Online Appendix to:

Crises and Recoveries in an Empirical Model of Consumption Disasters

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A Model Estimation

We employ a Bayesian MCMC algorithm to estimate our model. More specifically, we employ a Metropolized Gibbs sampling algorithm to sample from the joint posterior distribution of the unknown parameters and variables conditional on the data. This algorithm takes the following form in the case of our model.

The full probability model we employ may be denoted by

$$f(Y, X, \Theta) = f(Y, X|\Theta)f(\Theta),$$

where $Y \in \{C_{i,t}\}$ is the set of observable variables for which we have data,

$$X \in \{x_{i,t}, z_{i,t}, I_{W,t}, I_{i,t}, \phi_{i,t}, \theta_{i,t}\}$$

is the set of unobservable variables,

$$\Theta \in \{p_W, p_{CbW}, p_{CbI}, p_{Ce}, \rho_z, \theta, \sigma^2_\theta, \phi, \sigma^2_\phi, \mu_i, \sigma^2_{\epsilon,i,t}, \sigma^2_{\eta,i}, \sigma^2_{\nu,i}\}$$

is the set of parameters. From a Bayesian perspective, there is no real importance to the distinction between X and Θ. The only important distinction is between variables that are observed and those that are not. The function $f(Y, X|\Theta)$ is often referred to as the likelihood function of the model, while $f(\Theta)$ is often referred to as the prior distribution. Both $f(Y, X|\Theta)$ and $f(\Theta)$ are fully specified in sections 3 and 4 of the paper. The likelihood function may be constructed by combining equations (1)-(3), the distributional assumptions for the shocks in these equations and the distributional assumptions made about $I_{i,t}$ and $I_{W,t}$ in section 3. The prior distribution is described in detail in section 4.

The object of interest in our study is the distribution $f(X, \Theta|Y)$, i.e., the joint distribution of the unobservables conditional on the observed values of the observables. For expositional simplicity, let $\Phi = (X, \Theta)$. Using this notation, the object of interest is $f(\Phi|Y)$. The Gibbs sampler algorithm produces a sample from the joint distribution by breaking the vector of unknown variables into subsets and sampling each subvector sequentially conditional on the value of all the other unknown variables (see, e.g., Gelman et al., 2004, and Geweke, 2005). In our case we implement the Gibbs sampler as follows.

1. We derive the conditional distribution of each element of Φ conditional on all the other elements and conditional on the observables. For the ith element of Φ, we can denote this conditional distribution as $f(\Phi_i|\Phi_{-i}, Y)$, where Φ_i denotes the ith element of Φ and Φ_{-i} denotes all but the ith element of Φ. In most cases, $f(\Phi_i|\Phi_{-i}, Y)$ are common distributions such as normal distributions or gamma distributions for which samples can be drawn in a computationally efficient manner. For example, the distribution of potential consumption for a particular country in a particular year, $x_{i,t}$, conditional on all other variables is normal. This makes using the Gibbs sampler particularly efficient in our application. Only in the case of a (ρ_z, $\sigma^2_{\epsilon,i,t}$, $\sigma^2_{\eta,i}$, $\sigma^2_{\nu,i}$, ϕ, σ^2_ϕ, σ^2_θ) are the conditional distributions not readily recognizable. In these cases, we use the Metropolis algorithm to sample values of $f(\Phi_i|\Phi_{-i}, Y)$.[1]

2. We propose initial values for all the unknown variables Φ. Let Φ^0 denote these initial values.

3. We cycle through Φ sampling Φ_i^t from the distribution $f(\Phi_i|\Phi_{-i}^{t-1}, Y)$ where

$$\Phi_{-i}^{t-1} = (\Phi_1^t, ..., \Phi_{i-1}^t, \Phi_{i+1}^{t-1}, ..., \Phi_d^{t-1})$$

[1]The Metropolis algorithm samples a proposal Φ_i^* from a proposal distribution $J_t(\Phi_i^*|\Phi_i^{t-1})$. This proposal distribution must by symmetric, i.e., $J_t(x_a|x_b) = J_t(x_b|x_a)$. The proposal is accepted with probability $\min(r, 1)$ where $r = f(\Phi_i^*|\Phi_{-i}, Y)/f(\Phi_i^{t-1}|\Phi_{-i}, Y)$. If the proposal is accepted, $\Phi_i^t = \Phi_i^*$. Otherwise $\Phi_i^t = \Phi_i^{t-1}$. Using the Metropolis algorithm to sample from $f(\Phi_i|\Phi_{-i}, Y)$ is much less efficient than the standard algorithms used to sample from known distributions such as the normal distribution in most software packages. Intuitively, this is because it is difficult to come up with an efficient proposal distribution. The proposal distribution we use is a normal distribution centered at Φ_i^{t-1}.

and d denotes the number of elements in Φ. At the end of each cycle, we have a new draw Φ^t. We repeat this step N times to get a sample of N draws for Φ.

4. It has been shown that samples drawn in this way converge to the distribution $f(\Phi|Y)$ under very general conditions (see, e.g., Geweke, 2005). We assess convergence and throw away an appropriate burn-in sample.

In practice, we run four such "chains" starting two from one set of initial values and two from another set of initial values. We choose starting values that are far apart in the following way: The first set of starting values has $I_{i,t} = 0$ for all i and all t and sets $x_{i,t} = c_{i,t}$ and $z_{i,t} = 0$ for all i and all t. The second set of starting values is constructed as follows. $I_{i,t} = 1$ for all i and all t. We extract a smooth trend (with breaks in 1946 and 1973) from $c_{i,t}$. Denote this trend by $c^t_{i,t}$ and denote the remaining variation in consumption as $c^c_{i,t} = c_{i,t} - c^t_{i,t}$. We set $z_{i,t} = \min(\max(-0.5, c^c_{i,t}), 0)$ and $x_{i,t} = c_{i,t} - z_{i,t}$. The first set of starting values thus attributes all the variation in the data to $x_{i,t}$, while the second attributes the bulk of the variation in the data around a smooth trend to $z_{i,t}$.

Given a sample from the joint distribution $f(\Phi|Y)$ of the unobserved variables conditional on the observed data, we can calculate any statistic of interest that involves Φ. For example, we can calculate the mean of any element of Φ by calculating the sample analogue of the integral

$$\int \Phi_i f(\Phi_i|\Phi^{t-1}_{-i}, Y) d\Phi_i.$$

B Estimation with Breaks in 1951 Rather than 1946

Here we present results for an alternative estimation of our model in which we move that date of breaks in the average growth rate and volatility of transitory shocks. In our main estimation we assume that these breaks occur in 1946. However, one concern with this date is its proximity with the end of WWII. This may cause these breaks to absorb some of the recovery after WWII and thus bias the estimation of the permanent effect of this disaster. Here we move the date of these breaks to 1951 to assess the robustness of our main results to this concern.

Table A.1 - A.4 present our parameter estimates for this alternative estimation. These correspond to Tables 1 - 4 in the paper. The results are very similar to those for the baseline estimation. The short-run disaster shocks are estimated to be slightly larger in this case, while the long-run shocks are estimated to be somewhat smaller. The speed of mean reversion is also estimated to be slightly slower in this case. But all these differences are small. This can be seen more clearly in Figure A.1, which presents the response of consumption after a “typical” six year disaster. The figure compares this typical disaster for the baseline estimation in the paper and the estimation with breaks in 1951. This figure is analogous to Figure 2 in the paper. Figures A.2 and A.3 are analogous to Figures 3 and 5 in the paper. Again the results of both estimations are very similar.

Table A.5 presents results on the equity premium and the risk free rate for the estimation with breaks in 1951. With a CRRA = 6.4, the model generates an equity premium of 4.1%. This compares to 4.8% in for the baseline estimation in the paper. To match the equity premium given the parameter estimates from the estimation with breaks in 1951 we need to raise the CRRA to 6.8.

C Estimation Results for All Countries

Figure A.4 reports estimates of the key state variables in our model for each country. The following list is a key to the panels for each country in this figure:

1. The top-left figures plot consumption (black), the posterior mean of potential consumption (green) and the probability of disaster (red).

2. The top-right figures plot the posterior mean of the disaster gap (black) and 5% and 95% posterior probability bands (green and blue, respectively).

3. The middle-left figures plot the posterior mean of the size of the short run disaster shock (red) as well as consumption and potential consumption. More specifically, the red line is the posterior mean of $I_{i,t}\phi_{i,t}$, i.e., $E[I_{i,t}\phi_{i,t}|T]$.

4. The middle-right figures plot the posterior mean of the size of the long run disaster shock (red) as well as consumption and potential consumption. More specifically, the red line is the posterior mean of $I_{i,t}\theta_{i,t}$, i.e., $E[I_{i,t}\theta_{i,t}|T]$.

5. The bottom-left figures plot the size of the short run shocks conditional on a disaster, i.e., $E[I_{i,t}\phi_{i,t}|T]/E[I_{i,t}|T]$.

6. The bottom-right figures plot the size of the long run shocks conditional on a disaster, i.e., $E[I_{i,t}\theta_{i,t}|T]/E[I_{i,t}|T]$.

References

Gelman, A., J. B. Carlin, H. S. Stern, and D. B. Rubin (2004): *Bayesian Data Analysis*. John Wiley and Sons, Hoboken, New Jersey.

Geweke, J. (2005): *Contemporary Bayesian Econometrics and Statistics*. Chapman & Hall/CRC, Boca Raton, Florida.

TABLE A.I
Disaster Parameters

	Prior Dist.	Prior Mean	Prior SD	Post. Mean	Post SD.
p_W	Uniform	0.050	0.029	0.035	0.016
p_{CbW}	Uniform	0.500	0.289	0.621	0.077
p_{CbI}	Uniform	0.050	0.029	0.005	0.002
$1\text{-}p_{Ce}$	Uniform	0.500	0.289	0.828	0.027
ρ_z	Uniform	0.450	0.260	0.542	0.031
ϕ	Uniform*	-0.176	0.064	-0.119	0.008
θ	Normal	0.000	0.200	-0.011	0.010
σ_ϕ	Uniform*	0.098	0.047	0.089	0.006
σ_θ	Uniform	0.130	0.069	0.144	0.010

We specify uniform priors on φ^* and σ_φ^*, the mean and standard deviation of the underlying normal distribution (before truncation). These priors imply (non-uniform) priors on φ and σ_φ. The numbers in the table refer to the prior mean and standard deviation of φ and σ_φ.

TABLE A.II

Disaster Episodes

Country	Start Date	End Date	Max Drop	Perm Drop	Perm/Max	Country	Start Date	End Date	Max Drop	Perm Drop	Perm/Max
Argentina	1890	1908	-0.23	0.01	-0.06	Korea	1940	1959	-0.53	-0.43	0.80
Argentina	1914	1917	-0.13	-0.05	0.39	Korea	1997	2004	-0.23	-0.18	0.79
Argentina	1930	1933	-0.15	-0.09	0.60	Mexico	1911	1918	-0.17	0.28	-1.72
Australia	1914	1922	-0.29	-0.15	0.51	Mexico	1930	1935	-0.24	-0.05	0.21
Australia	1930	1934	-0.25	-0.15	0.62	Netherlands	1914	1919	-0.45	-0.04	0.08
Australia	1939	1955	-0.32	-0.05	0.16	Netherlands	1940	1951	-0.55	0.06	-0.10
Belgium	1913	1920	-0.40	0.06	-0.16	Norway	1914	1924	-0.15	-0.07	0.46
Belgium	1940	1948	-0.52	-0.02	0.03	Norway	1940	1944	-0.10	-0.08	0.77
Brazil	1930	1932	-0.12	-0.05	0.47	Peru	1930	1933	-0.17	-0.10	0.56
Brazil	1969	1975	-0.04	0.06	-1.76	Peru	1977	1993	-0.37	-0.33	0.88
Canada	1914	1926	-0.37	-0.18	0.49	Portugal	1914	1921	-0.29	-0.16	0.57
Canada	1930	1934	-0.29	-0.27	0.93	Portugal	1940	1942	-0.10	-0.06	0.66
Chile	1914	1934	-0.53	-0.35	0.66	Spain	1914	1919	-0.10	0.01	-0.05
Chile	1972	1987	-0.56	-0.52	0.93	Spain	1930	1961	-0.50	-0.38	0.77
Denmark	1914	1926	-0.16	-0.08	0.51	Sweden	1914	1923	-0.20	-0.14	0.70
Denmark	1940	1950	-0.28	-0.10	0.34	Sweden	1940	1949	-0.26	-0.10	0.39
Finland	1890	1893	-0.08	-0.02	0.21	Switzerland	1914	1921	-0.14	-0.08	0.57
Finland	1914	1921	-0.42	-0.22	0.52	Switzerland	1940	1950	-0.22	-0.09	0.40
Finland	1930	1934	-0.24	-0.12	0.49	Taiwan	1901	1912	-0.15	-0.01	0.03
Finland	1940	1946	-0.30	-0.10	0.32	Taiwan	1940	1950	-0.65	-0.30	0.46
France	1914	1921	-0.22	0.08	-0.36	United.Kingdom	1914	1921	-0.21	-0.10	0.50
France	1940	1946	-0.56	0.06	-0.11	United.Kingdom	1940	1946	-0.20	-0.04	0.21
Germany	1914	1933	-0.45	-0.21	0.47	United.States	1914	1922	-0.25	-0.14	0.56
Germany	1940	1949	-0.45	-0.18	0.40	United.States	1930	1934	-0.26	-0.14	0.51
Italy	1940	1947	-0.33	0.03	-0.10						
Japan	1914	1917	-0.05	0.11	-2.39	Average			-0.29	-0.11	0.26
Japan	1940	1949	-0.62	-0.22	0.36	Median			-0.25	-0.09	0.46

A disaster episode is defined as a set of consecudite years for a particular country such that: 1) The probability of a disaster in each of these years is larger than 10%, 2) The sum of the probability of disaster for each year over the whole set of years is larger than 1. Max Drop is the posterior mean of the maximum shortfall in the level of consumption due to the disaster. Perm Drop is the posterior mean of the permanent effect of the disaster on the level potential consumption. Perm/Max is the ratio of Perm Drop to Max Drop.

