Ht}(z)^{1-\theta}dz\right]^{\frac{1}{1-\theta}}, \quad P^T_{Ht} = \left[\int_0^1 P^T_{Ht}(z)^{1-\theta}dz\right]^{\frac{1}{1-\theta}}, \quad P^T_{Ft} = \left[\int_0^1 P^T_{Ft}(z)^{1-\theta}dz\right]^{\frac{1}{1-\theta}},$$

$$P_{Ht} = \left[\phi_N P^{N\ 1-\eta}_{Ht} + \phi_T \tau^H_{Ht} P^{T\ 1-\eta}_{Ht} + \phi_T \tau^F_{Ht} P^{T\ 1-\eta}_{Ft}\right]^{\frac{1}{1-\eta}}. \tag{6}$$

A.2 Firm Optimality Conditions

Firms in our model must satisfy demand. For firms in the non-tradeable sector, this implies that

$$\zeta C^N_{Ht}\left(\frac{P^N_{Ht}(z)}{P^N_{Ht}}\right)^{-\theta} \leq Z^N_{Ht} N^N_{Ht}(z).$$

Optimal choice of labor by the firm z implies that

$$W_{Ht} = S^N_{Ht}(z) Z^N_{Ht}, \tag{7}$$

where $S^N_{Ht}(z)$ is the firm's nominal marginal cost, i.e. the Lagrange multiplier on the constraint above.

Non-tradeable firms z that are able to reoptimize their price in period t set it to satisfy

$$\sum_{k=0}^{\infty} \alpha^k E_t \left[M_{Ht,t+k} Y^N_{H,t+k}(z)\left(P^{N*}_{Ht}(z) - \frac{\theta}{\theta-1} S^N_{H,t+k}(z)\right)\right] = 0, \tag{8}$$

where $P^{N*}_{Ht}(z)$ is the price the firm chooses. Intuitively, the firm sets its price equal to a constant markup over a weighted average of current and expected future marginal cost taking into account the probability that their price will remain unchanged in future periods. We can divide both sides of this equation by $P^N_{H,t-1}$ and rewrite it as

$$\sum_{k=0}^{\infty} \alpha^k E_t \left[M_{Ht,t+k} Y^N_{H,t+k}(z)\left(\frac{P^{N*}_{Ht}(z)}{P^N_{H,t-1}} - \frac{\theta}{\theta-1} MC^N_{H,t+k}(z)\frac{P^N_{H,t+k}}{P^N_{H,t-1}}\right)\right] = 0, \tag{9}$$

where $MC^N_{H,t+k}(z) = S^N_{H,t+k}(z)/P^N_{H,t+k}$ is real marginal cost in the non-tradeable sector.

Analogously to the non-tradeable sector, optimal firm labor demand in the tradeable sector is

$$W_{Ht} = S^T_{Ht}(z)Z^T_{Ht}(z). \tag{10}$$

where $S^T_{Ht}(z)$ is the tradeable goods firm's nominal marginal cost. Optimal choice of a new reset price by tradeable goods firms implies

$$\sum_{k=0}^{\infty}\alpha^k E_t\left[M_{Ht,t+k}Y^T_{H,t+k}(z)\left(P^T_{Ht}(z) - \frac{\theta}{\theta-1}S^T_{H,t+k}(z)\right)\right] = 0. \tag{11}$$

We can divide both sides of this equation by $P^T_{H,t-1}$ and rewrite it as

$$\sum_{k=0}^{\infty}\alpha^k E_t\left[M_{Ht,t+k}Y^T_{H,t+k}(z)\left(\frac{P^T_{Ht}(z)}{P^T_{H,t-1}} - \frac{\theta}{\theta-1}MC^T_{H,t+k}(z)\frac{P^T_{H,t+k}}{P^T_{H,t-1}}\right)\right] = 0 \tag{12}$$

where $MC^T_{H,t+k} = S^T_{H,t+k}(z)/P^T_{H,t+k}$ is the real marginal cost in the tradeable sector.

A.3 Zero Inflation Steady State

In the next section, we take a log-linear approximation of the equilibrium conditions of our model around a steady state with zero inflation and balanced trade. In this section, we solve for this steady state. The steady state of equations (7) and (10) are $W_H = S^N_H = S^T_H$. The steady state of equations (8) and (11) imply

$$\frac{W_H}{P_H} = \frac{\theta-1}{\theta}. \tag{13}$$

The steady state of equation (1) implies

$$\chi N_H^{\varphi^{-1}} = \mu^{-1}, \tag{14}$$

where N_H is the steady state per capita employment of households in the home region and $\mu = \theta/(\theta-1)$. The steady state of equation (2) implies $\beta R^n = 1$.

Since all firms face the same marginal cost in the steady state we consider, all prices will be equal and all relative goods prices will be one. This implies that in steady state the demand curves for home tradeable and non-tradeable goods—equations (3) and (4)—imply

$$C_H^N = \phi_N C_H, \tag{15}$$

$$C_H^{TH} = \phi_T \tau_H^H C_H, \tag{16}$$

$$C_H^{TF} = \phi_T \tau_H^F C_H. \tag{17}$$

We define total labor in the home non-tradeable and tradeable sectors as $N_{Ht}^N \equiv \int_0^1 N_{Ht}^N(z)dz$ and $N_{Ht}^T \equiv \int_0^1 N_{Ht}^T(z)dz$, respectively. In steady state, total non-tradeable employment, N_H^N, must equal total non-tradeable consumption, $\zeta\phi_N C$. This implies that

$$N_H^N = \zeta C_H^N = \zeta \phi_N C$$

where the second equality substitutes in equation (15). Similarly, steady state home tradeable employment, N_H^T, equals steady state total consumption for home tradeables. This implies that

$$N_H^T = \zeta C_H^{TH} + (1-\zeta)\, C_F^{TH}.$$

Using equations (16) and (17), we get that

$$N_H^T = \zeta \phi_T \tau_H^H C_H + (1-\zeta)\phi_T \tau_F^H C_F$$

Using that fact that $C_F = C_H = C$ in a symmetric steady state, we get that

$$N_H^T = \zeta \phi_T C \left(\tau_H^H + \frac{1-\zeta}{\zeta} \tau_F^H \right)$$

Finally, using the fact that $\tau_H^H = \tau_F^H = \zeta$ (no home bias in steady state), we get that

$$N_H^T = \zeta \phi_T C.$$

A.4 Derivation of Regional Phillips Curves

A first order Taylor-series expansion of equation (9) around the zero inflation and balanced trade steady state yields

$$p_{Ht}^{N*}(z) - p_{H,t-1}^{N} = (1-\alpha\beta)\sum_{k=0}^{\infty}(\alpha\beta)^k E_t\left[\widehat{mc}_{H,t+k}^{N} - \left(p_{H,t+k}^{N} - p_{H,t-1}^{N}\right)\right].$$

Rearranging this equation yields

$$p_{Ht}^{N*}(z) - p_{H,t-1}^{N} = \alpha\beta E_t\left[p_{H,t+1}^{N*}(z) - p_{Ht}^{N}\right] + (1-\alpha\beta)\,\widehat{mc}_{Ht}^{N} + \pi_{Ht}^{N}. \tag{18}$$

The expression for P_{Ht}^{N} given in the line above equation (6) implies

$$P_{Ht}^{N1-\theta} = \alpha P_{H,t-1}^{N1-\theta} + (1-\alpha)\,P_{Ht}^{N*1-\theta}$$

where P_{Ht}^{N*} denotes the reset price of firms that are able to change their price in period t. Here we exploit the fact that a random set of firms change their prices at time t and all of these firms set the same price. A first order Taylor series approximation of this last expression is

$$p_{Ht}^{N} = \alpha p_{H,t-1}^{N} + (1-\alpha)\,p_{Ht}^{N*}$$

which implies that

$$\pi_{Ht}^{N} = (1-\alpha)\left(p_{Ht}^{N*} - p_{H,t-1}^{N}\right). \tag{19}$$

Manipulation of equations (18) and (19) yields that

$$\pi_{Ht}^{N} = \beta E_t\pi_{H,t+1}^{N} + \lambda\widehat{mc}_{Ht}^{N} \tag{20}$$

where

$$\lambda = \frac{(1-\alpha)(1-\alpha\beta)}{\alpha}.$$

We can derive an analogous equation to equation (20) for the tradeable sector. In the tradeable sector we have that

$$\pi_{Ht}^{T} = \beta E_t\pi_{H,t+1}^{T} + \lambda\widehat{mc}_{Ht}^{T}, \tag{21}$$

where $\pi_{Ht}^{T} = p_{Ht}^{T} - p_{H,t-1}^{T}$ is producer price inflation in the home tradeable sector.

Taking logs of equation (7) implies that

$$\hat{mc}^N_{Ht} = \hat{w}_{Ht} - p^N_{Ht} - z^N_{Ht}.$$

Taking logs of labor supply—equation (1)—implies that

$$\hat{w}_{Ht} - p^N_{Ht} = \varphi^{-1}\hat{n}_{Ht}.$$

Combining these two equation yields

$$\hat{mc}^N_{Ht} = \varphi^{-1}\hat{n}_{Ht} + \left(p_{Ht} - p^N_{Ht}\right) - z^N_{Ht}. \tag{22}$$

We can substitute this equation into equation (21) to get that

$$\pi^N_{Ht} = \beta E_t \pi^N_{H,t+1} + \kappa \hat{n}_{Ht} - \lambda \hat{p}^N_{Ht} + \nu^N_{Ht} \tag{23}$$

where $\nu^N_{Ht} = -\lambda z^N_{Ht}$ and $\kappa = \lambda\varphi^{-1}$. This is the regional non-tradeable Phillips Curve in our model.

An analogous sequence of steps yields

$$\pi^T_{Ht} = \beta E_t \pi^T_{H,t+1} + \kappa \hat{n}_{Ht} - \lambda \hat{p}^T_{Ht} + \nu^T_{Ht}. \tag{24}$$

This is the regional tradeable Phillips Curve in our model.

