Worker Reallocation Over the Business Cycle:
Evidence from Canada

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Abstract

This paper studies the cyclical variability of job finding, separations, and employer-to-employer flows in Canada for 1978-2016, the longest such time series available. Our analysis is based on direct administrative records of job separations. Our measures provide a much cleaner record of gross worker flows than standard household surveys. They are not subject to time-aggregation bias or to the measurement error problems that plague standard household surveys on employment dynamics. Employer-to-employer flows are strongly pro-cyclical and are the dominant component of both job finding and separation. We document several additional facts regarding the role of job-to-job flows in labor market fluidity, the near-constancy of the ratio of hires coming from employment versus unemployment, and the roles of “ins” vs “outs” in the Canadian labor market.

Keywords: Unemployment, Separations, Hiring, Quits, Layoffs.
JEL Classification: E24, E32.

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1 Introduction

As new data have become available, there has been a renewed discussion of the cyclical behavior of gross worker flows. Early studies captured net flows instead of gross flows. However, gross flows are large relative to net flows. Since gross worker flows are a reallocation of resources, their behavior has a direct bearing on the extent to which there are cleansing or sullying effects from recessions (Caballero and Hammour, 2005; Foster et al., 2016). Cleansing effects arise if less productive job matches are destroyed during recessions making room for more productive matches (Hall, 1991; Davis and Haltiwanger, 1992; Mortensen and Pissarides, 1994). However, if reallocation is pro-cyclical there may be a sullying effect. Instead of increasing the average quality of job matches, recessions can cause lower quality matches to last longer since fewer job-to-job transitions occur (Shleifer, 1986; Stadler, 1990; Aghion and Saint-Paul, 1998; Barlevy, 2002).

Worker flows are also needed to estimate search and matching functions, which are a key input into macro-labor models. They provide a means to distinguish between different search and matching models. Moreover, as these flows are the result of decisions to work or search for work, they may provide insight into labor supply decisions and labor market frictions (Krusell et al., 2017). A growing literature studies employer-to-employer flows, which are vital in developing appropriate macro-labor models (Nagypal, 2004; Krause and Lubik, 2006; Kiyotaki and Lagos, 2007; Menzio and Shi, 2011), and understanding wage dynamics (Karahan et al., 2017).

We have four main results. First, we provide evidence on the cyclical behavior of gross worker flows, including employer-to-employer flows, based on direct administrative data on job separations for the Canadian labor market. Our time-series on employer-to-employer flows is the longest that we are aware of in the literature. We find that the rate of job finding is strongly pro-cyclical. We also find that employer-to-employer flows are pro-cyclical. Our work builds heavily on a previous analysis of the Canadian labor market by Picot et al. (1998).

Second, we compare the flows into new jobs from unemployment versus workers coming from other jobs and find that these flows are roughly proportional. This supports common assumptions in models of random, on-the-job search. In particular, this pattern arises in simple models where employed and unemployed job search are modeled as having the same search technology, and

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1 Also see Caballero et al. (1996), Gomes et al. (2001), and Hornstein et al. (2003).
2 Also see Van Zandwegrhe (2010) and Mukoyama (2014).
fixed differential search intensity.

Third, we decompose a measure of labor market fluidity into employer-to-employer flows, employment inflows, and employment outflows. Our results show that about 60% of the variation in labor market fluidity can be accounted for by employer-to-employer transitions, with the remainder almost equally split between employment inflows and outflows.

Fourth, we revisit the “ins” and “outs” debate for Canada. The question is how much time-variation in the unemployment rate can be explained by flows into unemployment versus flows out of unemployment. Shimer (2012) and Elsby et al. (2013) argue that flows out (i.e., the hiring margin) play a dominant role, and this has motivated models in which the layoff rate is constant. For Canada, we find that outflows can explain about 60-70% of the variation in the unemployment rate, with inflows explaining about 20-40%, depending on the model and decomposition method used. We find a somewhat larger role for ins relative to previous work for Canada by Elsby et al. (2013), partly because of the greater detail of the data we have access to.\(^3\)

Employer-to-employer flows account for a huge fraction of total worker flows and plausibly play a fundamental role in the functioning of the labor market. Several studies have found that these flows exhibit pro-cyclicality (Fallick and Fleischman, 2004; Mazumder, 2007; Bjelland et al., 2011). Yet, the empirical evidence on employer-to-employer flows is limited for several reasons. First, measuring employer-to-employer flows accurately requires either direct data on when a worker changes jobs or else high frequency data. However, standard data sets such as the CPS only measure labor market status – such as employment or unemployment at a monthly frequency. This introduces time aggregation bias, as multiple transitions cannot be captured between measurement periods. Moreover, the leading US data sources have short time-series or important measurement issues.\(^4\)

For example, the Current Population Survey (CPS) only permits estimation of employer-to-employer flows after the 1994 redesign, when the CPS began to ask returning employed respon-

\(^3\)Fujita and Ramey (2009), Elsby et al. (2009), and Shimer (2012) decompose the variance in the unemployment rate in the US to analyze whether unemployment inflows or outflows contribute more to cyclical unemployment rate fluctuations. Campolieti (2011) performs this decomposition for Canada. Petrongolo and Pissarides (2008) decompose unemployment variation for several European countries, and Elsby et al. (2013) perform the decomposition for OECD countries including Canada.

\(^4\)Employer-to-employer flows have essentially been included in search models as early as Burdett (1978) in the form of on-the-job-search. However, there does not appear to be any work estimating these flows until Blanchard and Diamond (1989). Blanchard and Diamond constructed a rough estimate of employer-to-employer flows based on the strong assumption that these flows were 40% of job quits, a statistic they took from Akerlof et al. (1988).
dents whether they still worked for the same employer from the previous period (Fallick and Fleischman, 2004).

The Longitudinal Employer Household Dynamics (LEHD) dataset provides direct quarterly matched worker-firm data from the unemployment insurance system that allows one to measure worker flows, but this begins only in 2000 for all states.\(^5\) Bjelland et al. (2011) exploit worker histories to estimate job-to-job flows by identifying workers whose employers changed between periods. However, time aggregation bias is a serious issue in the LEHD as it reports data quarterly. This makes it impossible to determine whether an employment change should be classified as a job-to-job transition or a transition with an intervening non-employment spell.

The Job Openings and Labor Turnover Survey (JOLTS) provides separation and hiring data from a survey of about 16,000 business establishments that could be used to estimate job-to-job flows if combined with other data. However, the data only goes back to the end of 2000.\(^6\)

The SIPP has asked about the identity of the employer from its inception of 1983, which Mazumder (2007) uses to estimate employer-to-employer flows. However, as is well-known, the SIPP is susceptible to important recall and response bias because respondents are only interviewed every four months. In particular, the SIPP exhibits seam bias: respondents are more likely to record a change between interview periods than within interview periods.\(^7\)

While a long time-series of such flows is available for Germany, we believe our estimates provide a useful complement to the international evidence because of the similarity between the Canadian and US labor markets.\(^8\) The US and Canada have comparable levels of labor fluidity, with both countries exhibiting worker reallocation rates of about 40% over the period 2000 to 2007 (Bassanini and Garnero, 2013). The two countries are also similar in their collective bargaining structure (Flanagan, 1999; OECD, 1994, 2017). Moreover, Canada’s business cycles are strongly correlated with the US at around 0.9 (Artis and Zhang, 1997, 1999).\(^9\)

\(^5\)Data for some states begin as early as 1990. The aggregate data are publicly available, however, richer data with granular firm and employee characteristics are confidential.
\(^6\)Davis et al. (2012) construct synthetic JOLTS data that extends the time-series back to 1990.
\(^7\)Moreover, use of the SIPP requires adjustments as data is unavailable for the year 2000, and households in poverty are oversampled for the years 1990, 1996, and 2001 (Mazumder, 2007).
\(^8\)Bachmann (2005) finds that employer-to-employer flows are pro-cyclical in West Germany (1975-2001). Other international evidence includes Gomes (2012), who finds evidence of pro-cyclicality within the UK (1993-2010).
\(^9\)Germany differs from the US and Canada in these regards. Germany’s average worker reallocation rate is about 30%. Collective bargaining in Germany is usually at the industry-level with strong coordination. In comparison, bargaining is usually at the plant-level for the US and Canada, with no coordination. German business cycles show a correlation of about 0.5 with US business cycles.
We describe our data sources in Section 2. In Section 3 we discuss our methodology. In Section 4 we give basic statistics on gross job finding and separations from the ROE data, and transitions under a three-state model from LFS data. In Section 5 we present our main results. We conclude in Section 6.

2 Data

We use two main datasets: the Record of Employment (ROE) and the Canadian Labour Force Survey (LFS). The ROE is an administrative dataset from the Employment Insurance program that we use to estimate job finding and separations. The LFS is a monthly survey similar to the Current Population Survey, focusing on working-age individuals. We use the public-use microdata files that are more readily available to researchers but are also de-identified. The LFS data are used to estimate flows between labor force states. Combining these datasets allows us to estimate employer-to-employer flows.