TABLE A.III

Mean Growth Rate of Potential Consumption

		Prior		Pre-1951		1951-1972		Post-1973	
	Prior Dist.	Prior Mean	Prior SD	Post. Mean	Post SD.	Post. Mean	Post SD.	Post. Mean	Post SD.
Argentina	Normal	0.02	1.00	0.017	0.009	0.016	0.012	0.007	0.010
Australia	Normal	0.02	1.00	0.015	0.006	0.022	0.005	0.020	0.003
Belgium	Normal	0.02	1.00	0.006	0.006	0.027	0.004	0.019	0.003
Brazil	Normal	0.02	1.00	0.024	0.008	0.039	0.010	0.016	0.009
Canada	Normal	0.02	1.00	0.027	0.004	0.026	0.005	0.018	0.004
Chile	Normal	0.02	1.00	0.019	0.009	0.024	0.011	0.038	0.011
Denmark	Normal	0.02	1.00	0.019	0.004	0.020	0.005	0.012	0.004
Finland	Normal	0.02	1.00	0.027	0.006	0.039	0.007	0.024	0.006
France	Normal	0.02	1.00	0.004	0.003	0.038	0.003	0.019	0.002
Germany	Normal	0.02	1.00	0.014	0.004	0.049	0.004	0.018	0.003
Italy	Normal	0.02	1.00	0.010	0.003	0.046	0.004	0.021	0.003
Japan	Normal	0.02	1.00	0.006	0.004	0.076	0.005	0.022	0.004
Korea	Normal	0.02	1.00	0.017	0.005	0.036	0.010	0.053	0.006
Mexico	Normal	0.02	1.00	0.005	0.007	0.028	0.008	0.016	0.007
Netherlands	Normal	0.02	1.00	0.010	0.004	0.035	0.006	0.015	0.004
Norway	Normal	0.02	1.00	0.017	0.004	0.026	0.005	0.025	0.004
Peru	Normal	0.02	1.00	0.023	0.005	0.025	0.007	0.011	0.009
Portugal	Normal	0.02	1.00	0.019	0.007	0.045	0.007	0.030	0.006
Spain	Normal	0.02	1.00	0.010	0.005	0.054	0.008	0.021	0.004
Sweden	Normal	0.02	1.00	0.025	0.003	0.024	0.004	0.013	0.003
Switzerland	Normal	0.02	1.00	0.013	0.003	0.028	0.003	0.009	0.002
Taiwan	Normal	0.02	1.00	0.008	0.006	0.056	0.008	0.056	0.006
United Kingdom	Normal	0.02	1.00	0.010	0.003	0.021	0.004	0.024	0.003
United States	Normal	0.02	1.00	0.019	0.003	0.025	0.004	0.022	0.003
Median				0.016	0.005	0.028	0.005	0.019	0.004
Simple Average				0.015	0.005	0.034	0.006	0.022	0.005

TABLE A.IV

Standard Deviation of Non-Disaster Shocks

	Priors			Permanent		Temporary Pre-1951		Temporary Post-1951	
	Dist.	Prior Mean	Prior SD	Post. Mean	Post SD.	Post. Mean	Post SD.	Post. Mean	Post SD.
Argentina	Uniform	0.075	0.04	0.054	0.008	0.021	0.015	0.013	0.009
Australia	Uniform	0.075	0.04	0.018	0.004	0.034	0.008	0.004	0.003
Belgium	Uniform	0.075	0.04	0.019	0.002	0.019	0.010	0.003	0.002
Brazil	Uniform	0.075	0.04	0.046	0.007	0.058	0.010	0.010	0.007
Canada	Uniform	0.075	0.04	0.021	0.003	0.030	0.007	0.002	0.002
Chile	Uniform	0.075	0.04	0.046	0.009	0.025	0.015	0.020	0.011
Denmark	Uniform	0.075	0.04	0.021	0.003	0.006	0.004	0.004	0.003
Finland	Uniform	0.075	0.04	0.032	0.004	0.016	0.008	0.004	0.003
France	Uniform	0.075	0.04	0.014	0.002	0.029	0.004	0.002	0.001
Germany	Uniform	0.075	0.04	0.018	0.002	0.013	0.006	0.002	0.002
Italy	Uniform	0.075	0.04	0.018	0.002	0.011	0.003	0.003	0.002
Japan	Uniform	0.075	0.04	0.022	0.003	0.018	0.005	0.003	0.002
Korea	Uniform	0.075	0.04	0.026	0.004	0.028	0.007	0.004	0.003
Mexico	Uniform	0.075	0.04	0.036	0.005	0.033	0.008	0.005	0.004
Netherlands	Uniform	0.075	0.04	0.024	0.003	0.017	0.006	0.003	0.002
Norway	Uniform	0.075	0.04	0.022	0.002	0.004	0.003	0.003	0.002
Peru	Uniform	0.075	0.04	0.034	0.004	0.006	0.004	0.005	0.004
Portugal	Uniform	0.075	0.04	0.033	0.004	0.022	0.008	0.005	0.004
Spain	Uniform	0.075	0.04	0.025	0.005	0.047	0.008	0.003	0.003
Sweden	Uniform	0.075	0.04	0.018	0.002	0.025	0.006	0.002	0.002
Switzerland	Uniform	0.075	0.04	0.012	0.002	0.039	0.006	0.001	0.001
Taiwan	Uniform	0.075	0.04	0.033	0.004	0.035	0.017	0.004	0.003
United Kingdom	Uniform	0.075	0.04	0.017	0.002	0.003	0.002	0.003	0.002
United States	Uniform	0.075	0.04	0.017	0.002	0.022	0.004	0.002	0.002
Median				0.022	0.003	0.022	0.006	0.003	0.002
Simple Average				0.026	0.004	0.023	0.007	0.005	0.003

TABLE A.V
Disasters and the Equity Premium

	Equity Premium	Risk-Free Rate
Baseline model with CRRA = 6.4	0.041	0.015
Baseline model with CRRA = 6.8	0.048	0.010

Both cases have IES = 2 and $\beta = \exp(-0.034)$. The return statistics are the log of the average gross return for each asset. The "Equity Premium" is the different between the average return on an unlevered equity claim and bills. The "Risk-Free Rate" is the average return on bills. These results are produced by simulating a long sample from the model with a representative set of disasters.

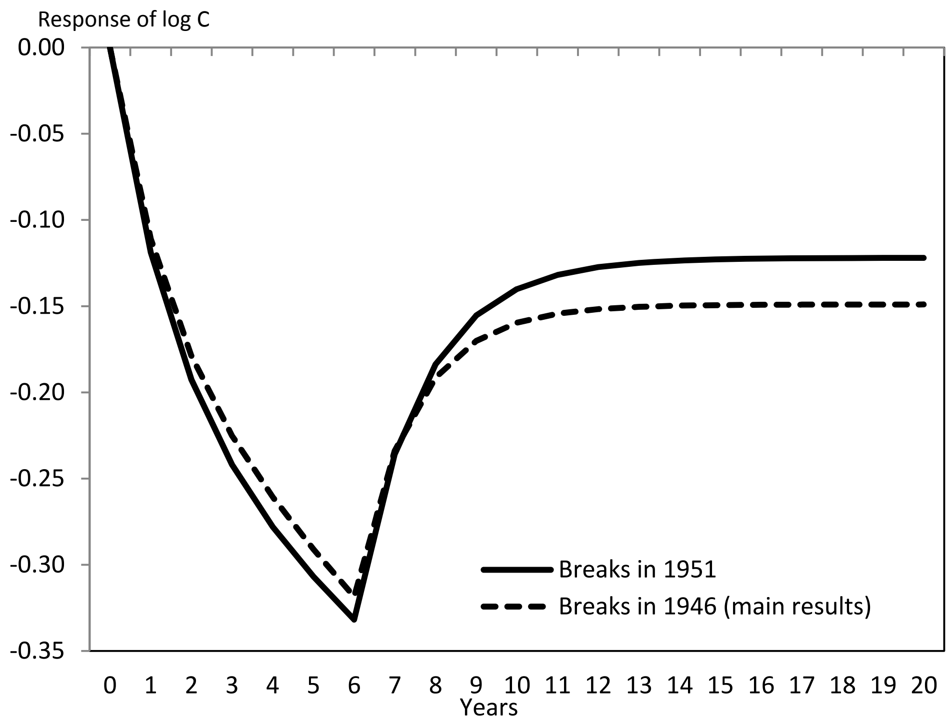


FIGURE A.I
A Typical Disaster

Note: The figure plots the evolution of log consumption during and after a disaster that strikes in period 1 and lasts for 6 years. This is plotted both for the version of the model presented in the main body of the paper (breaks in 1946) and the version of the model presented in the appendix (break in 1951). Over the course of the disaster, both ϕ and θ take values equal to their posterior means in each period. For simplicity, we abstract from trend growth and assume that all other shocks are equal to zero over this period.

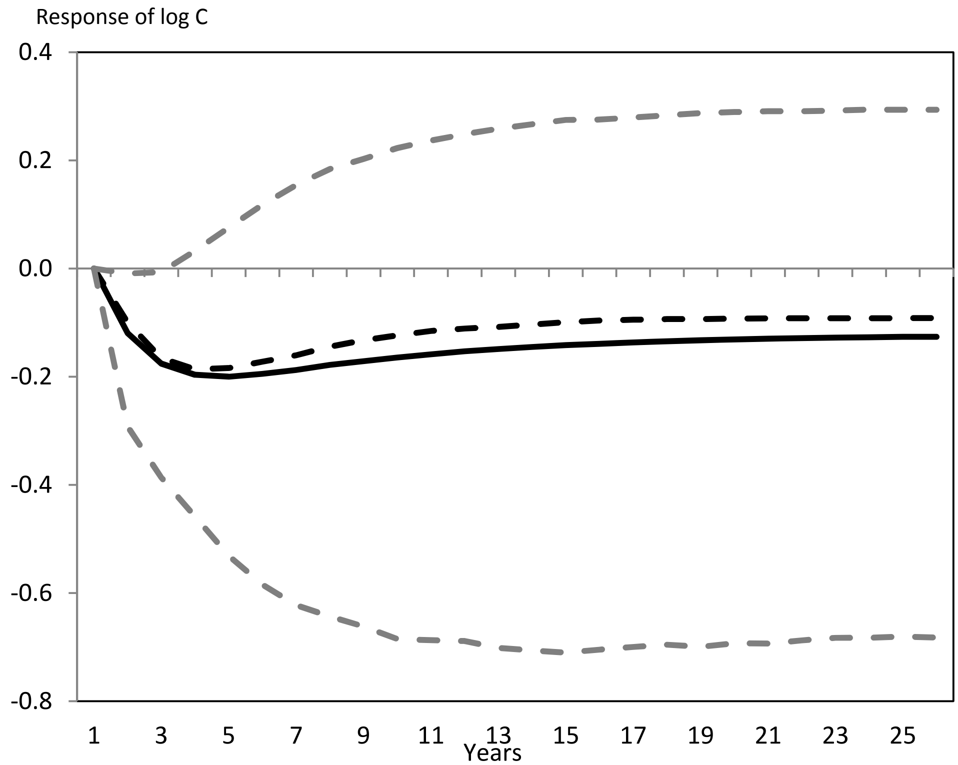


FIGURE A.II
Ex Ante Disaster Distribution

Note: The solid line is the mean of the distribution of the change in log consumption relative to its previous trend from the perspective of agents that have just learned that they have entered the disaster state but do not yet know the size or length of the disaster. The black dashed line is the median of this distribution. The grey dashed lines are the 5% and 95% quantiles of this distribution.

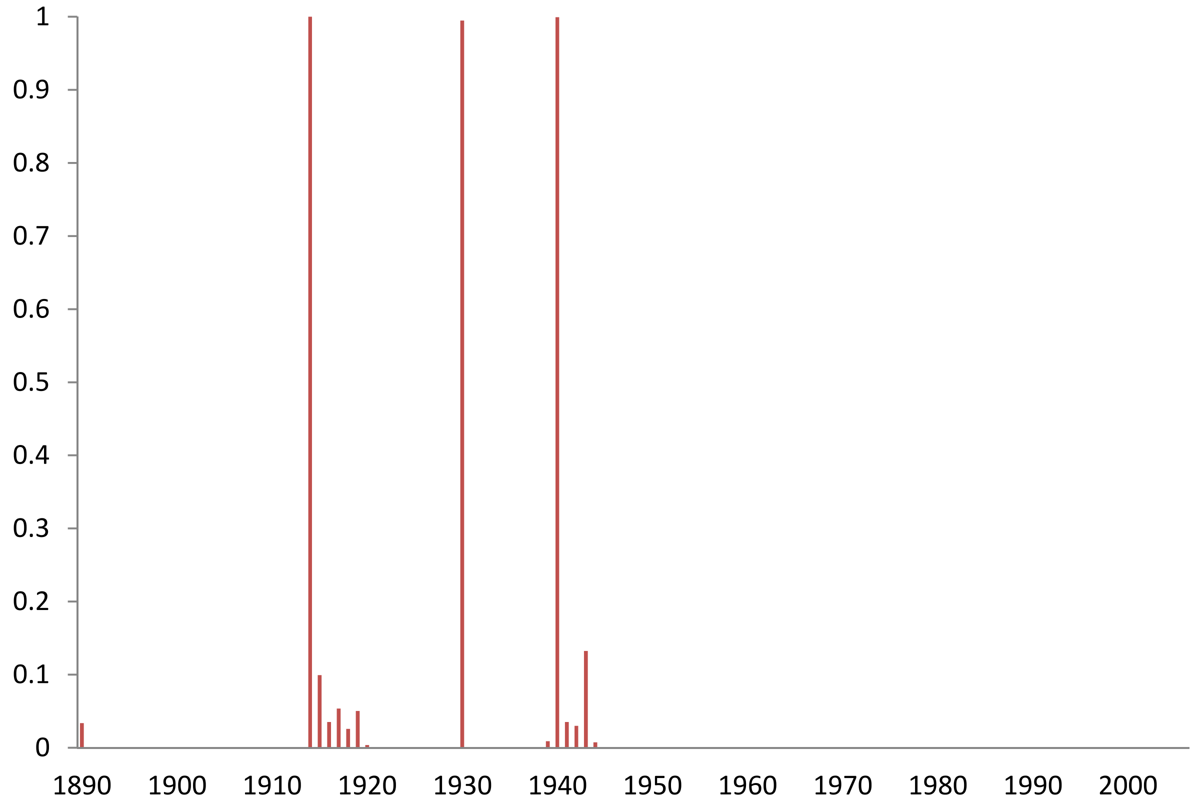


FIGURE A.III
World Disaster Probability

Note: The figure plots the posterior mean of $I_{W,t}$, i.e., the probability that the world entered a disaster in each year evaluated using data up to 2006.

