A Plucking Model of Business Cycles

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Abstract

In standard models, economic activity fluctuates symmetrically around a "natural rate" and stabilization policies can dampen these fluctuations but do not affect the average level of activity. An alternative view—labeled the "plucking model" by Milton Friedman—is that economic fluctuations are drops below the economy's full potential ceiling. We show that the dynamics of the unemployment rate in the US display a striking asymmetry that strongly favors the plucking model: increases in unemployment are followed by decreases of similar amplitude, while the amplitude of a decrease does not predict the amplitude of the following increase. In addition, business cycles last seven years on average and unemployment rises much faster during recessions than it falls during expansions. We augment a standard labor search model with downward nominal wage rigidity and show how it can fit the plucking property. We then show that additional non-standard features are required to match the level and asymmetry of the duration of contractions and expansions.

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1 Introduction

In the workhorse models currently used for most business cycle analysis, economic activity fluctuates symmetrically around a "natural rate" and stabilization policy does not appreciably affect the average level of output or unemployment. At best, stabilization policy can reduce inefficient fluctuations around the natural rate. As a consequence, in these models the welfare gains of stabilization policy are trivial (Lucas, 1987, 2003).

An alternative view is that economic contractions involve drops below the economy's full-potential ceiling or maximum level. Milton Friedman proposed a "plucking model" analogy for this view of business cycles: "In this analogy, [...] output is viewed as bumping along the ceiling of maximum feasible output except that every now and then it is plucked down by a cyclical contraction" (Friedman, 1964, 1993).[1] In the plucking model view of the world, improved stabilization policy that eliminates or dampens the "plucks"—i.e., contractions—increases the average level of output and decreases the average unemployment rate. Stabilization policy can therefore potentially raise welfare by substantial amounts (De Long and Summers, 1988; Benigno and Ricci, 2011; Schmitt-Grohe and Uribe, 2016).

We show that the dynamics of the US unemployment rate strongly favor the plucking model of business cycles. An implication of the plucking model—highlighted by Friedman (1964)—is that the dynamics of unemployment should display the following asymmetry: economic contractions are followed by expansions of a similar amplitude—as if the economy is recovering back to its maximum level—while the amplitude of contractions are not related to the previous expansion—each pluck seems to be a new event. We refer to this asymmetry as the plucking property. We present strong evidence that the US unemployment rate displays the plucking property: The increase in unemployment during a contraction forecasts the amplitude of the subsequent expansion one-for-one, while the fall in unemployment during an expansion has no explanatory power for the size of

[1]The term "plucking" originates in Friedman's image of a string (output) attached to the underside of a board (potential output): "Consider an elastic string stretched taut between two points on the underside of a rigid horizontal board and glued lightly to the board. Let the string be plucked at a number of points chosen more or less at random with a force that varies at random, and then held down at the lowest point reached." (Friedman, 1964)

the next contraction.

To match the facts about plucking that we document in the data, we introduce downward nominal wage rigidity into a tractable version of the Diamond-Mortenson-Pissarides model with endogenous separations analyzed by Fujita and Ramey (2012). We show that the model can quantitatively match the plucking dynamics of US unemployment. Our model reproduces the plucking property because good shocks mostly lead to increases in wages, while bad shocks mostly lead to increases in unemployment. The dynamics of the unemployment rate thus become asymmetric; the unemployment rate rises far above its steady state level in response to adverse shocks, but falls much less in response to favorable shocks.

The plucking dynamics of our model imply that fluctuations in unemployment are fluctuations above a resting point of low unemployment, not symmetric fluctuations around a natural rate. As a consequence, a reduction in the volatility of aggregate shocks not only reduces the volatility of the unemployment rate, but also reduces its average level, as in the models of Benigno and Ricci (2011) and Schmitt-Grohe and Uribe (2016). Eliminating all aggregate shocks in our calibrated model reduces the average unemployment rate from 5.7% to 3.1%.

An alternative interpretation of the "plucking" property of unemployment is that it derives from exogenous shocks to the determinants of unemployment rate that themselves have this property. We have chosen to model the exogenous shocks in our model as symmetric for two reasons. First, we are interested in exploring the ability of the DMP model to generate asymmetry within the labor market. Second, prior empirical work has found asymmetries to be more pronounced in the unemployment rate than in other macroeconomic data suggesting that the source of asymmetry is the labor market (e.g., McKay and Reis, 2008).

While our baseline model fits the plucking property, it fails to fit some other salient features of unemployment dynamics. Empirically, business cycles last around 7 years from peak to peak, and the unemployment rate rises much more rapidly during downturns than it falls during expansions. In principle, search models such as the one we introduce provide an intuitive mechanism for slow recoveries: firms can shed workers rapidly, but it takes time due to search and matching frictions to expand employment. In practice, however, the large number of workers who flow between employment and unemployment in every month implies that the DMP model does not have appreciable internal propagation (see, e.g., Cole and Rogerson, 1999). As a result, unemployment falls quickly once negative shocks dissipate. Our baseline model inherits this feature of standard search models, generating short business cycles.

Matching persistence is important in its own right. But it also interacts with explaining plucking (since a high degree of persistence leads expansions to be cut off before they have time to run their course). To match the persistence of unemployment cycles and assess whether our earlier conclusions about plucking carry over to a setting with empirically realistic persistence of unemployment cycles, we propose a new illustrative model with several non-standard features. In particular, this model features insecure short-term jobs and a hump-shaped driving process for productivity shocks (an AR(2) process).

The presence of insecure short-term jobs implies that most new matches turn out to be poor matches and separate quickly; however, some survive and become stable matches. As a consequence, workers who become unemployed cycle through several jobs before finding stable employment, a pattern emphasized by Hall (1995). This, in combination with the AR(2) shock process generates substantial internal propagation despite the large amount of churning observed in the data. This model fits not only the plucking property in the data but also the duration of business cycles and the asymmetry in the speed of contractions versus expansions.

Our work is related to several strands of existing literature. Petrosky-Nadeau, Zhang, and Kuehn (2018) show how a DMP model features asymmetries that can generate business-cycles disasters—large drops in production—despite symmetric shocks. This force generates some plucking. But when we consider business cycles of the size we have experience in our sample period, it generates much less plucking than we document motivating the role of DNWR. Kim and Nelson (1999) and Sinclair (2010) are two of the very few modern attempts to assess the specific asymmetry emphasized by Friedman. Caballero and Hammour (1998), Bordo and Haubrich (2012), and Fatás and Mihov (2015) explore related ideas. Ferraro (2017) provides an alternative explanation for the speed asymmetry in the unemployment rate.

The paper proceeds as follows. Section 2 presents our empirical results on the asymmetric dynamics of the unemployment rate. Section 3 lays out our plucking model of business cycles. Section 4 shows how this model can match the plucking property, and demonstrates that stabilizing fluctuations can reduce the average level of unemployment. Section 5 proposes a new model that can match the persistence of unemployment cycles. We analyze the quantitative implications of this model in section 6. Section 7 concludes.

2 The Plucking Property and the Dynamics of Unemployment Cycles

We start by demonstrating Friedman's plucking property for post-WWII US unemployment data; namely, the amplitude of a contraction forecasts the amplitude of the subsequent expansion, while the amplitude of an expansion does not forecast the amplitude of the subsequent contraction. We also demonstrate a speed asymmetry in unemployment: the unemployment rate rises more quickly than it falls, implying that the duration of recoveries is typically much longer than the duration of recessions, as emphasized by Neftçi (1984) among others. Finally, we review the empirical evidence on wage rigidity, and show that the incidence of wage rigidity is countercyclical.

To implement our empirical analysis, we define business cycle peaks and troughs such that they line up exactly with peaks and troughs of the unemployment rate. This yields business cycle dates that are very similar to but not identical to those identified by the NBER Business Cycle Dating Committee. Figure 1 plots the unemployment rate over our sample period—which runs from January 1948 to February 2020—with vertical lines indicating the times that we identify as business cycle peaks and troughs. For details on our algorithm, see Appendix A.[2]

[2] We identify ten peaks and ten troughs. To these we add a peak at the beginning of our sample. One might worry that the contraction at the beginning of our sample may have started earlier. We are however reassured on this point by the fact that the NBER identified November 1948 as a peak. We end our sample at the onset of the Covid-19 recession, another clear peak in the data.

2.1 The Plucking Property

Figure 2 presents scatter plots illustrating the plucking property for the unemployment rate. The left panel plots the amplitude of a contraction on the x-axis and the amplitude of the subsequent expansion on the y-axis. The amplitude of contractions is defined as the percentage point increase in the unemployment rate from the business cycle peak to the next trough. The amplitude of expansions is defined analogously. There is clearly a strong positive relationship between the amplitude of a contraction and the amplitude of the subsequent expansion in our sample period. In other words, the size of a contraction strongly forecasts the size of the subsequent expansion.

Table 1 reports the estimated coefficient from an OLS regression of the size of the subsequent expansion on the size of a contraction. The relationship is roughly one-for-one. For every percentage point increase in the amplitude of a contraction, the amplitude of the subsequent expansion increases by 1.1 percentage points on average. Despite the small number of data points, the relationship is highly statistically significant. Furthermore, the explanatory power of the amplitude of the previous contraction is large. The R^2 of this simple univariate regression is 0.59.

The right panel of Figure 2 plots the amplitude of an expansion on the x-axis and the amplitude of the subsequent contraction on the y-axis. In sharp contrast to the left panel, there is no statistically significant relationship in this case. The size of an expansion does not forecast the size of the next contraction. One cannot reject that, in Friedman's language, each contractionary pluck that the economy experiences is independent of what happened before. Table 1 reports the estimated coefficient from a linear regression of the size of the subsequent contraction on the size of an expansion. The relationship is actually slightly negative, but is far from statistically significant. Moreover, the R^2 of the regression is only 0.22.[3]

[3] Jackson and Tebaldi (2017) suggest that the *duration* (not size) of an expansion is predictive of the size of the following contraction. They motivate this idea by analogy to forest fires: the longer the expansion, the more "underbrush" builds up—e.g., low quality matches and entrants—that becomes fuel in the subsequent contraction. We find no evidence of the forest fire theory at the aggregate level: the duration of an expansion is no more predictive of the size of the following contraction than the size of the expansion is. The relationship is actually negative (but not significantly so), driven by the fact that the three longest post-WWII expansions (1961-1968, 1982-1989, 1992-2000) were followed by relatively mild recessions. Tasci and Zevanove (2019) confirm these results and also present state level results for the plucking model and

2.2 The Speed Asymmetry

Unemployment rises more quickly during contractions than it falls during expansions, a point made quantitatively in early work by Neftçi (1984).[4] Table 1 reports the average speed of expansions and contractions to illustrate this asymmetry. We measure the change in unemployment (in percentage points) over the spell and the length of time the spell lasts for. The speed for a expansion or contraction is the ratio of those two numbers. We then take a simple average across all expansions and separately a simple average across all contractions.

We find that the unemployment rate rises roughly twice as quickly during contractions (1.9 percentage points per year) as it falls during expansions (0.9 percentage points per year). This difference is highly statistically significant. We run a regression of the absolute value of the speed of expansions and contractions on a dummy variable for a spell being a contraction and find that the p-value for the dummy is 0.002.

Looking back at Figure 1, we can clearly see that when the unemployment rate starts falling, it usually falls relatively steadily for a long time. As a consequence, expansions are quite long. The average length of expansions in our sample is roughly 59 months, or almost five years. Contractions are also quite persistent, but less so. The average length of contractions in our sample is roughly 27 months, a bit more than two years. Perhaps most strikingly, in a few cases—the 1960s, 1980s, 1990s, and 2010s—the unemployment rate has fallen steadily for six to ten years without reversal.

forest fire theory. Their state level results are similar to our results at the aggregate level: There is strong evidence for the plucking property but no evidence for the forest fire theory.

[4] Sichel (1993) refers to this asymmetry as the "steepness" asymmetry, while McKay and Reis (2008) refer to it as the greater "violence" of contractions, in reference to Mitchell (1927). Given that contractions and expansions are of about the same average size (3.7 percentage points), the fact that contractions are "steeper" or "more violent" than expansions is equivalent to the fact that they are briefer. Awareness of this asymmetry dates back at least to the 1920s. Mitchell (1927) notes that "business contractions appear to be briefer and more violent than business expansions."

2.3 Wage Rigidity

A large literature has presented microeconomic evidence of downward nominal wage rigidity in US data.[5] Olivei and Tenreyro (2010) present evidence that such wage rigidity matters for real outcomes. A large theoretical literature has studied the implications of downward nominal wage rigidity for business cycles.[6]

An intuitive fact about the data is that wage rigidity has been countercyclical in recent US business cycles—i.e., wage freezes tend to occur during recessions. Figure 3 plots the fraction of "Wage Freezes" from the San Francisco Fed's Wage Rigidity Meter for the period 1997-2019 along with the unemployment rate (Left Panel) and the non-employment rate for the working age population (Right Panel).[7] The correlation is striking. The fraction of wage freezes rises rapidly in each of the three recessions that occur in this sample period. Before 1997 the data are less complete, and the correlation is weaker because of the confounding effects of changes in trend inflation.

3 A Plucking Model with Downward Nominal Wage Rigidity

Next, we present a model designed to fit the plucking property discussed in section 2. This model augments the workhorse DMP model with endogenous separation developed

[5]See, in particular, McLaughlin (1994); Kahn (1997); Card and Hyslop (1997); Bewley (1999); Altonji and Devereux (2000); Kurmann and McEntarfer (2017); Hazell and Taska (2018); Grigsby, Hurst, and Yildirmaz (2018). Correcting for measurement error might reveal an even higher prevalence of wage rigidity. Early work using data from the Panel Study of Income Dynamics (PSID) and the Current Population Survey (CPS) includes McLaughlin (1994), Kahn (1997), and Card and Hyslop (1997). Altonji and Devereux (2000) report larger amounts of downward nominal wage rigidity and virtually no wage cuts in the PSID after correcting for measurement error. Gottschalk (2005) and Barattieri, Basu, and Gottschalk (2014) report similar findings based on their adjustments for measurement error in the Survey of Income and Program Participation.

[6]Prominent contributions include, e.g., Akerlof, Dickens, and Perry (1996); Kim and Ruge-Murcia (2009); Benigno and Ricci (2011); Abbritti and Fahr (2013); Schmitt-Grohe and Uribe (2016); Chodorow-Reich and Wieland (2018). The importance of wage rigidity for generating realistic fluctuations in unemployment has been stressed by Shimer (2005), Hall (2005), Gertler and Trigari (2009), and Gertler, Huckfeldt, and Trigari (2016).

[7]The Wage Freeze measure reports the fraction of job-stayers whose wages are unchanged versus one year prior (Daly, Hobijn, and Wiles, 2011; Daly, Hobijn, and Lucking, 2012; Daly and Hobijn, 2014). See https://www.frbsf.org/economic-research/indicators-data/nominal-wage-rigidity/.

by Fujita and Ramey (2012) with downward nominal wage rigidity.