Since 1976, all Canadian employers have been required to issue a ROE when a worker separates from a full-time job. Workers employed less than 20 hours a week have also been included in the ROE since 1997, though this seems to have had little impact on the statistics. The self-employed do not receive a ROE; thus when we merge the ROE and LFS data, we subtract flows due to self-employment from the LFS.\(^\text{10}\)

While the ROE extends as far back as 1976, the extracts available to us begin in 1987. Thus, we extend the data back to 1978 using data from Picot et al. (1998). Picot, Lin and Pyper use the Longitudinal Worker File, which draws from the ROE, tax data, and the Longitudinal Employment Analysis Program data.

We use administrative data on the number of ROE’s issued each year over the period 1978-2016 to measure the separation rate. This measure of separations is not subject to time aggregation bias since all separations are recorded, even if the subsequent unemployment spell is very short.

\(^{10}\)We thank Roger Hubley and Lesle Wesa for providing us with administrative details regarding the ROE. Over time, the minimum coverage requirement for the ROE has changed somewhat. The ROE was initiated in 1976. Over 1972 - 1978, the minimum weekly earnings requirement was 20% of maximum weekly insurable earnings. For 1979 - 1980, the minimum requirement was set at 20 hours of work a week or 20% of maximum weekly insurable earnings. Over 1981 - 1986, 15 hours a week and 20% of maximum weekly insurable earnings became the minimum. Over 1987 - 1996, 15 hours a week or 20% of maximum weekly insurable earnings was the minimum. Persons not reaching at least one of these requirements would not receive an ROE. Effective January 1, 1997, the minimum requirement was abolished and every hour of work became insurable. All persons with a job separation from paid employment should receive a ROE.
Furthermore, the data distinguishes between different types of separations, allowing us to identify the flows of quits, layoffs, or other separations.

To estimate job finding, we combine the ROE data with data on total insurable employment excluding the self-employed, from the Canadian Employment Insurance Statistics (CEIS).\textsuperscript{11} The insurable employment stock is about 80% of the total employment stock estimated by the Canadian Labour Force Survey, on average. The CEIS data are also used to construct separation rates.

From the LFS microdata, we take variables on labor force state, duration of unemployment, duration of joblessness, class of a worker’s main job, and job tenure length. Using the weighted labor force status we estimate the stock of employed (including self-employed individuals), unemployed, and out of the labor force individuals. In addition, we measure short-term stocks, i.e. individuals who have recently flowed into a given state. We estimate the stock of short-term unemployment as the weighted number of individuals with a duration of unemployment that is four weeks or less. Short-term non-employment is estimated similarly, using the weighted number of individuals with a duration of joblessness that is one month or less. We also measure the intersection of short-term unemployment and short-term non-employment. These individuals will have flowed from employment to unemployment. Short-term employment is estimated as the weighted number of individuals with a job tenure that is one month or less. All of these estimated stocks are seasonally adjusted using the Census Bureau’s X-12-ARIMA program.

We also use unemployment and inactivity stocks that are not seasonally adjusted but are instead averaged annually. These are combined with the insured employment stock and ROE data to construct job finding rates and are only used for this purpose.

For comparisons against the US, we use separation levels and rates from the Job Openings and Labor Turnover Survey (JOLTS) and seasonally adjusted unemployment rate data from the Current Population Survey.

\textsuperscript{11}This series is referred to by Statistics Canada as “insured employment”, and is available on Statistics Canada’s public database in Table 14-10-0006-01 (Formerly Table 276-0011 in CANSIM). Insured employment is defined as all "employees", excluding the self-employed, as defined in the Canadian Labour Force Survey plus the members of the armed forces, who are not included in the Labor Force Survey. Our measure of job finding differs from the measure in Picot et al. (1998). They calculate job finding from the number of “person-jobs” per year measure recorded in the Longitudinal Worker File. This procedure double-counts the employment of workers who change jobs during the year, generating a substantially more volatile measure of employment.
3 Methodology

Our goal is to estimate employer-to-employer transitions (EE). This can be measured as the difference between total job finding and employment inflows. Since employment inflows capture all new entrants into employment, any job finding flows in excess of this must be from employer-to-employer transitions, as shown in Figure 1.\textsuperscript{12}

Employment inflows can be estimated using the LFS. We use aggregated stock data from the LFS to estimate transition probabilities using a three-state extension of the method proposed in Darby et al. (1985), where the three states are employment $E_t$, unemployment $U_t$, and out of the labor force (inactivity) $O_t$. Let $p^{XY}$ denote the probability of moving from state $X$ to state $Y$.

\textsuperscript{12}While our framework provides a relatively simple way to estimate employer-to-employer flows, it is subject to some limitations. Davis et al. (2006) warn that without longitudinal data, estimates will include spurious employer-to-employer flows. For example, with the data we have, it is impossible to distinguish whether an employer-to-employer transition has occurred, or if an individual has taken on multiple jobs. Moreover, we cannot adjust for the fact that insurable employment includes full-time members of the Armed Forces, while the employment from the LFS does not. However, this is likely to be a minor effect as there are only about 100,000 military members of the Armed Forces as of 2018, while average employment over the period 1978-2016 is about 14 million (http://www.forces.gc.ca/en/about/faq.page).
For example, $p^{EU}$ denotes the probability of moving from employment to unemployment. Now consider the following five equations:

$$U_{t+1} = U_t + E_t p^{EU}_t + O_t p^{OU}_t - U_t (p^{UE}_t + p^{UO}_t)$$ (1)

$$E_{t+1} = E_t + U_t p^{UE}_t + O_t p^{OE}_t - E_t (p^{EU}_t + p^{EO}_t)$$ (2)

$$U_{t+1}^s = E_t p^{EU}_t + O_t p^{OU}_t$$ (3)

$$N_{t+1}^s = E_t (p^{EU}_t + p^{EO}_t)$$ (4)

$$f^{EU}_t = E_t p^{EU}_t$$ (5)

Where $U_t^s$ is short-term unemployment, and $N_t^s$ is short-term non-employment, and $f^{EU}$ is the flow from employment to unemployment. We measure $f^{EU}$ as the next period intersection of short-term unemployment and short-term non-employment.

With these five equations, we only need one more condition to solve the system. We assume that the probability of a transition from labor force inactivity to employment, $p^{OE}$ is the midpoint of its boundary values in each period. We can bound $p^{OE}$ by noting that $p^{UO}$, $p^{UE}$, and $p^{OE}$ should be non-negative. The full solution for this model is shown in Appendix B. Note that our assumption on $p^{OE}$ is only needed to estimate $p^{UO}$ and $p^{UE}$. Since we can directly estimate $p^{EU}$ from the data, we can solve for $p^{OU}$ and $p^{EO}$. Moreover, our assumption on $p^{OE}$ has no impact on our estimate of the employer-to-employer rate as the assumption simply affects how employment inflows are apportioned between $p^{OE}$ and $p^{UE}$. As these flows will suffer from time-aggregation bias, we apply Mukoyama’s (2014) time-aggregation correction to the transition rates. We also calculate the transitions assuming $p^{OE}$ is constant as a check. We set $p^{OE} = 0.006$, which is the approximate midpoint of the constant boundary values that guarantee $p^{UO}$, $p^{UE}$, and $p^{OE}$ are negative in every time period.\(^{13}\)

To estimate total job finding, we turn to the ROE data. The ROE data allows us to directly measure total job separations, which can be disaggregated into quits, layoffs, and other separations.

\(^{13}\)Jones (1993) finds the average hazard rate for this flow is 3.7% using restricted LFS data, however, this leads to negative unemployment to employment transitions in our model.
We can then estimate total job finding from the ROE data using the following identity,

\[ E_i^t = E_i^{t-1} - S_{t-1} + H_{t-1}, \]  

(6)

where \( E_i^t \) is the number of employed persons covered by the ROE, \( S_{t-1} \) is the number of separations (measured by the number of ROE’s issued) and \( H_{t-1} \) is the number of jobs found.\(^{14}\) To make our rates from the ROE comparable to the rates from the LFS, we divide job finding and separation rates by 12, so that they are on a monthly basis.

We can then estimate employer-to-employer flows using the equation

\[ H_t = p^{EE} E_t + p^{UE} U_t + p^{OE} O_t - f_{se}^t, \]  

(7)

where \( p^{EE} \) is the employer-to-employer rate, and \( f_{se}^t \) is new entrants into self-employment. The latter must be subtracted from the employment inflows estimated from the LFS data as the self-employed are not covered by the ROE. We estimate \( f_{se}^t \) as the number of individuals whose main job is considered self-employment and whose self-employment tenure is one month or less. Since the ROE data are annual frequency while LFS data are monthly frequency, we adjust the data to make them compatible. In particular, we use the annual average of employment inflows and self-employment inflows in Equation 7. Thus, while \( p^{EE} \) is given on a monthly basis, there is only one unique value per year.