A.5 Aggregate Phillips Curve Derivation

Aggregate non-tradeable inflation can be written as $\pi^N_t = \zeta\pi^N_{Ht} + (1-\zeta)\,\pi^N_{Ft}$. Using the Phillips curve for home non-tradeable inflation—equation (23)—and its foreign counterpart, we get that satisfies

$$\pi^N_t = \beta E_t \pi^N_{t+1} + \kappa \hat{n}_t + \nu^N_t - \lambda\left[\zeta \hat{p}^N_{Ht} + (1-\zeta)\,\hat{p}^N_{Ft}\right], \tag{25}$$

Similarly, a weighted average of the Phillips curve for home tradeable inflation—equation (24)—and its foreign counterpart yields

$$\pi^T_t = \beta E_t \pi^T_{t+1} + \kappa \hat{n}_t + \nu^T_t - \lambda\left[\zeta \hat{p}^T_{Ht} + (1-\zeta)\,\hat{p}^T_{Ft}\right]. \tag{26}$$

First order expansions of equation 6 and its foreign counterpart around the zero inflation steady state yield that

$$p_{Ht} = \phi_N p_{Ht}^N + \phi_T \tau_H^H p_{Ht}^T + \phi_T \tau_H^F p_{Ft}^T \tag{27}$$

and

$$p_{Ft} = \phi_N p_{Ft}^N + \phi_T \tau_F^H p_{Ht}^T + \phi_T \tau_F^F p_{Ft}^T. \tag{28}$$

Then the aggregate price level, p_t satisfies $p_t = \zeta p_{Ht} + (1-\zeta) p_{Ft}$. Combining this equation with the previous two equations and using the fact that $\tau_H^H = \tau_F^H = \zeta$ yields

$$p_t = \phi_N \left(\zeta p_{Ht}^N + (1-\zeta) p_{Ft}^N\right) + \phi_T \left(\zeta\zeta p_{Ht}^T + \zeta (1-\zeta) p_{Ft}^T + (1-\zeta) \zeta p_{Ht}^T + (1-\zeta)(1-\zeta) p_{Ft}^T\right)$$

This equation simplifies to

$$p_t = \phi_N p_t^N + \phi_T p_t^T \tag{29}$$

where we use the notation $p_t^N = \zeta p_{Ht}^N + (1-\zeta) p_{Ft}^N$ and $p_t^T = \zeta p_{Ht}^T + (1-\zeta) p_{Ft}^T$.

Equation (29) implies that

$$\pi_t = \phi_N \pi_t^N + \phi_T \pi_t^T.$$

Combining this equation with equations (23) and (24) yields the aggregate Phillips curve

$$\pi_t = \beta E_t \pi_{t+1} + \kappa \hat{n}_t + \nu_t,$$

where $\nu_t \equiv \phi_N \nu_t^N + \phi_T \nu_t^T$ and we make use of the fact that

$$\left[\zeta \hat{p}_{Ht}^N + (1-\zeta) \hat{p}_{Ft}^N\right] + \left[\zeta \hat{p}_{Ht}^T + (1-\zeta) \hat{p}_{Ft}^T\right] = 0.$$

A.6 Deriving the Other Log-Linearized Equations

In this section, we will assume that supply shocks are zero for expositional simplicity. A log-linear approximation of the home consumption Euler equation—equation (2)—yields

$$\hat{c}_{Ht} + \frac{u_{cn}}{u_{cc}} \hat{n}_{Ht} = E_t \left[\hat{c}_{H,t+1} + \frac{u_{cn}}{u_{cc}} \hat{n}_{H,t+1}\right] + \frac{u_c}{u_{cc} C} \left(\hat{r}_t^n - E_t \pi_{H,t+1}\right), \tag{30}$$

where we use the fact that home per capita consumption and labor equal aggregate per capita consumption and labor at the steady state.

Next, we solve for the partial derivatives in the previous equation using the functional form for preferences, equation (8). We have that

$$\begin{aligned}
\frac{u_{cc}C}{u_c} &= \frac{-\sigma^{-1}C\left(C-\chi\frac{N^{1+\varphi^{-1}}}{1+\varphi^{-1}}\right)^{-\sigma^{-1}-1}}{\left(C-\chi\frac{N^{1+\varphi^{-1}}}{1+\varphi^{-1}}\right)^{-\sigma^{-1}}} \\
&= -\sigma^{-1}C\left(C-\chi\frac{N^{1+\varphi^{-1}}}{1+\varphi^{-1}}\right)^{-1} \\
&= -\sigma^{-1}\left(C^{-1}\left(C-\chi\frac{N^{1+\varphi^{-1}}}{1+\varphi^{-1}}\right)\right)^{-1} \\
&= -\sigma^{-1}\left(1-C^{-1}\chi N^{1+\varphi^{-1}}\left(1+\varphi^{-1}\right)^{-1}\right)^{-1} \\
&= -\sigma^{-1}\left(1-\left(\frac{C}{N}\right)^{-1}\chi N^{\varphi^{-1}}\left(1+\varphi^{-1}\right)^{-1}\right)^{-1} \\
&= -\sigma^{-1}\left(1-\left(\frac{C}{N}\right)^{-1}\mu^{-1}\left(1+\varphi^{-1}\right)^{-1}\right)^{-1} \\
&= -\sigma^{-1}\left(1-\mu^{-1}\left(1+\varphi^{-1}\right)^{-1}\right)^{-1},
\end{aligned}$$

where we use the steady state labor supply curve—equation (14)—and the fact that in the steady state $C = N$. Furthermore, we have that

$$\begin{aligned}
u_{cn} &= -\sigma^{-1}\left(C-\chi\frac{N^{1+\varphi^{-1}}}{1+\varphi^{-1}}\right)^{-\sigma^{-1}-1} \times -\frac{\chi}{1+\varphi^{-1}}\left(1+\varphi^{-1}\right)N^{\varphi^{-1}} \\
&= -u_{cc}\chi N^{\varphi^{-1}} \\
&= -u_{cc}\mu^{-1},
\end{aligned}$$

where we again make use of equation (14).

Combining this last to equations with equation (30) yields

$$\hat{c}_{Ht} - \mu^{-1}\hat{n}_{Ht} = E_t\left[\hat{c}_{H,t+1} - \mu^{-1}\hat{n}_{H,t+1}\right] - \sigma_c\left(\hat{r}^n_t - E_t\pi_{H,t+1}\right)$$

where $\sigma_c = \sigma\left(1-\mu^{-1}\left(1+\varphi^{-1}\right)^{-1}\right)$.

Solving this last equation forward yields

$$\begin{aligned}
\hat{c}_{Ht} - \mu^{-1}\hat{n}_{Ht} &= -\sigma_c E_t \sum_{j=0}^{\infty} \left(\hat{r}^n_{t+j} - E_t \pi_{H,t+1+j} \right) \\
&= -\sigma_c E_t \sum_{j=0}^{\infty} \hat{r}^n_{t+j} + \sigma_c E_t \sum_{j=0}^{\infty} \pi_{H,t+1+j} \\
&= -\sigma_c E_t \sum_{j=0}^{\infty} \hat{r}^n_{t+j} + \sigma_c E_t \sum_{j=0}^{\infty} \left(p_{H,t+1+j} - p_{H,t+j} \right) \\
&= -\sigma_c E_t \sum_{j=0}^{\infty} \hat{r}^n_{t+j} - \sigma_c p_{Ht}. \qquad (31)
\end{aligned}$$

Similarly, for foreign households we have

$$\hat{c}_{Ft} - \mu^{-1}\hat{n}_{Ft} = -\sigma_c E_t \sum_{j=0}^{\infty} \hat{r}^n_{t+j} - \sigma_c p_{Ft}. \qquad (32)$$

Combining equations (31) and (32) yields

$$\hat{c}_{Ht} - \mu^{-1}\hat{n}_{Ht} = \hat{c}_{Ft} - \mu^{-1}\hat{n}_{Ft} + \sigma_c \left(p_{Ft} - p_{Ht} \right),$$

which is the Backus-Smith condition for our model.

Define $\hat{\xi}_{Ht} = \log \tau_{Ht} - \log \tau_H$ and $\hat{\xi}_{Ft} = \log \tau_{Ft} - \log \tau_F$. With this notation, log-linear approximations of (3) and (4) as well as their foreign counterparts yields

$$\hat{c}^N_{Ht} = \hat{c}_{Ht} - \eta \left(p^N_{Ht} - p_{Ht} \right) \qquad (33)$$

$$\hat{c}^T_{Ht} = \hat{\xi}_{Ht} + \hat{c}_{Ht} - \eta \left(p^T_{Ht} - p^T_{Ht} \right) \qquad (34)$$

$$\hat{c}^T_{Ft} = \hat{\xi}_{Ft} + \hat{c}_{Ht} - \eta \left(p^T_{Ft} - p^T_{Ht} \right) \qquad (35)$$

$$\hat{c}^N_{Ft} = \hat{c}_{Ft} - \eta \left(p^N_{Ft} - p_{Ft} \right) \qquad (36)$$

$$\hat{c}^{TH}_{Ft} = \hat{c}_{Ft} - \eta \left(p^T_{Ht} - p_{Ft} \right) \qquad (37)$$

$$\hat{c}^{TF}_{Ft} = \hat{c}_{Ft} - \eta \left(p^T_{Ft} - p_{Ft} \right). \qquad (38)$$

Note that the expenditure share on tradeable and non-tradeable goods always sums to 1: $\tau_{Ht} +$

$\tau_{Ft} = 1$. This implies that

$$\hat{\xi}_{Ht}\tau_H^H + \hat{\xi}_{Ft}\tau_H^F = 0$$

which in turn implies that

$$\hat{\xi}_{Ft} = -\frac{\zeta}{1-\zeta}\hat{\xi}_{Ht}.$$

First differencing equations (27) and (28) implies

$$\pi_{Ht} = \phi_N \pi_{Ht}^N + \phi_T \tau_H^H \pi_{Ht}^T + \phi_T \tau_H^F \pi_{Ft}^T$$

and

$$\pi_{Ft} = \phi_N \pi_{Ft}^N + \phi_T \tau_F^H \pi_{Ht}^T + \phi_T \tau_H^F \pi_{Ft}^T.$$