3.1 Search and Matching with Endogenous Separation

The model consists of an infinite mass of atomistic firms, and a mass of workers with inelastic labor supply normalized to one. At the beginning of a period, workers are either already matched with a firm, or looking for a job.[8] If a firm and a job-seeker match in period t, we assume the worker starts working right away.[9] To match with a new worker, a firm must post a vacancy, at a cost c per period in which the vacancy remains open. An open vacancy fills with probability q_t, which is taken as exogenous by the firm. In the aggregate, the probability q_t is determined by a matching function $q(\theta_t)$, where $\theta_t = V_t/U_{0,t}$ denotes labor market tightness. Labor market tightness is the ratio of the number of vacancies posted V_t to the number of job-seekers $U_{0,t}$ at the beginning of the period. The matching function also determines the probability for a job-seeker of finding a job. This is equal to the ratio of matches $q(\theta_t)V_t$ to job-seekers $U_{0,t}$, $f(\theta_t) \equiv q(\theta_t)V_t/U_{0,t} = \theta_t q(\theta_t)$.

When a worker and a firm are matched, they produce output $A_t x_t$, where A_t and x_t are aggregate and match-specific productivity factors. We assume that both follow AR(1) exogenous processes in logs,

$$\log(A_t) = \rho^a \log(A_{t-1}) + \varepsilon_t^a, \tag{1}$$

$$\log(x_t) = \rho^x \log(x_{t-1}) + \varepsilon_t^x, \tag{2}$$

where ε_t^a and ε_t^x are Gaussian shocks with standard deviations σ_ε^a and σ_ε^x. All new matches start at the same match productivity level x^{hire}, which we take to be average match productivity $x = 1$.[10]

[8] Only unemployed workers look for jobs. Fujita and Ramey (2012) also develop a version of the model with on-the-job search. We consider their model with endogenous separation but no on-the-job search.

[9] This timing of the labor market follows, e.g., Blanchard and Gali (2010), but differs from Fujita and Ramey (2012). It is not important for our results, but avoids the unpalatable assumption that the number of job-seekers $U_{0,t}$ at the beginning of period t (which includes at least all the workers who separated from their jobs at the end of $t-1$) is the same as the number of unemployed workers U_t in period t, even when search frictions shrink to zero and the job finding rate is 1.

[10] Because Fujita and Ramey (2012) also consider a model with on-the-job search, they assume that new matches start at the highest productivity level to make sure all job offers are accepted. Since we do not consider on-the-job search, job offers are accepted even if the productivity in new matches is not the highest

An on-going match continues into the next period unless it is exogenously terminated—which occurs with probability δ—or endogenously terminated. Let $J_t(x_t)$ be the value to a firm of an on-going match with match-specific productivity x_t.[11] The firm can terminate the match if it yields a negative value. This implies that $J_t(x_t)$ is given by:

$$J_t(x_t) = \max\{J_t^c(x_t), 0\} \tag{3}$$

where J_t^c is the value of the match to the firm if it is continued, which solves the recursion

$$J_t^c(x_t) = x_t A_t - w_t(x_t) + \beta(1-\delta)E_t\left(J_{t+1}(x_{t+1})\right), \tag{4}$$

where $w_t(x_t)$ is the real wage paid in the match at period t.

Let $W_t(x_t)$ be the value to a worker of a match with match-specific productivity x_t, and U_t be the value to a worker of being unemployed. Since the worker can terminate the match if it yields it a negative value, the value of being employed in a match at x_t is given by

$$W_t(x_t) = \max\{W_t^c(x_t), U_t\} \tag{5}$$

where W_t^c is the value if the match is continued, which solves the recursion

$$W_t^c(x_t) = w_t(x_t) - \zeta + \beta E_t\left((1-\delta)W_{t+1}(x_{t+1}) + \delta(1-f_{t+1})U_{t+1} + \delta f_{t+1}W_{t+1}(x^{hire})\right), \tag{6}$$

where ζ is the dis-utility cost of working relative to being unemployed. The value of being unemployed solves the recursion

$$U_t = b + \beta E_t\left((1-f_{t+1})U_{t+1} + f_{t+1}W_{t+1}(x^{hire})\right), \tag{7}$$

where b is unemployment benefits. Subtracting (7) from (6), the value of being employed relative to being unemployed, $V_t = W_t - U_t$, solves the recursion

$$V_t^c(x_t) = w_t(x_t) - z + \beta(1-\delta)E_t\left(V_{t+1}(x_{t+1}) - f_{t+1}V_{t+1}(x^{hire})\right), \tag{8}$$

productivity level. Our process for match-specific shocks is also a more standard AR(1) process than the memoryless Poisson process with infrequent large shocks assumed in Fujita and Ramey (2012).

[11] We make explicit the dependence of J_t and other value functions only in the idiosyncratic state x_t. The dependence of J_t on the aggregate state is kept implicit in the t time-index of J_t.

where

$$V_t(x) = \max\{V_t^c(x), 0\} \tag{9}$$

and $z = b + \zeta$ is the flow value of unemployment in terms of both unemployment benefits and leisure.

Assuming free entry, the cost of posting a new vacancy must in equilibrium be equal to the benefit to the firm of posting a new vacancy. This equates the value of a new hire to the firm to the expected cost of a new hire:

$$J_t(x^{hire}) = \frac{c}{q_t}. \tag{10}$$

The model is closed with an assumption on wage-setting. Fujita and Ramey (2012) assume that wages are set according to Nash-bargaining, as is standard in search models. Define the total value of a continuing match to be $S_t^c(x_t) = J_t^c(x_t) + V_t^c(x_t)$. Under Nash-bargaining, $V_t^{c,Nash} = \gamma S_t^{c,Nash}$, where $\gamma \in [0,1]$ is the bargaining power of workers. Combining equations (4) and (8) and the Nash-bargaining assumption gives

$$J_t^{c,Nash}(x_t) = (1-\gamma)(A_t x_t - z) + \beta(1-\delta)E_t\left(J_{t+1}^{Nash}(x_{t+1}) - f_{t+1}^{Nash}\gamma J_{t+1}^{Nash}(x^{hire})\right). \tag{11}$$

Equations (10) and (11) allow us to solve for q_t and $J_t^{c,Nash}(x_t)$, as detailed in Appendix B.

Combining (4) and (11) allows us to recover the Nash wage as

$$w_t^{Nash}(x_t) = \Bigg(\gamma A_t x_t + (1-\gamma)z\Bigg) + \beta(1-\delta)E_t\Bigg(\gamma f_{t+1}^{Nash} J_{t+1}^{Nash}(x^{hire})\Bigg). \tag{12}$$

3.2 Downward Nominal Wage Rigidity

We extend the Fujita-Ramey model to allow for downward nominal wage rigidity (DNWR).[12] The presence of match heterogeneity implies that wages differ for new and ongoing matches. We assume DNWR for both groups. For an on-going match at time t, the nominal wage is either the flexible nominal wage—which we assume to be the wage

[12]We assume DNWR by directly specifying the wage-rule. Other work directly specifying a wage rule in a search and matching framework includes Blanchard and Gali (2010); Shimer (2010); Michaillat (2012). This approach has the advantage that we can more easily investigate what features the wage process needs to generate realistic unemployment dynamics.

that would obtain if the firm set wages by Nash bargaining in this and future periods—or, if this requires the nominal wage to fall, the nominal wage remains unchanged from the previous period, i.e.:

$$w_t(x_t, w_{t-1}) = \max\left\{w_t^{Nash}(x_t), \frac{w_{t-1}}{\bar{\Pi}_t}\right\}. \tag{13}$$

For new matches, we assume that the hiring wage at time t is either the flexible nominal wage—which we again assume to be the wage that would obtain under Nash bargaining—or, if this requires the nominal hiring wage to fall, the nominal hiring wage in the previous period, i.e.:

$$w_t^{new}(x^{hire}, w_{t-1}^{new}) = \max\left\{w_t^{Nash}(x^{hire}), \frac{w_{t-1}^{new}}{\bar{\Pi}_t}\right\}. \tag{14}$$

To generate fluctuations in firms' hiring decisions, wage rigidity in new matches is particularly important, as emphasized by Pissarides (2009). Assumption (14) is therefore essential to introducing DNWR in a search and matching model with match heterogeneity. Hazell and Taska (2018) document such DNWR in new hires' wages.[13]

In both equations (13) and (14), inflation relaxes the constraint on downward *real* wage adjustments: it greases the wheels of the labor market. We specify monetary policy as directly setting a path for the inflation rate Π_t, which we take to be constant at some target value $\overline{\Pi}$. Equations (10) and (11) still hold under DNWR, except that the firm's value function now depends on lagged wages. We first solve the model under Nash-bargaining to recover the Nash wage (12), then use equations (10) and (11) to solve for J_t^c and q_t, as detailed in Appendix B.

3.3 Worker Flow Accounting

Let s_t be the destruction rate, defined as the fraction of matches that get destroyed at the beginning of period t. Because matches can be endogenously terminated, the destruction rate s_t depends on the cross-sectional distribution of employment across the state of

[13]Our model does not feature preemptive wage moderation of the type that is present in the wage setting models (e.g., Kim and Ruge-Murcia, 2009; Elsby, 2009; Benigno and Ricci, 2011). Yet this does not mean firms in our model are myopic. They rationally maximize intertemporal profits. What they preemptively moderate in anticipation of a fall in productivity is hires, not wages. Either wages or hires can respond to concerns about the future. In our model it is hires that are moderated, because wages are not set by firms.

matches. Under Nash-bargaining, the state of a match reduces to match productivity x_t. Fujita and Ramey (2012) show how to keep track of the distribution of employment across matches to calculate the destruction rate in this case. Under DNWR, the state of a match includes both match productivity x_t and the lagged wage w_{t-1}. Appendix B shows how to keep track of the joint distribution of employment across the joint state (x_t, w_{t-1}) and calculate the destruction rate under DNWR.

Workers' job-finding rate f_t is a direct function of the vacancy-filling rate q_t through the matching function and can therefore be easily recovered from q_t. Since we assume that job-seekers who match with a firm at the beginning of period t start working at t, workers who separate from a firm between $t-1$ and t and join the pool of job-seekers may find a new job at t without spending time unemployed, instead transitioning directly from job to job. The exit rate from employment to unemployment is therefore distinct from the destruction rate s_t and equal to

$$\bar{s}_t = (1 - f_t)s_t. \tag{15}$$

The law of motion of the unemployment rate U_t is

$$U_t = (1 - f_t)U_{t-1} + \bar{s}_t(1 - U_{t-1}). \tag{16}$$

3.4 Calibration

Table 2 provides a summary of our calibration. We calibrate the model to a monthly frequency. We set the discount factor β to correspond to an annual interest rate of 4%. We assume a Cobb-Douglas matching function $q(\theta) = \mu\theta^{-\eta}$ and set the elasticity of the matching function to $\eta = 0.5$, in the middle of the range reported in Petrongolo and Pissarides (2001)'s survey. We calibrate the flow value of unemployment following Hagedorn and Manovskii (2008) to $z = 0.95$, so that the model generates significant fluctuations in the job-finding rate under Nash bargaining, which we will consider as a benchmark.[14] This

[14] Fujita and Ramey (2012) show that their model under Nash bargaining can generate fluctuations in the unemployment rate away from a high calibration of z by making unemployment fluctuate through fluctuations in separations. As they show however, away from a high calibration of z, the model cannot generate fluctuations in the job-finding rate.

calibration also generates the realistic prediction that firms' surplus is increasing in productivity under DNWR, as explained in Appendix F.1. We calibrate the exogenous destruction rate δ to match the average monthly share of quits in the non-farm sector in JOLTS between January 2000 and February 2020 of 1.9%.

The parameters μ and c jointly determine hiring costs. One of the two is redundant as only the composite parameter $c\mu^{\frac{-1}{1-\eta}}$ is relevant for the equilibrium. (See Appendix B.1 for further discussion of this point.) We normalize μ to 1. We calibrate c so that the cost of hiring a worker $\frac{c}{q} = J$ is 10% of monthly wages in a steady state with $u = 5.7\%$ and $\bar{s} = 2\%$, in line with what Silva and Toledo (2009) report based on the Employer Opportunity Pilot Project survey in the US. This yields $c = 0.30$.

We set the auto-regressive root of the aggregate productivity process ρ^a to 0.98 following Shimer (2010). We set the auto-regressive root of the match-specific productivity process ρ^x to 0.98, following Foster, Haltiwanger, and Syverson (2008)'s estimates of the persistence of plants' TFP (0.8 on an annual basis). When we consider the model under downward nominal wage rigidity, we set inflation to 2% per year. Inflation is immaterial in the version of the model without DNWR.

This leaves γ, σ_ε^a and σ_ε^x. We pick them to match the average level of the unemployment rate (5.7% in the data), the standard deviation of the unemployment rate (1.6% in the data), and the average of the rate of exit from employment $\bar{s}$ (2% as reported by Fujita and Ramey (2006, 2012)). We choose to match the standard deviation of unemployment exactly (as opposed to calibrating to the standard deviation of productivity in the data) so that we can apply our definition of expansions and contractions to our simulated samples in the same way as we do to the real world data. These choices yield $\gamma = 0.57$, $\sigma_a = 1.6\%$ and $\sigma_x = 2.1\%$ under Nash bargaining, and $\gamma = 0.43$, $\sigma_a = 1.5\%$ and $\sigma_x = 1.5\%$ under DNWR.

4 Quantitative Analysis of the Baseline Model

Given the asymmetries and non-linearities our model is intended to capture, we rely on global methods to numerically solve for the equilibrium. Appendix B discusses the algorithm we use in detail. We simulate 5000 samples of 866 periods (the length of our sample of real-world data) and calculate the statistics reported in the empirical Table 1 in each of these simulated samples. We then report the median estimate across samples for each statistic as a point estimate and the standard deviation of the estimates across samples in parentheses below each point estimate.

Table 3 reports the results. The top panel presents results on the plucking property. We first consider the model with flexible wages (Nash bargaining). This version of the model generates non-trivial plucking, but substantially less than in the data. The regression coefficient for the size of expansions on the previous contraction is 0.39 versus -0.03 for contractions on the previous expansion. These coefficients are 1.09 and -0.38 in the data. Next, we present results for the model with DNWR. This case produces substantially more plucking that the model with flexible wages. The regression coefficient for the size of expansions on the previous contraction is 0.61 versus -0.05 for contractions on the previous expansion.

Figure 4 illustrates these results graphically. It presents scatter plots for the regressions discussed in the last paragraph. These scatter plots reveal that there is a large amount of noise in the relationship between the amplitude of expansions and contractions in the model. This is particularly the case in the model with flexible prices. In this model, the R^2 of the size of expansions on the previous contraction is only 0.19, compared to 0.59 in the data. The model with DNRW does substantially better on this metric with an R^2 of 0.39.