Since we are interested in how much employer-to-employer transitions contribute to variation in the labor market, we need to construct a measure of labor market fluidity. One possible measure is the sum of job finding and separations. Davis and Haltiwanger (1990) call this measure ‘worker reallocation’. However, worker reallocation double-counts employer-to-employer transitions. Following Kiyotaki and Lagos (2007) we subtract employer-to-employer flows from worker reallocation. We refer to this measure as ‘total worker flows’, and use it as our measure of labor market fluidity.

As in Shimer (2012), we assume that all transitions follow a Poisson process such that the continuous rate \( \tilde{p}_t \) can be computed from the discrete rate \( p_t \) according to the formula \( \tilde{p}_t = -\log(1 - p_t) \). We apply this transformation to all rates we compute and only report the trans-

\(^{14}\)This procedure for estimating the number of people hired implicitly assumes that each worker has only a single job.
formed rates, although the difference is essentially negligible for our three-state results. Unlike Shimer, we do not make a distinction between ‘rates’ and ‘probabilities’ and use the terms interchangeably to refer to the continuous rates.

We detrend rates with a Hodrick-Prescott (HP) filter where noted to remove secular trends. Following Shimer (2012) we use a parameter of $\lambda = 10^4$ for the annual ROE data and a parameter of $\lambda = 10^6$ for the monthly Labour Force Survey data.

4 Basic Statistics

4.1 Canada vs. US

Business cycle fluctuations in the Canadian and US unemployment rates have mirrored one another closely over recent decades. Figure 2a plots the seasonally adjusted monthly unemployment rate for Canada and US over the period 1978-2016. We use data that have not been detrended for this figure. The business cycles in Canada and the US have been remarkably similar over this period. Figure 2b shows how the Canadian ROE separation rates compare to US separation rates from the JOLTS using data that have not been detrended.15 The Canadian and US series track each other closely and exhibit similar magnitudes.

4.2 Worker Flows

The levels of gross flows are shown in Figure 3. What is remarkable is the size of employer-to-employer flows relative to other flows in and out of employment. Moreover, the flat trend exhibited by employment inflows and outflows means that the increase in total job finding and separations comes almost solely from employer-to-employer flows. The only other flows that show an upward trend are flows between unemployment and inactivity.

The average flow rates for these transitions, as well as their correlations with the detrended unemployment rate, are displayed in Table 1.16 We compute the total job finding rate by dividing

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15 Our US results are similar to what Davis et al. (2006) find using the JOLTS.
16 Our results depart, in some cases substantially, from Jones (1993) who uses the panel component of the restricted-use LFS data to estimate worker flows. For example, Jones estimates an average of 1.3 million transitions between employment, unemployment, and inactivity per month. Our results suggest that on average during a similar time period just over 920,000 transitions are made. The direction of this difference is consistent with studies of response errors in US survey data. Poterba and Summers (1986) argue that response errors in labor force states cause spurious transitions to be recorded. This could cause Jones’ estimates to be inflated. Bowers and Horvath (1984) suggest that
Figure 2: Canada vs US


Figure 3: Worker Flow Levels (Thousands)

Notes: The figure plots annual worker flows for Canada over 1978-2016. These data are not detrended.
Table 1: Average Gross Worker Flow Rates

<table>
<thead>
<tr>
<th></th>
<th>Mean</th>
<th>Corr w/ U</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Job Finding</strong></td>
<td>0.030</td>
<td>-0.58</td>
</tr>
<tr>
<td>EE</td>
<td>0.030</td>
<td>-0.51</td>
</tr>
<tr>
<td>UE</td>
<td>0.11</td>
<td>-0.79</td>
</tr>
<tr>
<td>OE</td>
<td>0.012</td>
<td>0.21</td>
</tr>
<tr>
<td><strong>Job Separations</strong></td>
<td>0.053</td>
<td>-0.28</td>
</tr>
<tr>
<td>EE</td>
<td>0.030</td>
<td>-0.51</td>
</tr>
<tr>
<td>EU</td>
<td>0.008</td>
<td>0.35</td>
</tr>
<tr>
<td>EO</td>
<td>0.007</td>
<td>0.20</td>
</tr>
<tr>
<td>Layoffs</td>
<td>0.022</td>
<td>0.59</td>
</tr>
<tr>
<td>Quits</td>
<td>0.012</td>
<td>-0.70</td>
</tr>
<tr>
<td>Other</td>
<td>0.017</td>
<td>-0.07</td>
</tr>
<tr>
<td>OU</td>
<td>0.048</td>
<td>-0.02</td>
</tr>
<tr>
<td>UO</td>
<td>0.28</td>
<td>-0.49</td>
</tr>
</tbody>
</table>

Notes: The first column reports time-series averages of each worker transition probability in Canada. The second column reports the correlation with the detrended unemployment rate. The unemployment rate is detrended using an HP-filter with $\lambda = 10^6$. The sample period is 1978-2016.

Job finding flows by the working-age population minus uninsurable employment. This procedure results in an average rate of 0.030. The job finding rate can then be decomposed into EE flows (with an average rate of 0.030), UE flows (0.11), and OE flows (0.012). The total job finding rate is negatively correlated with the detrended unemployment rate (-0.58). The EE rate (-0.51), and UE rate (-0.79) are also pro-cyclical, while OE flows are counter-cyclical (0.21).

The average job separation rate out of insurable employment is 0.053. The separation rate can be decomposed in two ways: into EE, EU (0.008), and EO (0.007), or into layoffs (0.022), quits (0.012), and other separations (0.017). The total separation rate is pro-cyclical (-0.28). Under the first decomposition, the pro-cyclicality is due to EE flows, which overwhelm the counter-cyclicality of employment outflows (0.35 for EU and 0.20 for EO). Under the second decomposition, the pro-cyclicality predominately comes from quits (-0.70), which bury the counter-cyclicality of layoffs (0.59). Other types of separations are nearly acyclical (-0.07).

Flows between unemployment and inactivity are both large, with an average rate of 0.048 for OU flows, and 0.28 for UO flows. The OU rate is effectively acyclical (-0.02). However, the UO rate exhibits pro-cyclicality (-0.49).
Assuming that the OE rate is constant increases the average UE flow to 0.15, and decreases the UO rate to 0.20. This decreases the pro-cyclicality of UE flows (-0.64), as well as UO flows (-0.30).\textsuperscript{17}

Much of the literature (e.g. Elsby et al., 2009; Fujita and Ramey, 2009; Shimer, 2012) has used a two-state model.\textsuperscript{18} How much difference does this make? It affects the level of the flows a bit (0.43 vs 0.38 for unemployment outflows, 0.022 vs 0.021 for inflows), but has almost no effect on the cyclicality. The level of unemployment outflows in the two-state model has a correlation of 0.99 with unemployment outflow levels from the 3-state model (UE + UO), and the correlation is 0.99 for the unemployment inflow levels.

5 Main Results

5.1 Cyclicality of Employer-to-Employer Transitions

We show the basic statistics for employer-to-employer flow rates in Table 1. The average monthly rate is 3\%, which is substantially larger than the other flows originating from employment. It is comforting that the average rate we find is similar to those found for the US by Fallick and Fleischman (2004) and Nagypal (2004) who estimate monthly rates of 2.6\% and 2.75\% respectively for the US using CPS data.\textsuperscript{19} Figure 4 shows that employer-to-employer flows exhibit a strong negative correlation with the unemployment rate (-0.51).

5.2 Proportionality of EE and UE Flows

We compare EU flows against our direct measure of layoffs, and UE flows against EE flows in Figure 5. Our measure of the EU rate closely tracks layoffs. In contrast, there is almost no correlation between the ROE measure of separations and the EU rate. This reflects the massive role of employer-to-employer transitions (which are pro-cyclical and dwarf the counter-cyclicality of layoffs) in total separations. Similarly, the UE rate closely tracks EE flows What does this tell us about labor market search models?

Consider a simple model of random, on-the-job search based off of Petrongolo and Pissarides

\textsuperscript{17}When calculating the UE and UO rates under a constant OE, we do not correct for time-aggregation bias.

\textsuperscript{18}We show details of the two-state model and its results in Appendix D.

\textsuperscript{19}Our results are substantially above those of Bjelland et al. (2011), who estimate average quarterly rates of 3.9\% for the US using the Longitudinal Employer Household Dynamics database. This is not unexpected as Bjelland et al. (2011) focus on stable employer-to-employer transitions, and thus do not address transitions between short-term jobs.
Figure 4: Employer-to-Employer Rate

Notes: The figure plots the imputed monthly rate of employer-to-employer transitions, alongside the unemployment rate, for Canada over 1978-2016. The employer-to-employer flows are estimated using data from the Record of Employment and Labour Force Survey, while the unemployment rate is drawn from the Labour Force Survey. All data are detrended using an HP Filter with $\lambda = 10^6$.