Figure A.IV

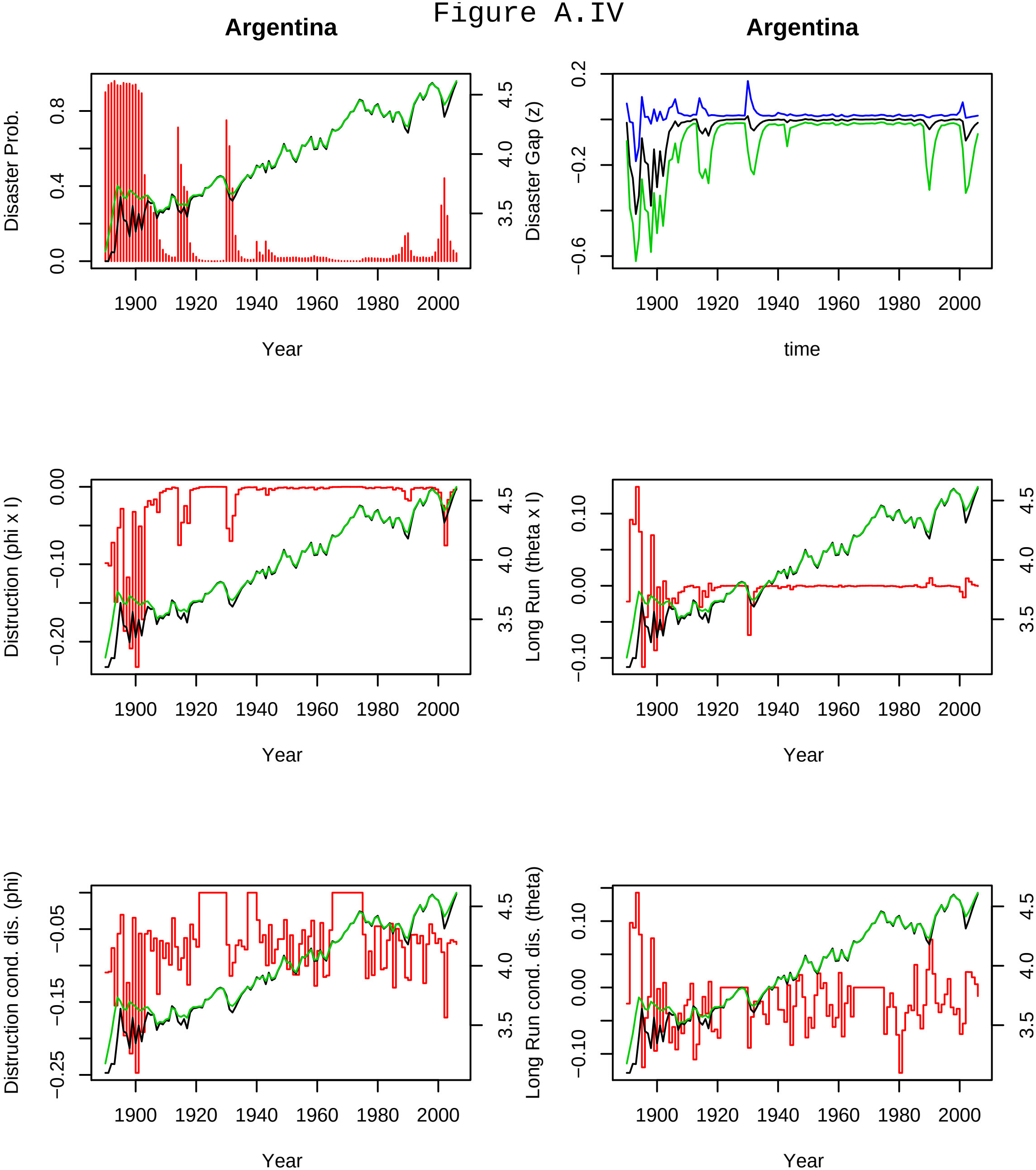

Argentina
Disaster Prob.
0.0
0.4
0.8
Disaster Gap (z)
3.5
4.0
4.5
1900
1920
1940
1960
1980
2000
Year
Argentina
0.2
−0.2
−0.6
time
Distruction (phi x l)
0.00
−0.10
−0.20
Long Run (theta x l)
0.10
0.00
−0.10
Distruction cond. dis. (phi)
−0.05
−0.15
−0.25
Long Run cond. dis. (theta)

Figure A.IV (cont.)

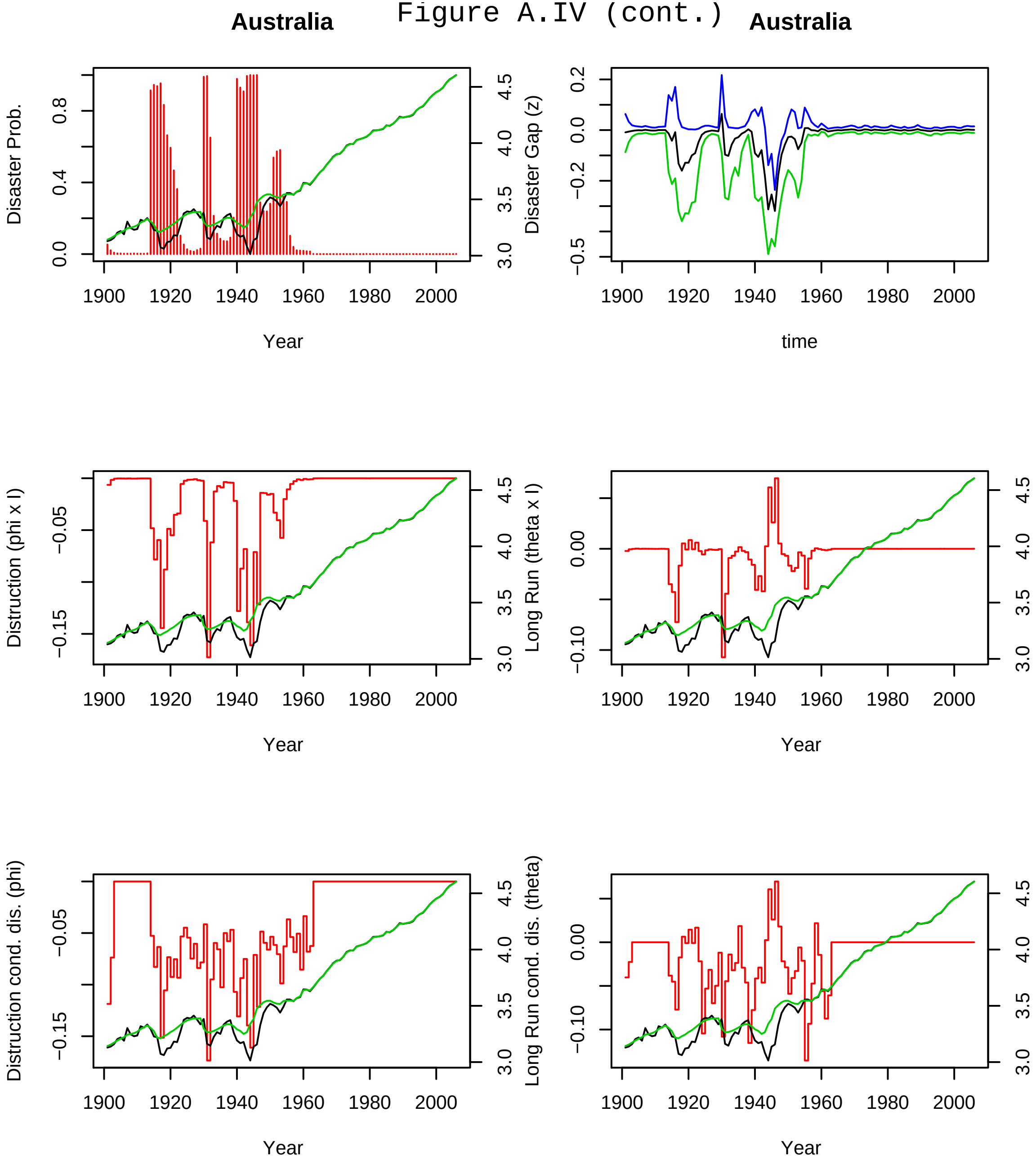


Figure A.IV (cont.)

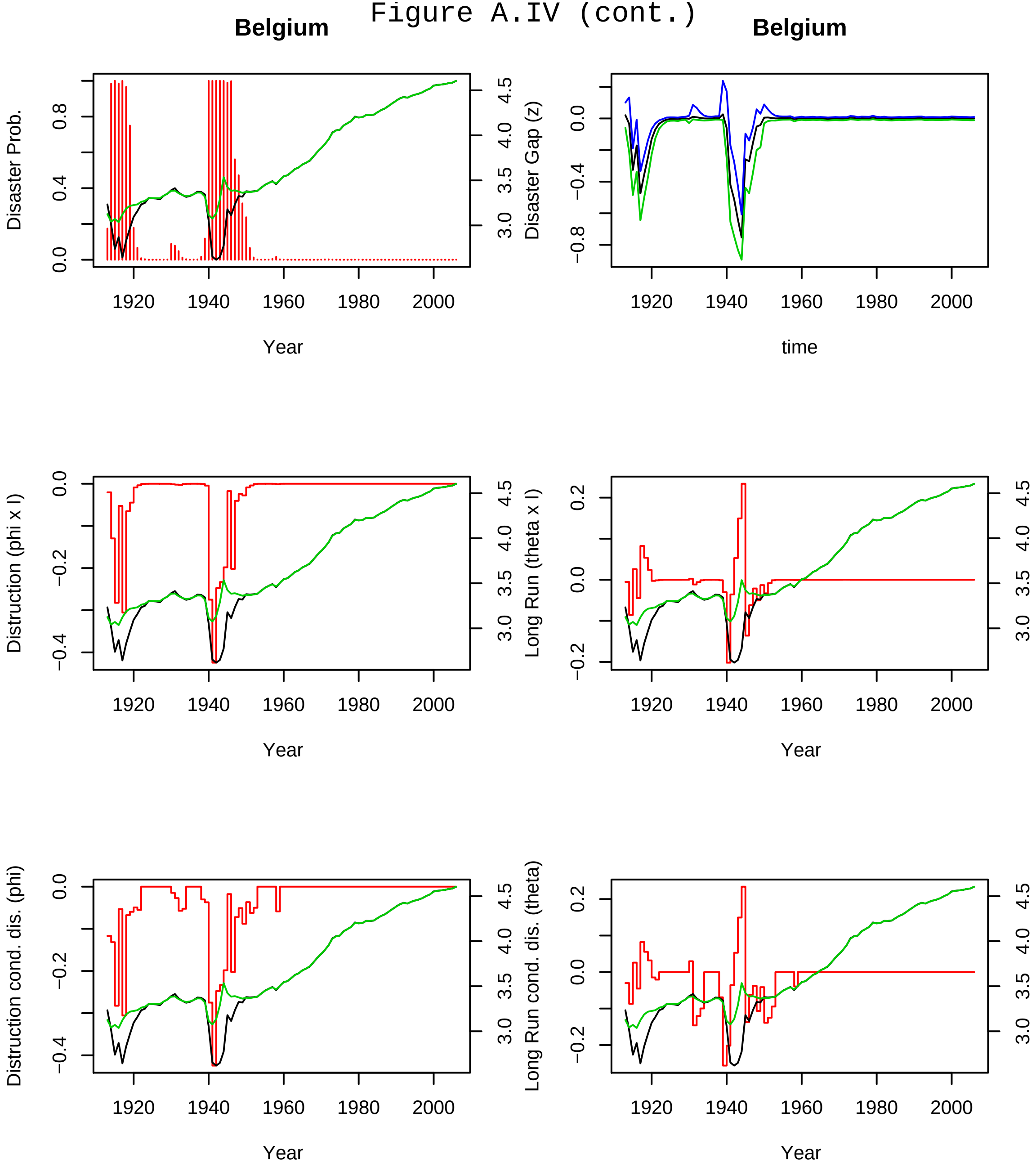
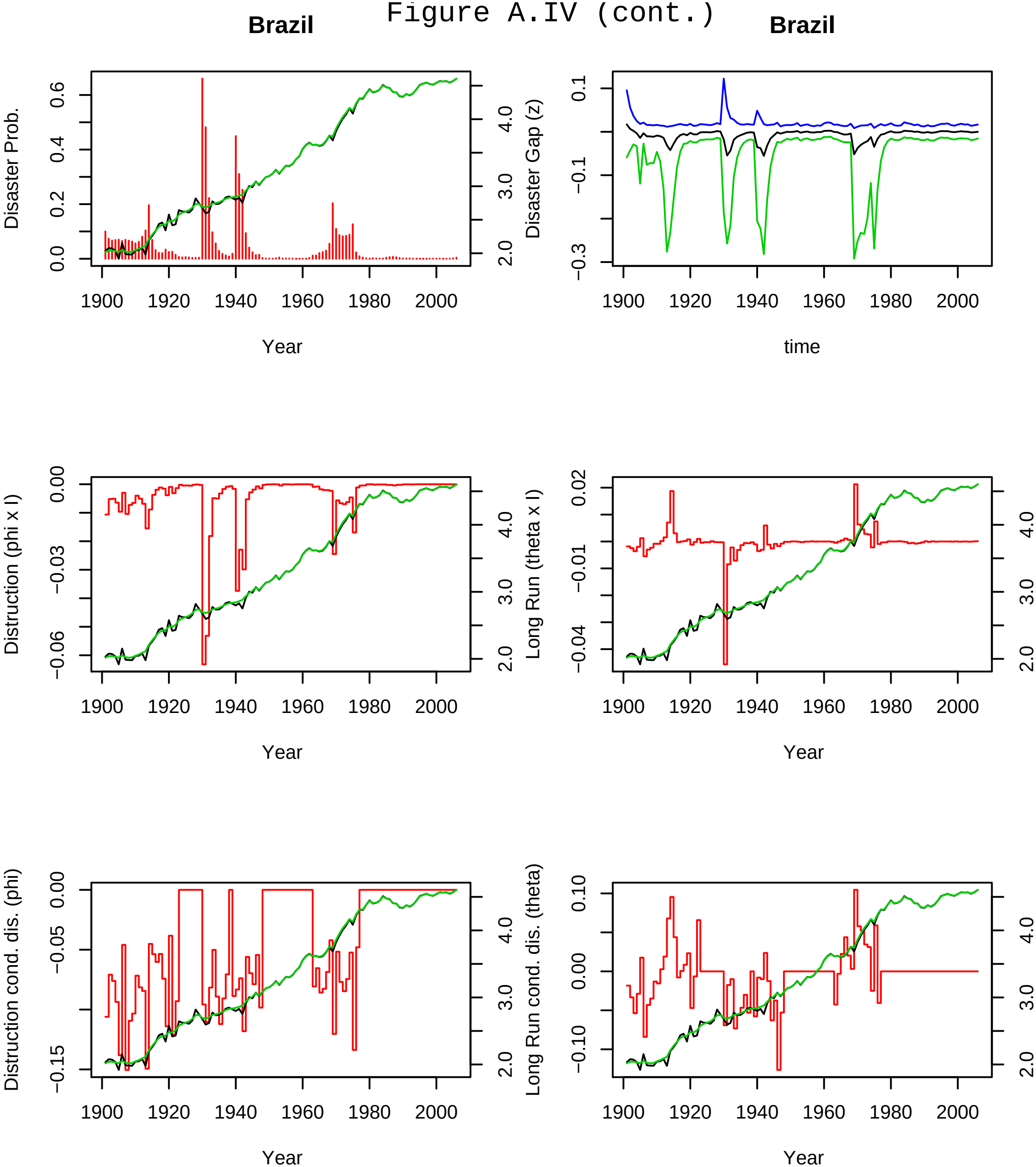


Figure A.IV (cont.)