Note that without supply shocks, output and employment are equal. This implies that

$$Y_{Ht}^N = N_{Ht}^N, \quad Y_{Ht}^T = N_{Ht}^T, \quad Y_{Ft}^N = N_{Ft}^N, \quad Y_{Ft}^T = N_{Ft}^T.$$

We furthermore have that

$$\zeta N_{Ht} = N_{Ht}^N + N_{Ht}^T.$$

This equation says that total labor supplied by households in the home region equals total labor demanded by firms. The ζ on the left-hand-side reflects the fact that N_{Ht} is per capita labor supply. A log-linear approximation of this last expression around the symmetric steady state yields

$$\begin{aligned}\hat{n}_{Ht} &= \frac{N^N}{N^N + N^T}\hat{n}_{Ht}^N + \frac{N^T}{N^N + N^T}\hat{n}_{Ht}^T \\ &= \phi_N \hat{n}_{Ht}^N + \phi_T \hat{n}_{Ht}^T\end{aligned}$$

Similarly, in the foreign region we have that

$$\hat{n}_{Ft} = \phi_N \hat{n}_{Ft}^N + \phi_T \hat{n}_{Ft}^T.$$

Aggregate employment is

$$N_t = \zeta N_{Ht} + (1-\zeta) N_{Ft}.$$

Log-linearizing this equation around the symmetric steady state yields

$$\begin{aligned}\hat{n}_t &= \frac{\zeta N}{\zeta N + (1-\zeta) N}\hat{n}_{Ht} + \frac{(1-\zeta) N}{\zeta N + (1-\zeta) N}\hat{n}_{Ft} \\ &= \zeta \hat{n}_{Ht} + (1-\zeta)\,\hat{n}_{Ft},\end{aligned}$$

where N is steady state household labor supply, equal across the two regions at the symmetric steady state.

Market clearing conditions in the non-tradeable sector implies that $N_{Ht}^{N} = \zeta C_{Ht}^{N}$. A log-linear approximation of this expression yields that

$$\hat{n}_{Ht}^{N} = \hat{c}_{Ht}^{N}.$$

Using equation (33), we get that

$$\hat{n}_{Ht}^{N} = \hat{c}_{Ht} - \eta\left(\hat{p}_{Ht}^{N} - \hat{p}_{Ht}\right)$$

A similar set of steps for the foreign region yields

$$\hat{n}_{Ft}^{N} = \hat{c}_{Ft} - \eta\left(p_{Ft}^{N} - p_{Ft}\right).$$

In the tradeable sector market clearing implies

$$\begin{aligned}N_{Ht}^{T} &= \zeta C_{Ht}^{TH} + (1-\zeta)\, C_{Ft}^{TH} \\ &= \zeta \phi_T \tau_{Ht}^{H} C_{Ht}\left(\frac{P_{Ht}^{T}}{P_{Ht}}\right)^{-\eta} + (1-\zeta)\,\phi_T \tau_{Ft}^{H} C_{Ft}\left(\frac{P_{Ft}^{T}}{P_{Ft}}\right)^{-\eta}\end{aligned}$$

where the second line follows from equations (3) and (4). Log-linearizing around the symmetric steady state implies

$$\hat{n}_{Ht}^{T} = \zeta\left[\hat{c}_{Ht} - \eta\left(p_{Ht}^{T} - p_{Ht}\right) + \hat{\xi}_{Ht}\right] + (1-\zeta)\left[\hat{c}_{Ft} - \eta\left(p_{Ht}^{T} - p_{Ft}\right)\right].$$

Similarly, in the foreign region we have

$$\hat{n}_{Ft}^{T} = \zeta\left[\hat{c}_{Ht} - \eta\left(p_{Ft}^{T} - p_{Ht}\right) + \hat{\xi}_{Ft}\right] + (1-\zeta)\left[\hat{c}_{Ft} - \eta\left(p_{Ft}^{T} - p_{Ft}\right)\right].$$

Finally, we define deviations of unemployment from the steady state as

$$\hat{n}_{Ht} = \log N_{Ht} - \log N_H \approx (N_{Ht} - 1) - (N_H - 1) = -(u_{Ht} - u_H) = -\hat{u}_{Ht}.$$

A.7 Log-Linearized Equations of the Model

For convenience we repeat the full set of log-linearized equilibrium conditions of the model.

- Parameters:
  - $\sigma_c = \sigma \left(1 - \mu^{-1} \left(1 + \varphi^{-1}\right)^{-1}\right)$
  - $\kappa = \lambda \varphi^{-1}$
  - $\lambda = (1 - \alpha)(1 - \alpha\beta)/\alpha$
  - $\mu = \theta / (\theta - 1)$
  - $\tau_H^H = \tau_F^H = \zeta$
- The law of motion for tradeable demand is

$$\hat{\xi}_{Ht} = \rho_\xi \hat{\xi}_{Ht} + \varepsilon_t$$

  and

$$\hat{\xi}_{Ft} = -\frac{\zeta}{1-\zeta} \hat{\xi}_{Ht}.$$

- The home non-tradeable Phillips Curve is:

$$\pi_{Ht}^N = \beta E_t \pi_{H,t+1}^N - \kappa \hat{u}_{Ht} - \lambda \hat{p}_{Ht}^N + \nu_{Ht}^N$$

- The home tradeable Phillips Curve is:

$$\pi_{Ht}^T = \beta E_t \pi_{H,t+1}^T - \kappa \hat{u}_{Ht} - \lambda \hat{p}_{Ht}^T + \nu_{Ht}^T.$$

- The home Euler equation is:

$$\hat{c}_{Ht} - \mu^{-1} \hat{n}_{Ht} = E_t \left[\hat{c}_{H,t+1} - \mu^{-1} \hat{n}_{H,t+1}\right] - \sigma_c \left(\hat{r}_t^n - E_t \pi_{H,t+1}\right)$$

- The Backus-Smith condition is:

$$\hat{c}_{Ht} - \mu^{-1}\hat{n}_{Ht} = \hat{c}_{Ft} - \mu^{-1}\hat{n}_{Ft} + \sigma_c\left(p_{Ft} - p_{Ht}\right)$$

- The foreign non-tradeable Phillips Curve is:

$$\pi_{Ft}^{N} = \beta E_t \pi_{F,t+1}^{N} + \kappa\hat{n}_{Ft} - \lambda\hat{p}_{Ft}^{N} + \nu_{Ft}^{N}$$

- The foreign tradeable Phillips Curve is:

$$\pi_{Ft}^{T} = \beta E_t \pi_{F,t+1}^{T} + \kappa\hat{n}_{Ft} - \lambda\hat{p}_{Ft}^{T} + \nu_{Ft}^{T}$$

- Definitions of inflation:

$$\pi_{Ht} = p_{Ht} - p_{H,t-1}$$

$$\pi_{Ft} = p_{Ft} - p_{F,t-1}$$

$$\pi_{Ht}^{N} = p_{Ht}^{N} - p_{H,t-1}^{N}$$

$$\pi_{Ht}^{T} = p_{Ht}^{T} - p_{H,t-1}^{T}$$

$$\pi_{Ft}^{N} = p_{Ft}^{N} - p_{F,t-1}^{N}$$

$$\pi_{Ft}^{T} = p_{Ft}^{T} - p_{F,t-1}^{T}$$

$$\pi_{Ht} = \phi_N \pi_{Ht}^{N} + \phi_T \tau_H^H \pi_{Ht}^{T} + \phi_T \tau_H^F \pi_{Ft}^{T}$$

$$\pi_{Ft} = \phi_N \pi_{Ft}^{N} + \phi_T \tau_F^H \pi_{Ht}^{T} + \phi_F \tau_F^F \pi_{Ft}^{T}$$

- The home resource constraint in the non-tradeable sector is:

$$\hat{n}_{Ht}^{N} = \hat{c}_{Ht} - \eta\left(p_{Ht}^{N} - p_{Ht}\right)$$

- The foreign resource constraint in the non-tradeable sector is:

$$\hat{n}_{Ft}^{N} = \hat{c}_{Ft} - \eta\left(p_{Ft}^{N} - p_{Ft}\right)$$

- The home resource constraint in the tradeable sector is:

$$\hat{n}_{Ht}^{T} = \zeta \left[\hat{c}_{Ht} - \eta \left(p_{Ht}^{T} - p_{Ht} \right) + \hat{\xi}_{Ht} \right] + (1 - \zeta) \left[\hat{c}_{Ft} - \eta \left(p_{Ht}^{T} - p_{Ft} \right) \right]$$

- The foreign resource constraint in the tradeable sector is:

$$\hat{n}_{Ft}^{T} = \zeta \left[\hat{c}_{Ht} - \eta \left(p_{Ft}^{T} - p_{Ht} \right) + \hat{\xi}_{Ft} \right] + (1 - \zeta) \left[\hat{c}_{Ft} - \eta \left(p_{Ft}^{T} - p_{Ft} \right) \right]$$

- Aggregate labor in the home region then satisfies the log-linear equations

$$\hat{n}_{Ht} = \phi_N \hat{n}_{Ht}^{N} + \phi_T \hat{n}_{Ht}^{T}$$

- Aggregate labor in the foreign region satisfies

$$\hat{n}_{Ft} = \phi_N \hat{n}_{Ft}^{N} + \phi_T \hat{n}_{Ft}^{T}$$

- Monetary policy is

$$\hat{r}_t^n = \varphi_\pi \left(\pi_t - \bar{\pi}_t \right) + \varphi_n \left(\hat{n}_t - \bar{n}_t \right) + \varepsilon_{rt}$$

- Aggregate employment satisfies

$$\hat{n}_t = \zeta \hat{n}_{Ht} + (1 - \zeta) \hat{n}_{Ft}$$

- Aggregate inflation satisfies

$$\hat{\pi}_t = \zeta \hat{\pi}_{Ht} + (1 - \zeta) \hat{\pi}_{Ft}$$

- The deviation of unemployment from its steady state value is

$$\hat{u}_t = -\hat{n}_t.$$

A.8 The Importance of Non-Tradeable Inflation

Here, we show that the slope of the regional Phillips Curve for overall regional consumer price inflation is smaller than the slope of the aggregate Phillips Curve, by a factor equal to the expenditure share on non-tradeable goods. For simplicity, we present this derivation with all supply shocks ν_t set to zero.