Let the meeting technology for both employed and unemployed job searchers be described by $m(sE_t + U_t, V_t)$, where $E_t$ denotes the employment stock, $U_t$ denotes the unemployment stock, $V_t$ is the level of vacancies, and $s$ is the search intensity of employed searchers relative to unemployed searchers. The match efficiency multiplier for unemployed searchers is exogenously given as $A_U$, while the match efficiency for employed searchers is exogenously given as $A_E$. Within match efficiency we include the acceptance probability that a meeting will result in a match. We also assume that any trends in search intensity will be captured by match efficiency, as search intensity and match efficiency cannot be separately identified without additional data (Hall and Schulhofer-Wohl, 2015). Next, assume that total meetings between searchers and vacancies are proportionately distributed between employed and unemployed searchers according to their contribution to total search effort. Then the UE flow rate and employer-to-employer flow rates will be given by:

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20 We ignore search from outside the labor force.
21 It is simple to extend the model to allow unemployed searchers to receive more or less than their ‘fair’ proportion of meetings, as in Burgess (1993).
Notes: Panel 5a compares layoffs with a three-state measure of employment to unemployment transitions for Canada over 1978-2016. Layoffs are based on annual Record of Employment data, detrended using an HP filter with $\lambda = 10^4$. The EU measure is the quarterly average of monthly series constructed using Labour Force Survey data, detrended with $\lambda = 10^6$.

Panel 5b compares the employer-to-employer transition rate with the UE rate. Both are detrended using an HP-filter with $\lambda = 10^6$. 

Figure 5: Layoffs vs EU and Job Finding vs UE
\[ UE = A_U \frac{1}{sE_t + U_t} m(sE_t + U_t, V_t) \]  
\[ EE = sA_E \frac{1}{sE_t + U_t} m(sE_t + U_t, V_t) \]

The ratio of UE flows to EE flows will be:

\[ \frac{UE}{EE} = \frac{A_U}{sA_E} \]

Thus, the proportionality result we find will be generated if the match efficiencies remain stable over the business cycle, or if they change proportionally. Studies of match efficiency for unemployed searchers have found that it exhibits pro-cyclicality (Barlevy, 2011; Veracierto, 2011; Barnichon and Figura, 2015). Our result is consistent with Hall and Schulhofer-Wohl (2015), who estimate match efficiency among heterogeneous job searchers, including employed searchers. They find an aggregate measure of match efficiency that includes employer-to-employer flows shows the same movement as an aggregate measure that excludes those flows.\textsuperscript{22} This suggests that the match efficiencies for employed and unemployed searchers comove.

In addition to the assumption that the ratio of aggregate matching efficiencies is constant, these results require two further assumptions. First, the structure and parameters of the meeting technology are identical. That is, the search behavior of the employed and unemployed can be modeled as being the same. Second, the total number of meetings between job searchers and vacancies is distributed based on their contribution to total search effort.

These two assumptions are common in random, on-the-job search models that endogenize the offer arrival rate (i.e. the meeting technology) (e.g. Burgess, 1993; Pissarides, 1994, 2000), including state-of-the-art models such as Lise and Robin (2017) that allow for heterogeneity among job searchers and firms. The assumption of a constant ratio between match efficiencies depends on the structural details of the model. For example, Lise and Robin allow for the possibility that this ratio will be constant depending on the implementation of the model.\textsuperscript{23}

\textsuperscript{22}Hall and Schulhofer-Wohl (2015) estimate job finding probabilities for different classes of searchers using a logit function with CPS data. Then using vacancy data from the JOLTS, they estimate detrended match efficiencies as the residual of a regression of job finding probability on a measure of labor market tightness.

\textsuperscript{23}In Lise and Robin (2017) the acceptance function is based on match surplus (the difference between the value of a match and the outside option). Match surplus varies over time only through aggregate productivity shocks that affect
Notes: This plots the EE / UE ratio against the detrended unemployment rate. The unemployment rate is detrended using an HP-filter with $\lambda = 10^6$.

Moscarini and Postel-Vinay (2019) offer another interpretation of proportionality between UE and EE flows. They argue that the ratio of EE to UE can be viewed as a measure of mismatch or misallocation for employed workers. The EE rate will increase relative to the UE rate when more employed workers are mismatched, such that they can easily be poached away from their current employer. In their model, misallocation will be greatest after recessions. This is because workers who move from unemployment to employment are more likely to be poorly matched. As the economy expands, firms find it increasingly difficult to poach workers since each potential EE transition moves worker wages closer to the value of the marginal product of their labor. This proceeds until a recession strikes, increasing the pool of unemployed workers and beginning the cycle anew.

Is counter-cyclical mismatch borne out in the Canadian data? Figure 6 plots the EE / UE both match values and the outside option. If aggregate productivity shocks affect match values and the outside option in the same way the model will generate strict proportionality of UE and EE flows. However, in their implementation of the model, only match value is a function of aggregate shocks, while the outside option is not. How closely UE and EE comove depends, therefore, on how much the ratio of acceptance probabilities of employed versus unemployed workers varies over the business cycle.

24Moscarini and Postel-Vinay (2019) ignore heterogenous outside options (reservation wages) of unemployed workers.
ratio against the detrended unemployment rate. The EE / UE ratio tracks the unemployment rate remarkably closely with a correlation of 0.68 using annual data. The main divergence between the two is during the tech bubble, when the EE / UE ratio recovers even as the unemployment rate continues to fall. The strong counter-cyclicality of the EE / UE ratio is particularly noteworthy given that both the EE and UE rate are pro-cyclical.

5.3 Role of EE Flows in Labor Market Fluidity

We have described gross worker flows, but how do these various flows contribute to labor market fluidity? We can approach this using total worker flows ($w_f$) as our measure of labor market fluidity, which we base off of Kiyotaki and Lagos (2007).\footnote{Kiyotaki and Lagos (2007) note that worker turnover = job finding + separations, and that $w_f$ = worker turnover – EE, from which our Equation 11 can be inferred.} This can be decomposed into three components: employer-to-employer transitions (EE), employment inflows (NE), and employment outflows (EN).\footnote{Haltiwanger et al. (2015) also decompose worker flows into employer-to-employer flows, employment inflows, and unemployment outflows. Our paper performs this decomposition for gross worker flows, while they decompose net flows.}

$$w_f = EE + NE + EN$$

(11)

The latter two flows are simply job finding less employer-to-employer flows, and separations less employer-to-employer flows, all of which we detrend using the HP filter with $\lambda = 10^4$.\footnote{We construct these measures using the ROE data instead of our three-state flows from the LFS. This is because our measure of employer-to-employer flows excludes self-employment.}

As these components are additively separable, we can easily decompose the variance of total worker flows by following the logic of Fujita and Ramey (2009).

$$Var(w_f) = Var(EE) + Var(NE) + Var(EN)$$

$$+ 2Cov(EE, NE) + 2Cov(EE, EN) + 2Cov(NE, EN)$$

(12)

$$= Cov(EE, w_f) + Cov(NE, w_f) + Cov(EN, w_f)$$

Since the variance of total worker flows will be the sum of the covariances between it and its components, we can determine the contribution of each component by dividing both sides of the
Table 2: Total Worker Flows Decomposition

<table>
<thead>
<tr>
<th>EE</th>
<th>Employment Inflows</th>
<th>Employment Outflows</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.62</td>
<td>0.20</td>
<td>0.18</td>
</tr>
</tbody>
</table>

Notes: The table presents decompositions of the variance of total worker flows for Canada over 1978-2016. These flows are estimated using data from the Record of Employment and Labour Force Survey, and are detrended using an HP filter with $\lambda = 10^4$.

equation by the variance of $w_f$. For example, the contribution from EE flows will be,

$$
\beta_{EE} = \frac{\text{Cov}(EE, w_f)}{\text{Var}(w_f)} \tag{13}
$$

This allows us to rewrite Equation 12 as follows.$^{28,29}$

$$
1 = \beta_{EE} + \beta_{NE} + \beta_{EN} \tag{14}
$$

Table 2 shows the contributions from each component. We find that about 60% of the cyclical variation in total worker flows can be accounted for by employer-to-employer transitions. Employment inflows and outflows each explain about another 20%. The dominant role of employer-to-employer transitions, coupled with its pro-cyclicality, suggest that recessions have a sullying effect on the Canadian Labor Market. The reduction in worker reallocation during recessions potentially leads to lower quality job matches.