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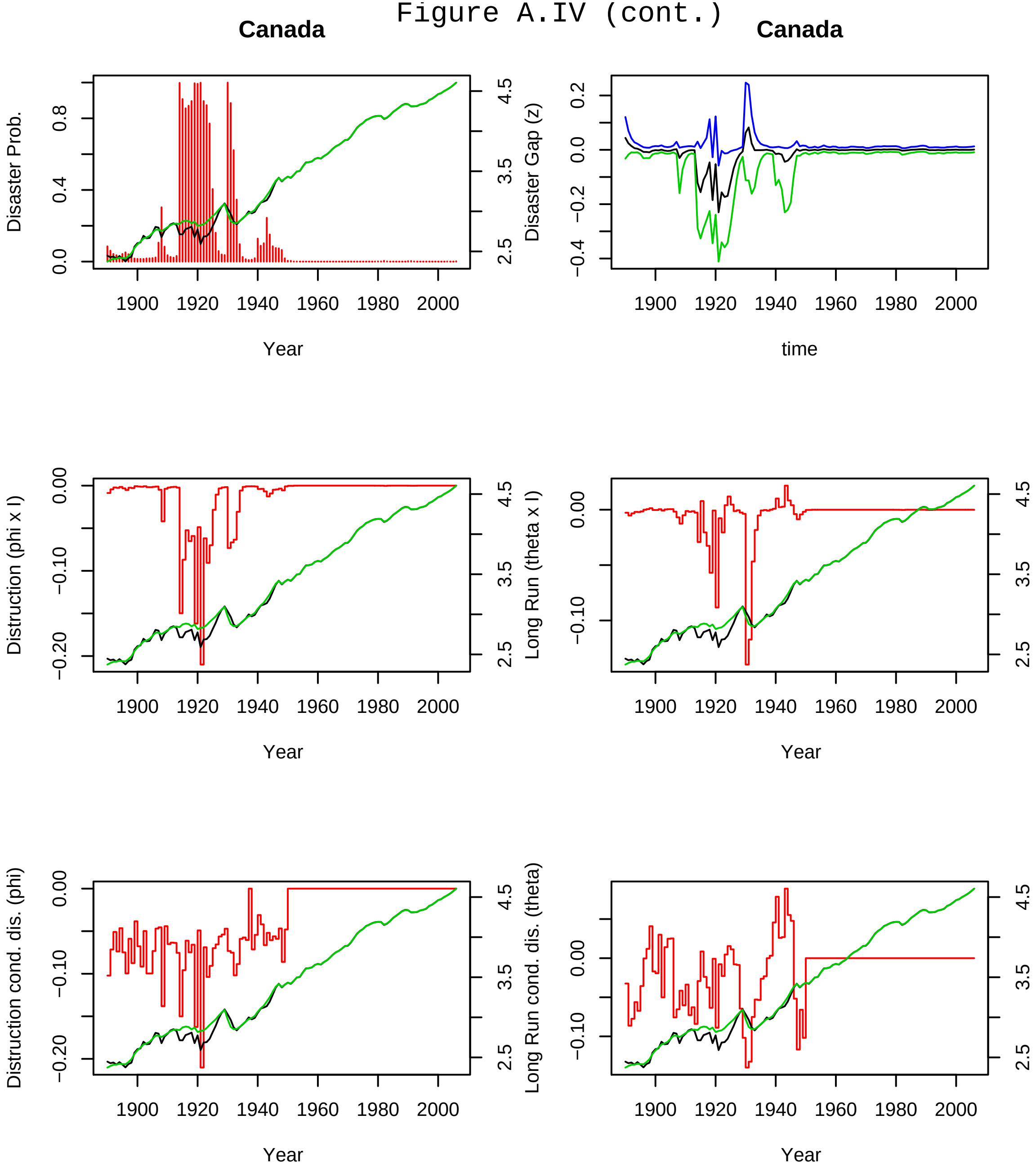

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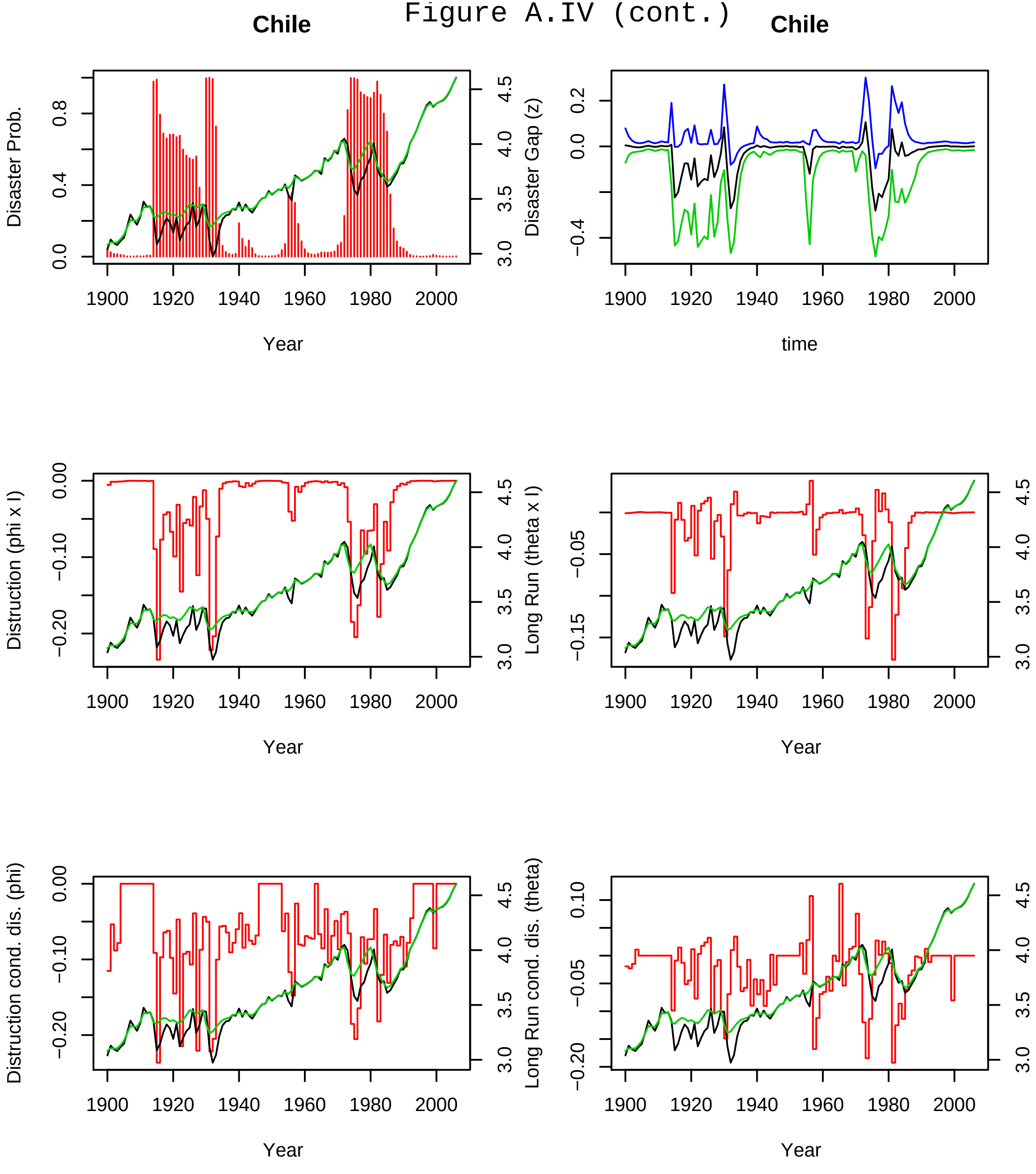


Figure A.IV (cont.)

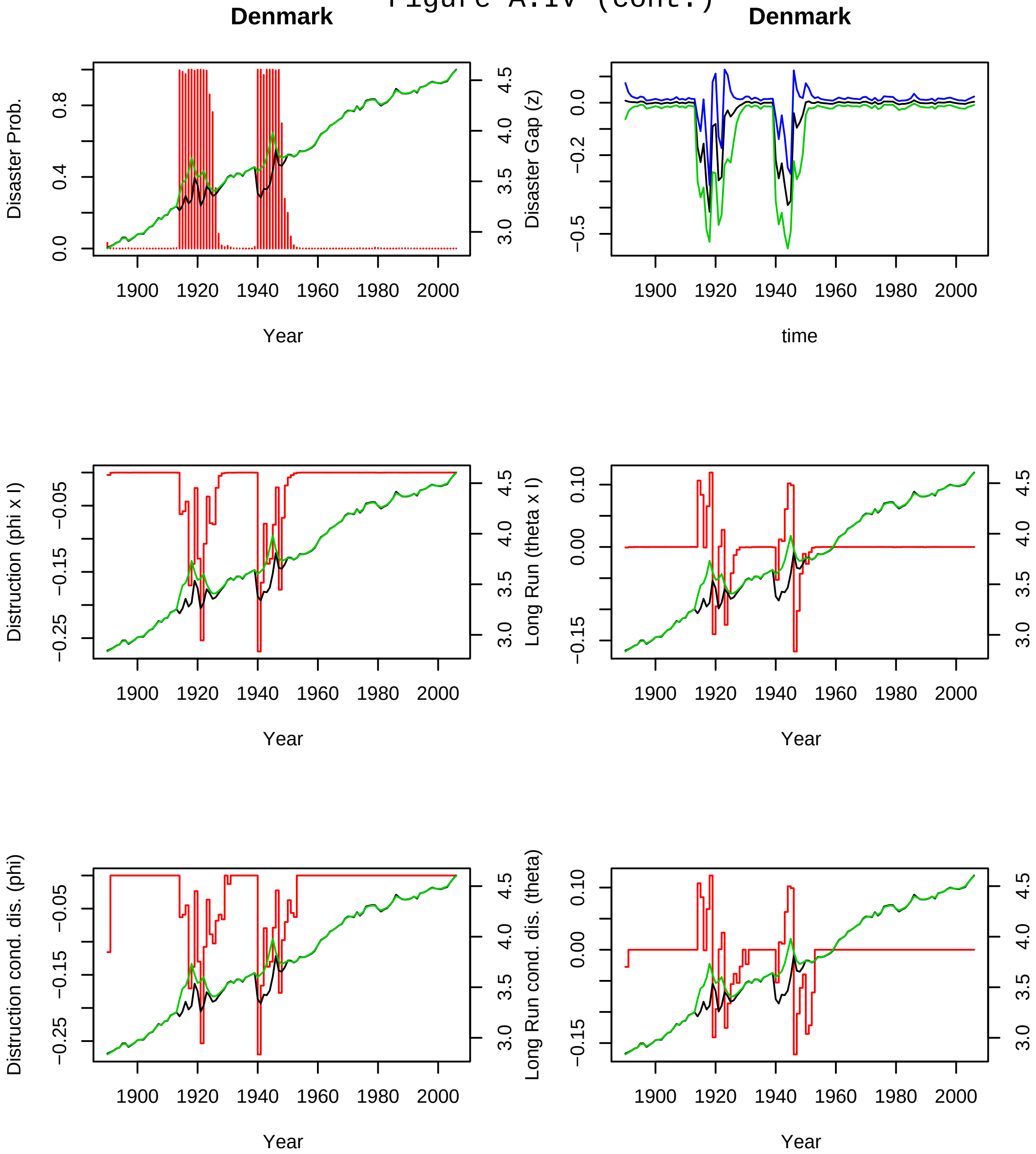


Figure A.IV (cont.)