Consider the Phillips curves for home non-tradeables, home tradeables, and foreign tradeables:

$$\pi^N_{Ht} = \beta E_t \pi^N_{H,t+1} - \kappa \hat{u}_{Ht} - \lambda \hat{p}^N_{Ht}$$

$$\pi^T_{Ht} = \beta E_t \pi^T_{H,t+1} - \kappa \hat{u}_{Ht} - \lambda \hat{p}^T_{Ht}$$

$$\pi^T_{Ft} = \beta E_t \pi^T_{F,t+1} - \kappa \hat{u}_{Ft} - \lambda \hat{p}^T_{Ft}.$$

Substituting these three equations into the definition for home consumer price inflation

$$\pi_{Ht} = \phi_N \pi^N_{Ht} + \phi_T \tau^H_H \pi^T_{Ht} + \phi_T \tau^F_H \pi^T_{Ft}$$

yields

$$\pi_{Ht} = \beta E_t \pi_{H,t+1} - \left(\phi_N + \phi_T \tau^H_H\right) \kappa \hat{u}_{Ht} - \lambda \left(\phi_N \hat{p}^N_{Ht} + \phi_T \tau^H_H \hat{p}^T_{Ht}\right) - \phi_T \tau^F_H \kappa \hat{u}_{Ft} - \lambda \phi_T \tau^F_H \hat{p}^T_{Ft}.$$

An analogous derivation yields the following Phillips curve for foreign consumer prices

$$\pi_{Ft} = \beta E_t \pi_{F,t+1} - \left(\phi_N + \phi_T \tau^F_F\right) \kappa \hat{u}_{Ft} - \lambda \left(\phi_N \hat{p}^N_{Ft} + \phi_T \tau^F_F \hat{p}^T_{Ft}\right) - \phi_T \tau^H_F \kappa \hat{u}_{Ht} - \lambda \phi_T \tau^H_F \hat{p}^T_{Ht}.$$

Subtracting the second of these last two equations from the first (and using the fact that $\tau^H_H = \tau^H_F = \zeta$) yields

$$\pi_{Ht} - \pi_{Ft} = \beta \left(E_t \pi_{H,t+1} - E_t \pi_{F,t+1}\right) - \phi_N \kappa \left(\hat{u}_{Ht} - \hat{u}_{Ft}\right) - \phi_N \lambda \left(\hat{p}^N_{Ht} - \hat{p}^N_{Ft}\right). \tag{39}$$

The coefficient in a regional panel regression corresponds to the coefficient in a differenced equation like this one. Notice that the coefficient on unemployment is $\phi_N \kappa$ rather than κ. In other words, the coefficient differs from the coefficient in the aggregate Phillips curve by the factor ϕ_N.

A.9 The Role of GHH Preferences

The key feature of GHH preferences that we exploit is that, with GHH preferences, there are no wealth effects on labor supply either at the aggregate or the regional level. In contrast, with separable preferences, wealth effects on labor supply are an important determinant of marginal cost and therefore influence the Phillips curve.

To see this more clearly, consider the non-tradeable regional Phillips curve under separable preferences:

$$\pi_{Ht}^N = \beta E_t \pi_{H,t+1}^N - \kappa \hat{u}_{Ht} + \lambda\sigma^{-1}\hat{c}_{Ht} - \lambda\hat{p}_{Ht}^N + \nu_{Ht}^N, \tag{40}$$

and the aggregate Phillips Curve under separable preferences:

$$\pi_t = \beta E_t \pi_{t+1} - \kappa \hat{u}_t + \lambda\sigma^{-1}\hat{c}_t + \nu_t. \tag{41}$$

Relative to the GHH case, both the non-tradeable regional Phillips curve and aggregate Phillips curve include a consumption term. These terms appear because of wealth effects on labor supply affect marginal cost in this model. These wealth effects complicate the comparison between the regional and aggregate Phillips curve because the relationship between employment and consumption is different at the aggregate level than at the regional level. At the aggregate level, $\hat{c}_t = \hat{n}_t + z_t$. This implies that we can replace the $\hat{c}_t$ term with $\hat{n}_t + z_t$ in equation (41) and get a consolidated coefficient of $\kappa + \lambda\sigma^{-1}$ on unemployment. At the regional level, however, this is not possible because risk-sharing across regions implies that $\hat{c}_{Ht} \neq \hat{n}_{Ht} + z_{Ht}$. This difference implies that the slope of the non-tradeable regional Phillips curve will differ from the slope of the aggregate Phillips curve when preferences are separable.

A.10 Relaxing Full Information Rational Expectations

To derive the solved forward Phillips Curve, equation (13) in the main text, we manipulated the Phillips curve under the standard assumption of full-information rational expectations. However, this type of derivation actually only relies on the weaker assumption that the law of iterated expectations holds. Let's consider the aggregate Phillips curve—equation (12)—for simplicity. Under the assumption that the law of iterated expectations holds and the additional simplifying assumption that the unemployment rate follows an AR(1) process, we can solve this equation forward to get that

$$\pi_t = -\frac{\kappa}{1 - \rho_{\tilde{u}}^F \beta}\tilde{u}_t + F_t \pi_{t+\infty} + \tilde{\omega}_t, \tag{42}$$

where F_t denotes agents' expectations conditional on information at time t, $F_t \pi_{t+\infty}$ is the agent's subjective forecast about the inflation target, $\tilde{\omega}_t \equiv F_t \sum_{j=0}^{\infty} \beta^j \nu_{t+j}$, and $\rho_{\tilde{u}}^F$ is agents' subjective belief about the autoregressive coefficient governing the persistence of fluctuations in unemployment. Notice that if $\rho_{\tilde{u}}^F < \rho_{\tilde{u}}$, the Phillips Curve is less forward looking than the rational expectations Phillips curve. Rational expectations is the special case where $\rho_{\tilde{u}}^F = \rho_{\tilde{u}}$ and $F_t\pi_{t+\infty} = E_t\pi_{t+\infty}$. Coibion and Gorodnichenko (2012, 2015) provide evidence consitent with the law of iterated expectations holding but full information rational expectations not holding. See Adam and Padula (2011), Coibion and Gorodnichenko (2015), Coibion, Gorodnichenko, and Kamdar (2018) for further discussion of these issues.

A.11 Time Aggregation

Here, we show how time aggregation associated with using four-quarter inflation as our dependent variable implies that we should divide our estimate of κ by 4 since our model in section 3 is written in terms of quarterly inflation. Consider the non-tradeable regional Phillips Curve—equation (13) from the main text:

$$\pi_{Ht}^N = -E_t \sum_{j=0}^{\infty} \beta^j \left(\kappa \tilde{u}_{H,t+j} + \lambda \hat{p}_{H,t+j}^N\right) + E_t\pi_{t+\infty}, \tag{43}$$

where for simplicity we have set the supply shock ω_{Ht}^N equal to zero. We can rewrite this last equation as

$$p_{Ht}^N - p_{H,t-1}^N = -\kappa PV_{Ht}^u - \lambda PV_{Ht}^p + E_t\pi_{t+\infty}$$

where

$$PV_{Ht}^u = E_t \sum_{j=0}^{\infty} \beta^j \tilde{u}_{H,t+j}$$

$$PV_{Ht}^p = E_t \sum_{j=0}^{\infty} \beta^j \hat{p}_{H,t+j}^N.$$

This same equation hold for periods t, $t-1$, $t-2$, and $t-3$:

$$\begin{aligned}
p_{Ht}^N - p_{H,t-1}^N &= -\kappa PV_{Ht}^u - \lambda PV_{Ht}^p + E_t\pi_{t+\infty} \\
p_{H,t-1}^N - p_{H,t-2}^N &= -\kappa PV_{H,t-1}^u - \lambda PV_{H,t-1}^p + E_{t-1}\pi_{t+\infty} \\
p_{H,t-2}^N - p_{H,t-3}^N &= -\kappa PV_{H,t-2}^u - \lambda PV_{H,t-2}^p + E_{t-2}\pi_{t+\infty} \\
p_{H,t-3}^N - p_{H,t-4}^N &= -\kappa PV_{H,t-3}^u - \lambda PV_{H,t-3}^p + E_{t-3}\pi_{t+\infty}
\end{aligned}$$

Summing the preceding four equations together yields

$$
\begin{aligned}
p_{Ht}^{N} - p_{H,t-4}^{N} = & -\kappa \left(PV_{Ht}^{u} + PV_{H,t-1}^{u} + PV_{H,t-2}^{u} + PV_{H,t-3}^{u} \right) \\
& - \lambda \left(PV_{Ht}^{p} + PV_{H,t-1}^{p} + PV_{H,t-2}^{p} + PV_{H,t-3}^{p} \right) \\
& + E_t \pi_{t+\infty} + E_{t-1} \pi_{t+\infty} + E_{t-2} \pi_{t+\infty} + E_{t-3} \pi_{t+\infty}.
\end{aligned}
$$

Taking expectations at time $t-4$ then yields

$$
\begin{aligned}
E_{t-4} p_{Ht}^{N} - p_{H,t-4}^{N} = & -\kappa \left(E_{t-4} PV_{Ht}^{u} + E_{t-4} PV_{H,t-1}^{u} + E_{t-4} PV_{H,t-2}^{u} + E_{t-4} PV_{H,t-3}^{u} \right) \\
& - \lambda \left(E_{t-4} PV_{Ht}^{p} + E_{t-4} PV_{H,t-1}^{p} + E_{t-4} PV_{H,t-2}^{p} + E_{t-4} PV_{H,t-3}^{p} \right) \\
& + 4 E_{t-4} \pi_{t+\infty}.
\end{aligned}
$$

Adding and subtracting p_{Ht}^{N} yields

$$
\begin{aligned}
p_{Ht}^{N} - p_{H,t-4}^{N} = & -\kappa \left(E_{t-4} PV_{Ht}^{u} + E_{t-4} PV_{H,t-1}^{u} + E_{t-4} PV_{H,t-2}^{u} + E_{t-4} PV_{H,t-3}^{u} \right) \\
& - \lambda \left(E_{t-4} PV_{Ht}^{p} + E_{t-4} PV_{H,t-1}^{p} + E_{t-4} PV_{H,t-2}^{p} + E_{t-4} PV_{H,t-3}^{p} \right) \\
& + 4 E_{t-4} \pi_{t+\infty} - \left(E_{t-4} p_{Ht}^{N} - p_{Ht}^{N} \right).
\end{aligned}
$$