5.4 The Ins and Outs of Unemployment in Canada

Variance decompositions have also been used to study whether unemployment inflows or outflows contribute more to the variation in the unemployment rate. A recent generation of search models has included only endogenous job creation, as opposed to endogenous separation, motivated by the notion that endogenous separations are acyclical (Hall, 2006; Shimer, 2012). However, contemporary papers that have decomposed the unemployment rate have found a more nuanced picture.$^{30}$

\footnote{\(\beta_{EE}\) can be easily computed as the slope coefficient from a simple regression of $EE$ on $w_f$.}
\footnote{As Fujita and Ramey note, these terms are defined identically to the betas used in finance.}
\footnote{Earlier papers in the literature framed the argument in terms of unemployment incidence (inflows) versus unemployment duration (outflows). Sider (1985) and Baker (1992) argue that outflows play a more prominent role in understanding unemployment dynamics. Darby et al. (1985, 1986), Blanchard and Diamond (1990), Davis and Haltiwanger (1990, 1992), Burgess (1992), and Burgess and Turon (2005) support the position that inflows are the dominant}
Table 3: Unemployment Rate Decomposition

<table>
<thead>
<tr>
<th></th>
<th>UE</th>
<th>EU</th>
<th>UO</th>
<th>OU</th>
<th>EO</th>
<th>OE</th>
</tr>
</thead>
<tbody>
<tr>
<td>Canada</td>
<td>0.55</td>
<td>0.26</td>
<td>0.19</td>
<td>0.001</td>
<td>0.07</td>
<td>-0.04</td>
</tr>
<tr>
<td>US Shimer (2012)</td>
<td>0.49</td>
<td>0.22</td>
<td>0.17</td>
<td>0.12</td>
<td>-0.05</td>
<td>0.08</td>
</tr>
</tbody>
</table>

Notes: The table presents various decompositions of the variability in the steady-state unemployment rate for Canada over 1978-2016, and for the US over different periods. The Canadian data are detrended using an HP filter with $\lambda = 10^6$. Canadian rates are detrended using an HP Filter with $\lambda = 10^6$. Canadian data are drawn from the Labour Force Survey. The US results are drawn from Shimer (2012), covering the period 1967-2010. The US results are drawn from data that have been detrended using the HP Filter.

Shimer (2012) decomposes the variance in unemployment rate fluctuations by computing hypothetical steady-state unemployment rates where only one type of transition is allowed to vary, with the others held at their average value. He derives an expression for steady-state unemployment under a three-state setting by assuming that employment, unemployment, and labor force inactivity are all at their steady-state levels,

$$u^*_t = \frac{p_t^{EO} + p_t^{OU} + p_t^{EU} + p_t^{OE} + p_t^{EU}}{(p_t^{EO} + p_t^{OU} + p_t^{EU} + p_t^{OE} + p_t^{EU})} + (p_t^{EO} + p_t^{OU} + p_t^{EU} + p_t^{OE} + p_t^{EU})$$

These hypothetical steady-states are then regressed on the actual next-period unemployment rate. The slope coefficient thus shows the proportion of the total variation in the actual unemployment rate that can be accounted for by the covariance between the actual unemployment rate and the hypothetical rates. Full details can be found in Appendix E. This is not an exact decomposition, so the contributions will not sum to one. However, this method gives a quantitative measure of the comovement between unemployment rate variation, and the variation in worker flows when using a two-state model or a three-state model.

Steady-state decompositions are only valid if the correlation between the steady-state and actual unemployment rate is high. Following Shimer (2012), we look at the correlation between the steady-state unemployment rate and the actual unemployment rate in the following period. We use quarterly averaged data as Shimer does, resulting in a correlation of 0.93.31

Table 3 shows the coefficients from Shimer’s regression-based decomposition method. We find that the UE rate accounts for 55% of unemployment rate variation. The EU rate accounts for...
another 26%. The UO rate is the next largest contributor at just under 20%. The other flows in and out of the labor force collectively contributing about 3%.

Table 3 also shows Shimer’s results for the US over 1967-2010. He finds that UE, EU, and UO rates respectively account for 49%, 22%, and 17% of unemployment rate variation. This is very similar to what we find for Canada. However, Shimer finds a much larger contribution from the OU rate (12% vs 0.1%), and his signs and magnitudes for the EO and OE rates are almost the opposite of what we observe.

5.5 Comparison to Literature

Within the literature, variance decompositions of the unemployment rate have differed along two dimensions: whether the decomposition takes place under a three-state or two-state model, and the method used for the decomposition.\textsuperscript{32} In addition to Shimer’s regression-based decomposition method, Elsby et al. (2009) and Fujita and Ramey (2009) derive a log-differentiation decomposition method. This method decomposes the steady-state unemployment rate into unemployment inflow and outflow components. The advantage of their method is that the contributions approximately sum to one. However, it is not obvious how to extend the log-differentiation method to a three-state setting, which is why we only present the regression-based decomposition above.\textsuperscript{33} Details of the log-differentiation decomposition method can be found in Appendix F.

To compare against the existing literature, including results for Canada, we show the decomposition results using Shimer’s (2012) two-state model in Table 4.\textsuperscript{34} Under the log-differentiation method, we find that unemployment outflows account for about 60% of the variation in the steady-state unemployment rate, while the inflow rate accounts for the remaining 40%. The regression-based method attributes a noticeably smaller role for unemployment inflows: just over 20%, while about 70% of unemployment rate variation is explained by outflows.

Our two-state unemployment rate decompositions for Canada are similar to earlier estimates\textsuperscript{34}A third dimension exists that considers whether the decompositions take place in steady-state or out of steady-state. Elsby et al. (2013) suggest the difference is negligible for Canada. We only consider steady-state decompositions. \textsuperscript{33}Smith (2011) has made progress in extending this decomposition model to three-states, but the role of flows in and out of the labor force are still difficult to interpret under her extension.\textsuperscript{34}The correlation between the steady-state and actual unemployment rate is 0.95 for the two-state model, using monthly data. The difference in correlations between the two and three-state settings appears to exist because deriving a steady-state formula for the unemployment rate requires stronger assumptions in the three-state model than the two-state model. The three-state model assumes that employment, unemployment, and inactivity are all at their steady-state values. The two-state model only assumes that unemployment and the total of employment and inactivity are at their steady-state. Thus it allows the proportion of employment and inactivity to vary.
Table 4: Two-State Unemployment Rate Decomposition

<table>
<thead>
<tr>
<th></th>
<th>Canada</th>
<th>US</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Outflows</td>
<td>Inflows</td>
</tr>
<tr>
<td>Log-Differentiation Decomposition</td>
<td>0.59</td>
<td>0.41</td>
</tr>
<tr>
<td>Regression-based Decomposition</td>
<td>0.69</td>
<td>0.22</td>
</tr>
</tbody>
</table>

Notes: The table presents various decompositions of the variability in the steady-state unemployment rate for Canada over 1978-2016, and for the US over different periods. The Canadian data are detrended using an HP filter with $\lambda = 10^6$. Canadian rates are detrended using an HP filter with $\lambda = 10^6$. Canadian data are drawn from the Labour Force Survey. The US results are drawn from Shimer (2012) for the regression-based method, which covers the period 1976-2010, and Fujita and Ramey (2009) for the log-differentiation method, which covers 1976-2005. All US results are drawn from data that have been detrended using the HP Filter.

for the US when holding the method constant. Shimer (2012) finds an outflow contribution of 85% and an inflow contribution of 15% for the US using his regression-based method. Fujita and Ramey (2009) attribute about 60% of variation to unemployment outflows and 40% to inflows for the US, using a log-differentiation decomposition. Notably, these are almost identical to our results for Canada when using their decomposition method.

Using a log-differentiation decomposition, we find a somewhat larger role for unemployment inflows than previous studies that look at Canada, such as Elsby et al. (2013) and Campolieti (2011). The differences between our results and Elsby et al. arise from limitations in their data, as well as small methodological differences. Similar small definitional changes are likely to explain the gap between our results and Campolieti as well.

6 Conclusion

We have four main results. First, using Record of Employment data and the Labour Force Survey, we provide a complete decomposition of worker flows for Canada, including employer-to-employer transitions. Employer-to-employer flows are large and strongly pro-cyclical. Second, we find that unemployment to employment transitions and gross job finding flows tend to be proportional, which supports common assumptions used in random search models. Third, we decompose the variation of a measure labor market fluidity into employer-to-employer transi-

---

35Shimer uses duration-based flow data from the CPS over 1976-2010, detrended with an HP filter with the non-standard parameter of $\lambda = 10^5$.