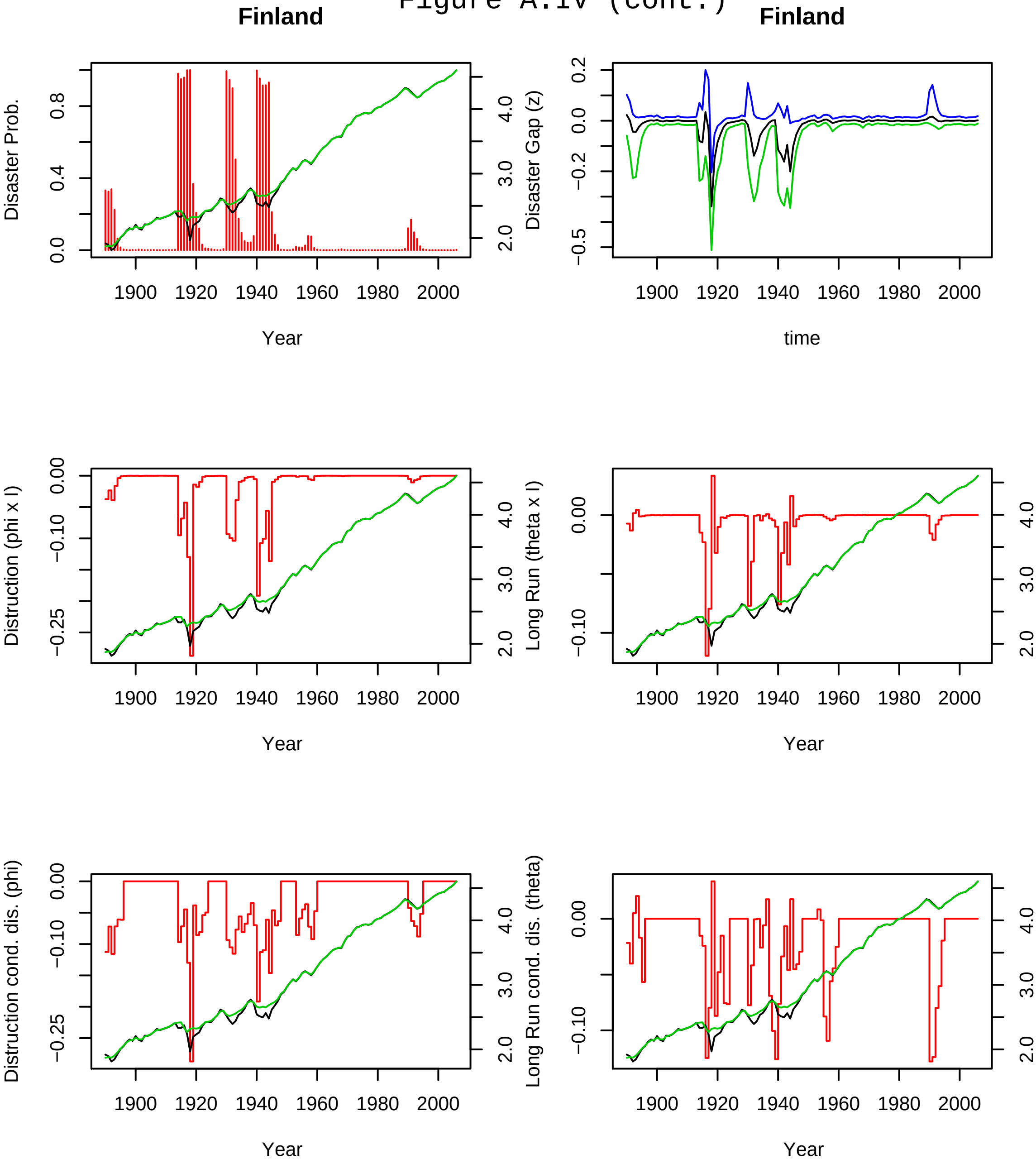
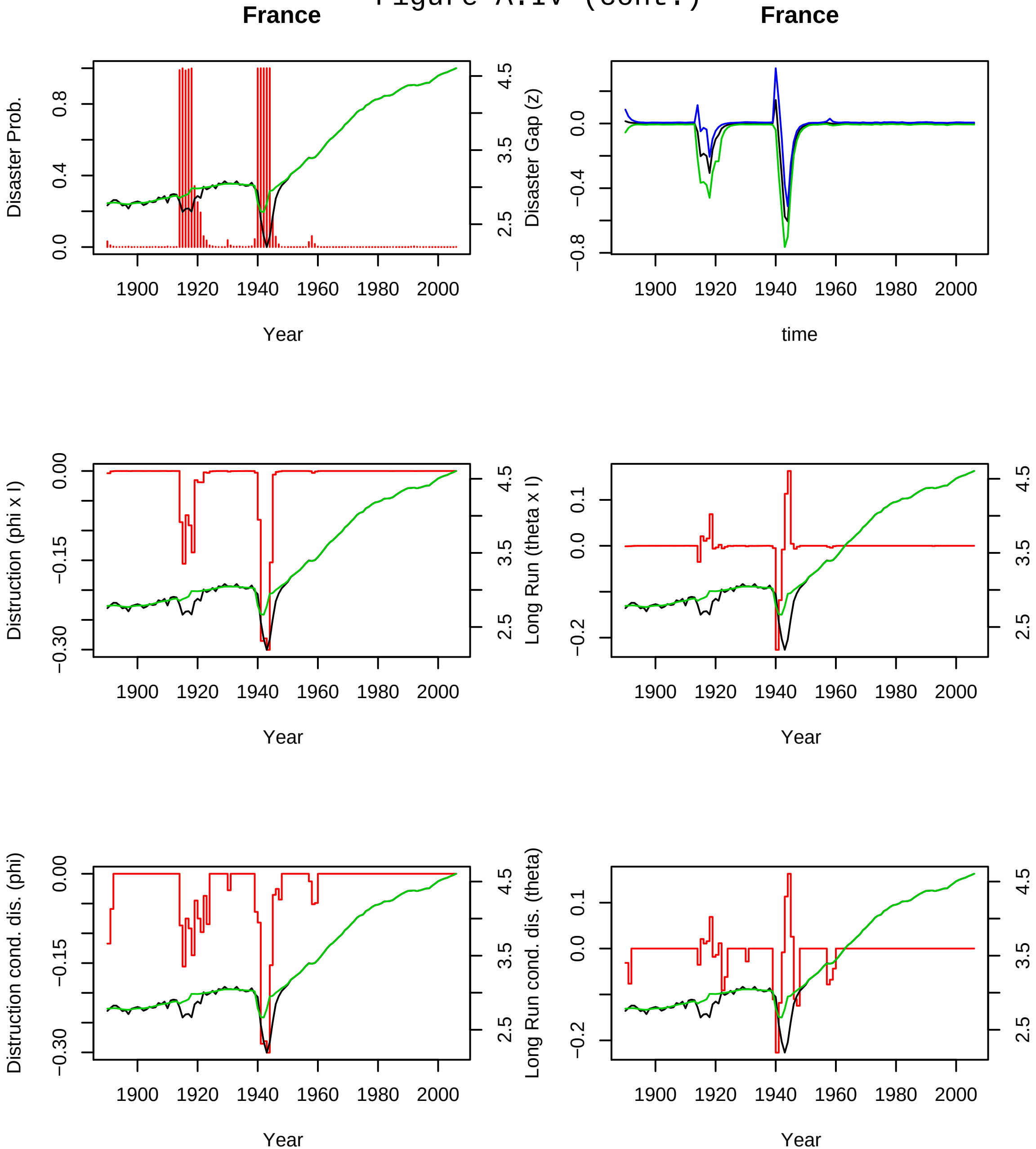


Figure A.IV (cont.)



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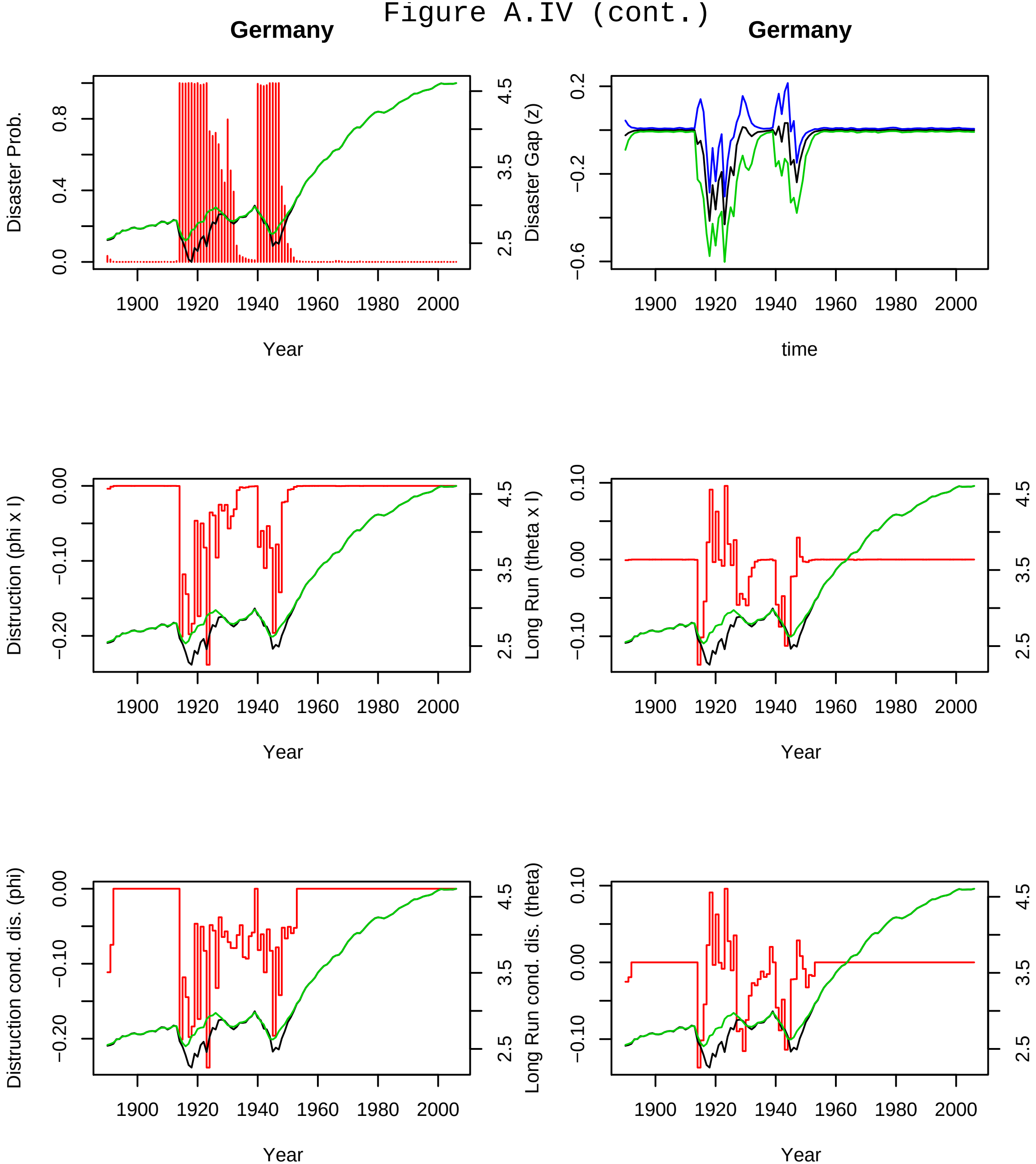

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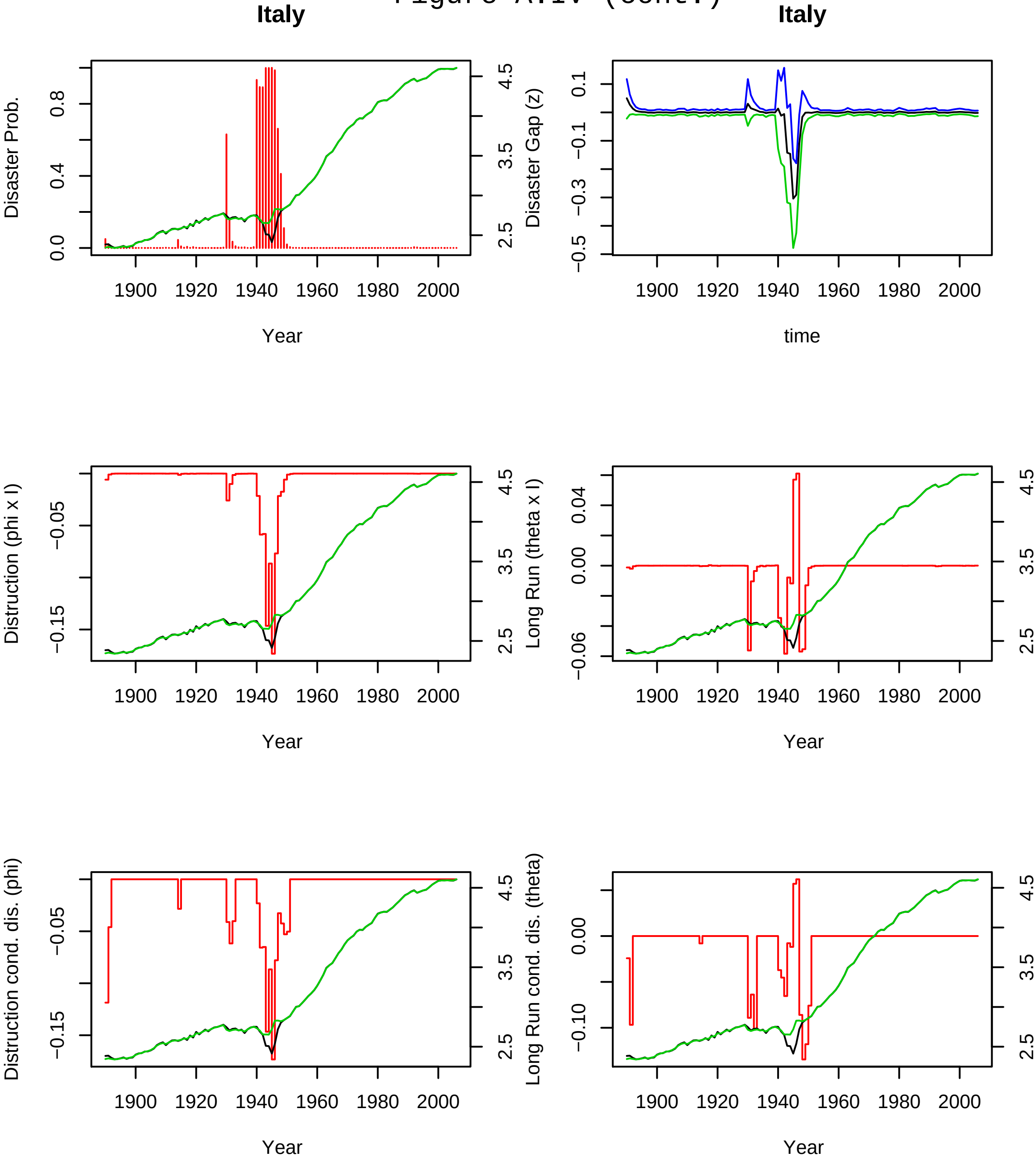