We now assume that PV_{Ht}^{u} and PV_{Ht}^{p} are well approximated by univariate driftless random walks. We present empirical evidence supporting this assumption in section A.11.1 below. Given this assumption, the preceding equation simplifies to

$$
\begin{aligned}
p_{Ht}^{N} - p_{H,t-4}^{N} &= -4\kappa E_{t-4} PV_{Ht}^{u} \\
&\quad - 4\lambda E_{t-4} PV_{Ht}^{p} \\
&\quad + 4E_{t-4}\pi_{t+\infty} - \left(E_{t-4} p_{Ht}^{N} - p_{Ht}^{N}\right) \\
&= -4\kappa \sum_{j=0}^{\infty} \beta^{j} \tilde{u}_{H,t+j} - 4\lambda \sum_{j=0}^{\infty} \beta^{j} \hat{p}_{H,t+j}^{N} + 4E_{t-4}\pi_{t+\infty} \\
&\quad - 4\kappa \left(E_{t-4} \sum_{j=0}^{\infty} \beta^{j} \tilde{u}_{H,t+j} - \sum_{j=0}^{\infty} \beta^{j} \tilde{u}_{H,t+j} \right) \\
&\quad - 4\lambda \left(E_{t-4} \sum_{j=0}^{\infty} \beta^{j} \hat{p}_{H,t+j}^{N} - \sum_{j=0}^{\infty} \beta^{j} \hat{p}_{H,t+j}^{N} \right) \\
&\quad - \left(E_{t-4} p_{Ht}^{N} - p_{Ht}^{N}\right) \\
&= -4\kappa \sum_{j=0}^{\infty} \beta^{j} \tilde{u}_{H,t+j} - 4\lambda \sum_{j=0}^{\infty} \beta^{j} \hat{p}_{H,t+j}^{N} + 4E_{t-4}\pi_{t+\infty} + \upsilon_{Ht}
\end{aligned}
$$

where υ_{Ht} is a rational expectations error uncorrelated with variables at time $t-4$. This last equation shows that estimating equation (17) with the dependent variable defined as $\pi_{it}^{N} = p_{it}^{N} - p_{i,t-4}^{N}$ yields estimates of 4κ and 4λ provided that the forward sums in equation (17) are well approximated by a random walk.

A.11.1 The Dynamics of the Present Value of Unemployment and Relative Prices

The derivation above relied on the simplifying assumption that $\sum_{j=0}^{\infty} \beta^{j} \tilde{u}_{H,t+j}$ and $\sum_{j=0}^{\infty} \beta^{j} \hat{p}_{H,t+j}^{N}$ follow univariate random walks. We can assess the accuracy of this assumption by running the regressions

$$
\sum_{j=0}^{T} \beta^{j} u_{i,t+j} = \alpha_i + \gamma_t + \rho_{uu} \sum_{j=0}^{T} \beta^{j} u_{i,t+j-1} + \rho_{up} \sum_{j=0}^{T} \beta^{j} p_{i,t+j-1}^{N}
$$

$$
\sum_{j=0}^{T} \beta^{j} p_{i,t+j}^{N} = \alpha_i + \gamma_t + \rho_{pu} \sum_{j=0}^{T} \beta^{j} u_{i,t+j-1} + \rho_{pp} \sum_{j=0}^{T} \beta^{j} p_{i,t+j-1}^{N}.
$$

As in the main text, we truncate the infinite sums at $T = 20$. Table A.1 presents results for these regression. In both regressions, the coefficient on the lag of the dependent variable is very close to 1, whereas the coefficient on the lag of the other future sum is near zero. This shows that both the present value of both unemployment and the present value of relative prices are well

Table A.1: Random Walks in the Present Values of Unemployment and Relative Prices

	PV of Unemployment		PV of Relative Prices	
	(1)	(2)	(3)	(4)
Lag PV of Unemployment	0.999	0.997	0.020	0.007
	(0.001)	(0.002)	(0.005)	(0.008)
Lag PV of Rel. Prices	-0.002	-0.001	0.994	0.992
	(0.001)	(0.002)	(0.001)	(0.005)
State Effects	✓	✓	✓	✓
Time Effects		✓		✓

Note: In the first two columns, we regress $\sum_{j=0}^{T} \beta^j \tilde{u}_{i,t+j}$ on its quarterly lag and the quarterly lag of $\sum_{j=0}^{T} \beta^j p_{i,t+j}^N$, where $\tilde{u}_{it}$ is unemployment in state i in quarter t and p_{it}^N is relative non-tradeable prices. In the last two columns we repeat the exercise with $\sum_{j=0}^{T} \beta^j p_{i,t+j}^N$ as the outcome. The sample period is 1978-2018. We set T = 20. Unemployment is in percentage points and relative prices are in 100 x log points. The regression is unweighted. Standard errors are in parentheses. These are two-way clustered by date and state. The number of observations is 3495.

approximated by univariate random walk processes at quarterly frequency.

A.12 Applying Our Estimation Procedure to Model Generated Data

Our empirical specification—equation (17)—approximates the structural regional Phillips curve in the model we present in section 3—equation (14). The approximation is that we truncate the present sums of unemployment and relative prices. We can assess the error associated with this approximation by estimating κ with equation (17) using data generated by our model. This will also more generally verify that our empirical method is able to identify κ when applied to data generated from our model.

To simulate data from the model, we adopt a quarterly calibration. We simulate the model for a wide range of values for the slope of the Phillips curve κ. We vary κ across these simulations by varying the frequency of price change α. There are 12 remaining parameters in the model for which we must select values. The values that we choose for these parameters are listed Table A.2. We first calibrate 9 parameters to standard values, using external sources. We set the quarterly discount factor to $\beta = 0.99$ consistent with an annual riskless real interest rate of 4 percent. We set the elasticity of intertemporal substitution equal to $\sigma = 1$. We set the Frisch elasticity of labor supply to $\varphi = 1$ roughly in line with the mix of micro- and macro-economic evidence in Chetty et al. (2011). We follow Itskhoki and Mukhin (2017) and Feenstra et al. (2018) in setting the elasticity of substitution between tradeables and non-tradeables to $\eta = 1.5$. We set the elasticity of substitution across varieties to $\theta = 4$. We set the size of the home region to $\zeta = 0.05$. This results in a home

Table A.2: Calibrated Parameters

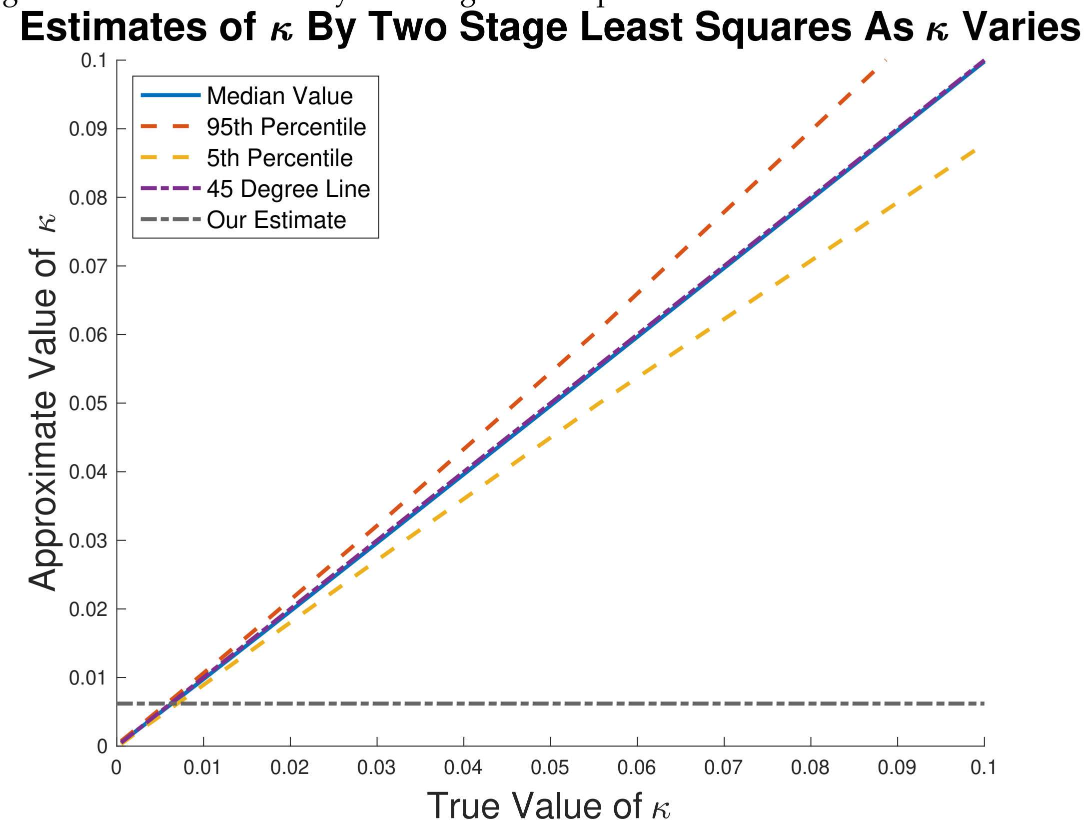
Externally Calibrated Parameters		
Intertemporal Elasticity of Substitution	σ	1
Discount factor	β	0.99
Elasticity of substitution between tradeables and non-tradeables	η	1.5
Elasticity of substitution between varieties	θ	4
Size of home region	p	0.05
Steady state consumption share of non-tradeables	ϕ_H^N	0.66
Taylor Rule coefficient on inflation	φ_π	1.5
Taylor Rule coefficient on unemployment	φ_u	1.5
Frisch elasticity of labor supply	φ	1
Parameters Calibrated From the Data		
Persistence of tradeable demand	ρ_ξ	0.9
Parameters Targeting Moments from Data		
Standard deviation of tradeable demand innovation	σ_ξ	2.1
Standard deviation of supply shock	σ_ν	2.1
Targeted Moments	Model	Data
Standard deviation of annual unemployment	2.1	2.1

regions that is in between the size of New York and Pennsylvania. We set the steady state share of consumption of non-tradeables to $\phi_N = 0.66$ following Nakamura and Steinsson (2014). We set the parameters of the central bank's interest rate rule to $\varphi_\pi = 1.5$ and $\varphi_u = 1.5$.