36Fujita and Ramey perform this analysis on gross flows data from the CPS over 1976-2005 detrended with an HP filter with a standard parameter of $\lambda = 1600$.

tions, employment inflows, and employment inflows. Employer-to-employer transitions account for about 60% of the variation. Fourth, we find that unemployment outflows contribute more to unemployment rate variation than inflows. However, the contribution from inflows is also substantial, in line with the previous literature.
# A Key Variables

The key variables we estimate, their data sources, and their formula are shown in Table A.1. Since our primary data sources (the ROE and LFS) are reported at different frequencies (annually and monthly, respectively), we make adjustments so that they are compatible. Once the job finding and separation rates are computed, they are divided by 12 so that they are on a monthly basis. When computing the EE rate, we take the annual averages of short-term unemployment inflows, UE flows, and OE flows. Thus, there is only one unique value for the EE rate in a given year.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Data sources</th>
<th>Formula</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Total Separation Rate</strong></td>
<td>ROE, CEIS</td>
<td>$S_t / E_t^i$</td>
</tr>
<tr>
<td><strong>Layoff Rate</strong></td>
<td>ROE, CEIS</td>
<td>Layoffs_t / $E_t^i$</td>
</tr>
<tr>
<td><strong>Quit Rate</strong></td>
<td>ROE, CEIS</td>
<td>Quits_t / $E_t^i$</td>
</tr>
<tr>
<td><strong>Other Separation Rate</strong></td>
<td>ROE, CEIS</td>
<td>Other Separations_t / $E_t^i$</td>
</tr>
<tr>
<td><strong>Job Finding Rate</strong></td>
<td>ROE, CEIS, LFS</td>
<td>$E_{t+1} - E_t + St$</td>
</tr>
<tr>
<td><strong>EU Rate</strong></td>
<td>LFS</td>
<td>$f^EU_t / E_t$</td>
</tr>
<tr>
<td><strong>EO Rate</strong></td>
<td>LFS</td>
<td>$N_t^{s} / E_t - p_t^{EU}$</td>
</tr>
<tr>
<td><strong>OU Rate</strong></td>
<td>LFS</td>
<td>$U_{t+1} - E_{t+1}^{EU}$</td>
</tr>
<tr>
<td><strong>OE Rate</strong></td>
<td>LFS</td>
<td>$0.006$</td>
</tr>
<tr>
<td><strong>UE Rate</strong></td>
<td>LFS</td>
<td>$U_t - U_{t+1} + E_t - E_{t+1} - N_{t+1}^s + O_t p_t^{OE}$</td>
</tr>
<tr>
<td><strong>UO Rate</strong></td>
<td>LFS</td>
<td>$H_t + E_{t+1}^{se} - U_{t+1}^{U} - O_t p_t^{OE}$</td>
</tr>
<tr>
<td><strong>EE Rate</strong></td>
<td>ROE, LFS</td>
<td>$H_t + E_{t+1}^{se} - U_{t+1}^{U} - O_t p_t^{OE}$</td>
</tr>
</tbody>
</table>

*Notes:* The table presents the variables of interest, and the data sources and formula used to generate the variables.

- ROE refers to the Record of Employment.
- CEIS refers to the Canadian Employment Insurance Statistics.
- LFS refers to the Canadian Labour Force Survey.
- $S_t$ denotes gross separation flows, which are directly estimated from the data.
- $E_t^i$ denotes the insurable employment stock.
- $E_t$, $U_t$, and $O_t$ denote the employment, unemployment, and inactivity stock, respectively.
- $f_{EU}^t$ denotes the flow from employment to unemployment, which is estimated as the next period flow of individuals into short-term unemployment and short-term non-employment simultaneously.
- $U_{t+1}^s$ and $N_{t+1}^s$ refer to next period short-term unemployment and non-employment, respectively.
- $p_t$ denotes the probability of transitioning between given labor force states (e.g. $p_t^{EU}$ is the probability of transitioning from employment to unemployment.
- $H_t$ denotes gross job finding flows.
- $E_{t+1}^{se}$ denotes inflows into self-employment.

The above formula are all for discrete rates. In all our results we convert these to continuous rates by assuming that the transitions follow a Poisson process so that the continuous rate can be computed as $\tilde{p}_t = -\ln(1 - p_t)$ as in Shimer (2012).
B Three-State Model Solution

As Equation 5 implies, we can estimate the level of employment to unemployment transitions probability \( f_{EU}^t \) directly from the data. This allows us to determine the probability of this transition by dividing by the employment stock, \( E_t \).

From there we can solve for the probability of employment to inactivity transitions \( p_{EO}^t \) using Equation 4, and the probability of inactivity to unemployment transitions \( p_{OU}^t \) using Equation 3.

\[
p_{EO}^t = \frac{N_{t+1}^s}{E_t} - p_{EU}^t
\]

(16)

\[
p_{OU}^t = \frac{U_{t+1}^s - E_t p_{EU}^t}{O_t}
\]

(17)

Where \( N_{t+1}^s \) denotes the next month’s level of individuals who have been jobless for one month or less given previous employment, \( U_{t+1}^s \) denotes the next month’s level of individuals who have been unemployed for four weeks or less, and \( O_t \) is the stock of out-of-labor force individuals.

Once a value is assumed for the inactivity to employment flow rate \( p_{OE}^t \), we can solve for the unemployment to employment probability \( p_{UE}^t \) by combining and rearranging Equations 2 and 4:

\[
E_{t+1} = E_t - N_{t+1}^s + U_t p_{UE}^t + O_t p_{OE}^t
\]

(18)

\[
p_{UE}^t = \frac{1}{U_t} \left( E_{t+1} - E_t + N_{t+1}^s - O_t p_{OE}^t \right)
\]

(19)

Likewise, we can solve for the unemployment to out-of-the-labor force probability \( p_{UO}^t \) by combining and rearranging Equations 1 through 4:

\[
U_{t+1} + E_{t+1} + N_{t+1}^s - U_{t+1}^s = U_t - U_t (p_{UE}^t + p_{UO}^t) + E_t + U_t p_{UE}^t + O_t p_{OE}^t
\]

(20)

\[
p_{UO}^t = \frac{1}{U_t} \left( U_t - U_{t+1} + E_t - E_{t+1} - N_{t+1}^s + U_{t+1}^s + O_t p_{OE}^t \right)
\]

(21)

Where \( U_t \) denotes the stock of the unemployed individuals.
We show these flow rates assuming that the flow rate from out-of-the-labor force to employment is constant at 0.006 in Figure B.1.

C Three-State Model Assumptions

We assume that the inactivity to employment flow rate is 0.006. We use this assumption as most flow rates result in some negative unemployment to employment or unemployment to inactivity flows. Using numerical analysis, we find that all flows are non-negative when the inactivity to employment flow rate is between about 0.002 and 0.0105. Thus, we select 0.006 as the approximate midpoint between these boundaries. In Figure C.2 we show the flow rates if the boundaries are used. We also show basic statistics in Table C.2

<table>
<thead>
<tr>
<th></th>
<th>OE = 0.002</th>
<th>OE = 0.0105</th>
<th>OE = 0.002</th>
<th>OE = 0.0105</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
<td>0.18</td>
<td>0.12</td>
<td>0.16</td>
<td>0.23</td>
</tr>
<tr>
<td>Std. Dev.</td>
<td>0.035</td>
<td>0.030</td>
<td>0.053</td>
<td>0.064</td>
</tr>
<tr>
<td>Unemployment Rate Corr.</td>
<td>-0.72</td>
<td>-0.53</td>
<td>-0.24</td>
<td>-0.35</td>
</tr>
</tbody>
</table>

Notes: The table presents average monthly statistics UE rate and UO rate under the boundary value assumptions for OE for Canada over 1978-2016, using Labour Force Survey data. The unemployment rate is detrended using an HP Filter with $\lambda = 10^9$.

D Shimer Two-State Model

We denote the probability of these two-state transitions as $p_{t}^{ue}$ and $p_{t}^{eu}$. Note that we use uppercase superscripts to denote three-state models, and lowercase superscripts to denote two-state models.

Assume that workers are homogeneous with identical unemployment inflow and outflow probabilities. The continuous unemployment outflow probability can be calculated as:

$$\tilde{p}_{t}^{ue} = -\ln \left( \frac{U_{t+1} - U_{t+1}^s}{U_t} \right)$$

(22)

Calculating the continuous inflow rate to unemployment, $\tilde{p}_{t}^{eu}$ is slightly more involved. Again
Figure B.1: Complete Decomposition of Worker Flows Between States

Notes: The figure plots imputed flow rates between employment, unemployment, and out-of-the-labor-force for Canada over 1978-2016 based on monthly data from the Labour Force Survey. All data are detrended using an HP Filter with $\lambda = 10^6$. 
Figure C.2: Three-State Flow Bounds

Notes: The figure plots imputed flow rates between unemployment and inactivity, and unemployment and employment under different inactivity to employment flow rates for Canada over 1978-2016 based on monthly data from the Labour Force Survey.
following Shimer, we derive an implicit function of the inflow rate to unemployment.

\[ U_{t+1} = \frac{(1 - e^{-\bar{p}_{ue}^{ue} - \bar{p}_{eu}^{ue}})\bar{p}_{eu}^{ue}}{\bar{p}_{ue}^{ue} + \bar{p}_{eu}^{ue}}(E_t + U_t) + e^{-\bar{p}_{ue}^{ue} - \bar{p}_{eu}^{ue}} U_t \]  

(23)

where \( E_t \) is the average employment stock over period \( t \). We assume that the working-age non-institutionalized population does not change within a period.