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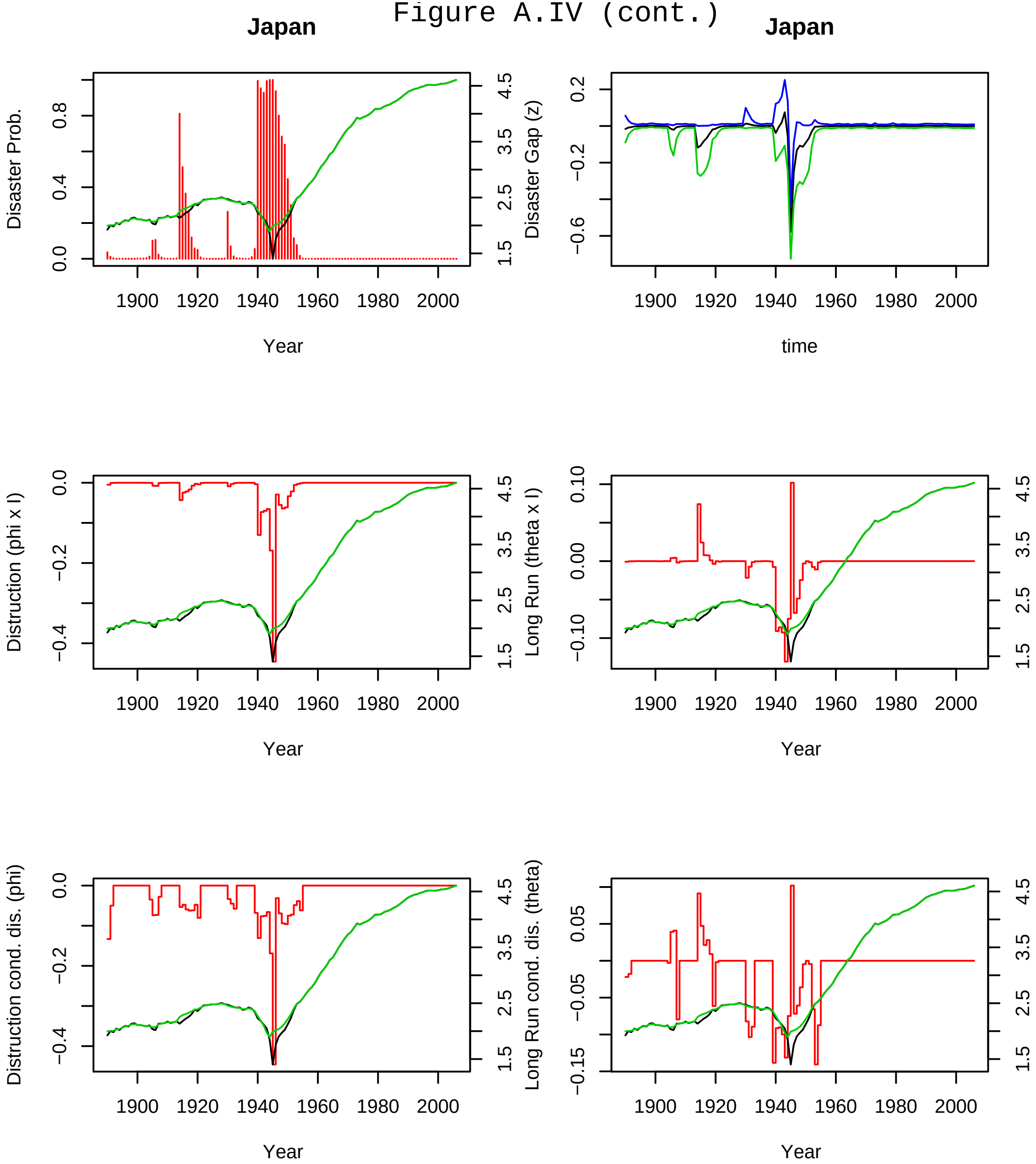

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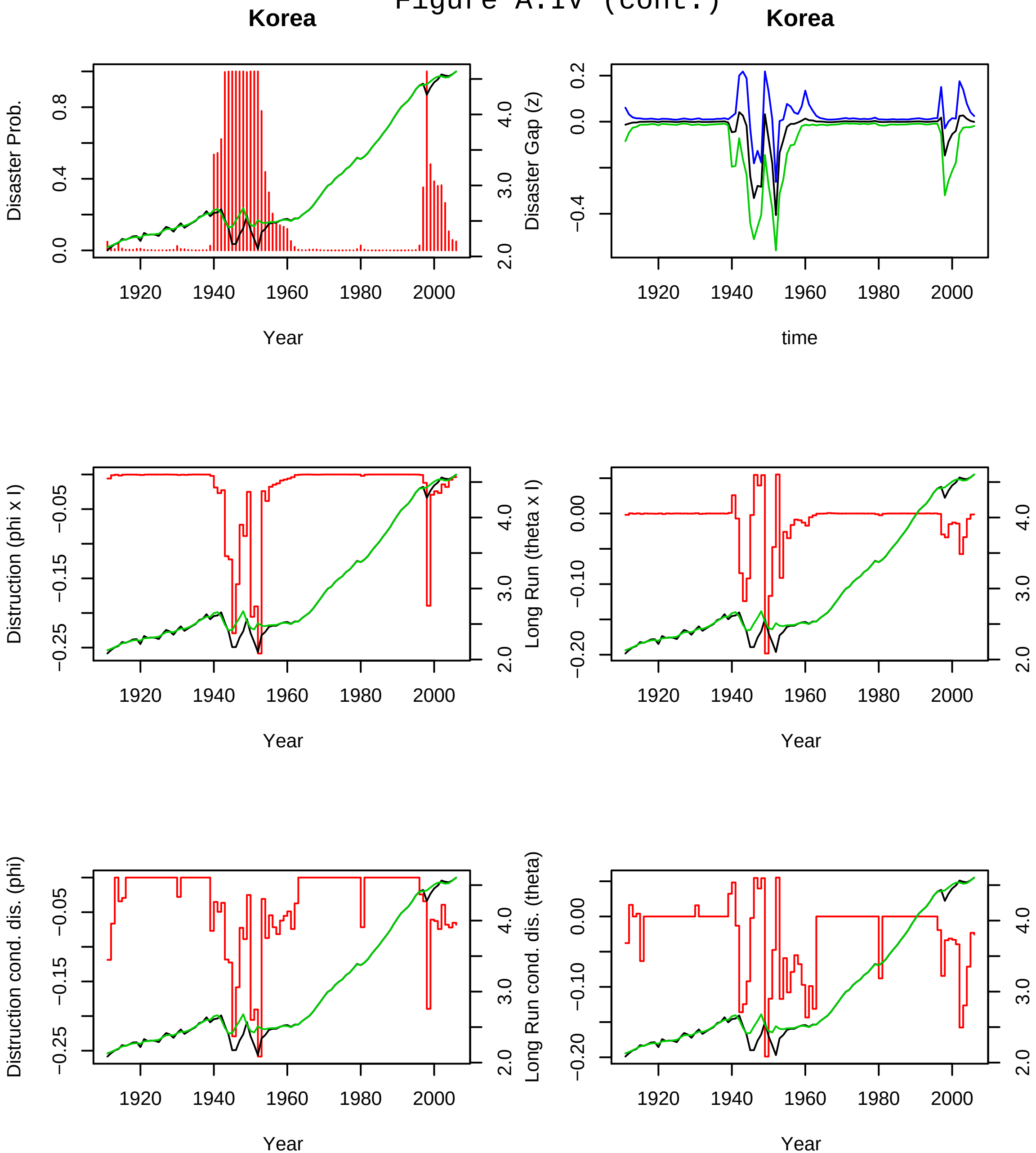

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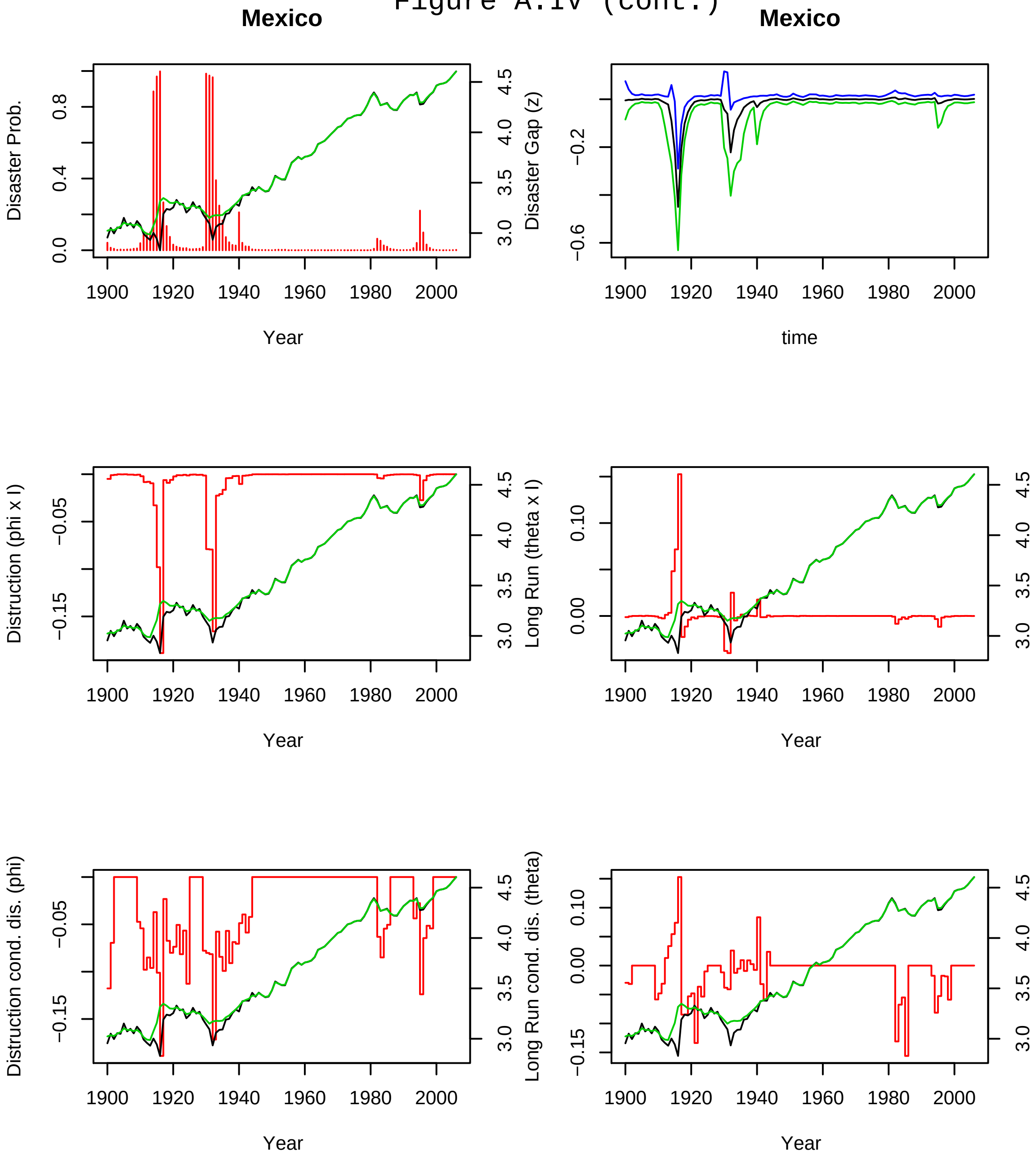

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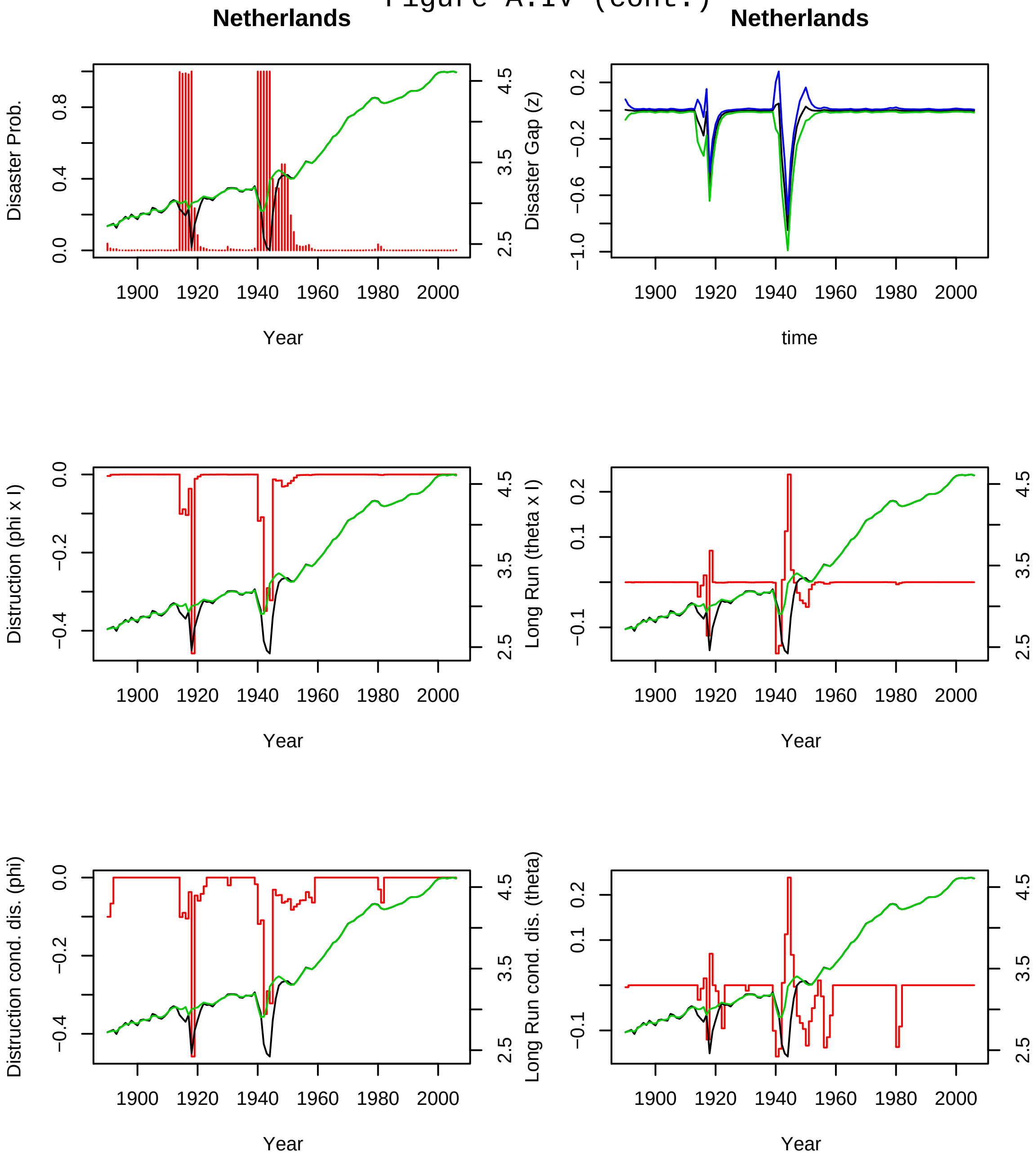