4.1 Mechanisms for Plucking

Two features of the model contribute to the plucking property. First, non-linearity in the worker-flow equations (15)-(16) generates plucking. Figure 6 plots the steady-state relationship between the job finding rate and unemployment implied by the worker-flow relationship (15)-(16) taken in steady-state.[15] The relationship is convex because it

[15]The analytical expression is $u = s/(s + (f/(1 - f))$ under our assumption that workers who separate from a firm get a chance to find a new job right away and spend no time unemployed. Assuming instead that workers must necessarily spend one period unemployed before finding a new job, the relationship would be $u = s/(s + f)$. In this case too, the steady-state worker-flow relationship is convex but the non-

gets harder and harder to lower the unemployment rate the lower it gets. Earlier work has emphasized this non-linearity (Petrosky-Nadeau and Zhang, 2017; Petrosky-Nadeau, Zhang, and Kuehn, 2018).[16]

The reason why this non-linearity generates only modest amounts of plucking is that over the empirically relevant range of unemployment rates – 2.5% and 10.8% for the sample period 1948-2019 – the degree of non-linearity is modest. This is illustrated in Figure 6. In Table 3, we make sure not the overstate the strength of this non-linearity by presenting results for a "top-truncated" sample that only includes expansions and contractions that are less than 6.5 percentage points in size—i.e., the size of the 2009-2019 expansion which is the largest one in our sample.

The second feature of our model that contributes to plucking is DNWR. When wages are downward rigid, negative shocks typically result in higher unemployment while positive shocks yield wage increases. As a result, unemployment sometimes rises far above its steady state, but rarely falls below. Table 3 shows that this feature can generate a substantial amount of plucking.

But even the model with DNWR produces less plucking than the data. The top panel of Figure 5 helps us understand why. This panel plots a simulated path of unemployment in our model with DNWR. A prominent feature of this simulated series is that expansions are often interrupted by a new contraction before unemployment has had enough time to fully recover. This can introduce a great deal of noise in our measure of plucking (as in Figure 4). Some expansions will be cut short. Others will appear excessively long because their length is governed not only by the size of the previous contraction but also by earlier contractions the recovery from which were cut short.

A second weakness of the model with DNWR is that the unemployment rate is much less persistent than in the data. The low persistence in the model is documented in the lower half of Table 3. The model generates unemployment cycles that are far too short. The average duration of expansions is 25.1 months versus 59.1 in the data, and the average duration of contraction is 16.1 months, versus 26.9 in the data.

linearities it induces are quantitatively small.

[16] Hairault, Langot, and Osotimehin (2010), Jung and Kuester (2011), and Lepetit (2018) also emphasize this source of non-linearity.

The model with flexible wages does considerably better on these metrics. With our Hagedorn and Manovskii (2008) calibration, movements in real wages are muted and the model's endogenous variables inherit the behavior of the shock process. This yields quite a bit of persistence. With DNWR, however, it is nominal wages that are rigid in downturns. Real wages fall due to increases in the price level. This implies that gaps between A_t and w_t get eroded faster than in the flexible wage model with the Hagedorn and Manovskii (2008) calibration. In section 5, we present a model with several features designed match both plucking and persistence simultaneously.

4.2 Stabilization Policy and Output Gaps in the Plucking Model

In a thought-provoking exercise, Lucas (1987, 2003) showed that the welfare benefits of eliminating all economic fluctuations are trivial in a simple benchmark model. Crucially, Lucas assumed in this thought experiment that the average level of economic activity was unaffected by the elimination of business cycles. Our plucking model violates this assumption.

In a plucking model, recessions are asymmetrically periods when the economy drops below potential. Eliminating these business cycle fluctuations raises the average level of economic activity. The top panel of Figure 5 shows that in our model, eliminating all fluctuations reduces the average unemployment rate from 5.7% to 3.1%. Conversely, increasing the standard deviation of aggregate shocks by 50% (from 1.5% to 2.25%) increases the average unemployment rate to 12.2%. Figure C.1 in Appendix C plots the average level of the unemployment rate in our plucking model as a function of the volatility of aggregate shocks.

The plucking model also implies that standard measures of the output gap are biased. Aiyar and Voigts (2019) show that common estimation methods of the output gap implicitly assume a zero-mean output gap. In the plucking model, however, output does not fluctuate symmetrically around a natural rate. Standard methods systematically underestimate the amount of slack because the output gap is on average negative.

4.3 Entry and Exit over the Business Cycle

The bottom two panels of Figure 5 give simulated paths for the job-finding rate f_t and the rate of exit from employment $\bar{s}_t$ in our model. Both the hiring and separation margins contribute to the sharp rise in unemployment during recessions, consistent with evidence documented by Elsby, Michaels, and Solon (2009) for US data.[17] Negative shocks decrease the job-finding rate and increase the rate of exit from employment, while positive shocks mostly lead to increases in wages.

We quantify the relative importance of hiring versus separations by calculating their contributions to the volatility of unemployment. For the contribution of the job-finding rate f, we simulate the unemployment rate from equation (16) with $\bar{s}$ fixed at its mean, then calculate the standard deviation of the resulting counterfactual unemployment rate, and divide it by the standard deviation of actual unemployment. Analogously, for the employment exit rate, we do the same analysis with the job-finding rate fixed at its mean. According to this metric, fluctuations in the job-finding rate explain 42% of fluctuations in unemployment, while fluctuations in the employment exit rate explain 55%.

Table 4 reports additional statistics: the volatility, the auto-correlation, and the correlation with productivity of the unemployment rate, of the job-finding rate, and of the employment exit rate. We calculate these statistics for the HP-filtered log of simulated series from the model. Relative to the model with Nash bargaining, the model with DNWR lowers the correlation of all three variables with productivity and brings it closer to the data. The auto-correlation statistics are however substantially smaller than in the data—a manifestation of the lack of persistence also captured with the speed and duration statistics in Table 3.

[17] The separation margin is not necessary to replicate the plucking property however. Appendix F presents a simplified version of the model introduced in section 5 with a constant exogenous separation rate, and shows that it, too, generates the plucking property (see Table F.5.)

5 A New Model with Decreasing Returns to Labor, Insecure Jobs, and AR(2) Shocks

We next propose a model with several "non-standard" features, aimed at generating substantially more persistence in unemployment fluctuations relative to the model used earlier in the paper. The non-standard features are: 1) decreasing returns to labor, 2) insecure short-term jobs and 3) AR(2) shocks. Our model builds on Michaillat (2012), who also emphasizes the importance of decreasing returns in understanding unemployment dynamics.

The new features come at a cost. We are no longer able to solve the model with match-specific productivity shocks of the type we had in section 3. This limits the model's realism when it comes to the hiring and separation margins. We do, however, incorporate sectoral heterogeneity in productivity which implies that downward nominal wage rigidity binds in sectors with stagnant productivity, even if aggregate productivity growth is positive.

5.1 Decreasing Returns to Labor and Heterogeneous Labor Inputs

The model consists of a continuum of sectors $i \in [0, 1]$, each employing a distinct type of labor i. In each sector, firms have access to a decreasing-returns production function that uses labor i as its single input, $Y_t^i = A_t^i F(N_t^i)$, where Y_t^i is output, N_t^i is employment, and A_t^i is an exogenous productivity shifter. For simplicity, we restrict sectoral heterogeneity to labor markets: consumers perceive goods produced in different sectors as identical and therefore value them equally. All goods are sold in a competitive product market at a common price P_t.

A given worker provides a particular type of labor, and can therefore only seek to work at a firm in one sector. This implies that there is a distinct labor market for every type of labor and workers cannot flow across labor markets. We think of these labor types as occupations in a particular location, e.g., lawyers in Houston. Switching occupations is difficult due to occupation-specific human capital. Mobility constraints limit the willingness of workers to switch locations.[18]

5.2 Labor Demand with Insecure Short-Term Jobs

A firm in sector i starts period t with the stock of workers it inherits from the previous period, denoted by M_t^i. These workers are securely attached to the firm: at the end of each period only a small fraction $\delta \in (0,1)$ of them separate from the firm for exogenous reasons. Before starting production, the firm hires H_t^i workers. These workers start working at time t. The level of employment at the firm at t is then $N_t^i = M_t^i + H_t^i$.

Newly-hired workers separate from the firm at the end of period t at a higher rate $d \geqslant \delta$. If they do not separate at the end of period t, however, they join the pool of securely attached workers and face the low exogenous separation rate δ from period $t+1$ onward. The stock of workers attached to the firm at the beginning of the next period is then

$$M_{t+1}^i = (1-\delta)M_t^i + (1-d)H_t^i \quad \text{if} \quad H_t^i \geqslant 0. \tag{17}$$

The difference between the separation rates of securely attached workers and newly-hired workers captures the fact that many newly formed matches turn out to be poor matches for various reasons and are therefore terminated quickly. An unemployed worker will typically transition between several jobs with intervening unemployment spells before one of these jobs turns out to be a good match and thus turns into a long-term, secure position. The fact that newly hired workers face much higher separation rates than workers with longer tenure has been emphasized by Hall (1995), Pries (2004), Krolikowski (2017), Jung and Kuhn (2019), Jarosch (2021), and Hall and Kudlyak (2021), among others. The fact that there are effectively only two types of jobs in our model—very short-term jobs that last one period and very secure jobs—is a simplifying assumption that we make for tractability. This assumption alleviates the need to keep track of the stock of short-term workers working at the firm.

[18] The assumption of such differentiated labor inputs is standard in the New-Keynesian literature. There, differentiated labor inputs are an important source of strategic complementarity in price setting. See, e.g., Woodford (2003, ch. 3).

We allow the firm not to hire in period t and instead endogenously lay off securely-attached workers, above and beyond the fraction δ that exogenously left at the end of period $t-1$. In this case, $H_t^i < 0$ and all the workers N_t^i who work at the firm in period t are securely attached, so that the firm starts the next period with a number of workers

$$M_{t+1}^i = (1-\delta)N_t^i \quad \text{if} \quad H_t^i \leqslant 0. \tag{18}$$

We assume that the firm pays its securely attached and newly-hired workers the same wage. One rationale for this assumption is that paying them differently would adversely affect morale at the firm (Bewley, 1999). We denote the real wage in sector i by w_t^i. To hire workers, the firm must post vacancies. Posting a vacancy costs cA_t^i units of goods, where c is a constant.[19] A vacancy translates into a hire if it matches with a job-seeker. A match happens with probability q_t^i, which firm i takes as given. Hiring one worker has the expected cost $A_t^i c/q_t^i$.

Firm i's real profits at time t are revenues net of the cost of labor and hiring costs

$$A_t^i F(N_t^i) - w_t^i N_t^i - \frac{A_t^i c}{q_t^i} H_t^i I|_{H_t^i \geqslant 0}. \tag{19}$$

Like all agents in the model, the firm is risk-neutral and discounts the future with a factor $\beta \in (0,1)$. The firm is forward-looking and chooses how many workers to hire to maximize intertemporal real profits

$$\Omega_t^i(M_t^i) = \max_{H_t^i}\{A_t^i F(M_t^i + H_t^i) - w_t^i \times (M_t^i + H_t^i) - \frac{A_t^i c}{q_t^i} H_t^i 1_{H_t^i \geqslant 0} + \beta E_t(\Omega_{t+1}^i(M_{t+1}^i)\}, \tag{20}$$

subject to the law-of-motion of its workforce $M_{t+1}^i(N_t^i)$ given by equations (17)-(18).

In this model with decreasing returns to labor, the level of employment at the firm N_t^i affects the marginal productivity of all workers and therefore the bargaining position of the firm when negotiating wages with all its workers. The firm can therefore in principle internalize the effect of its chosen employment level on the wage it will be able to bargain with its workers, leading to intrafirm bargaining (Stole and Zwiebel, 1996; Brügemann,

[19] We make the cost of posting a vacancy proportional to productivity as in e.g. Blanchard and Gali (2010), because it allows us to consider non-stationary sectoral productivity shocks without increasing the size of the state-space beyond what is computationally feasible.

Gautier, and Menzio, 2019). With DNWR however, intrafirm bargaining becomes much harder to solve. We therefore abstract from intrafirm bargaining and assume that firms do not internalize the effect of their chosen employment level of the wage they will be able to bargain.[20]

Let $J_t^{i*}(M_t^i) = \frac{\partial \Omega_t^i(M_t^i)}{\partial M_t^i}$ denote the equilibrium marginal value to the firm of a long-term worker inherited from the previous period. For a level of employment N_t^i—not necessarily equal to M_t^i—the marginal value to the firm of a worker already attached to the firm is

$$J_t^i(N_t^i) \equiv A_t^i F'(N_t^i) - w_t^i + \beta(1-\delta) E_t \left(J_{t+1}^{i*}(M_{t+1}^i(N_t^i)) \right) \tag{21}$$

and the marginal value to the firm of a newly-hired worker is

$$J_t^{i,new}(N_t^i) \equiv A_t^i F'(N_t^i) - w_t^i + \beta(1-d) E_t \left(J_{t+1}^{i*}(M_{t+1}^i(N_t^i)) \right). \tag{22}$$

Each is equal to the marginal product of labor net of the real wage, plus the continuation value discounted with the appropriate separation rate. When already-hired workers and newly-hired workers face the same separation rate $\delta = d$, then $J_t^{i,new} = J_t^i$ and the model reduces to its version without insecure short-term jobs which we consider in Appendix F.

Consider the demand for labor of a firm that starts off period t with M_t^i workers inherited from the previous period. If at the inherited level of employment M_t^i, the marginal value of a newly-hired worker is greater than the hiring cost $J_t^{i,new}(M_t^i) > \frac{A_t^i c}{q_t^i}$, the firm will hire additional workers $N_t^i > M_t^i$ up to a point where

$$J_t^{i,new}(N_t^i) = \frac{A_t^i c}{q_t^i}. \tag{23}$$

If at the inherited level of employment M_t^i, the marginal value of an already-attached worker is positive but the marginal value of a newly-hired worker is less than the hiring

[20] Absent DNWR, the bargaining between the firm and its worker yields a differential equation for the wage that can be solved in closed form for a constant-elasticity production function (Cahuc, Marque, and Wasmer, 2008; Elsby and Michaels, 2013). Under DNWR however, the derivative of the wage is discontinuous at employment levels at which the DNWR constraint starts binding, preventing a simple closed-form solution. Cahuc, Marque, and Wasmer (2008) and Elsby and Michaels (2013) show that the Nash-bargained wage taking into account intrafirm bargaining only differs from the Nash-bargained wage that abstracts from it (30) by a multiplicative coefficient in front of the marginal product of labor $F'(N)$.

cost, i.e., $J_t^i(M_t^i) \geqslant 0$ and $J_t^{i,new}(M_t^i) \leqslant \frac{A_t^i c}{q_t^i}$, the firm will freeze employment at

$$N_t^i = M_t^i. \tag{24}$$

If at the inherited level of employment M_t^i, the marginal value of an already-attached worker is negative $J_t^i(M_t^i) < 0$, the firm will endogenously fire some of its existing workers $N_t^i < M_t^i$ up to

$$J_t^i(N_t^i) = 0. \tag{25}$$

Appendix D.1 provides derivations. Note that firms cannot simultaneously hire and fire endogenously at a sectoral level, since we do not allow for match heterogeneity.

