CONSUMPTION-BASED ASSET PRICING

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Why is the return on the stock market so high?  
(Relative to the "risk-free rate")

Why is the stock market so volatile?

What does this tell us about the risk and risk aversion?
Consumption-based asset pricing starts from the Consumption Euler equation:

\[ U'(C_t) = E_t[\beta U'(C_{t+1})R_{i,t+1}] \]

Where does this equation come from?

- Consumer $1 less today
- Invest in asset \( i \)
- Use proceeds to consume $ \( R_{it+1} \) tomorrow

Two perspectives:

- Consumption Theory: Conditional on \( R_{it+1} \), determine path for \( C_t \)
- Asset Pricing: Conditional on path for \( C_t \), determine \( R_{it+1} \)
\[ U'(C_t) = \beta E_t[U'(C_{t+1})R_{i,t+1}] \]

- A little manipulation yields:

\[ 1 = E_t \left[ \frac{\beta U'(C_{t+1})}{U'(C_t)} R_{i,t+1} \right] \]

\[ 1 = E_t[M_{t+1} R_{i,t+1}] \]

- Stochastic discount factor:

\[ M_{t+1} = \beta \frac{U'(C_{t+1})}{U'(C_t)} \]
Stochastic Discount Factor

- Fundamental equation of consumption-based asset pricing:
  \[1 = E_t[M_{t+1} R_{i,t+1}]\]

- Stochastic discount factor prices all assets!!

- This (conceptually) simple view holds under the rather strong assumption that there exists a complete set of competitive markets
**Prices, Payoffs, and Returns**

- Return is defined as payoff divided by price:

\[
R_{i,t+1} = \frac{X_{i,t+1}}{P_{i,t}}
\]

where \(X_{i,t+1}\) is (state contingent) payoff from asset \(i\) in period \(t + 1\) and \(P_{i,t}\) is price of asset \(i\) at time \(t\)

- Fundamental equation can be rewritten as:

\[
P_{i,t} = E_t[M_{t+1}X_{i,t+1}]
\]
Multi-Period Assets

- Assets can have payoffs in multiple periods:

\[ P_{i,t} = E_t[M_{t+1}(D_{i,t+1} + P_{i,t+1})] \]

where \( D_{i,t+1} \) is the dividend, and \( P_{i,t+1} \) is (ex dividend) price

- Works for stocks, bonds, options, everything.
Suppose $P_{s,t+1}$ is the price of the Arrow security that pays $1$ if state $s$ occurs at time $t + 1$ and zero otherwise.

The price of any security can be written two ways:

$$P_{i,t} = \sum_s P_{s,t+1} X_{s,t+1}, \quad P_{i,t} = E_t[M_{t+1} X_{t+1}]$$

which implies

$$M_{s,t+1} = \frac{P_{s,t+1}}{\pi_{s,t+1}}$$

where $\pi_{s,t+1}$ is the probability of state $s$ in period $t + 1$.

This is why you sometimes see $E_t[M_{t+1} X_{t+1}]$ type terms in budget constraints.
**Modigliani-Miller Theorem**

- Value of an asset is the sum of its parts:

  \[ P_{i,t} = \sum_s P_{s,t+1} X_{s,t+1} \]

- Doesn’t matter how the asset is sliced up!
  (as long as the total payoff is not changed)
  - Capital structure of a firm doesn’t matter for its value!
  - Dividend policy of a firm doesn’t matter for its value!
  - Whether a firm buys insurance (hedges a risk) doesn’t matter!