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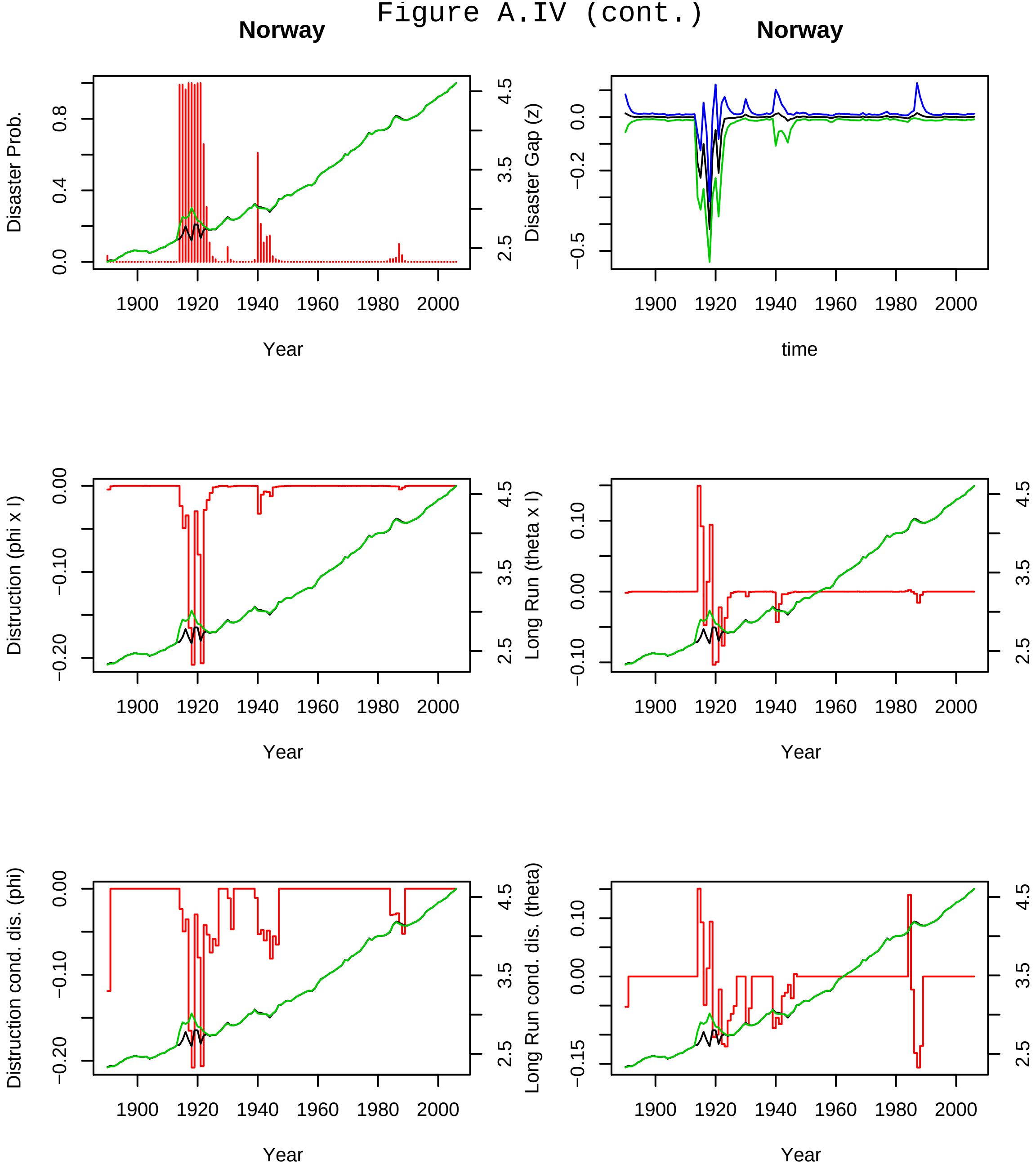

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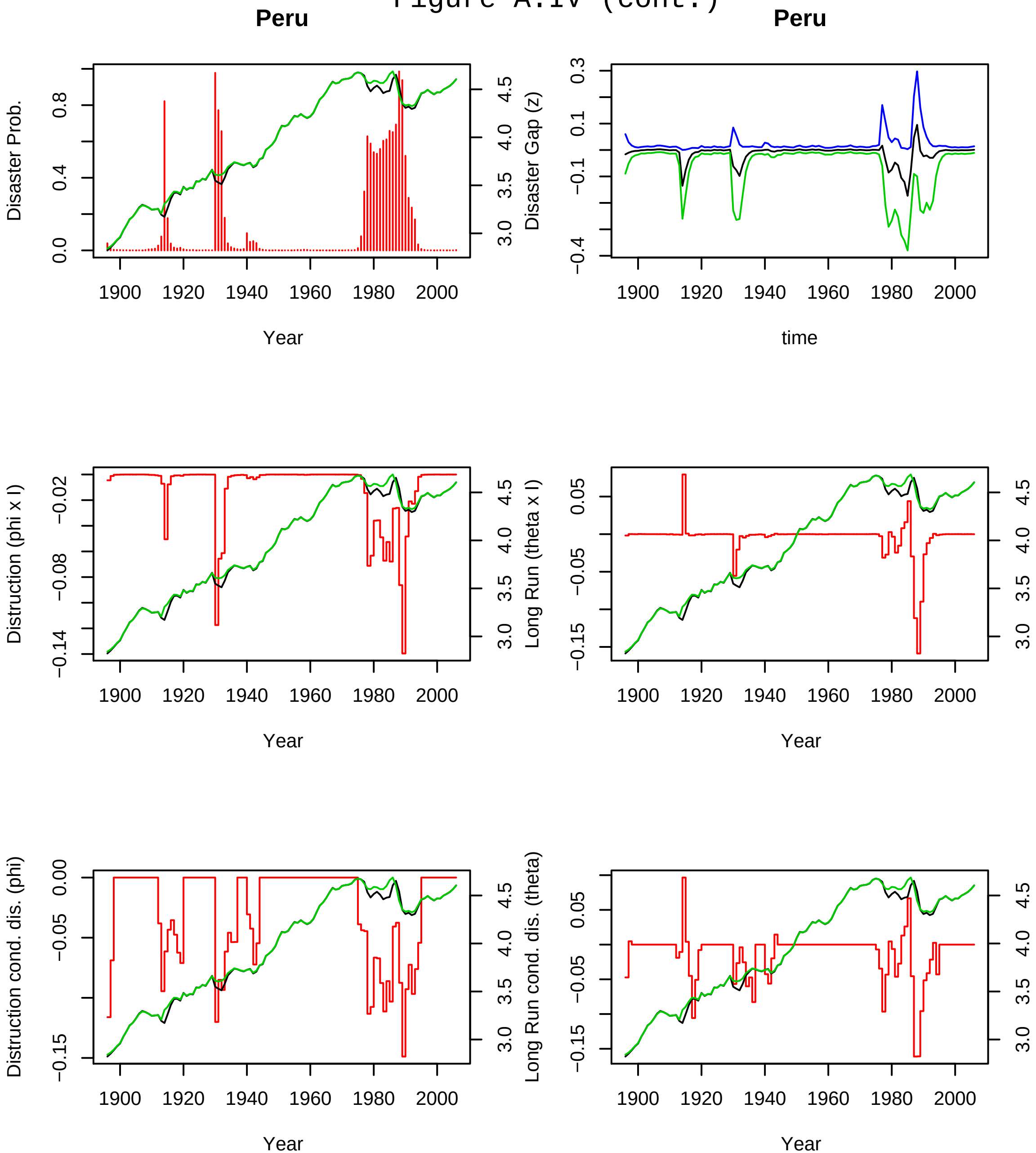

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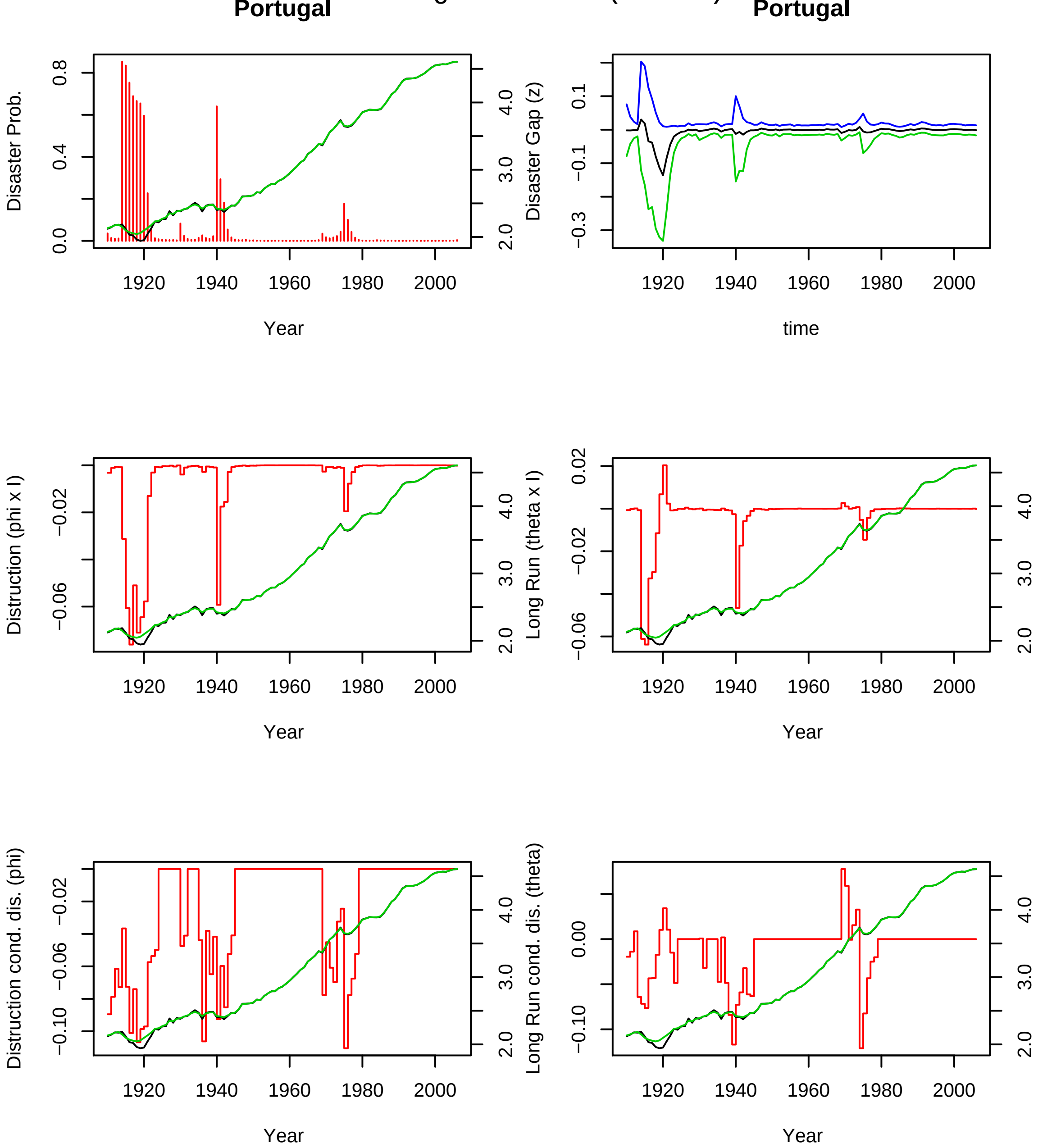

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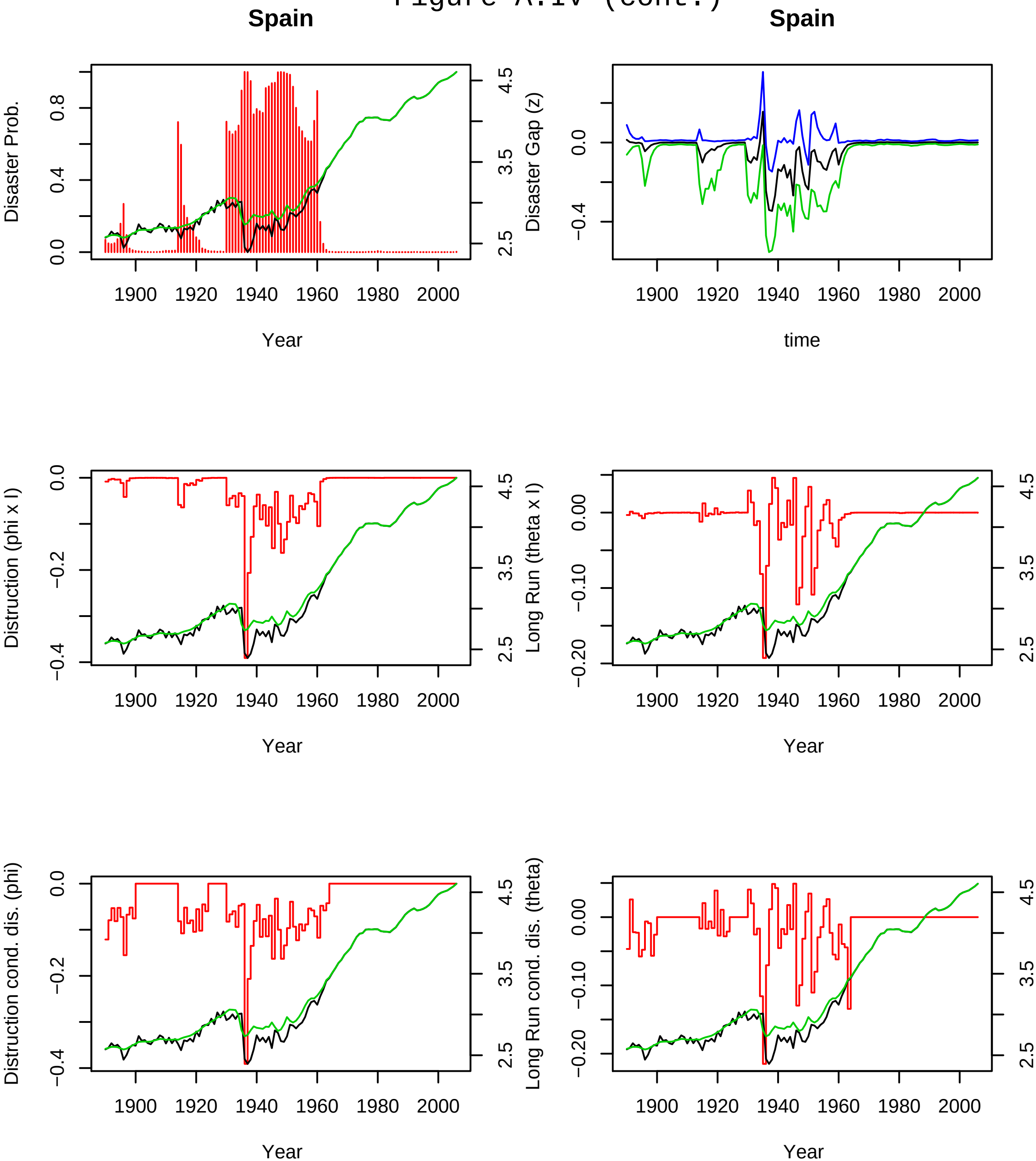