5.3 Workers

There is a fixed supply of workers in each sector, common across sectors. Workers supply an exogenous quantity of labor which we normalize to 1. Workers in sector i can be in one of three states at t. A number $N_{0,t}^i \equiv \min(N_t^i, M_t^i)$ are employed in a secure job. A number $N_t^i - N_{0,t}^i$ are newly employed. The remaining $U_t^i \equiv 1 - N_t^i$ are unemployed. We denote $U_{t,0}^i \equiv 1 - N_{t,0}^i$ the number of workers who are either unemployed or employed in a short-term job, and refer to it as the "broad" unemployment rate. $U_{t,0}^i$ is also the number of job-seekers (see below). The N_t^i employed workers—either in short-term jobs or secure jobs—earn the real wage w_t^i and forego leisure relative to being unemployed, which they value as $A_t^i \zeta$ units of consumption. Unemployed workers earn unemployment benefits $A_t^i b$.[21]

Unemployed workers transition to being newly employed with the job-finding probability f_t^i, and remain unemployed with probability $1 - f_t^i$. We assume that employed workers who lose their jobs between periods $t-1$ and t get a chance to find a new job at the beginning of period t and therefore to work in period t, spending no time unemployed. Securely employed workers separate from their jobs with a probability δ_t^i, which

[21]Making unemployment benefits and the utility of leisure proportional to productivity allows us to consider non-stationary sectoral productivity shocks without increasing the size of the state-space beyond what is computationally feasible.

may be higher than δ due to endogenous layoffs. They therefore transition to unemployment with a probability $\delta_t^i(1-f_t^i)$, transition to being newly hired with probability $\delta_t^i f_t^i$, and remain securely employed with a probability $(1-\delta_t^i)$. Newly employed workers separate from their jobs with a probability d_t^i, which can be higher than d due to endogenous layoffs. They therefore transition to being securely employed with a probability $1-d_t^i$, transition to unemployment with a probability $d_t^i(1-f_t^i)$, and remain newly employed—in a new job—with probability $d_t^i f_t^i$.

Workers, like firms, are risk-neutral. The values they derive from being unemployed U_t^i, securely employed W_t^i, and newly employed $W_t^{i,new}$ solve the recursive equations

$$U_t^i = A_t^i b + \beta E_t\bigg((1-f_{t+1}^i)U_{t+1}^i + f_{t+1}^i W_{t+1}^{i,new}\bigg), \tag{26}$$

$$W_t^i = w_t^i - A_t^i\zeta + \beta E_t\bigg(\delta_{t+1}^i(1-f_{t+1}^i)U_{t+1}^i + \delta_{t+1}^i f_{t+1}^i W_{t+1}^{i,new} + (1-\delta_{t+1}^i)W_{t+1}^i\bigg), \tag{27}$$

$$W_t^{i,new} = w_t^i - A_t^i\zeta + \beta E_t\bigg(d_{t+1}^i(1-f_{t+1}^i)U_{t+1}^i + d_{t+1}^i f_{t+1}^i W_{t+1}^{i,new} + (1-d_{t+1}^i)W_{t+1}^i\bigg). \tag{28}$$

5.4 Matching Function

The probability of filling a vacancy q_t^i is determined in equilibrium by a matching function $q(\theta_t^i)$, where $\theta_t^i = \max(0, H_t^i/q_t^i)/U_{0,t}^i$ denotes labor market tightness in labor-market i. Labor market tightness is the ratio of the number of vacancies posted $\max(0, H_t^i/q_t^i)$ to the number of job-seekers at the beginning of the period. Our assumption that workers who separate from their jobs at $t-1$ get a chance to find a new job at the beginning of t implies that the pool of job-seekers at the beginning of period t is all workers not securely employed at t, $U_{0,t}^i$. The matching function also determines the probability for an unemployed worker of type i of finding a job: it is equal to the ratio of hires to job-seekers $f(\theta_t^i) \equiv \max(0, H_t^i)/U_{0,t}^i = \theta_t^i q(\theta_t^i)$.

When firms hire in sector i, they do not endogenously lay off workers so $N_{0,t}^i = M_t^i$ and the number of job-seekers is equal to $U_{0,t}^i = 1 - M_t^i$. The job-finding rate in sector i is therefore the following function of employment N_t^i and the inherited stock of employment M_t^i

$$f_t^i = \max\left(0, \frac{N_t^i - M_t^i}{1 - M_t^i}\right). \tag{29}$$

Appendix D.2 provides details on worker flow accounting in the model.

5.5 Wage-Setting

Downward nominal wage rigidity is defined relative to a flexible-wage benchmark defined as the wage bargained by the firm and a securely attached worker, assuming Nash bargaining prevails today and in all subsequent periods. Appendix D.3 shows that this wage satisfies

$$w_{Nash,t}^i = A_t^i\Big(\gamma F'(N_{Nash,t}^i) + (1-\gamma)z\Big) + \beta E_t\Big((1-\delta)(1-\gamma)f_{Nash,t+1}^i V_{Nash,t+1}^{i,new}\Big), \tag{30}$$

where $\gamma \in [0,1]$ parameterizes the bargaining power of workers.

We assume that the nominal wage is set to the flexible wage—given by equation (30)—except if this requires the nominal wage to fall. Expressed in terms of real wages, the wage-setting equation is

$$w_t^i = \max\left\{w_{Nash,t}^i, \frac{w_{t-1}^i}{\overline{\Pi}_t}\right\}. \tag{31}$$

5.6 Equilibrium

We again specify monetary policy as setting inflation to some target value $\overline{\Pi}$. An equilibrium is given by a process for employment N_t^i, labor market tightness θ_t^i, and the real wage w_t^i for each sector $i \in [0,1]$ such that in all sectors i, firm i is on its labor demand schedule—equation (23), (24), or (25) depending on the situation—the job-finding rate satisfies equation (29), and wages are set according to equation (31).

We assume that sectoral productivity $\log(A_t^i)$ in sector i is the sum of a time trend g, an aggregate component $\log(A_t)$, and an idiosyncratic component $\log(Z_t^i)$: $\log(A_t^i) = g \times t + \log(A_t) + \log(Z_t^i)$, where all these processes are independent: $\log(A_t) \perp\!\!\!\perp \log(Z_t^i), \log(Z_t^i) \perp\!\!\!\perp \log(Z_t^j)$ for $i \neq j$. We assume the idiosyncratic component follows an AR(1) in growth rates $\Delta\log(Z_t^i) = \rho_{\Delta z}\Delta\log(Z_{t-1}^i) + \varepsilon_t^{\Delta z,i}$, with Gaussian innovations: $\varepsilon_t^{\Delta z,i} \sim \mathcal{N}(0, \sigma_\varepsilon^{\Delta z})$.

We assume the aggregate component follows an AR(2) in levels $\log(A_t) = (I - \rho_1^a L)^{-1}(I - \rho_2^a L)^{-1}\varepsilon_t^a$, with Gaussian innovations: $\varepsilon_t^a \sim \mathcal{N}(0, \sigma_\varepsilon^a)$. Here, we are motivated by the DSGE literature, which often includes equations that yield AR(2) dynamics–e.g., investment adjustment costs, habits in consumption, and lagged terms in the price and wage Phillips curves combined with AR(1) shocks. This feature of our model generates high persistence at business cycle frequencies without extreme levels of persistence at very low frequencies, and also helps fit the fact that the dynamic responses of economic activity to many shocks is hump-shaped (e.g. Romer and Romer, 2004; Christiano, Eichenbaum, and Evans, 2005).[22]

6 Quantitative Analysis of New Model

Given the complexity of the model presented in section 5, we focus on an illustrative calibration as opposed to a full quantitative analysis. Appendix E describes the calibration we rely on. Like in the previous section, we rely on global methods to numerically solve for the equilibrium. Appendix D.5 discusses the algorithm we use in detail.

Table 3 presents our results for the new model in the rightmost column. The main way in which the new model improves on the DMP model discussed earlier in the paper is that it generates realistically slow unemployment cycles. Moreover, the model also generates a substantial amount of asymmetry in the speed and duration of recessions versus expansions, which the workhorse DMP model also failed to match. The average duration of expansions is 64.0 months in the new model, while the average duration of contractions is 36.6 months. As in the data, expansions are about twice as long as contractions. The unemployment rate rises much more quickly than it falls in the new model: on average, it rises by 1.44 percentage points per year and falls by 0.81 percentage points per year. In the data, these number are 1.89 and 0.87. Figure 7 plots a simulated path for the unemployment rate U_t, as well as the for "broad" unemployment U_t^0 (i.e., all workers not in secure jobs). The greater persistence and asymmetric persistence yields a simulated

[22] Fujita and Ramey (2007) explore an alternative mechanism for increasing the propagation of shocks in the labor market. They assume that the cost of opening a vacancy is non-zero and increasing in the number of new vacancies opened. This makes vacancy creation sluggish.

unemployment rate that resembles the behavior of the real-world unemployment.

The new model also generates slightly larger amounts of plucking than the DMP model discussed earlier in the paper. The overall fit of the new model is therefore substantially better. It can both generate realistic plucking and persistence, while the earlier DMP model cannot.

In appendix F, we show that it is the addition of short-term insecure jobs and AR(2) shocks that generate persistence in the new model. Plucking in the new model is generated by the same forces as in our earlier DMP model. In particular, the new model with DNWR generates a great deal of plucking with and without the addition of short-term insecure jobs and AR(2) shocks, and it generates very little plucking when we drop DNWR independent of whether we include short-term insecure jobs and AR(2) shocks.

The analysis in appendix F also shows that the asymmetry in persistence between expansions and contractions requires the combination of DNWR, short-term insecure jobs, and AR(2) shocks. With DNWR, a string of negative labor demand shocks eventually results in significant endogenous separations of securely attached workers—i.e., a burst of separations. These bursts of separations are short-lived and modest in size. But they contribute to speeding up unemployment contractions.[23] These laid-off workers then cycle through several short-term jobs before they eventually find new stable employment. This process of cycling through short-term jobs contributes to preventing the unemployment rate from quickly returning to its steady state level.

As Benigno and Ricci (2011) emphasize, sectoral shocks are an important source of volatility in labor demand. The fact that sectoral shocks are non-stationary allows the Nash wage to rise for many consecutive periods in a sector, to a high level that then durably constrains wages when sectoral productivity falls for many consecutive periods. Decreasing returns to labor make firms able to withstand such large and persistent shocks without laying off all their workers, as we explain in Appendix F.1. These factors together imply that the DNWR constraint continues to bind in the presence of sectoral shocks

[23]These bursts of separations do not arise in a model with symmetric real wage rigidity. In a version of the model with symmetric real wage rigidity shown in appendix F, the monthly rate of separation from secure employment rises above 0.5% once every 43,300 months (3,600 years). With DNWR, this rate rises above 0.5% once every 253 months (21 years).

despite the combined effect of inflation (2% per year) and growth (2.3% per year) that "grease the wheels of the labor market".

7 Conclusion

We build a plucking model of the business cycle that captures the asymmetry in the predictive power of contractions and expansions emphasized by Milton Friedman. We show that a workhorse labor search model augmented with downward nominal wage rigidity can fit these facts. In this model, eliminating business cycles lowers the average unemployment rate. Since output is more often below than above the natural rate, standard methods systematically underestimate the amount of slack in the economy.

While our benchmark model model with match heterogeneity and downward nominal wage rigidity succeeds in fitting the plucking property, it fails to match the overall duration of unemployment cycles, and the fact that expansions are on average twice as long as contractions. We therefore introduce a second model with several non-standard features—decreasing returns to labor, insecure jobs and AR(2) shocks—on top of downward nominal wage rigidity. These new features interact to slow down unemployment cycles, and explain why unemployment rises faster than it falls. However, they come at the cost of a less realistic hiring and separation margin since we are no longer able to solve the model with match-specific productivity shocks. A full integration of these features is a promising topic for future research.

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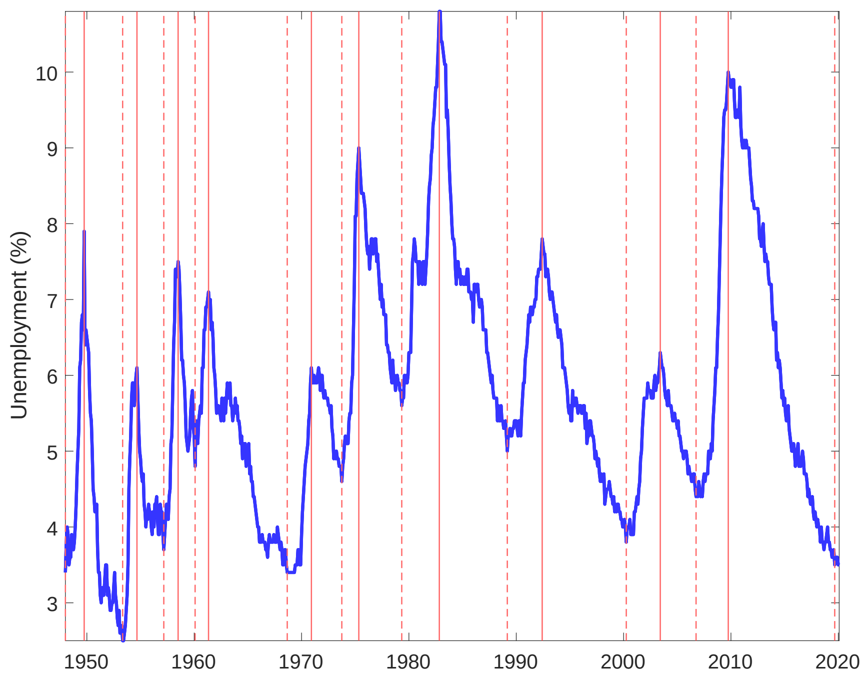


Figure 1: Peaks and Troughs in the Unemployment Rate

Note: The unemployment rate is plotted in blue. Business cycle peaks are denoted by dashed red vertical lines, while business cycle troughs are denoted by solid red vertical lines.