We then calibrate the first order autocorrelation of tradeable demand to 0.9. We obtain this value by regressing tradeable demand on its 16th lag, and taking the 16th root of the regression coefficient. We assume that the supply shock is i.i.d. We choose the value of the remaining two parameters, the standard deviations of the innovations to the supply and tradeable demand shocks, to jointly match two criteria. First, we require that the standard deviation of annual unemployment generated from the model equals its value in the data conditional on time and state fixed effects. Second, we require that the contribution of supply and demand shocks to the variance of unemployment is equal. To calculate the standard deviation of unemployment, we simulate data from the model at a quarterly frequency, setting $\kappa = 0.0062$ (the estimate of κ in Column (4) of Table 1). We time-aggregate the simulated data from the model, in order calculate the standard deviation of unemployment in simulated data in the same way as in real world data.

We now use the model to show that our GMM procedure consistently estimates κ. We simulate the model for a range of values for κ between 0.0005 and 0.1. For each value of κ, we simulate the model 1,000 times. For each of these simulation, we generate data for 8,000 periods, roughly

Figure A.1: Estimates of κ By Two Stage Least Squares on Simulated Data from our Model



the size of our quarterly dataset. We then estimate κ for each simulation using our two stage least squares procedure.

Figure A.1 plots the median estimated κ as a function of the true value of κ along with the 5th and 95th quantile of the distribution of the estimated κ's as function of the true value of κ. For visual aid, we also plot the 45 degree line and a horizontal line at our estimated value of κ.

Figure A.1 shows that our two stage least squares procedure consistently estimates κ. The median estimate of κ always lies very close to the 45 degree line. This implies that inconsistency due to truncating the present values of unemployment and relative prices is not important. Also, our procedure is quite precise. In the region of the value of κ that we have estimated from the data, the 5th and 95th percentiles are close to one another and to the median estimate of κ.

Table B.1: Slope of the Aggregate Phillips Curve

	Pre-1990	Post-1990
	(1)	(2)
Core CPI	0.796	0.120
	(0.120)	(0.026)
Median CPI	0.386	0.247
	(0.136)	(0.032)
Shelter CPI	1.624	0.397
	(0.350)	(0.048)
PCE	0.416	0.040
	(0.078)	(0.019)
Core less Shelter CPI	0.221	-0.069
	(0.103)	(0.026)
Core CPI RS	0.182	0.149
	(0.108)	(0.027)

B Data Appendix

B.1 Sensitivity of the Phillips Curve Slope using Aggregate Data

Table B.1 presents estimates of the slope of the Phillips curve using aggregate data for several different measures of inflation. We present estimates separately for the period 1978-1990 and 1991-2018. In each case, we run the regression

$$\pi_t - E_t \pi_{t+\infty} = \alpha + \psi \tilde{u}_{t-4} + \epsilon_t, \tag{44}$$

with 10-year ahead inflation expectations from the Survey of Professional Forecasters serving as a proxy for $E_t \pi_{t+\infty}$, and the 4 quarter moving average of the CBO unemployment gap serving as a proxy for $\tilde{u}_t$. We present results for six measures of inflation: the Core CPI, the Median CPI produced by the Federal Reserve Bank of Cleveland, the CPI for shelter, the PCE, the Core CPI less shelter, and the Core CPI research series. These regressions are run on quarterly data.

The results in B.1 show that the slope of the Phillips curve estimated using aggregate data is highly sensitive to seemingly minor changes in the inflation measure used. This is particularly the case in the pre-1990 sample where the slope estimates vary by roughly a factor of 10 from 0.182 to 1.624. The estimates for the post-1990 sample also vary a great deal, but somewhat less than the pre-1990 estimates.

Table B.1 also illustrates that inference about the degree to which the Phillips curve flattens based on aggregate data is highly sensitive to the inflation measure used. For some measures, the Phillips curve flattens a great deal (e.g., Core CPI and CPI for shelter). But for others it does not flatten much at all (e.g., median CPI and Core CPI research series).

There has been extensive discussion in the literature behind this. Stock and Watson (2019) discuss how certain sub-indices — such as shelter — are more cyclical than others. Ball and Mazumder (2019) argue that the Median CPI has advantages arising from the elimination of large fluctuations in certain components of the CPI. The difference between CPI inflation and PCE inflation arises to a significant degree from differences in the treatment of housing services in the early 1980s and the fact that the BLS does not revise the CPI, while the PCE is revised.

B.2 CPI Inflation Using Pre- and Post-1983 Housing Methodology

The BLS made a significant change to the methods used to calculate inflation for owner-occupied housing in 1983. This was important given the sizable weight of owner-occupied housing in the CPI (22.8%). Before 1983, the component of the CPI having to do with owner-occupied housing was constructed from a weighted average of changes in house prices and mortgage costs (i.e., interest rates). More specifically, it was made up of home purchases (9.9 percentage points); mortgage interest cost (6.5 percentage points), other financing, taxes and insurance (2.7 percentage points); and maintenance and repairs (3.7 percentage points). For further discussion, see Bureau of Labor Statistics (1982) and Poole, Ptacek, and Verbrugge (2005).

In 1983, the BLS shifted to using changes in rents as a proxy for inflation of owner occupied housing. Figure B.1 plots CPI inflation from 1972 to 2018 (gray line). It also plots our attempt at estimating what CPI inflation would have been had the BLS not changed the methodology for calculating the shelter component in 1983 (black line). Evidently, the pre-1983 methodology yields a much more variable (and cyclical) measure of inflation over the last few decades. The difference between the gray line and the black line in Figure B.1 prior to 1983 gives a sense for how accurately we can replicate the BLS's pre-1983 methodology.

B.3 Price Index Construction

Here we discuss several details of our procedure for constructing state-level price indexes.

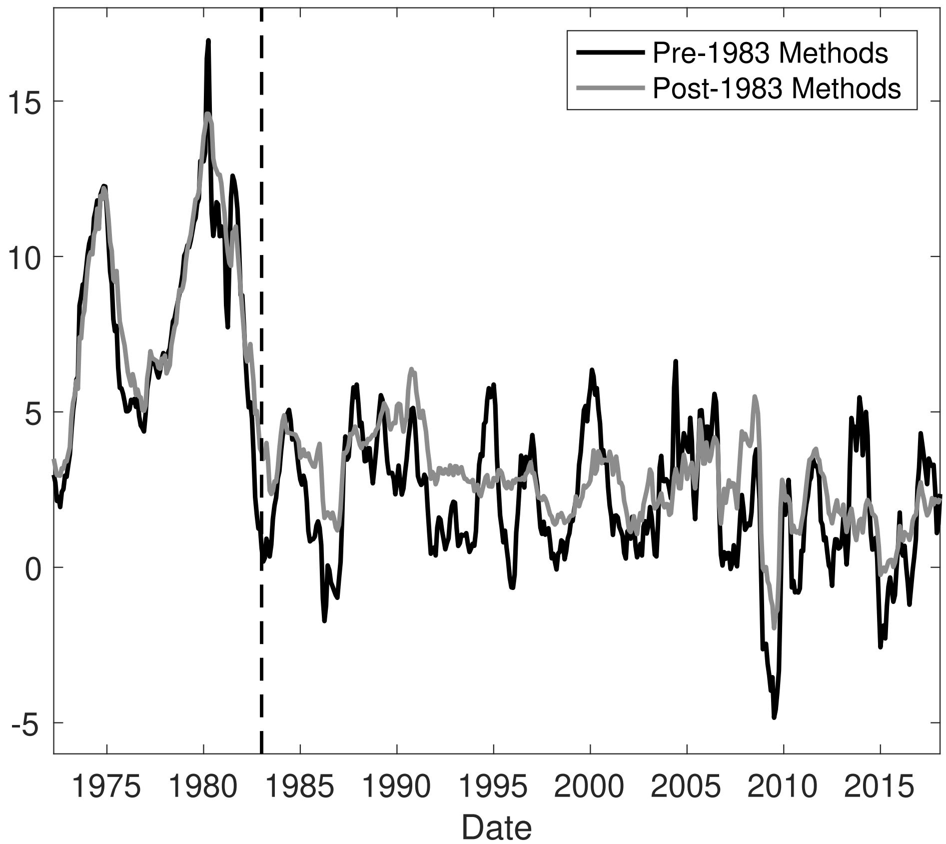


Figure B.1: CPI Inflation Using Pre- and Post-1983 Housing Methodology

Note: This figure plots overall CPI inflation in the US (gray line) and our attempt at estimating what CPI inflation would have been had the BLS not changed the methodology for calculating the shelter component in 1983 (black line). We present these results for the sample period 1972 to 2018. The difference between the gray and the black line before 1983 gives a sense for how accurately we can replicate the BLS's pre-1983 methodology.

B.3.1 Sample Restrictions

We restrict the sample we use in several ways. First, we exclude from our sample price relatives involving a product replacement when the size of the new product is unobserved. This reduces sampling error in our price indexes. Second, we Winsorize price relatives that are larger than 10 or smaller than 0.1. Third, we drop quote lines that include collected prices that are smaller than a tenth of a cent. A quote line includes all versions of a particular "quote-outlet" pair. Recall that a "quote-outlet" pair represents a specific product in a specific location, such as a 2L bottle of Diet Coke from the Westside Market at 110th Street in New York City.

Fourth, we drop observations associated with clearance sales at the end of a quote line. Intuitively, if products systematically go on sale, and then disappear from the data, this can lead to a sharply declining price index (e.g., for women's dresses) unless the product that exits is linked with a new comparable product (next season's similar women's dress). To be precise, we drop observations when they are flagged as on temporary sale and are not observed with a regular price afterwards. In contrast, if we observe a price for the same quote line at a later point following the sale, we will include the sale observations even if there has been a version change. In the case of a version change, we compute the effective price change by adjusting for quality as in equation (45) below.