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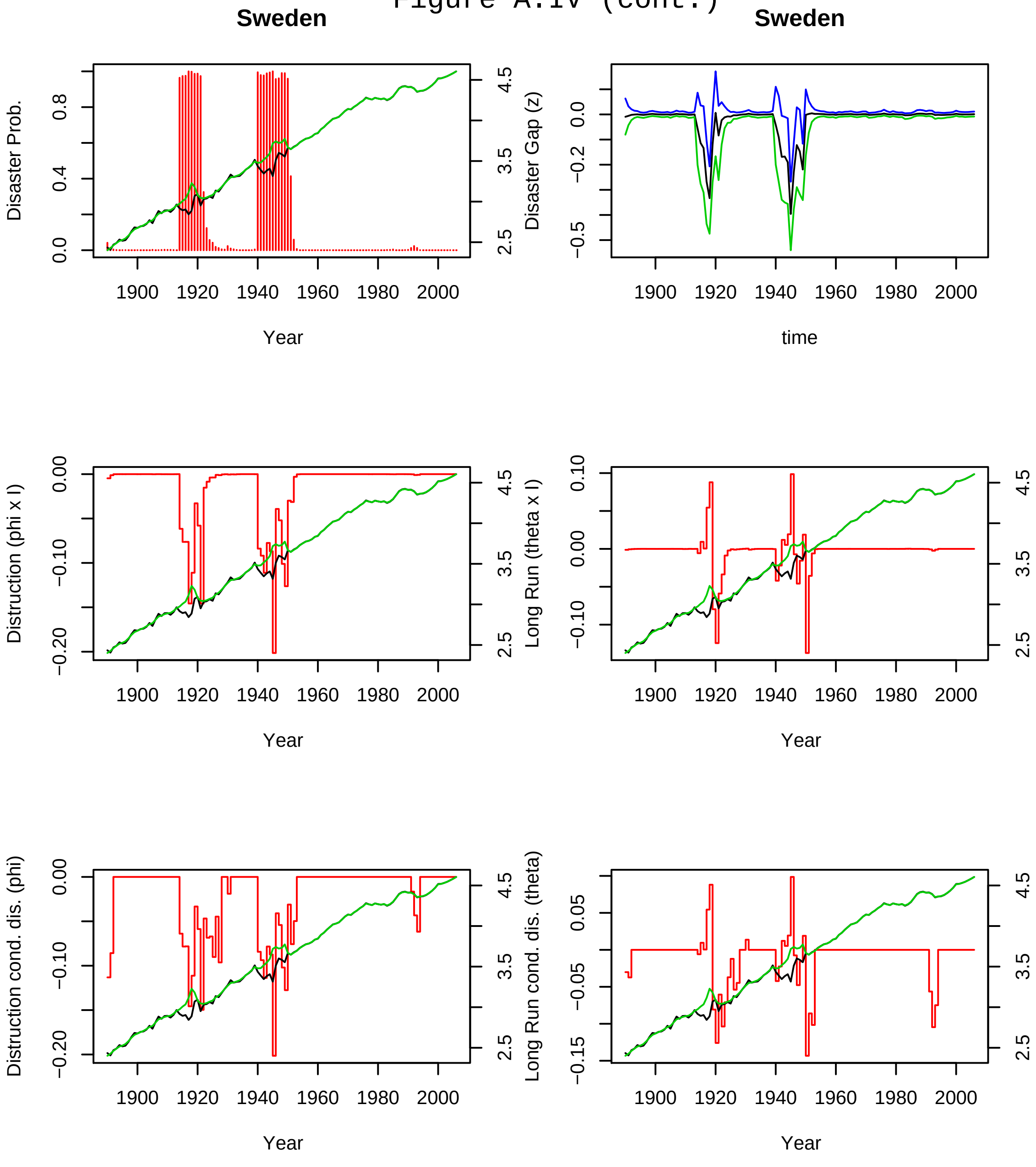

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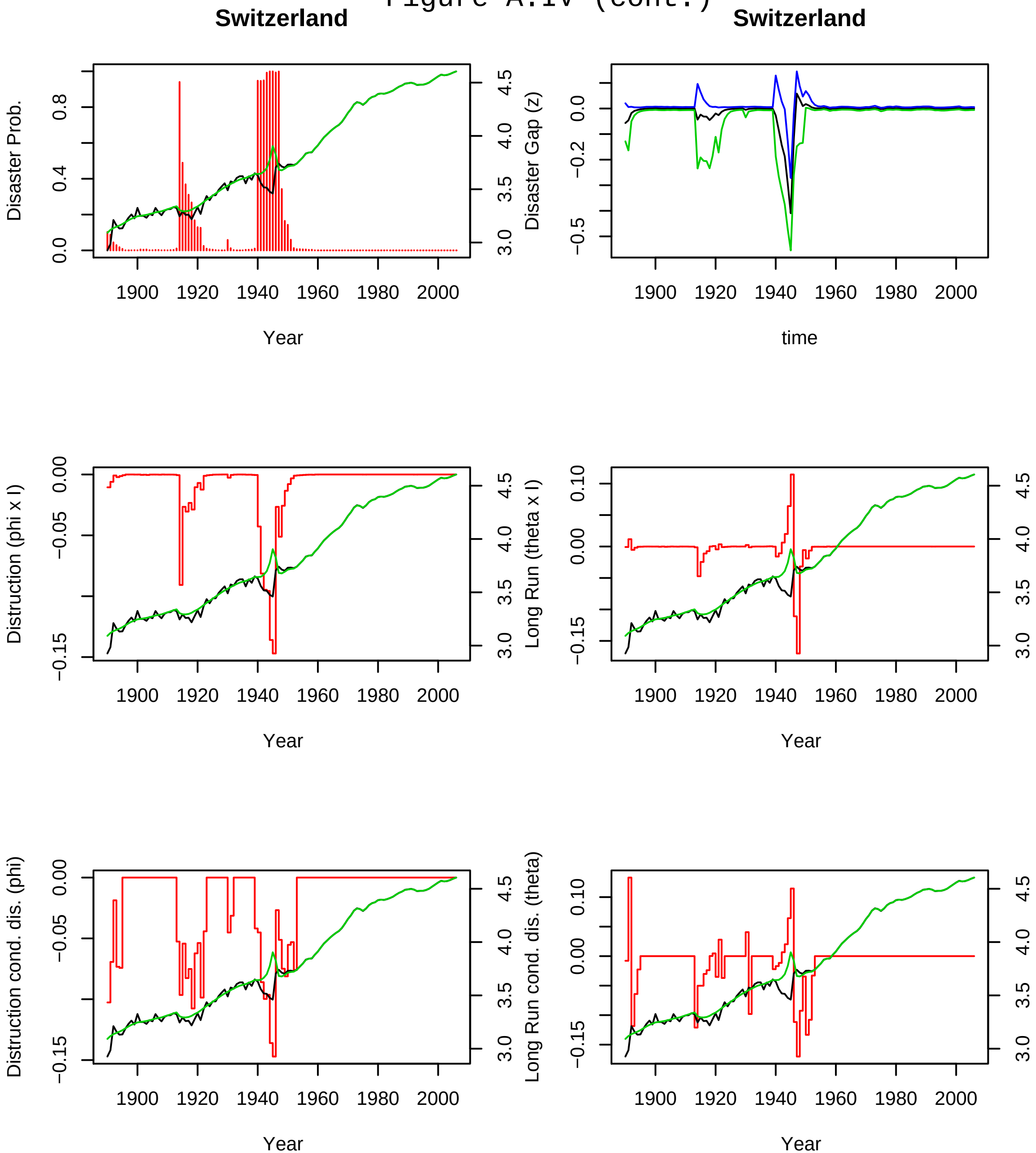

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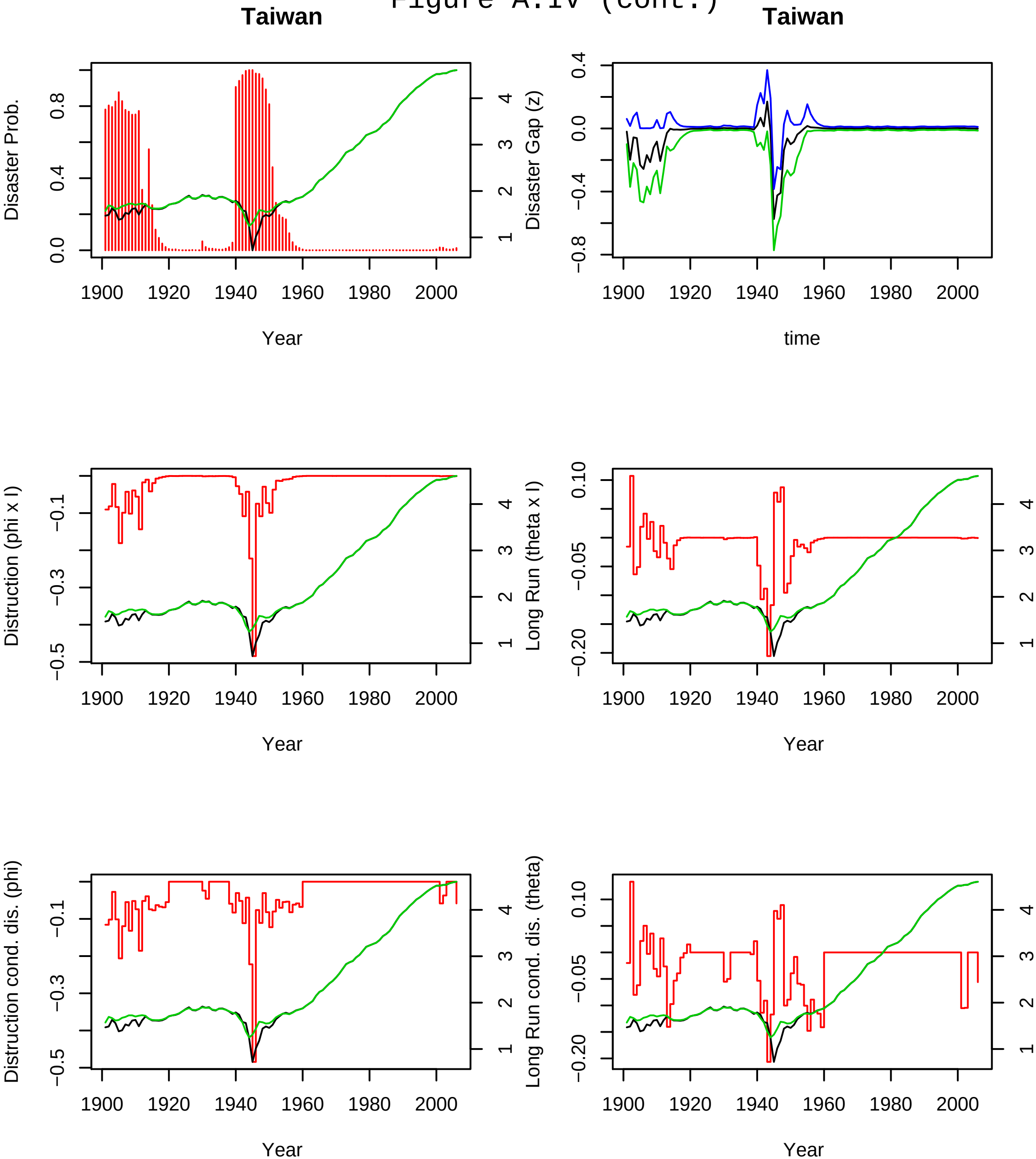

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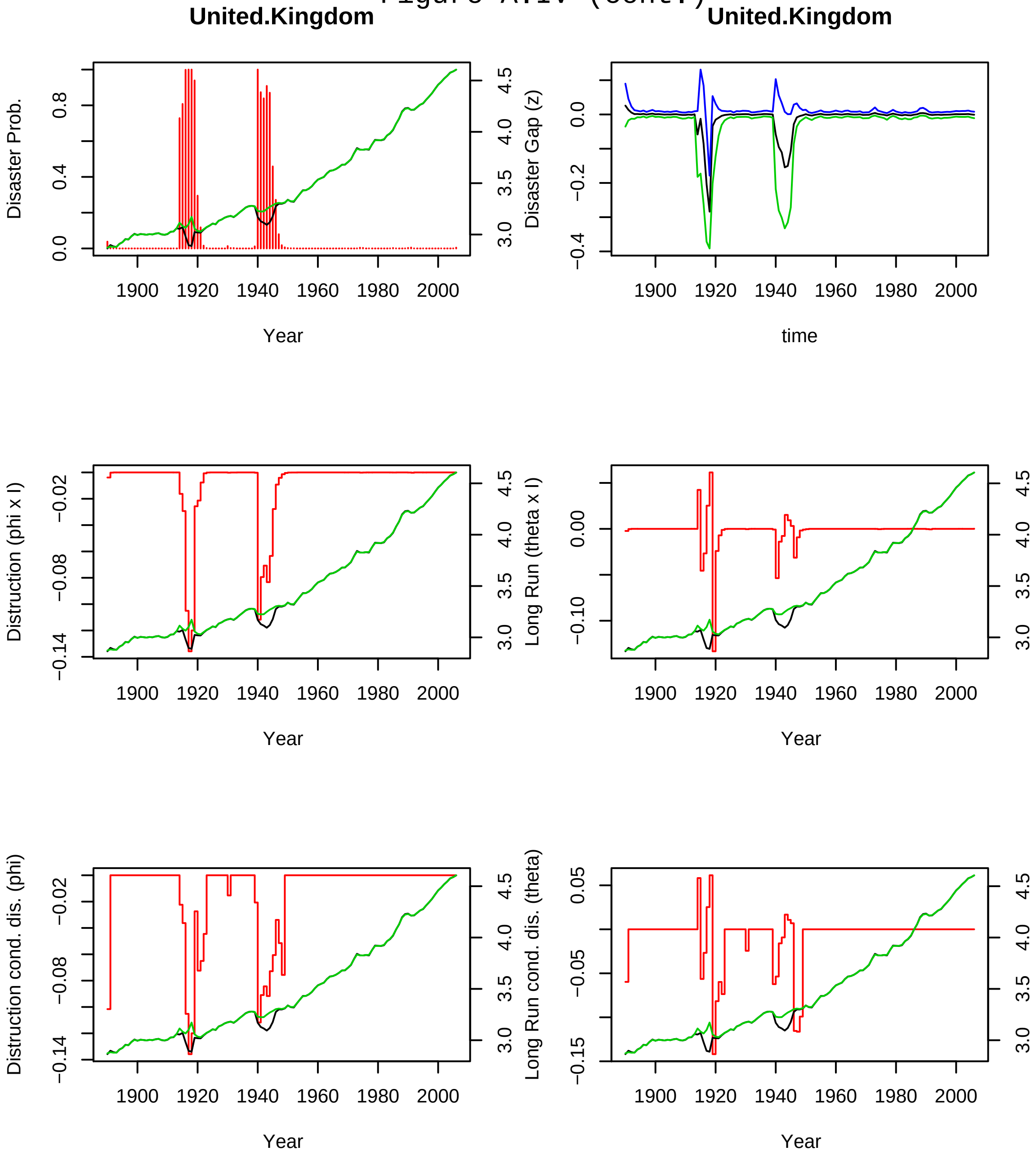

Figure A.IV (cont.)