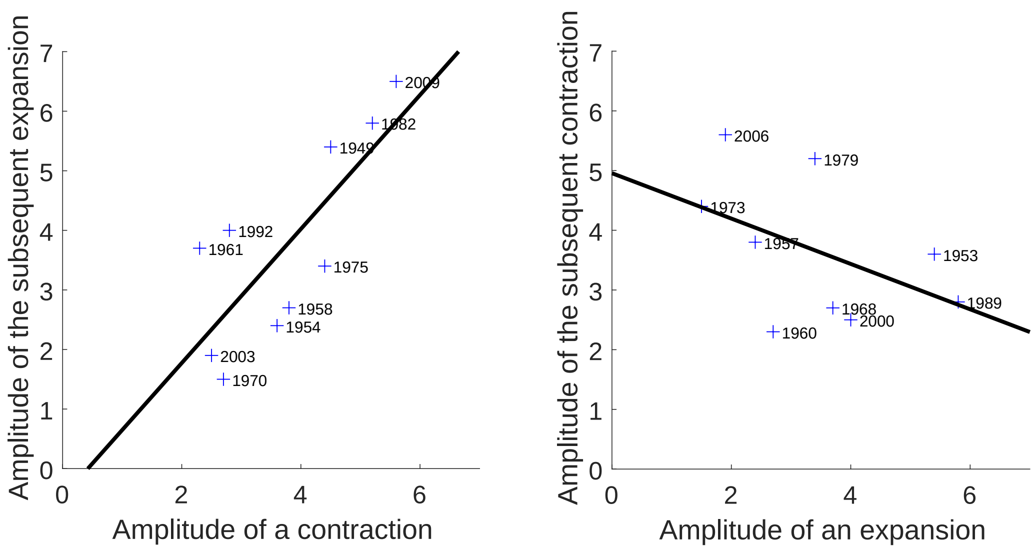
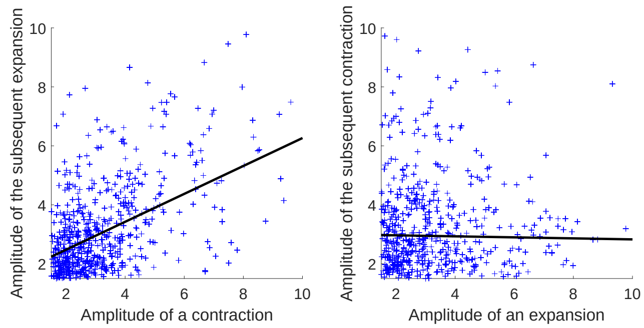


Figure 2: The Plucking Property of the Unemployment Rate

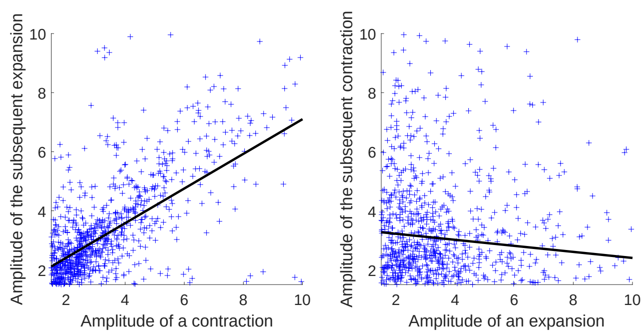
Note: The points in the left panel are labeled with the year the contraction in question ended and expansion in question began. The points in the right panel are labeled with the year the expansion in question ended and contraction in question began. OLS regression lines are plotted in each panel.

Figure 3: San Francisco Fed Wage Rigidity Meter

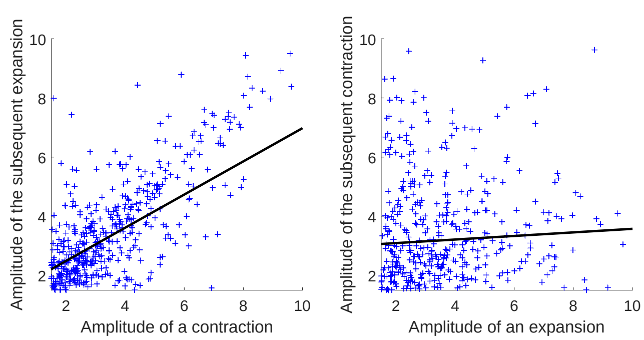
Note: The figure plots the share of wage freezes of all job-stayers (paid by the hour or not) with respect to the wage one year prior, with no correction for measurement errors. This series is constructed by the Federal Reserve Bank of San Francisco using data used from the Current Population Survey.



(a) Fujita-Ramey Model under Flexible Wages (Nash Bargaining)



(b) Fujita-Ramey Model under DNWR



(c) New Model under DNWR

Figure 4: Plucking Scatter Plots

Note: The figure displays the scatter plots associated to the plucking regressions for the Fujita-Ramey model under flexible wages (panel a), the Fujita-Ramey model under DNWR (panel b) and the new model of section 5 under DNWR. The plots feature all the expansion/contraction pairs obtained by pooling together 500 samples of 866 months. OLS regression lines are plotted in each panel.

Figure 5: Simulated Paths for the Unemployment Rate, Job-Finding Rate and Employment Exit Rate

Note: The figure plots sample path of 72 years (the same length as our empirical sample) for the unemployment rate u, the job-finding rate f and employment exit rate $\bar{s}$ in the Fujita-Ramey model with downward nominal wage rigidity of section 3. The dashed lines indicate the steady-state level of each variable.

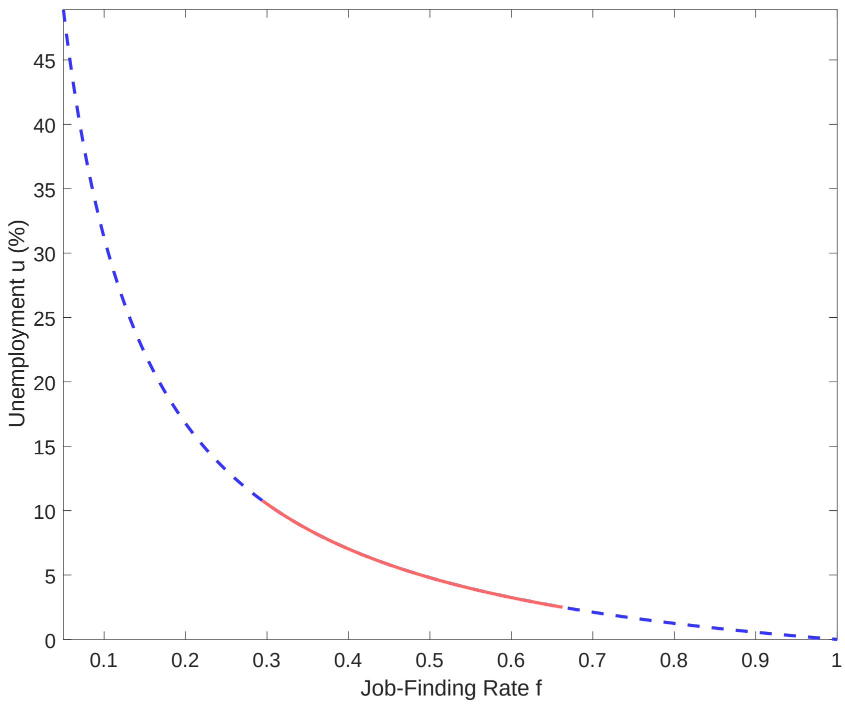


Figure 6: Steady-State Worker-Flow Relationship

Note: The figure plots the steady-state relationship between the job-finding rate and the unemployment rate implied by the worker-flow equation (15)-(16). In plain line the relationship is plotted over the range of unemployment rates observed in the US between 1948 and 2019: from 2.5% to 10.8%.

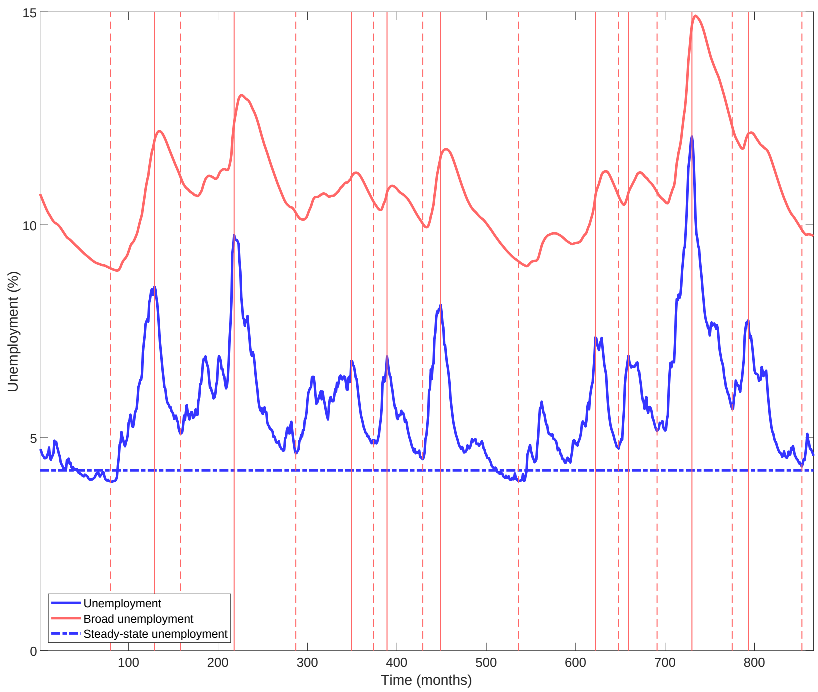


Figure 7: Simulated Paths for the Unemployment Rate: New Model

Note: The figure plots sample paths of 72 years (the same length as our empirical sample) for the unemployment rate and broad unemployment rate in our model with downward nominal wage rigidity, secure and insecure jobs, multiple sectors and an AR(2) aggregate productivity process. The solid vertical lines identify business-cycle troughs, while the dashed vertical lines identify peaks.

Table 1: Plucking Property of Unemployment and Speed Asymmetries

	β	R^2
Subsequent expansion on contraction	1.12	0.59
	(0.33)	
Subsequent contraction on expansion	-0.38	0.22
	(0.27)	
Speed of expansions (pp/year)	0.87	
Speed of contractions (pp/year)	1.89	
P-value for equal speed	0.002	
Duration of expansions (months)	59.1	
Duration of contractions (months)	26.9	

Note: The first row reports the coefficient in an OLS regression of the size of the subsequent expansion (percentage point fall in unemployment rate) on the size of a contraction (percentage point increase in unemployment rate). The second row reports the coefficent in an analogous regression of the size of the subsequent contraction on the size of an expansion. The speed of expansions and contractions in the third and fourth rows is measured in percentage points of unemployment per year.

Table 2: Calibration

β	$0.96^{1/12}$	
η	0.5	
c	0.30	
z	0.95	
δ	1.9%	
ρ^a	0.98	
ρ^x	0.98	
	Nash	DNWR
$\overline{\Pi}$	–	$1.02^{1/12}$
γ	0.57	0.43
σ_ε^a	st. $\sigma^a = 1.6\%$	st. $\sigma^a = 1.5\%$
σ_ε^x	st. $\sigma^x = 2.1\%$	st. $\sigma^x = 1.5\%$

Note: The abbreviation "st." stands for "such that."

Table 3: Simulation Results: Plucking Property, Speed, and Duration

	Data	Fujita-Ramey Model		New Model
		Nash	DNWR	DNWR
Subsequent expansion	1.12	0.44	0.63	0.68
on contraction, β		(0.40)	(0.26)	(0.42)
Subsequent contraction	-0.38	-0.06	-0.11	-0.08
on expansion, β		(0.49)	(0.32)	(0.67)
Subsequent expansion	0.59	0.19	0.45	0.53
on contraction, R^2		(0.22)	(0.25)	(0.31)
Subsequent contraction	0.22	0.07	0.04	0.13
on expansion, R^2		(0.19)	(0.11)	(0.29)
Speed of expansions	0.87	1.56	2.53	0.81
(pp / year)		(0.57)	(0.74)	(0.22)
Speed of contractions	1.89	1.66	4.93	1.44
(pp / year)		(0.69)	(1.49)	(0.35)
Duration of expansions	59.1	36.2	27.9	64.0
(months)		(12.4)	(7.0)	(16.0)
Duration of contractions	26.9	36.6	18.1	36.6
(months)		(11.7)	(6.1)	(12.6)

Note: The table compares data from the Fujita-Ramey model of section 3, both with Nash bargaining and with downward nominal wage rigidity (DNWR), and from the new model of section 5. The first (third) row reports the coefficient (R^2) in an OLS regression of the size of an expansion (percentage point fall in unemployment rate) on the size of the previous contraction (percentage point increase in unemployment rate). The second (fourth) row report the coefficient (R^2) in an analogous regression of the size of a contraction on the size of the previous expansion. The next two rows report the average speed of expansion and contractions, measured in percentage points of unemployment per year. The final two rows report the average duration of expansions and contractions, measured in months. For the models, the reported point estimate is the median value of the statistic over 5000 samples of 866 periods each (the length of our sample of real-world data). Expansions and contractions of more than 6.5 percentage points are excluded from the samples. Appendix **??** provides results including all expansions and contractions. The standard error reported in parentheses is the standard deviation of the estimates across the 5000 samples.

Table 4: Second Order Moments

X_t	u_t	f_t	$\bar{s}_t$
Data			
$\sigma(X)$	0.096	0.077	0.058
$corr(A_t, X_t)$	-0.460	0.369	-0.535
$corr(X_t, X_{t-1})$	0.926	0.803	0.631
Nash			
$\sigma(X)$	0.115	0.063	0.081
$corr(A_t, X_t)$	-0.964	0.965	-0.877
$corr(X_t, X_{t-1})$	0.814	0.767	0.510
DNWR			
$\sigma(X)$	0.131	0.072	0.116
$corr(A_t, X_t)$	-0.80	0.779	-0.525
$corr(X_t, X_{t-1})$	0.654	0.555	0.191

Note: The table reports the second order moments of the Fujita-Ramey model of section 3 under Nash bargaining and Downward Nominal Wage Rigidity. The moments are reported after data has been turned quarterly, logged, and HP-filtered. The data panel uses the data from Fujita and Ramey (2012) and is therefore identical to the results they report. For the model panels, the table reports medians of each statistics over our 5000 samples of 866 months each.

A Defining Expansions and Contractions

A.1 Defining Expansions and Contractions

Since our empirical analysis is based on the amplitude and speed of cyclical movements in unemployment, we define business cycle peaks in troughs in such a way that they line up exactly with peaks and troughs of the unemployment rate. This yields business cycle dates that are very similar to but not identical to those identified by the NBER Business Cycle Dating Committee (because the NBER Business Cycle Dating Committee uses a wide variety of cyclical indicators beyond unemployment to date turning points).

We develop a simple algorithm that defines business cycle peak and trough dates for the unemployment rate. The basic idea is to find local minima and maxima of the unemployment rate. However, we ignore small "blips" or "wiggles" in the unemployment rate and focus instead on delineating substantial swings in the unemployment rate in a similar manner as the peaks and troughs identified by the NBER Business Cycle Dating Committee.

Table A.1 presents the peak and trough dates we identify and compares them with the peak and trough dates identified by the NBER.

A.2 An Algorithm for Defining Expansions and Contractions

Let u_t denote the unemployment rate at time t. The algorithm begins by taking the first month of our sample as a candidate for a business cycle peak, cp. If, in all the following months until unemployment becomes X percentage points higher than u_{cp}, unemployment is higher than u_{cp}, we confirm that cp is a business cycle peak. If, instead, the unemployment rate falls below u_{cp} before it is confirmed as a peak, the month in which this happens becomes the new candidate peak. Once we have identified a peak, we switch to looking for a trough (in the analogous manner) and so on until we reach the end of the sample. Formally, starting with $t = 1$ the algorithm is:

1. Set $cp = t$ and set $t = t + 1$ (i.e., move to the next time period).

2. If $u_t < u_{cp}$ go back to step 1

3. If $u_{cp} \leqslant u_t < u_{cp} + X$ set $t = t + 1$ and go back to step 2

4. If $u_t \geqslant u_{cp} + X$ add cp to the set of peaks

5. Set $ct = t$ and set $t = t + 1$

6. If $u_t > u_{ct}$ go back to step 5

7. If $u_{ct} \geqslant u_t > u_{ct} - X$ set $t = t + 1$ and go back to step 6

8. If $u_t \leqslant u_{ct} - X$ add ct to the set of troughs, and go back to step 1

We set $X = 1.5$ percentage points. With this value, our algorithm generates the same set of expansions and contractions as the NBER Business Cycle Dating Committee with one exception: Our algorithm considers the 1979-1982 double-dip recession as a single contraction as opposed to two contractions interrupted by a brief and small expansion (unemployment decreased by 0.6 percentage points in 1980-1981). Values for X between 0.8 and 1.5 percentage points identify exactly the same cycles. Values of X larger than 1.5 drop the 1970-1973 expansion.