Table D.3: Two-State Model Statistics

<table>
<thead>
<tr>
<th></th>
<th>Unemployment Outflows</th>
<th>Unemployment Inflows</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
<td>0.38</td>
<td>0.034</td>
</tr>
<tr>
<td>Std. Dev.</td>
<td>0.052</td>
<td>0.0027</td>
</tr>
<tr>
<td>Unemployment Rate Corr.</td>
<td>-0.82</td>
<td>0.45</td>
</tr>
</tbody>
</table>

Notes: The table presents average monthly statistics for worker flows for Canada over 1978-2016, calculated using a two-state Shimer model from Labour Force Survey data. All series are detrended using an HP Filter with \( \lambda = 10^6 \).

When comparing results from the two-state model and three-state model, we calculate the three-state unemployment inflow rate as the sum of EU and OU levels divided by the sum of employment and inactivity, with the continuous transformation applied to the result. The three-state outflow rate is computed under a similar procedure.

If we use a two-state measure of job finding instead of a three-state measure to compute employer-to-employer flows, the level of the employer-to-employer rate rises slightly (3.2%), though the standard deviation and cyclicality remain substantially the same. Further, the employer-to-employer rate we estimate is not sensitive to our normalizing assumption for the three-state model, as job finding under the three-state model is \( UE + UO \), and the three-state model computes essentially the same job finding flows regardless of our normalizing assumption. The normalizing assumption simply changes the proportion of job finding flows that come from UE versus UO.

E Regression-based Decomposition

Shimer (2012) proposes the following decomposition. Suppose \( p_{eu}^{ue} \) is the unemployment outflow rate, \( p_{ue}^{ue} \) is the inflow rate, and \( u_t \) is the unemployment rate, \( \bar{p}_{ue}^{ue} \) is the average inflow rate and \( \bar{p}_{eu}^{ue} \) is the average outflow rate. The steady-state unemployment can be expressed as:
The “outflow” component of steady-state unemployment is \( \frac{p_{EU}^t}{p_{EU}^t + p_{UE}^t} \), and the “inflow” component is \( \frac{p_{UE}^t}{p_{EU}^t + p_{UE}^t} \). The fraction of the variation in the unemployment rate due to outflows is the covariance between \( \frac{p_{EU}^t}{p_{EU}^t + p_{UE}^t} \) and \( u_{t+1} \) divided by the variance of \( u_{t+1} \), which is conveniently the coefficient of a regression of \( \frac{p_{EU}^t}{p_{EU}^t + p_{UE}^t} \) on \( u_{t+1} \). The fraction of the variation due to outflows is the covariance of \( \frac{p_{EU}^t}{p_{EU}^t + p_{UE}^t} \) with \( u_{t+1} \) divided by the variance of \( u_{t+1} \). As Shimer (2012) discusses, this is not an exact decomposition so the two parts need not sum to one.38

This method can also be applied to three-state models, using the conditions that employment, unemployment, and inactivity are all at steady state.

\[
E_t(p_{EU}^t + p_{EO}^t) = U_t(p_{UE}^t + p_{OE}^t) = O_t(p_{UE}^t + p_{OU}^t) = E_t(p_{EO}^t + p_{OU}^t) = U_t(p_{OE}^t + p_{EU}^t)
\]

Manipulating these equations, the steady-state unemployment rate can be expressed as:

\[
u_t^* = \frac{p_{EO}^t p_{OU}^t + p_{OE}^t p_{EU}^t + p_{OU}^t p_{UE}^t + p_{OE}^t p_{EU}^t}{(p_{EO}^t p_{OU}^t + p_{OE}^t p_{EU}^t + p_{OU}^t p_{UE}^t) + (p_{EO}^t p_{OE}^t + p_{OU}^t p_{EU}^t + p_{OE}^t p_{UE}^t)}
\]

By constructing hypothetical steady-states where only one transition is allowed to vary, while the others are held constant at their mean value, we can determine how much that transition contributes to variation in the unemployment rate. E.g. the EU component of steady-state unemployment rate is

\[
\frac{p_{EO}^t p_{OU}^t + p_{OE}^t p_{EU}^t + p_{OU}^t p_{UE}^t + p_{OE}^t p_{EU}^t}{(p_{EO}^t p_{OU}^t + p_{OE}^t p_{EU}^t + p_{OU}^t p_{UE}^t) + (p_{EO}^t p_{OE}^t + p_{OU}^t p_{EU}^t + p_{OE}^t p_{UE}^t)}.
\]

F Log-Differentiation Decomposition

Begin with Shimer’s steady-state unemployment rate equation under a two-state model:

\[
u_t^* = \frac{p_{EU}^t}{p_{EU}^t + p_{UE}^t}
\]

Log differentiation of both sides yields:

\[
\text{Nevertheless, in Shimer’s (2012) application, the two parts sum almost exactly to one.}
\]

\[38\]
\[ d \ln u_t^* = d \ln p_t^{eu} - d \ln (p_t^{eu} + p_t^{ue}) \] (28)

\[ d \ln u_t^* = d \ln p_t^{eu} - \frac{1}{p_t^{eu} + p_t^{ue}} d(p_t^{eu} + p_t^{ue}) \] (29)

\[ d \ln u_t^* = d \ln p_t^{eu} - \frac{1}{p_t^{eu} + p_t^{ue}} dp_t^{eu} - \frac{1}{p_t^{eu} + p_t^{ue}} dp_t^{ue} \] (30)

\[ d \ln u_t^* = d \ln p_t^{eu} - u_t^* d \ln p_t^{eu} - (1 - u_t^*) d \ln p_t^{ue} \] (31)

\[ d \ln u_t^* = (1 - u_t^*) [d \ln p_t^{eu} - d \ln p_t^{ue}] \] (32)

Fujita and Ramey (2009) then write this as a generic equation.

\[ du_t^* = du_t^{eu} + du_t^{ue} \] (34)

Taking the variance of the generic equation is akin to taking the variance of the sum of correlated variables. Thus the variance of the generic equation will equal the sum of the covariances.

\[ \text{Var}(du_t^*) = \text{Cov}(du_t^*, du_t^{eu}) + \text{Cov}(du_t^*, du_t^{ue}) \] (35)

Define:

\[ \beta^{eu} = \frac{\text{Cov}(du_t^*, du_t^{eu})}{\text{Var}(du_t^*)} \] (36)

As Fujita and Ramey (2009) notes, this is equivalent to the betas in finance.

The decomposition can then be written as:

\[ 1 = \beta^{eu} + \beta^{ue} \] (37)
\( \beta_{cu} \) can then be estimated as the coefficients of simple linear regressions of \((1 - u_t^*) \Delta \ln p_t^{cu} \) on \( \Delta \ln u_t^* \), with a similar procedure being used to estimate \( \beta_{ue} \).

**G  Differences between our study and Elsby, Hobijn, and Şahin (2013)**

We summarize the key differences between our analysis and Elsby et al. (2013) [EHS] in Table G.4, and show the cumulative effect of incorporating details of the EHS analysis. Rows 1-4 are differences caused by data limitations, while rows 5-8 are methodological differences. The data limitations are caused by their use of OECD harmonized data. The OECD data only have annual data; hence, EHS have to approximate the model solution to allow for time aggregation. The OECD data also define short-term unemployment as people who have become unemployed in the last 0-3 weeks rather than 0-4 weeks, as one would wish to use in a monthly model. In contrast, we are able to use monthly data with short-term unemployed defined as people who became unemployed in the last 0-4 weeks (we calculate this directly from the LFS). The largest methodological difference is that we include all unemployed people in calculating the fraction of short-term unemployed, so we are assuming those who do not report a duration have durations greater than 1 month (whereas Elsby et al. assume they are drawn equally from all durations). Furthermore, EHS perform their decomposition using the actual unemployment rate, while we decompose the steady-state unemployment rate.

**H  Three State Model and Decomposition with non-constant OE**

We estimate and upper and lower bound for the OE rate at each time period by setting UE and UO to zero, respectively. In the latter case, this leads to negative values of OE, so we set these at zero. We then set OE as the midpoint of the boundary values, and then calculate UE and UO based on this value. These flow rates are then adjusted for time-aggregation bias using Mukoyama’s (2014) procedure. Basic statistics for these flows are shown in Table I.7.

**I  Extra Figures and Tables**
Table G.4: Differences between Elsby et al. (2013) and Our Analysis

<table>
<thead>
<tr>
<th></th>
<th>Outflow Contribution</th>
<th>Inflow Contribution</th>
</tr>
</thead>
<tbody>
<tr>
<td>(1) Baseline</td>
<td>0.59</td>
<td>0.41</td>
</tr>
<tr>
<td>(2) Data Averaged Annually&lt;sup&gt;a&lt;/sup&gt;</td>
<td>0.61</td>
<td>0.38</td>
</tr>
<tr>
<td>(3) EHS Annual Formula Used&lt;sup&gt;b&lt;/sup&gt;</td>
<td>0.66</td>
<td>0.32</td>
</tr>
<tr>
<td>(4) ( U^s: 0-3 \text{ Weeks} )</td>
<td>0.69</td>
<td>0.30</td>
</tr>
<tr>
<td>(5) ( U^s ) Incidence Definition&lt;sup&gt;c&lt;/sup&gt;</td>
<td>0.73</td>
<td>0.25</td>
</tr>
<tr>
<td>(6) No Detrending</td>
<td>0.75</td>
<td>0.24</td>
</tr>
<tr>
<td>(7) Time Period Ends in 2009</td>
<td>0.75</td>
<td>0.23</td>
</tr>
<tr>
<td>(8) Actual vs Steady-State Decomposition</td>
<td>0.78</td>
<td>0.25</td>
</tr>
<tr>
<td>(9) EHS results</td>
<td>0.80</td>
<td>0.24</td>
</tr>
</tbody>
</table>

Notes: The table presents the differences between the analysis of Elsby et al. (2013) and our analysis, as well as the cumulative effect of adopting elements of the analysis of EHS.