- Holds when:
  - Complete set of competitive markets exists
    (no bankruptcy costs, no agency costs, etc.)
  - No taxes
Larry Summers (1985) critique of (then) finance

Two kinds of research on ketchup market

General Economists:
- Ask what the fundamental determinants of the price of ketchup is
- Analyze messy data on supply and demand
- Tough question, modest progress

Ketchup Economists:
- Analyze hard data on transactions
- Two quart sized ketchup bottles invariably sell for twice as much as one
- No bargains from storing ketchup or mixing ketchup, etc.
- Conclude from this that ketchup market is efficient
Everything Log-Normal

\[ 1 = E_t[M_{t+1} R_{i,t+1}] \]

- Let’s assume for simplicity that everything is log-normal
- If \( X_{t+1} \) is log-normal, then

\[ \log E_t X_{t+1} = E_t \log X_{t+1} + \frac{1}{2} \text{Var}_t \log X_{t+1} \]

- Now let’s take logs of asset pricing equation:

\[ 0 = \log E_t[M_{t+1} R_{i,t+1}] \]

\[ 0 = E_t r_{i,t+1} + E_t m_{t+1} + \frac{1}{2}(\sigma_i^2 + \sigma_m^2 + 2\sigma_{im}) \]

where \( r_{i,t+1} = \log R_{i,t+1}, m_t = \log M_{t+1}, \sigma_i^2 = \text{Var}_t \log R_{i,t+1}, \text{etc.} \)
\[ 0 = E_t r_{i,t+1} + E_t m_{t+1} + \frac{1}{2} \left( \sigma_i^2 + \sigma_m^2 + 2 \sigma_{im} \right) \]

- This equation should hold for all assets
- What does it imply about the risk-free asset?

\[ r_{f,t} = -E_t m_{t+1} - \frac{1}{2} \sigma_m^2 \]

- Or in levels:

\[ 1 = E_t [M_{t+1} R_{f,t}] \quad \Rightarrow \quad R_{f,t} = \frac{1}{E_t M_{t+1}} \]

- Since risk free return is risk free, it is determined at time \( t \)
Combining:

\[ 0 = E_t r_{i,t+1} + E_t m_{t+1} + \frac{1}{2}(\sigma_i^2 + \sigma_m^2 + 2\sigma_{im}) \]

\[ r_{f,t} = -E_t m_{t+1} - \frac{1}{2}\sigma_m^2 \]

we get

\[ E_t r_{i,t+1} - r_{f,t} + \frac{1}{2}\sigma_i^2 = -\sigma_{im} \]

Equity premium is negative of covariance of return on equity and the stochastic discount factor.
$$E_t r_{i,t+1} - r_{f,t} + \frac{1}{2} \sigma_i^2 = -\sigma_{im}$$

- What is with the $\frac{1}{2} \sigma_i^2$ term?
\[ E_t r_{i,t+1} - r_{f,t} + \frac{1}{2} \sigma^2_i = -\sigma_{im} \]

- What is with the \( \frac{1}{2} \sigma^2_i \) term?

\[ \log E_t R_{i,t+1} - \log R_{f,t} = E_t r_{i,t+1} - r_{f,t} + \frac{1}{2} \sigma^2_i \]

- Geometric mean: \( E_t r_{i,t+1} \)
- Arithmetic mean: \( E_t R_{i,t+1} \)

\[ \log E_t R_{i,t+1} = E_t r_{i,t+1} + \frac{1}{2} \text{Var}_t \log R_{i,t+1} \]

- Standard deviation annual real return on stocks is roughly 18%

\[ \frac{1}{2} \text{Var}_t \log R_{i,t+1} = \frac{1}{2} \sigma^2_i = 1.5\% \]
Two ways to write equity premium equation:

\[ E_t r_{i,t+1} - r_{f,t} + \frac{1}{2} \sigma_i^2 = -\sigma_{im} \]

\[ \log E_t R_{i,t+1} - \log R_{f,t} = -\sigma_{im} \]

Also recall that log of expected gross return is approximately equal to the expected net return:

\[ \log (1 + x) \approx x \]

for small \( x \)
Now suppose that

\[ U(C_t) = \frac{C_t^{1-\gamma} - 1}{1 - \gamma} \]

This utility function is sometimes called CRRA utility for “constant relative risk aversion”

Relative risk aversion:

\[ \frac{U''(C)C}{U'(C)} = \gamma \]

Why do we think that this utility function is reasonable?
Now suppose that

$$U(C_t) = \frac{C_t^{1-\gamma} - 1}{1 - \gamma}$$

This utility function is sometimes called CRRA utility for “constant relative risk aversion”

Relative risk aversion:

$$\frac{U''(C)C}{U'(C)} = \gamma$$

Why do we think that this utility function is reasonable?