An advantage of our algorithm is that it does not impose a duration upon expansions and contractions but only a size X, in contrast to other algorithms based on turning points like the Bry and Boschan (1971) routine. Our algorithm can therefore also be used to define expansions and contractions in our model simulations, even for models that do not match the duration of expansions and contractions in the real-world data.

A.3 Peak and Trough Dates from 1948 to 2020

Table A.1 presents the peak and trough dates we identify. For comparison purposes, we also present the peak and trough dates identified by the NBER. We identify the same set of expansions and contractions as the NBER Business Cycle Dating Committee with one exception: we consider the 1979-1982 double-dip recession as a single contraction as opposed to two contractions interrupted by a brief and small expansion (unemployment decreased by 0.6 percentage points in 1980-1981). The exact timing of the NBER peaks and troughs do not line up exactly with ours for the reasons discussed above. However,

Table A.1: Business Cycle Peaks and Troughs

	Unemployment		NBER	
	Peak	Trough	Peak	Trough
1	[1/1948]	10/1949	11/1948	10/1949
2	5/1953	9/1954	7/1953	5/1954
3	3/1957	7/1958	8/1957	4/1958
4	2/1960	5/1961	4/1960	2/1961
5	9/1968	12/1970	12/1969	11/1970
6	10/1973	5/1975	11/1973	3/1975
7a	5/1979		1/1980	7/1980
7b		11/1982	7/1981	11/1982
8	3/1989	6/1992	7/1990	3/1991
9	4/2000	6/2003	3/2001	11/2001
10	10/2006	10/2009	12/2007	6/2009
11	9/2019		2/2020	

Note: Business cycle peaks and troughs defined solely based on the unemployment rate and, for comparison, business cycle peaks and troughs as defined by the Business Cycle Dating Committee of the National Bureau of Economic Research.

in most cases, our dates are quite similar to theirs. The NBER peaks tend to lag our peaks by a few months and the NBER troughs tend to precede our troughs by a few months. This implies that our estimate of the average duration of contractions is about one year longer than what results from the NBER's dating procedure. We identify September 2019 as a peak as opposed to February 2020 because the unemployment rate first hit 3.5% in September 2019. When several months are tied for the lowest unemployment rate at the end of an expansion, our algorithm picks the first of these months as the peak (and similarly for troughs). Table A.2 lists the duration of all expansions and contractions over our sample period.

B Solution Method

B.1 Normalization of μ

Recall that the matching function is Cobb-Douglas. The vacancy-filling rate is therefore $q_t = \mu\theta^{-\eta}$. Furthermore, the job finding rate is $f_t = \theta_t q(\theta_t)$. Combining these equations

Table A.2: The Duration of Expansions and Contractions

	Dates		Length in Months	
	Peak	Trough	Expansion	Contraction
1	[1/1948]	10/1949		21
2	5/1953	9/1954	43	16
3	3/1957	7/1958	30	16
4	2/1960	5/1961	19	15
5	9/1968	12/1970	88	27
6	10/1973	5/1975	34	19
7	5/1979	11/1982	48	42
8	3/1989	6/1992	76	39
9	4/2000	6/2003	94	38
10	10/2006	10/2009	40	36
11	9/2019		119	
Mean			59.1	26.9

allows us to express the vacancy-filling rate as a function of the job-finding rate:

$$q_t = \mu^{\frac{1}{1-\eta}} (f_t)^{\frac{-\eta}{1-\eta}}. \tag{B.1}$$

We can now see that there is a one-to-one mapping between the cost of hiring a worker $C_t \equiv c/q_t$ and the job-finding rate f_t:

$$C_t \equiv \frac{c}{q_t} = \left(c\mu^{\frac{-1}{1-\eta}} \right) (f_t)^{\frac{\eta}{1-\eta}}. \tag{B.2}$$

This mapping can be used to write the equilibrium conditions of the model in terms of either the cost of hiring a worker or the job-finding rate, without reference to the other (and without reference to labor market tightness). When the model is written in this way (e.g., in terms of the cost of hiring a worker), the parameters c and μ only enter the model though the composite term $c\mu^{\frac{-1}{1-\eta}}$. This implies that we can normalize either c or μ without loss of generality. We choose to normalize $\mu = 1$. Intuitively, only the cost of hiring a worker matters to a firm. It is immaterial to the firm whether this cost consists of posting few vacancies that fill with a high probability but are expensive to post, or of posting many vacancies that fill with a low probability but are inexpensive to post.

B.2 Solving for the Policy Functions

To solve for the policy function under Nash-bargaining, we follow the solution method of Fujita and Ramey (2012) to solve for the functions $J^{Nash}(A, x)$ and $q(A)$. The state-space consists of the two exogenous states A and x. We discretize the AR(1) process for A_t using the Rouwenhorst (1995) method with 11 grid points, and the AR(1) process for x_t using the Tauchen (1986) method with 201 grid points. Combining equations (10) and (11), we can solve for the functions J^{Nash} and $1/q$ by iteration on the policy functions. Specifically, given guesses on the functions J and $1/q$ (and therefore f), we use these guesses to calculate the expected terms on the RHS of equation (11) and update J^{Nash}. We then update $1/q$ using equation (10). We iterate until convergence.

Under DNWR, we first solve for the Nash wage as a function of the state (A, x) by solving the model under Nash-bargaining. This gives the Nash wage under the assumption that wages will be flexible at all future dates, including wages in new matches that are relevant to determine the outside option of workers. Under DNWR, the value function J depends on the two exogenous states x and A, and the new endogenous state of the lagged real wage w_{-1}, $J(A, x, w_{-1})$. Under the assumption on the Nash-bargained wage, J is independent of the state w_{-1}^{new}, so that for numerical considerations, the state-space is only three-dimensional.

The recursion on J is the same as (4), up to the new dependence of the value function on the new state w_{-1}:

$$J(A, x, w_{-1}) = \max\{J^c(A, x, w_{-1}), 0\}, \tag{B.3}$$

where J^c is the value if the match is continued, which solves the recursion

$$J^c(A, x, w_{-1}) = xA - w(A, x, w_{-1}) + \beta(1 - \delta)E\left(J(A', x', w)\right), \tag{B.4}$$

where the real wage w is given by equation (13). Equations (B.3)-(B.4) allow to solve for J by iteration. We again use 11 points on the A dimension, 201 on the x dimension, and 401 grid points on the new endogenous dimension w_{-1}. When iterating on equation (B.4), calculating the expected term on the RHS requires to evaluate the value function J at values of the endogenous state w that are not on the grid. We rely on linear interpolation to do so.

Once J is solved for, we can obtain $1/q(A, w_{-1}^{new})$ from J and the free-entry condition (10) which now depends on the new state w_{-1}^{new},

$$J(A, x^{hire}, w_{-1}^{new}) = \frac{c}{q(A, w_{-1}^{new})}. \tag{B.5}$$

B.3 Calculating Separation

Because matches can be endogenously terminated, the destruction rate s_t depends on the cross-sectional distribution of employment across matches' states. Calculating the destruction rate in simulations of the model therefore requires us to keep track of the distribution of employment across matches'states. Under Nash-bargaining, matches' states reduce to match productivity x_t. We follow the method in Fujita and Ramey (2012) to keep track of the distribution of employment across x_t and calculate the destruction rate, only adapting it to any Markovian process for x so that it can accommodate our AR(1) assumption (2) on x_t.

Define $n_t(x)$ the number of employed workers at productivity x, and n_t the vector of $n_t(x)$. (Note that our n_t is the density of the distribution of employment, while Fujita-Ramey's e_t on p.75-77 is the CDF.) We therefore have:

$$u_t = 1 - \sum_x n_t(x). \tag{B.6}$$

Define $n_t^0(x)$ the number of workers employed at productivity x at the beginning of period t, after shocks and exogenous separation have occurred, but before endogenous separation has occurred. Denote n_t^0 the vector of $n_t^0(x)$. We have:

$$n_t^0 = (1 - \delta)(T^x)' n_{t-1} \tag{B.7}$$

where T^x is the transition matrix of the Markovian process of x. Define $fired_t$ the number of workers fired at t. It solves:

$$fired_t = \sum_x n_t^0(x) 1_{J_t(x)=0}. \tag{B.8}$$

The job-destruction rate s_t is given by:

$$s_t = \delta + \frac{fired_t}{1 - u_{t-1}}. \tag{B.9}$$

The new distribution of employment at t solves the recursion:

$$n_t(x) = n_t^0(x)1_{J_t(x)>0} + f_t(s_t + (1-s_t)u_{t-1})1_{x=x^{hire}}. \tag{B.10}$$

Under DNWR, calculating the destruction rate requires to keep track of the distribution of employment along both match productivity x and wages w. We do so in the following way. Let $m_{t-1}(x^*, w^*) = P(x_{t-1} = x^*, \frac{w_{t-1}}{\Pi} \leqslant w^*)$ be the number of matches at $t-1$ with idiosyncratic productivity $x_{t-1} = x^*$ and a real wage less than Πw^*. Considering the number of real wages below Πw^* instead of below w^* is for convenience: This way it gives the number of matches with real wages below w^* at the beginning of period t, after inflation from $t-1$ to t has eroded lagged real wages. Note that $m_{t-1}(x^*, \infty)$ is the number of firms with idiosyncratic productivity $x_{t-1} = x^*$ at $t-1$, and $\sum_{x^*} m_{t-1}(x^*, \infty)$ is employment at $t-1$.

Denote $m_t^0(x^*, w^*) = P(x_t = x^*, \frac{w_{t-1}}{\Pi} \leqslant w^*)$ the number of matches with idiosyncratic productivity $x_t = x^*$ and inherited real wage less than w^* at the beginning of period t, after match-specific productivity shocks and exogenous separation shocks have hit but before any wage-adjustment. It is given by

$$m_t^0 = (1-\delta)T_x' m_{t-1} \tag{B.11}$$

where T_x is again the transition matrix of the Markovian process of x.

We now keep track of how wage adjustments change the distribution of wages under DNWR. Denote $m_t^1(x^*, w^*)$ the number of wages with $x_t = x^*$ and inherited real wage less than w^* after wage adjustments. It is the same as m_t^0, except that all wages below $w_t^{Nash}(x^*)$ are reset to $w_t^{Nash}(x^*)$, i.e.

$$m_t^1(x^*, w^*) = 0 \text{ for all } w^* \leqslant w_t^{Nash}(x^*). \tag{B.12}$$

We now calculate the number of endogenously terminated matches, and keep track of how it affects the distribution of wages. Denote $w^{thresh}(x^*)$ the threshold on wages above which matches with productivity x^* are terminated. It is defined as the lowest wage w such that $J(x^*, w) = 0$. The number $fired_t(x^*)$ of matches with productivity x^* that are terminated is $m_t^1(x^*, \infty) - m_t^1(x^*, w^{thresh}(x^*))$. Knowing the number of exogenously and endogeneously separated matches we can calculate the separation rate as:

$$s_t = \frac{\sum_x fired_t(x)}{N_{t-1}} + \delta. \tag{B.13}$$

Denote $m_t^2(x^*, w^*)$ the number of wages with $x_t = x^*$ and inherited real wage less than w^* of wages after endogenous separation. It is the same as m_t^1, except that it no longer includes wages above $w^{thresh}(x^*)$, i.e.

$$m_t^2(x^*, w^*) = m_t^1(x^*, w^{thresh}(x^*)) \text{ for all } w^* \geqslant w^{thresh}(x^*). \tag{B.14}$$

We now keep track of how new hires affect the distribution of wages. Denote $m_t^3(x^*, w^*)$ the number of wages with $x_t = x^*$ and inherited real wage less than w^* after hiring. It is the same as m_t^2, except that it adds the number of new hires at productivity x^{hire} and hiring wage w_t^{hire}, i.e.

$$m_t^3(x^{hire}, w^*) = m_t^2(x^{hire}, w^*) + f_t(1 - (1 - s_t)N_{t-1}) \text{for all} w^* \geqslant w_t^{hire}(x^{hire}). \tag{B.15}$$

This is the distribution of effective real wage in period t. Employment at t is therefore given by $\sum_{x^*} m_t^3(x^*, \infty)$.

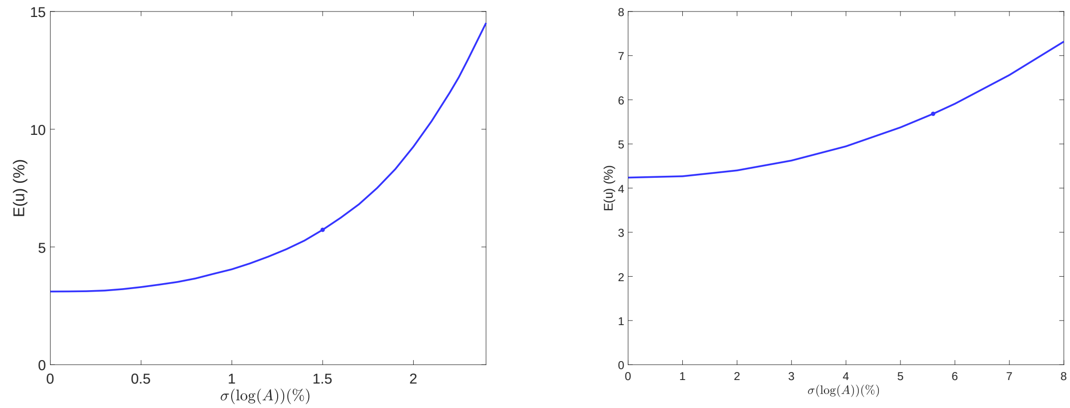
Finally, we keep track of the eroding effect of inflation and growth from t to $t+1$ to get $m_t(x^*, w^*)$ and be able to start the whole process in period $t+1$. We have that

$$m_t(x^*, w^*) = m_t^3(x^*, \Pi \times w^*). \tag{B.16}$$

We calculate it by linear interpolation.

C The Volatility of Aggregate Shocks and the Average Level of Unemployment

Figure C.1 plots the average level of the unemployment rate in our plucking model as a function of the volatility of aggregate shocks. Both models have the property that average unemployment increases with the volatility of the aggregate shocks, from a steady-state level of 2.9% in the model of section 3 and 4.2% in the model of section 5. Average unemployment increases less steeply with the volatility of shocks in the new model of section 5 because decreasing returns to scale make firms able to withstand larger shocks under DNWR without being willing to lay off all their workers, as explained in Appendix F.1.