B.3.2 Quality Adjustments

When a BLS price collector identifies a version change of a particular product (e.g., a new version of the same rain coat), they determine whether the substitution is "comparable." If they deem it to be comparable, they assess whether a quality adjustment is necessary. Specifically, the price collector uses the code CP for a comparable substitution, the code QC for a substitution that is considered comparable after quality adjustment, and SR for non-comparable substitutions. For observations that are considered QC, the analyst will record a quality adjustment factor. This information is then used in the construction of the price relative for that product.

We follow an analogous procedure. We include price relatives at the time of version changes in our index construction only if the version change is comparable (i.e., CP or QC). In the case of QC substitutions, we make use of the reported quality adjustment using the formula

$$r_{it} = \left(\frac{P_{it}}{P_{i,t-\tau} + QA_{i,t-\tau,t}} \right)^{1/\tau}, \tag{45}$$

where $QA_{i,t-\tau,t}$ is the quality adjustment entered for the substitution.

B.3.3 Aggregation

Armed with these price relatives, we first aggregate to the product category level (ELI) within each state using a simple geometric average

$$R_{j,x,t} = \prod_{i \in j,x} r_{i,t},$$

where j is an ELI and x is a state.

Finally, we aggregate the ELI price relatives $R_{j,x,t}$ within sectors in each state using a weighted geometric average

$$R_{s,x,t} = \prod \left[(R_{j,x,t})^{W_j / \sum_{m \in s,x} W_m} \right],$$

where s denotes sector, and W_j is the expenditure weight of each ELI. These sectors can be defined broadly as all of non-tradeables or even the entire non-shelter CPI. We use expenditure weights that are constant across states and time. Specifically, we use the CPI expenditure weights for 1998.

B.4 Definition of Non-Tradeables Inflation

Below we list the ELIs that we categorize as non-tradeables. We define non-tradeables in a relatively conservative manner since including tradeable goods in our definition of what constitutes a non-tradeable good can lead to attenuation in the slope of the Phillips curve (if tradeable goods are price nationally). Our definition of non-tradeables is similar to the BLS service aggregation. It differs in two ways. First, we include ELIs in the Food Away from Home category as non-tradeables. Second we exclude several ELIs in Transportation Services, Utilities, and Truck Rentals. An important example is airline tickets. These have highly variable prices and are collected using a different procedure than other services in the CPI Research Database. See Nakamura and Steinsson (2008) for more discussion of the behavior of transportation services prices.

- education services
  - college tuition and fixed fees
  - elementary and high school tuition and fixed fees
  - day care and nursery school
  - technical and business school tuition and fixed fees
- telephone services
  - main station charges
  - interstate telephone services
- food away from home
  - lunch
  - dinner
  - candy, gum, etc.
  - breakfast or brunch
  - full service meals and snacks

  - limited service meals and snacks
  - food at employee sites and schools
  - food from vending machines and mobile vendors
  - board, catered events, and other food away from home
  - beer, ale, and other alcoholic malt beverages away from home
- other personal services
  - beauty parlor services for females
  - legal fees
  - funeral expenses
  - household laundry and dry cleaning, excluding coin-operated
  - shoe repair and other shoe services
  - clothing rental
  - replacement of setting for women's rings
  - safe deposit box rental
  - ax return preparation and other accounting fees
  - care of invalids, elderly and convalescents in the home
- housing services
  - housing at school, excluding board
  - lodging while out of town
  - tenants' insurance
  - electricity
  - utility natural gas service
  - residential water and sewer service
  - garbage/trash collection
  - gardening or lawn care services
  - moving, storage, freight express

  - repair of household appliance
  - reupholstery of furniture
  - inside painting and/or papering

- medical services
  - general medical practice
  - dentures, bridges, crowns, implants
  - optometrists/opticians
  - services by other medical professionals
  - hospital room inpatient
  - nursing and convalescent home care

- recreational services
  - community antenna or cable tv
  - prerecorded - video tapes and discs
  - other entertainment services
  - pet services
  - veterinarian services
  - photographer's fees
  - film processing
  - fees for participant sports
  - admission to movies, theaters, and concerts
  - admission to sporting events
  - fees for lessons or instructions

- transportation services
  - used cars
  - truck rental
  - other vehicle rental

- painting entire automobile
- vehicle inspection
- automotive brake work
- automobile insurance
- drivers license
- local automobile registration
- vehicle tolls
- automobile service clubs
- intercity bus fare
- intercity train fare
- passenger ship fares
- intracity mass transit
- taxi fare

B.5 Definition of Tradeable Employment Shares

We follow Mian and Sufi (2014) in defining the tradeable employment share as the share associated with the following sectors: "agriculture, forestry, fishing and hunting," "mining, quarrying, and oil and gas extraction," and manufacturing (SIC sectors A, B and D; and NAICS sectors 11, 21, and 31-33). The QCEW censors data if there are fewer than three establishments in the industry-state, or if one firm constitutes more than 80 percent of industry-state employment. 5% of NAICS 3 digit state-by-industry cells are censored, while 10% of SIC 2 digit state-by-industry cells are censored. If an industry-state observation is missing or censored in a given quarter, we exclude this observation when we calculate the instrument.

Anthracite mining is discontinued after 1987 in the SIC. We drop this industry. We also drop observations from California before 1978, due to the exceptionally volatile share of agricultural employment in California during 1976-1978.

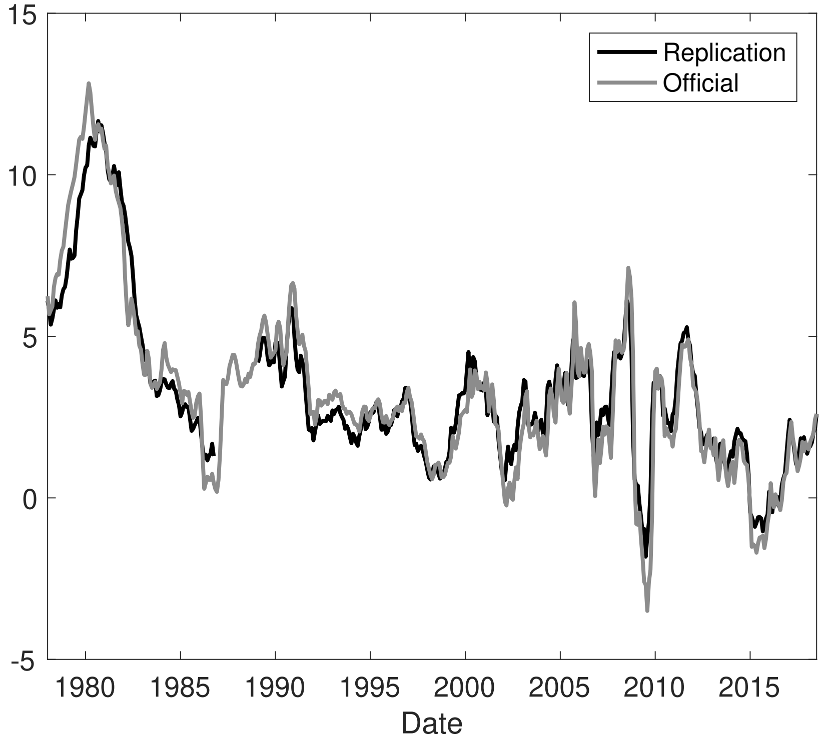


Figure C.1: Aggregate Non-Shelter Inflation

Note: The figure plots the 12-month non-shelter inflation rate for the US published by the Bureau of Labor Statistics (official) as well as the corresponding inflation rate using our methods (replication).

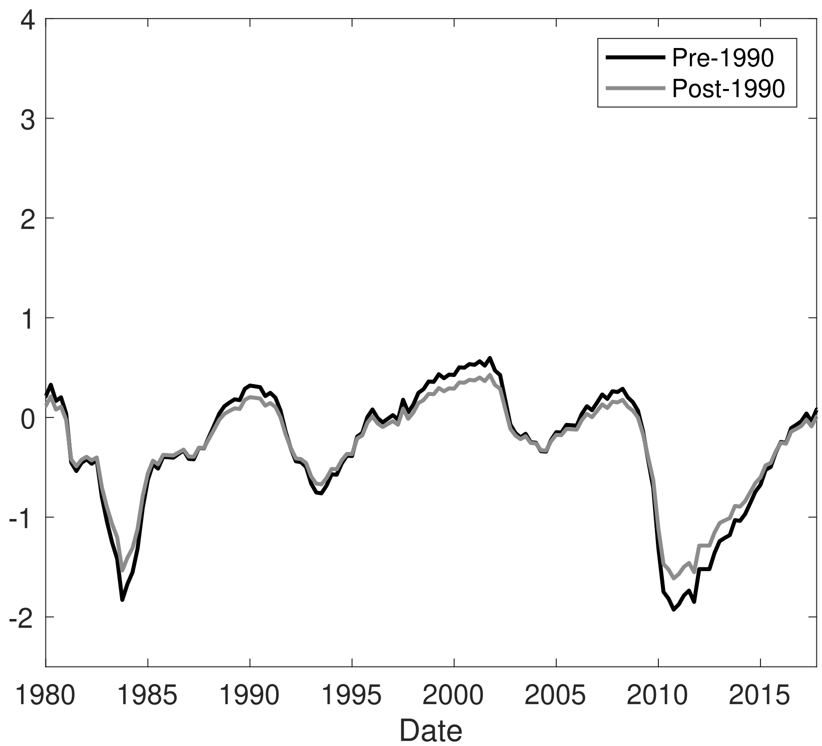


Figure C.2: Effect of Flattening on Aggregate Fit

Note: This figure plots the variation in inflation caused by changes in unemployment working through the slope of the Phillips curve according to our pre-1990 and post-1990 estimates of κ. In both cases we weight our non-shelter estimates of κ with our estimate of κ for rents.

