Appendix to "Accounting for Incomplete Pass-Through"

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Computational Algorithm

We solve for equilibrium prices in the dynamic pricing model using the following iterative procedure. For expositional simplicity, we present the algorithm for the case of two firms $j = 1, 2$. It is, however, easy to see how the algorithm can be generalized to the case of n firms. We will begin by describing the value function iteration procedure used to solve each individual firm's dynamic pricing problem. Suppose we start with an initial value for firm j's expected value EV_j at time $t-1$,

$$EV_j(p^w_{1t-1}, p^w_{2t-1}, C_{t-1}) = E_{t-1}V_j(p^w_{1t-1}, p^w_{2t-1}, C_t, \gamma_{jt}), \tag{1}$$

where V_j is the value function described in section 6 and E_{t-1} is the expectation conditional on all information known by firm j at time $t-1$.

The value function iteration procedes by iteratively updating EV_j until a fixed point is obtained. We next describe the procedure we use to update EV_j in the value function iteration. The first step is to calculate the value from different possible prices excluding the menu cost,

$$W'(p^w_{1t}, p^w_{2t}, c_t) = \pi_{jt}(p^w_{1t}, p^w_{2t}, C_t) + \beta EV_j(p^w_{1t}, p^w_{2t}, C_t). \tag{2}$$

This expression depends on the current prices of the firm's competitors as well as current costs.

The second step in updating the value function is to calculate the expectation of W' over competitors' prices. The menu cost model implies a simple structure for this expectation since firm j' has probability $1 - \mathrm{pr}_{j'}$ of maintaining its current price, and probability $\mathrm{pr}_{j'}$ of changing its price. Let us denote the firm's price conditional on adjusting by p^{w*}_{jt}. A given firm's pricing strategy depends on the entire vector of past prices (p^w_{1t-1}, p^w_{2t-1}). Denoting the expectation over competitors' prices as W'' we have,

$$W''(C_t, p^w_{1t}; p^w_{1t-1}, p^w_{2t-1}) = (1 - \mathrm{pr}_2)W'(p^w_{1t}, p^w_{2t-1}, C_t) + \mathrm{pr}_2 W'(p^w_{1t}, p^{w*}_2, C_t). \tag{3}$$

Third, we must calculate the firm's optimal pricing policy. There are two relevant cases. The expectation if the firm does not adjust its price is

$$W_{nch}(p^w_{1t-1}, p^w_{2t-1}, C_t) = W''(C_t, p^w_{1t-1}; p^w_{1t-1}, p^w_{2t-1}), \tag{4}$$

while the expectation if it does adjust its price is

$$W_{ch}(p^w_{1t-1}, p^w_{2t-1}, C_t) = \max_{p^w_{1t}} W''(C_t, p^w_{1t}; p^w_{1t-1}, p^w_{2t-1}). \tag{5}$$

The firm's decision about whether to adjust its price depends on the difference between its payoffs when it adjusts and when it does not adjust,

$$\Delta W = W_{ch}(p^w_{1t-1}, p^w_{2t-1}, C_t) - W_{nch}(p^w_{1t-1}, p^w_{2t-1}, C_t). \tag{6}$$

The firm adjusts its price when $\Delta W > \gamma_{jt}$ while it maintains a fixed price when $\Delta W <= \gamma_{jt}$. Recall that we assume that the menu cost γ_{jt} is independent and identically distributed with an exponential distribution; i.e., $F(\gamma_{jt}) = 1 - \exp(-\frac{1}{\sigma}\gamma_{jt})$. The probability of price adjustment is therefore $Pr_{ch} = F(\Delta W)$, where $F(x) = 1 - \exp(-\frac{1}{\sigma}x)$.

Fourth, in order to update the firm's value, we must calculate the expected menu cost if the firm changes its price. The expected menu cost differs from the mean of the menu cost distribution since the firm is more likely to adjust its price when it faces a low menu cost. The optimal pricing policy implies that the firm adjusts only when $\Delta W > \gamma_{jt}$. Since we assume that the menu cost is distributed exponentially, the firm's expected menu cost takes the form,

$$E(\gamma_{jt}|\gamma_{jt} < \Delta W) = \sigma - \frac{\Delta W \exp \frac{-1}{\sigma}\Delta W}{\exp \frac{-1}{\sigma}\Delta W}. \tag{7}$$

The expected value is a weighted average of its value conditional on adjusting and not adjusting,

$$W = (1 - Pr_{ch})W_{nch} + Pr_{ch}[W_{ch} - E(\gamma_{jt}|\gamma_{jt} < \Delta W)]. \tag{8}$$

Finally, we use the stochastic process for costs to take an expectation over future commodity costs at time $t-1$. We discretize the process for costs given by (20) using the method of Tauchen (1986). This implies a discrete Markov process with the transition matrix Λ. Applying this Markov transition matrix to W we have,

$$EV_j = \Lambda W. \tag{9}$$

We solve for the firm's optimal policy by repeatedly applying this procedure to update EV_j until a fixed point is found.

This value function iteration procedure is nested within an "outer loop" that searches for a fixed point in the firms' dynamic pricing policies. In this outer loop, we first solve for firm 1's optimal policy, conditional on an initial value for the pricing policy of firm 2; and use the results to update firm 1's policy rule. We then solve for firm 2's optimal policy, conditional on the updated pricing policy of firm 1. We use the results of this exercise to update firm 2's policy rule. We repeat this exercise until the maximum differences in firm pricing policies between successive iterations are sufficiently small. Once this point is reached, we run our algorithm for an additional 1500 iterations to check that the equilibrium does not change.

One interesting feature of the dynamic model is that only the size of the menu cost relative to the market size, γ_{jt}/M, matters in determining firm behavior. This can be seen by the following argument. Let us assume that the value function V scales with M. By the definitions above, ΔW and W'' also scale with M in this case, implying that the firm's optimal price conditional on adjusting is invariant to M. Moreover, since ΔW scales with M, the probability of adjustment, $Pr_{ch} = 1 - \exp(-\frac{1}{\sigma}\Delta W)$ depends only on γ_{jt}/M. Thus, given our assumptions, the firm's pricing policy depends only on γ_{jt}/M. Since the value function is the discounted expected sum of future profits (which scale with M conditional on prices), this allows us to verify our original claim that the value function scales with M.