(a) Fujita Ramey Model of Section 3 (b) New Model of Section section 5

Figure C.1: Average Unemployment and the Volatility of Aggregate Shocks

Note: The figure gives the average rate of unemployment as a function on the standard deviation of aggregate shocks in the DNWR model of section 3 (panel a) and the the DNWR model of section 5 (panel b).

D Appendix for the Model in Section 5

D.1 Labor Demand

The employment decision of the firm is

$$\Omega_t^i(M_t^i) = \max_{H_t^i}\{A_t^i F(M_t^i + H_t^i) - w_t^i(M_t^i + H_t^i) - \frac{A_t^i c}{q_t^i} H_t^i 1_{H_t^i \geqslant 0} + \beta E_t(\Omega_{t+1}^i(M_{t+1}^i)\}, \quad \text{(D.1)}$$

subject to the law-of-motion of its workforce $M_{t+1}^i(N_t^i)$ given by equations (17)-(18).

The envelope condition is

$$J_t^{i*}(M_t^i) = A_t^i F'(N_t^{i*}) - w_t^i + \beta(1-\delta) E_t\left(J_{t+1}^{i*}(M_{t+1}^i(N_t^{i*}))\right). \quad \text{(D.2)}$$

where we define $J_t^{i*}(M_t^i) = \frac{\partial \Omega_t^i}{\partial M_t^i}(M_t^i)$, the equilibrium marginal value to the firm of a securely attached worker inherited from the previous period.

The first-order conditions give:

- If $H_t^{i*} > 0$ (i.e., if the firm hires):

$$\frac{A_t^i c}{q_t^i} = A_t^i F'(N_t^{i*}) - w_t^i + \beta(1-d) E_t\left(J_{t+1}^{i*}(M_{t+1}^i(N_t^{i*}))\right). \quad \text{(D.3)}$$

This happens when $A_t^i F'(M_t^i) - w_t^i + \beta(1-d) E_t\left(J_{t+1}^{i*}(M_{t+1}^i(M_t^i))\right) > \frac{A_t^i c}{q_t^i}$.

- If $H_t^{i*} < 0$ (i.e., if the firm fires):

$$0 = A_t^i F'(N_t^{i*}) - w_t^i + \beta(1-\delta)\left(J_{t+1}^*(M_{t+1}^i(N_t^{i*}))\right). \tag{D.4}$$

  This happens when $A_t^i F'(M_t^i) - w_t^i + \beta(1-\delta)E_t\left(J_{t+1}^{i*}(M_{t+1}^i(M_t^i))\right) < 0$.

- When neither condition is satisfied, i.e. when

$$A_t^i F'(M_t^i) - w_t^i + \beta(1-d)E_t\left(J_{t+1}^{i*}(M_{t+1}^i(M_t^i))\right) \leqslant \frac{A_t^i c}{q_t^i}, \tag{D.5}$$

$$A_t^i F'(M_t^i) - w_t^i + \beta(1-\delta)E_t\left(J_{t+1}^{i*}(M_{t+1}^i(M_t^i))\right) \geqslant 0, \tag{D.6}$$

  then the firm freezes hires $H_t^{i*} = 0$.

To simplify and interpret this labor demand schedule, equation (B.3) defines the marginal value to the firm of a worker already attached to the firm and equation (22) defines the marginal value to the firm of a newly-hired worker. The labor-demand schedule can then be rewritten as in equations (23)-(25).

In equilibrium, because all firms in sector i are identical, if firms are freezing employment then $A_t^i c/q_t^i = 0$. Therefore, the aggregate labor demand schedule in sector i is:

- If $J^{i,new}(M_t^i) > 0$, then firms hire up to:

$$J^{i,new}(N_t^{i*}) = \frac{A_t^i c}{q(N_t^{i*})}. \tag{D.7}$$

- If $J^i(M_t^i) < 0$ then firms fire up to:

$$J^i(N_t^{i*}) = 0. \tag{D.8}$$

- If $J^i(M_t^i) \geqslant 0 \geqslant J^{i,new}(M_t^i)$ firms freeze employment:

$$N_t^{i*} = M_t^i. \tag{D.9}$$

D.2 Worker Flow Accounting

From economy-wide worker flows, we can define the economy-wide rate of job destruction s_t, the economy-wide rate of inflow from employment into unemployment $\bar{s}_t$, and the economy-wide rate of outflow from unemployment to employment f_t (equal to the economy-wide job-finding rate). In sector i at time t, there are $N^i_{0,t} = \min(N^i_t, M^i_t)$ workers who work in the same job as at time $t-1$. The number of workers who find a new job at the beginning of time t in sector i is therefore $N^i_t - N^i_{0,t}$. In the overall economy it is $N_t - N_{0,t}$, where the aggregates N_t and $N_{0,t}$ are defined as sums across sectors, $N_t = \int_i N^i_t di$ and $N_{0,t} = \int_i N^i_{0,t} di$. The economy-wide job-finding rate is therefore

$$f_t = \frac{N_t - N_{0,t}}{1 - N_{0,t}}. \tag{D.10}$$

The number of workers who separate from their jobs between $t-1$ and t in sector i is $N^i_{t-1} - N^i_{0,t}$. In the total economy it is $N_{t-1} - N_{0,t}$. The economy-wide job destruction rate is therefore:

$$s_t = \frac{N_{t-1} - N_{0,t}}{N_{t-1}}. \tag{D.11}$$

Our assumption that a worker who separates from his job at the end of period $t-1$ has a chance f_t of finding a new job at the beginning of period t implies that the job-destruction rate s_t is not equal to the rate of inflow from employment into unemployment $\bar{s}_t$, or employment exit rate. Among the workers who separates from their jobs at the end of $t-1$, the fraction f_t that starts a new job at the beginning of t does not transition from employment to unemployment but from job to job. Only the fraction $1 - f_t$ transitions to unemployment. The economy-wide rate of inflow from employment into unemployment is therefore:

$$\bar{s}_t = (1 - f_t)s_t. \tag{D.12}$$

From combining equations (D.10), (D.11), and (D.12), the law of motion of economy-wide unemployment $U_t \equiv 1 - N_t$ is still given by equation (16).

D.3 The Nash-Bargained Wage

We solve for the Nash-bargaining equilibrium in order to obtain the Nash wage. We take the Nash wage to be the one that prevails when wages are Nash-bargained between the firm and securely attached workers, both today and in all subsequent periods. For workers, we get from equations (26)-(28) that the values of having a secure job relative to being unemployed $V_t^i \equiv W_t^i - U_t^i$ and the value of having a short-term job relative to being unemployed $V_t^{i,new} \equiv W_t^{i,new} - U_t^i$ solve:

$$V_t^{i,new} = w_t^i - A_t^i z + \beta E_t\left((1 - d_{t+1}^i)\left(V_{t+1}^i - f_{t+1}^i V_{t+1}^{i,new}\right)\right), \tag{D.13}$$

$$V_t^i = w_t^i - A_t^i z + \beta E_t\left((1 - \delta_{t+1}^i)\left(V_{t+1}^i - f_{t+1}^i V_{t+1}^{i,new}\right)\right), \tag{D.14}$$

where $z = b + \zeta$ is the flow value of unemployment in terms of both unemployment benefits and more leisure.

For firms, we get from equations (B.3)-(22) that:

$$J_t^{i,new} = A_t^i F'(N_t^i) - w_t^i + \beta E_t((1-d)J_{t+1}^i), \tag{D.15}$$

$$J_t^i = A_t^i F'(N_t^i) - w_t^i + \beta E_t((1-\delta)J_{t+1}^i). \tag{D.16}$$

Summing up to get the total surpluses of a secure job $S_t^i \equiv J_t^i + V_t^i$ and of a temporary job $S_t^{i,new} \equiv J_t^{i,new} + V_t^{i,new}$:[24]

$$S_t^{i,new} = A_t^i(F'(N_t^i) - z) + \beta E_t((1-d)(S_{t+1}^i - f_{t+1}^i V_{t+1}^{i,new})), \tag{D.17}$$

$$S_t^i = A_t^i(F'(N_t^i) - z) + \beta E_t((1-\delta)(S_{t+1}^i - f_{t+1}^i V_{t+1}^{i,new})). \tag{D.18}$$

Nash-bargaining between the firm and long-term workers implies that $J_t^i = (1-\gamma)S_t^i$ and $J_{t+1}^i = (1-\gamma)S_{t+1}^i$. Combined with equation (D.18) this implies:

$$J_{Nash,t}^i = (1-\gamma)A_t^i(F'(N_{Nash,t}^i) - z) + \beta E_t\left((1-\delta)J_{Nash,t+1}^i - (1-\gamma)(1-\delta)f_{Nash,t+1}^i V_{Nash,t+1}^{i,new})\right). \tag{D.19}$$

Combining equations (D.16) and (D.19) to eliminate $J_{Nash,t}^i$ gives the expression of the wage given in equation (30).

Injection of the expression for the Nash wage—equation (30)—into equation (D.15) gives:

$$J_{Nash,t}^{i,new} = (1-\gamma)A_t^i(F'(N_{Nash,t}^i) - z) + \beta E_t\left((1-d)J_{Nash,t+1}^i - (1-\gamma)(1-\delta)f_{Nash,t+1}^i V_{Nash,t+1}^{i,new}\right). \tag{D.20}$$

[24] We use the fact that whenever $\delta_{t+1}^i \neq \delta$ and $d_{t+1}^i \neq d$ there are endogenous layoffs so $S_{t+1}^i = J_{t+1}^i = 0$ and $f_{t+1}^i = 0$, so we can replace δ_{t+1}^i with δ and d_{t+1}^i with d.

Injecting the expression for the Nash wage (30) into equation (D.13) gives:

$$V^{i,new}_{Nash,t} = \gamma A^i_t(F'(N^i_{Nash,t}) - z) + \beta E_t\Bigg((1-d)\frac{\gamma}{1-\gamma}J^i_{Nash,t+1} - \Bigg(\gamma(1-\delta)+\delta-d\Bigg)f^i_{Nash,t+1}V^{i,new}_{Nash,t+1}\Bigg). \tag{D.21}$$

An equilibrium under Nash bargaining is then processes for N^i_t, J^i_t, $J^{i,new}_t$ and $V^{i,new}_t$ that solve (D.19), (D.20) and (D.21), and the labor demand schedule (23)-(25), and where f^i_t is the function of N^i_t given by equation (29). This can be calculated recursively as follows. Given J^i_{t+1}, f^i_{t+1} and $V^{i,new}_{t+1}$ the expressions (D.19) and (D.20) solve for employment N^i_t when intersected with the labor demand schedule (23)-(25), and so for f^i_t. Equations (D.19) and (D.21) then allow us to calculate J^i_t and $V^{i,new}_t$. The Nash wage can then be recovered, e.g., by equation (D.16).

D.4 Steady-State

In a non-stochastic steady-state equilibrium with with $A_t = 1$ and $\Delta\log(Z^i_t) = 0$, equations (B.3) and (22) reduce, for $\tilde{J} = J^i/A^i$, $\tilde{J}^{new} = J^{i,new}/A^i$, and $\tilde{w} = w^i/A^i$, to

$$\tilde{J}(N,\tilde{w}) = \frac{1}{1-\beta e^g(1-\delta)}(F'(N)-\tilde{w}), \tag{D.22}$$

$$\tilde{J}^{new}(N,\tilde{w}) = (1-\beta e^g(d-\delta))\tilde{J}(N,\tilde{w}). \tag{D.23}$$

In steady-state, firms hire workers such that the law of motion of the stock of attached workers is given by equation (17). Combined with the definition of hires—$N^i_t = M^i_t + H^i_t$—this gives the steady-state relationship between M and N

$$M(N) = \frac{1-d}{1-d+\delta}N. \tag{D.24}$$

Combined with equation (29), this gives $f(N)$ as a function of N alone, and through equation (B.1) it gives the hiring costs $c/q(N)$ as a function of N alone.

In steady-state, firms hire workers such that labor demand is given by equation (23). Combined with equation (D.23), this gives

$$\frac{c}{q(N)} = (1-\beta e^g(d-\delta))\tilde{J}(N,\tilde{w}). \tag{D.25}$$

To calibrate the model, we assume that hiring costs $\frac{c}{q}$ are 10% of $\tilde{J}$, and obtain the steady-state wage as the only one consistent with equations (D.25)-(D.22) and a level of employment $N = 1 - U = 1 - 5.7\%$.

In steady-state, the wage is equal to the Nash-bargained wage. (Under downward nominal wage rigidity, we consider cases where $\log(\bar{\Pi}) + g \geqslant 0$ to make sure this can be the case.) We back out the value of a workers' bargaining power γ as the only value that make the steady-state Nash wage equal to the steady-state wage we have obtained.

D.5 Solution Method

The hiring decision of a firm in sector i is a function of four or five state variables depending on the process for aggregate productivity. These are: aggregate productivity A (and lagged aggregate productivity A_{-1} if aggregate productivity follows an AR(2)), idiosyncratic productivity growth ΔZ^i, the lagged wage w^i_{-1}, and the stock of workers inherited from the previous period, M^i. We have introduced sectoral heterogeneity in such a way that a firm does not need to forecast any endogenous aggregate variable in order to decide how many workers to hire. Therefore, we do not need to keep track of the endogenous aggregate state of the economy in order to solve for the hiring decision of a firm.

A solution to the model can be described as a pair of policy functions for N^i and $\tilde{J}^i = J^i/A^i$ over this (four or) five dimensional state space. We make the following change of variables. First, we define the AR(1) process:

$$\eta_t \equiv (1 - \rho^a_2 L)^{-1} \varepsilon^a_t, \tag{D.26}$$

so that:

$$\log(A_{t+1}) = \rho^a_1 \log(A_t) + \eta_{t+1}. \tag{D.27}$$

Second, we define $\zeta^i_{-1} \equiv w^i_{-1}/A^i$ the ratio of the lagged wage to current total productivity. Given these definitions, the five-dimensional state can be parameterized as $(\log(A), \eta, \Delta\log(Z^i), \zeta^i_{-1}, M^i)$.

We form a discrete grid of the state-space with 11 points along the exogenous dimensions $(\log(A), \eta, \Delta\log(Z^i))$ and 21 along the endogenous dimensions (ζ^i_{-1}, M^i). We approximate the AR(1) processes for the exogenous variables η_t and $\Delta\log(Z^i_t)$ using the

Rouwenhorst (1995) discretization method. The Rouwenhorst method is more accurate than the Tauchen (1986) method for persistence processes. Petrosky-Nadeau and Zhang (2017) emphasize this point in the context of the DMP model. Our approach to adapting the Rouwenhorst method to an AR(2) process is close to the one used by Galindev and Lkhagvasuren (2010), who consider the more general case of a VAR(1).

We solve for the policy functions at each point on the grid by policy function iteration. Specifically, we start from an initial guess for the policy functions for N^i and $\tilde{J}^i$. At each point of the grid, we then use this guess to calculate the expectation term in equation (B.3)-(22). In calculating the expectation term, we need to evaluate the policy function for $\tilde{J}^i$ at points that are not on the grid. We do so through linear interpolation. Having calculated values for the expectation term, we compute the values of N^i and $\tilde{J}^i$ that solve equations (23)-(25), (D.10), and (B.1), and store the resulting values in new policy functions. Once this has been done for all points on the grid, we have a new set of guesses for the policy functions. We repeat this process until the policy functions converge. In simulating the heterogeneous model, we assume 1000 sectors.