A gap remains between our results and EHS because we do not account for all differences, such as differences in the numerical method used to solve for unemployment inflow rates.

<sup>a</sup> The data are averaged annually, but remain on a monthly basis.

<sup>b</sup> The use of annual data means EHS have to approximate the model solution to allow for time aggregation.

<sup>c</sup> EHS define short-term unemployment incidence as the proportion of short-term unemployed over the number of unemployed who report a duration, instead of dividing by the total number of unemployed.

Table H.5: Worker Flows

(a) Worker Flows – Variable OE with Time-Aggregation Bias Correction

<table>
<thead>
<tr>
<th></th>
<th>UE</th>
<th>EU</th>
<th>UO</th>
<th>OU</th>
<th>EO</th>
<th>OE</th>
<th>EE</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
<td>0.11</td>
<td>0.0076</td>
<td>0.28</td>
<td>0.048</td>
<td>0.0075</td>
<td>0.012</td>
<td>0.030</td>
</tr>
<tr>
<td>Std. Dev.</td>
<td>0.020</td>
<td>0.0018</td>
<td>0.063</td>
<td>0.0064</td>
<td>0.0011</td>
<td>0.0024</td>
<td>0.0025</td>
</tr>
<tr>
<td>Unemployment Rate Corr.</td>
<td>-0.79</td>
<td>0.35</td>
<td>-0.49</td>
<td>-0.024</td>
<td>0.20</td>
<td>0.21</td>
<td>-0.51</td>
</tr>
</tbody>
</table>

(b) Worker Flows – OE = 0.006

<table>
<thead>
<tr>
<th></th>
<th>UE</th>
<th>EU</th>
<th>UO</th>
<th>OU</th>
<th>EO</th>
<th>OE</th>
<th>EE</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
<td>0.15</td>
<td>0.006</td>
<td>0.196</td>
<td>0.039</td>
<td>0.008</td>
<td>0.006</td>
<td>0.030</td>
</tr>
<tr>
<td>Std. Dev.</td>
<td>0.032</td>
<td>0.0016</td>
<td>0.057</td>
<td>0.0045</td>
<td>0.0011</td>
<td>-</td>
<td>0.0025</td>
</tr>
<tr>
<td>Unemployment Rate Corr.</td>
<td>-0.64</td>
<td>0.41</td>
<td>-0.30</td>
<td>0.17</td>
<td>0.17</td>
<td>-</td>
<td>-0.50</td>
</tr>
</tbody>
</table>

Notes: Panel I.7b presents average monthly statistics for worker flows for Canada over 1978-2016, calculated using a three-state model with data from the Labour Force Survey and the Record of Employment. No series have been detrended, except the unemployment rate. The unemployment rate is detrended using an HP Filter with \( \lambda = 10^6 \).
Table H.6: Unemployment Rate Decomposition

<table>
<thead>
<tr>
<th></th>
<th>UE</th>
<th>EU</th>
<th>UO</th>
<th>OU</th>
<th>EO</th>
<th>OE</th>
</tr>
</thead>
<tbody>
<tr>
<td>Canada – OE = 0.006</td>
<td>0.69</td>
<td>0.26</td>
<td>0.054</td>
<td>0.022</td>
<td>0.066</td>
<td>-</td>
</tr>
<tr>
<td>CAN – Variable OE w/ Time-agg corr</td>
<td>0.55</td>
<td>0.26</td>
<td>0.19</td>
<td>0.0014</td>
<td>0.073</td>
<td>-0.04</td>
</tr>
<tr>
<td>Experimental Log-diff decomp</td>
<td>0.47</td>
<td>-0.02</td>
<td>0.15</td>
<td>0.39</td>
<td>-0.0082</td>
<td>0.0165</td>
</tr>
<tr>
<td>US Shimer (2012)</td>
<td>0.49</td>
<td>0.22</td>
<td>0.17</td>
<td>0.12</td>
<td>-0.05</td>
<td>0.08</td>
</tr>
</tbody>
</table>

Notes: The table presents various decompositions of the variability in the steady-state unemployment rate for Canada over 1978-2016, and for the US over different periods. The Canadian data are detrended using an HP filter with \( \lambda = 10^6 \). Canadian rates are detrended using an HP Filter with \( \lambda = 10^6 \). Canadian data are drawn from the Labour Force Survey. The US results are drawn from Shimer (2012), covering the period 1967-2010. The US results are drawn from data that have been detrended using the HP Filter.

Table I.7: Empirical Results – Common Denominator

(a) Separation and Job Finding Statistics

<table>
<thead>
<tr>
<th></th>
<th>Job Finding</th>
<th>Separation</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>All</td>
<td>Layoffs</td>
</tr>
<tr>
<td>Mean</td>
<td>0.027</td>
<td>0.026</td>
</tr>
<tr>
<td>Std. Dev.</td>
<td>0.0019</td>
<td>0.0019</td>
</tr>
<tr>
<td>Unemployment Rate Corr.</td>
<td>-0.58</td>
<td>-0.49</td>
</tr>
</tbody>
</table>

(b) Worker Flows

<table>
<thead>
<tr>
<th></th>
<th>UE</th>
<th>EU</th>
<th>UO</th>
<th>EO</th>
<th>OU</th>
<th>OE</th>
<th>EE</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
<td>0.0071</td>
<td>0.0037</td>
<td>0.0089</td>
<td>0.0047</td>
<td>0.012</td>
<td>0.0019</td>
<td>0.018</td>
</tr>
<tr>
<td>Std. Dev.</td>
<td>0.0015</td>
<td>0.00084</td>
<td>0.0017</td>
<td>0.00058</td>
<td>0.0012</td>
<td>0.000062</td>
<td>0.0019</td>
</tr>
<tr>
<td>Unemployment Rate Corr.</td>
<td>0.08</td>
<td>0.32</td>
<td>0.26</td>
<td>-0.04</td>
<td>0.18</td>
<td>-0.07</td>
<td>-0.56</td>
</tr>
</tbody>
</table>

Notes: Panel I.7a presents statistics for the imputed monthly job finding rate and separation rate taken over the working age population for Canada over 1978-2016 using gross flow data from the Record of Employment. Panel I.7b presents average monthly statistics for worker flows as rates over the working age population for Canada over 1978-2016, calculated using a three-state model with data from the Labour Force Survey and the Record of Employment. No series have been detrended.
Table I.8: Average Gross Worker Flows over the Working Age Population

<table>
<thead>
<tr>
<th></th>
<th>Mean</th>
<th>Corr w / U</th>
</tr>
</thead>
<tbody>
<tr>
<td>Job Finding</td>
<td>0.027</td>
<td>-0.58</td>
</tr>
<tr>
<td>EE</td>
<td>0.018</td>
<td>-0.50</td>
</tr>
<tr>
<td>UE</td>
<td>0.007</td>
<td>-0.64</td>
</tr>
<tr>
<td>OE</td>
<td>0.002</td>
<td>-</td>
</tr>
<tr>
<td>Job Separations</td>
<td>0.026</td>
<td>-0.28</td>
</tr>
<tr>
<td>EE</td>
<td>0.018</td>
<td>-0.50</td>
</tr>
<tr>
<td>EU</td>
<td>0.004</td>
<td>0.41</td>
</tr>
<tr>
<td>EO</td>
<td>0.005</td>
<td>0.17</td>
</tr>
<tr>
<td>Layoffs</td>
<td>0.011</td>
<td>0.59</td>
</tr>
<tr>
<td>Quits</td>
<td>0.006</td>
<td>0.70</td>
</tr>
<tr>
<td>Other</td>
<td>0.009</td>
<td>-0.07</td>
</tr>
<tr>
<td>OU</td>
<td>0.012</td>
<td>0.17</td>
</tr>
<tr>
<td>UO</td>
<td>0.009</td>
<td>-0.30</td>
</tr>
</tbody>
</table>

Notes: The first column reports time-series averages of each worker flow as a fraction of the working age population in Canada. The second column reports the correlation of the corresponding transition probabilities with the unemployment rate. We have detrended the unemployment rate using an HP-filter with $\lambda = 10^6$. The sample period is 1978-2016.
References