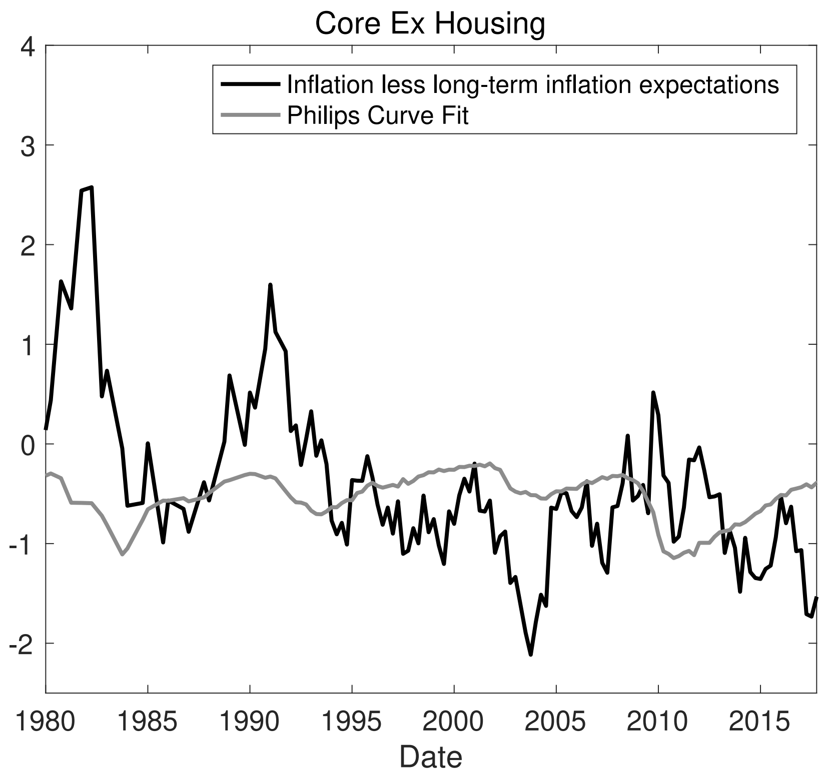


Figure C.3: Aggregate Phillips Curve Excluding Housing

Note: This figure shows the fit of the aggregate Phillips curve for core inflation excluding housing.

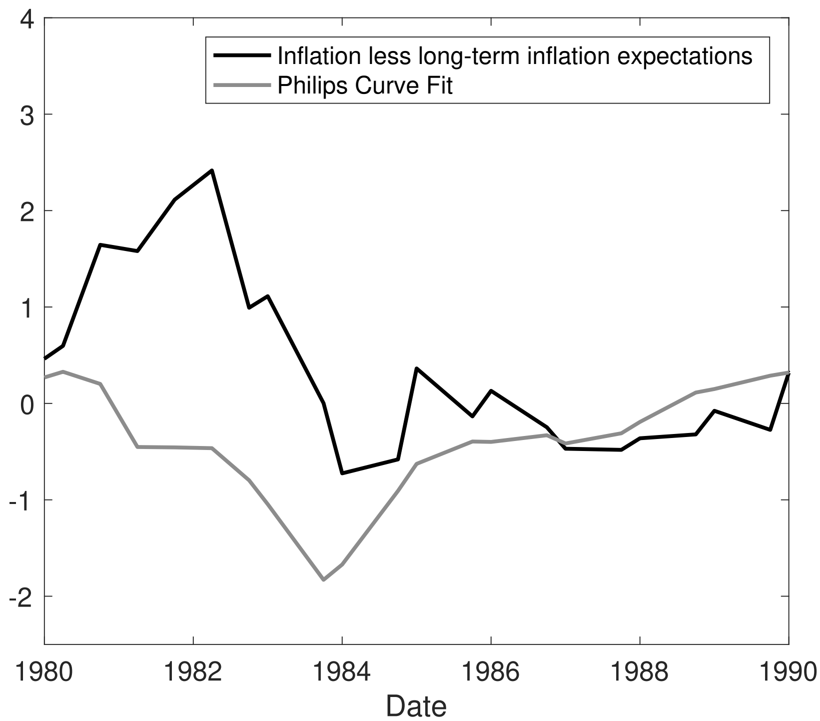


Figure C.4: Fit of the Aggregate Phillips Curve During the Volcker Disinflation

Note: This figure shows the fit of the aggregate Phillips curve for core inflation over the period 1980-1990. The gray line uses a weighted average of our pre-1990 non-shelter estimate for κ and our estimate of κ for rents.

Table C.1: First Stage Regressions with Future Sum of Unemployment and Relative Prices

	(1)	(2)	(3)	(4)
	Panel A: Future Sum of Unemployment			
Lagged Unemployment	7.029	3.661	5.477	
	(0.635)	(0.474)	(0.510)	
Lagged Tradeable Demand				-4.465
				(0.594)
Lagged Relative Price	0.181	-0.178	0.259	0.833
	(0.160)	(0.202)	(0.565)	(0.516)
	Panel B: Future Sum of Relative Price of Non-Tradeables			
Lagged Unemployment	0.520	1.713	-1.973	
	(0.889)	(0.684)	(1.096)	
Lagged Tradeable Demand				1.011
				(1.028)
Lagged Relative Price	18.572	16.895	13.081	13.710
	(0.194)	(0.288)	(1.000)	(1.283)
State Effects		✓	✓	✓
Time Effects			✓	✓

Note: This table presents results for the first stage regressions for our estimation of κ. In Panel A, the outcome is the discounted future sum of quarterly state unemployment, in percentage points, truncated at 20 quarters. In Panel B the outcome is the discounted future sum of the relative price of non-tradeables, in 100 x log points, truncated at 20 quarters. In the first three columns, the regressors are the fourth lags of unemployment and the relative price of non-tradeables, in 100 x log points. In the final column, the regressors are the fourth lags of tradeable demand and the relative price of non-tradeables. Standard errors are reported in parentheses, clustered by state. The fixed effects included for each column are reported at the bottom of the table. All regressions are unweighted and have 3323 observations.

Table C.2: Estimates of λ from Regression (17)

	No Fixed Effects	No Time Effects	Lagged Unempl.	Tradeable Demand IV
	(1)	(2)	(3)	(4)
λ	0.0010	0.0022	0.0029	0.0020
	(0.0001)	(0.0002)	(0.0009)	(0.0007)
State Effects		✓	✓	✓
Time Effects			✓	✓

Note: This table presents estimates of λ, the coefficient on the present value of relative prices from regression equation (17). The outcome variable is cumulative non-tradeable inflation over four quarters, measured in percentage points. The regressors are discounted future sums of quarterly state unemployment, in percentage points, and the relative price of non-tradeables, in 100 x log points. Each of these is truncated at 20 quarters. In the first three columns we instrument using the fourth lags of quarterly state unemployment and the relative price of non-tradeables (this is OLS for ψ). In the fourth column, we replace lagged unemployment with our tradeable demand instrument among the instruments. In all columns, we estimate λ by two-sample two stage least squares, and apply the correction to our standard errors from Chodorow-Reich and Wieland (2019). The sample period is 1978-2018. Standard errors are reported in parentheses, clustered by state. Fixed effects for each column are reported at the bottom of the table. All regressions are unweighted. The number of observations is 3323 in the first three columns with slightly fewer in the last column due to differencing.

Table C.3: Estimate of κ as Calibrated Value of β Varies

	$\beta = 0.99$	$\beta = 0.95$	$\beta = 0.90$
	(1)	(2)	(3)
κ	0.0062	0.0084	0.0116
	(0.0025)	(0.0033)	(0.0046)
State Effects	✓	✓	✓
Time Effects	✓	✓	✓

Note: This table presents estimates of κ from regression equation (17), with different calibrated values of β. The outcome variable is cumulative non-tradeable inflation over four quarters, measured in percentage points. The regressors are discounted future sums of quarterly state unemployment, in percentage points, and the relative price of non-tradeables, in 100 x log points. Both sums are truncated at 20 quarters. In all columns, we estimate κ by two-sample two stage least squares, and apply the correction to our standard errors from Chodorow-Reich and Wieland (2019). We include time and state fixed effects. The sample period is 1978-2018. Standard errors are reported in parentheses, clustered by state.

Table C.4: Estimates of κ for Different Truncation Lengths of Discounted Sums

	$T = 10$	$T = 20$	$T = 30$	$T = 40$
	(1)	(2)	(3)	(4)
κ	0.0100	0.0062	0.0044	0.0051
	(0.0038)	(0.0025)	(0.0019)	(0.0021)
State Effects	✓	✓	✓	✓
Time Effects	✓	✓	✓	✓

Note: This table presents estimates of κ from regression specification (17), for different truncation lengths of the discounted sums on the right-hand-side. The outcome variable is cumulative non-tradeable inflation over four quarters, measured in percentage points. The regressors are the present values of quarterly state unemployment, in percentage points, and the relative price of non-tradeables, in 100 x log points. We vary to truncation point for these sums between T=10 and t=40 across the columns in the table. We include time and state fixed effects. The sample period is 1978-2018. Standard errors are reported in parentheses, clustered by state. We use a two sample two stage least squares regression and apply the correction to our standard errors from Chodorow-Reich and Wieland (2019).

Table C.5: Slope of the Regional Phillips Curve: Rents

	No Fixed Effects	No Time Effects	Lagged Unempl.
	(1)	(2)	(3)
	Panel A: Estimates of κ from equation (17)		
κ	0.0074	0.0179	0.0243
	(0.0006)	(0.0014)	(0.0053)
	Panel B: Estimates of ψ from equation (17)		
ψ	0.268	0.356	0.603
	(0.041)	(0.044)	(0.124)
State Effects		✓	✓
Time Effects			✓

Note: The table presents estimates of ψ, and κ for rents. The outcome variable is the state-level annual rent inflation rate, measured in percentage points from the American Community Survey for the years 2001 to 2017 that we gathered from IPUMS USA . In Panel A, the regressor of interest is the discounted future sum of annual state unemployment, measured in percentage points. In Panel B, the regressor of interest is lagged state unemployment, measured in percentage points. We estimate κ by two-sample two stage least squares, and apply the correction to our standard errors from Chodorow-Reich and Wieland (2019). Standard errors are reported in parentheses and clustered by state. Controls for each column are reported at the bottom of the table. All regressions are unweighted.

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