In solving for the model under wage-rigidity, we need to first solve for the Nash wage for each point of the grid. We do so by first solving the model under the assumption of Nash-bargaining using the same iterative method. The policy function under Nash bargaining does not depend on ζ^i_{-1}.

E Calibration of the Model in Section 5

Table E.3 provides a summary of the calibration we use for the new model presented in section 5. In appendix F, we present a few variants on this model to understand which features drive which results. The bottom half of Table E.3 breaks out a case with symmetric real wage rigidity along side the baseline assumption of DNWR for certain parameters.

We calibrate the model to a monthly frequency. We set the discount factor β to correspond to an annual interest rate of 4%. We set the growth rate of productivity g to 2.3% annually, the average growth of US labor productivity from 1948 to 2018. We set the rate of inflation to 2% per year.

We assume a constant-elasticity production function $F(N) = N^{\alpha}$ and set $\alpha = 2/3$. We assume a Cobb-Douglas matching function $q(\theta) = \mu\theta^{-\eta}$ and set the elasticity of the matching function to $\eta = 0.5$, in the middle of the range reported in Petrongolo and Pissarides (2001)'s survey. We calibrate the flow value of unemployment z to 70% of the wage (specifically, 70% of the labor share α) following Hall and Milgrom (2008): 25% through unemployment benefits b and 45% through less foregone leisure ζ.

We calibrate the monthly separation rates δ and d so that, based on steady-state relationships, the average separation rate is $s = 4.95\%$. This is the separation rate consistent in steady-state with an unemployment rate of 5.7% and an average job-finding rate f of 45%, as estimated by Shimer (2012) based on CPS data, since in steady-state $1/s = (1/u - 1)(1/f - 1)$. The average separation rate in steady-state satisfies $s = \frac{\delta}{\delta+1-d}$. We set $\delta = 0.2\%$, implying $d = 96.2\%$. This calibration allows us to generate a slow rebuilding of long-term firm-worker relationships during expansions, even though insecure short-term jobs last only a month.

The parameter μ is still redundant and normalized to 1 (See Appendix B.1 for further discussion of this point.) We set c so that the cost of hiring a worker $\frac{c}{q} = J$ is 10% of the monthly steady-state wage $\bar{w}$ in a steady state with $u = 5.7\%$, in line with what Silva and Toledo (2009) report based on the Employer Opportunity Pilot Project survey in the US. This yields $c = 6.2 \times 10^{-3}$.

We estimate the two roots of the aggregate productivity process from the BLS quarterly series on labor productivity. We first apply a three-period moving-average filter to smooth out high-frequency variations including measurement errors. We then detrend the series by removing a quadratic trend. The quadratic trend allows us to capture the productivity slowdown from the 1970s onward. We then estimate an AR(2) on the cycle component of labor productivity. After converting the roots to a monthly frequency, this yields $\rho_1^a = 0.985$ and $\rho_2^a = 0.88$.[25] We calibrate the volatility of the aggregate shocks to match the volatility of the unemployment rate—see below.

We calibrate the persistence of the idiosyncratic productivity process based on KLEMS

[25] The autoregressive coefficients of the AR(2) estimation are $\phi_1^a = 1.64$ and $\phi_2^a = -0.65$. They are related to the roots ρ_1^a and ρ_2^a through the equation $I - \phi_1^a L - \phi_2^a L^2 = (I - \rho_1^a L)(I - \rho_2^a L)$. This gives roots 0.96 and 0.68 on a quarterly frequency. The monthly roots are the quarterly roots raised to the power $1/3$.

Table E.3: Calibration Full Model

β	$0.96^{1/12}$	
α	$2/3$	
η	0.5	
z	0.47	
δ	0.2%	
d	96.2%	
c	6.2×10^{-3}	
ρ_a^1	0.985	
ρ_a^2	0.88	
$\rho_{\Delta z}$	0.96	
$\sigma_{\Delta\varepsilon}^z$	7×10^{-4}	
	SRWR	DNWR
g	$0.023/12$	$0.023/12$
$\overline{\Pi}$	–	$1.02^{1/12}$
ρ	0.9	–
γ	0.99	0.97
σ_ε^a	st. $\sigma^a = 3.4\%$	st. $\sigma^a = 5.6\%$

Note: "DRL" stands for "Decreasing Returns to Labor". The abbreviation "st." stands for "such that."

annual data on US sectoral productivity from 1947 to 2010 (Jorgenson, Ho, and Samuels, 2012). The KLEMS dataset provides labor productivity series (value added per hour) for 31 sectors. We take $\log(Z_t^i)$ to be the log difference between the sectoral labor productivity series and the BLS series for aggregate labor productivity. Here again, we first apply a three-period moving-average filter to the level of these series to smooth out high-frequency variations in $\log(Z_t^i)$. We then first-difference the resulting series and estimate AR(1) models for $\Delta \log(Z_t^i)$ in each sector. The average estimated autoregressive root across sectors is $\rho_{\Delta z} = 0.62$ at an annual frequency. We therefore calibrate $\rho_{\Delta z} = 0.62^{\frac{1}{12}} = 0.96$ in our monthly calibration. We calibrate the volatility of idiosyncratic productivity growth $\sigma_\varepsilon^{\Delta z}$ to roughly match the average of the fraction of constrained firms in the data as measured by the San Francisco Fed's Wage Rigidity Meter: 13% (see Figure 3). The value we use is $\sigma_\varepsilon^{\Delta z} = 7 \times 10^{-4}$.

We are left with calibrating the bargaining power of workers γ and the standard deviation of innovations to the aggregate productivity process σ_ε^a. We calibrate them so that the average and standard deviation of the unemployment rate in simulations of the model match their values in the data (5.7% and 1.6 percentage points). Like for the model of section 3, we choose to match the standard deviation of unemployment exactly (as opposed to calibrating to the standard deviation of productivity in the data) so that we can apply our definition of expansions and contractions to our simulated samples in the same way as we do to the real world data. The resulting bargaining power of workers is $\gamma = 0.97$. The resulting value for the standard deviation of the productivity process is $\sigma^a = 0.056$.

F Decomposing Plucking in the Model of Section 5

The new model presented in section 5 has several non-standard features in addition to DNWR: decreasing returns to labor, insecure short-term jobs, and AR(2) aggregate shocks. Here we shed light on the role of these different features by presenting results for simplified versions of this model that more closely approximate the model used in section 3. In particular, we present results for a version of our new model in which we eliminate the insecure short-term jobs (by setting $d = \delta$) and AR(2) aggregate shocks (replacing them with AR(1) shocks (1)). In this version we also eliminate sectoral shocks and trend growth. This version of our new model then only differs from the model in section 3 by having decreasing returns to labor, and not having match-specific shocks. The calibration for this simplified model is presented in Table F.4.[26]

We compare both the full model of section 5 and this simpler version to the same models under symmetric wage rigidity instead of DNWR. Since we calibrate the model away from the Hagedorn-Manovskii calibration, we cannot assume Nash bargaining as a benchmark, as it would fail to generate fluctuations in unemployment (Shimer, 2005). We therefore instead assume symmetric real wage rigidity. Following Shimer (2010), we

[26] We set $\delta = d = 4.95\%$ to match the steady state separation rate. We set $c = 0.15$ so that the cost of hiring a worker is 10% of the monthly steady-state wage. We set the auto-regressive root of the aggregate productivity process ρ_1^a to 0.98 following Shimer (2010). The parameters γ and σ_ε^a are set to match the average and standard deviation of the unemployment rate as in the full model.

Table F.4: Calibration Simple DRL Model

β	$0.96^{1/12}$	
α	$2/3$	
η	0.5	
z	0.47	
δ	4.95%	
d	4.95%	
c	0.15	
ρ_a	0.98	
	SRWR	DNWR
g	$0.023/12$	0
$\overline{\Pi}$	–	$1.02^{1/12}$
ρ	0.9	–
γ	0.87	0.80
σ_ε^a	st. $\sigma^a = 2.0\%$	st. $\sigma^a = 3.1\%$

Note: "DRL" stands for "Decreasing Returns to Labor". The abbreviation "st." stands for "such that."

assume the real wage is a weighted average of the past real wage and the present flexible-wage target given by equation (30):

$$\log(w_t^i) = \rho \log(e^g w_{t-1}^i) + (1-\rho)\log(w_{Nash,t}^i), \tag{F.1}$$

where $\rho \in [0,1]$ is a weight that we set to 0.9 following Shimer (2010), and g is the rate of trend growth in productivity.[27] Again, we set the parameters γ and σ_ε^a to match the average and standard deviation of the unemployment rate.

Table F.5 presents results that are analogous to Table 3 in the main paper for these cases. This table shows clearly that the addition of insecure short-term jobs and AR(2) shocks has very little bearing on plucking. These features primarily add persistence to the model. This can be seen by comparing the two columns for the "Simple DRL" model to the two columns for the "Full" model. The plucking statistics are large with DNWR in both models and small with SRWR in both models. What differs strongly across these

[27]Since we assume a symmetric process for the logarithm of A_t^i, we take the average to be geometric—arithmetic for the logarithm of wages—in order not to introduce an ad hoc source of asymmetry in the model. Also, in some cases wages in a sector may be so low relative to productivity in that sector in the model with symmetric real wage rigidity that firms are willing to hire more workers than there are in the sector. In these cases, we assume that firms hire all workers but no more.

models is the persistence statistics in the bottom half of the table. The Full model generates a great deal more persistence than the Simple DRL model.

In addition to this, we see from Table F.5 that the sources of plucking are similar in our new model as in the DMP model we analyze in sections 3-4. The new model with DNRW produces slightly more plucking than the DMP model with DNWR. However, the new model with SRWR actually produces somewhat less plucking than the DMP model with Nash bargained wages.

F.1 The Role of Decreasing Returns to Labor

Table F.5 makes clear what the role of insecure short-term jobs and AR(2) shocks are in the new model. But what about decreasing returns to labor (DRL)? Why do we include this feature in our new model? The main reason is to allow the model to replicate the fraction of wage freezes in the data (on average 13%). With constant returns to labor, replicating this turns out to be impossible, as firms react to large negative shocks by laying off all their workers rather than maintaining them at a frozen wage.

With constant returns to labor, the firm's flow value of labor is $A - w$, which is independent of the number of employees working for the firm. If the firm faces a large negative shock to A that is quite persistent (as shocks are in the new model) the firm will want to lay off not just a few workers but all its workers. With DRL, the marginal product of workers rises as the firm lays off workers. DRL therefore leads firms to lay off a portion of its workforce after negative shocks as opposed to all workers.

This implies that with constant returns to scale, the average unemployment rate increases sharply with the volatility of sectoral shocks, but the share of matches in which the DNWR constraint binds remains far below our calibration target. In contrast, with DRL, most workers are retained even when the DNWR constraint binds and it is possible to match a 13% fraction of wage freezes in steady-state.

Decreasing returns to labor also allow us to move away from the (Hagedorn and Manovskii, 2008) calibration while keeping hiring pro-cyclical. Recall that in the standard DMP model low values of the flow value of unemployment z imply that the firms' value

Table F.5: Plucking Property, Speed, and Duration: New Model, Additional Results

	Data	Simple DRL		Full	
		SRWR	DNWR	SRWR	DNWR
Subsequent expansion on contraction, β	1.12	0.24	0.80	0.28	0.68
		(0.22)	(0.20)	(0.37)	(0.42)
Subsequent contraction on expansion, β	-0.38	0.13	-0.04	0.12	-0.08
		(0.22)	(0.21)	(0.47)	(0.67)
Subsequent expansion on contraction, R^2	0.59	0.06	0.62	0.11	0.53
		(0.10)	(0.24)	(0.20)	(0.31)
Subsequent contraction on expansion, R^2	0.22	0.03	0.02	0.08	0.13
		(0.08)	(0.06)	(0.17)	(0.29)
Speed of expansions (pp/year)	0.87	3.59	3.76	1.45	0.81
		(0.37)	(0.52)	(0.22)	(0.22)
Speed of contractions (pp/year)	1.89	3.41	3.96	1.45	1.44
		(0.33)	(0.57)	(0.21)	(0.35)
Duration of expansions (months)	59.1	14.2	12.8	34.9	64.0
		(1.8)	(1.7)	(6.8)	(16.0)
Duration of contractions (months)	26.9	14.8	16.0	34.2	36.6
		(1.8)	(2.6)	(6.5)	(12.6)

Note: The table compares real world data with data from four versions of the model of section 5. The first column—labeled "Data"—reports empirical results based on data from the U.S. economy from section 2. The second and third columns—labeled "Simple DRL"—report results for the simple version of the model with decreasing returns to labor but a single type of job ($\delta = d$), no sectoral heterogeneity, and aggregate productivity following an AR(1) process. The second column reports results under symmetric real wage rigidity ("SRWR") and the third column reports results under downward nominal wage rigidity ("DNWR"). The fourth and fifth columns—labeled "Full"—report results for the full model with decreasing returns to labor, two types of jobs ($\delta < d$), sectoral heterogentiy, and aggregate productivity following an AR(2) process. The fourth column reports results under symmetric real wage rigidity ("SRWR") and the fifth column reports results under downward nominal wage rigidity ("DNWR"). The first (third) row reports the coefficient (R^2) in an OLS regression of the size of an expansion (percentage point fall in unemployment rate) on the size of the previous contraction (percentage point increase in unemployment rate). The second (fourth) row report the coefficient (R^2) in an analogous regression of the size of a contraction on the size of the previous expansion. The next two rows report the spell-weighted average speed of expansions and contractions, measured in percentage points of unemployment per year. The final two rows report the average duration of expansions and contractions, measured in months. For the models, the reported point estimate is the median value of the statistic over 5000 samples of 866 periods each (the length of our sample of real-world data). Expansions and contractions of more than 6.5 percentage points are excluded from the samples. The standard error reported in parentheses is the standard deviation of the estimates across the 5000 samples. The full model is simulated with 1000 sectors.

function increases little with productivity, and hiring is therefore close to acyclical—the Shimer (2005) unemployment volatility puzzle. With DNWR this problem can become even worse, making hiring *counter-cyclical*. In this case higher productivity has two effects: it increases the current flow value to firms $A - w$, but it also increases wages, increasing the probability that the DNWR constraint will bind in the future. For a low value of z, the first effect is small—the root of the Shimer puzzle—and the second effect can dominate.

Decreasing returns bring a third effect into play. While higher productivity today still increases wages and makes it more likely that the DNWR constraint will bind in the future, the flow value of firms is now $AF'(N) - w$. If the DNWR constraint will be binding in the future, employment N will also be lower at that point, raising the marginal productivity $F'(N)$ of workers. The job-finding rate is therefore procyclical in our calibration of the model of section 5 with decreasing returns to labor, even though we calibrate the value of unemployment to 70% of wages as in Hall and Milgrom (2008).