Consider agent with CRRA utility and wealth $W$ facing portfolio choice between risky and risk-free asset. Fraction allocated to risky asset is independent of wealth.

(CARA utility: Dollar amount invested in risky asset is independent of wealth)

Consistent with stable interest rate and risk premia in the presence of long-run growth
Consider the following gamble: I flip a coin and ...
- If it comes up heads, I multiply your lifetime income by 1 million
- If it comes up tails, I reduce your lifetime income by 10%
Consider the following gamble: I flip a coin and ...

- If it comes up heads, I multiply your lifetime income by 1 million
- If it comes up tails, I reduce your lifetime income by 10%

If you accept, your CRRA is less than 10
Introspection about $\gamma$

- Consider the following gamble: I flip a coin and...
  - If it comes up heads, I multiply your lifetime income by 1 million
  - If it comes up tails, I reduce your lifetime income by 10%
- If you accept, your CRRA is less than 10
- What about 20% reduction? If yes, CRRA < 5
Consider the following gamble: I flip a coin and ...

- If it comes up heads, I multiply your lifetime income by 1 million
- If it comes up tails, I reduce your lifetime income by 10%

If you accept, your CRRA is less than 10

What about 20% reduction? If yes, CRRA < 5

What about 30% reduction? If yes, CRRA < 3
Introspection about $\gamma$

- Consider the following gamble: I flip a coin and ...
  - If it comes up heads, I multiply your lifetime income by 1 million
  - If it comes up tails, I reduce your lifetime income by 10%
- If you accept, your CRRA is less than 10
- What about 20% reduction? If yes, CRRA < 5
- What about 30% reduction? If yes, CRRA < 3
- What about 50% reduction? If yes, CRRA < 2
What fraction of your lifetime wealth would you be willing to pay to avoid a 50/50 risk of gaining or losing a share $\alpha$ of your lifetime wealth

- $\alpha = 0.10$
- $\alpha = 0.30$
Table 2.1
Relative risk premium \( \hat{\pi} \) associated to the risk of gaining or losing (\( \alpha \% \) of wealth)

<table>
<thead>
<tr>
<th>RRA</th>
<th>( \alpha = 10% )</th>
<th>( \alpha = 30% )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \gamma = 0.5 )</td>
<td>0.3</td>
<td>2.3</td>
</tr>
<tr>
<td>( \gamma = 1 )</td>
<td>0.5</td>
<td>4.6</td>
</tr>
<tr>
<td>( \gamma = 4 )</td>
<td>2.0</td>
<td>16.0</td>
</tr>
<tr>
<td>( \gamma = 10 )</td>
<td>4.4</td>
<td>24.4</td>
</tr>
<tr>
<td>( \gamma = 40 )</td>
<td>8.4</td>
<td>28.7</td>
</tr>
</tbody>
</table>

Source: Gollier (2001)
Power Utility

\[ U(C_t) = \frac{C_t^{1-\gamma} - 1}{1 - \gamma} \]

- With time separable power utility, \( \gamma \) is also the inverse of the intertemporal elasticity of substitution

\[
\frac{d \log(C_{t+1}/C_t)}{d \log(P_{t+1}/P_t)} = \frac{d \log(C_{t+1}/C_t)}{d \log R_{ft}} = \frac{1}{\gamma}
\]

- Only one parameter. So, it plays many roles.
  (Also governs strength of wealth effect on labor supply)
**Power Utility**

\[
U(C_t) = \frac{C_t^{1-\gamma} - 1}{1 - \gamma}
\]

- Implies:

\[
U'(C_t) = C_t^{-\gamma}
\]

\[
M_{t+1} = \frac{\beta U'(C_{t+1})}{U'(C_t)} = \beta \left( \frac{C_{t+1}}{C_t} \right)^{-\gamma}
\]
**Euler Equation with Power Utility**

\[ 1 = E_t[(1 + R_{i,t+1})M_{t+1}] \]

\[ 1 = E_t \left[ (1 + R_{i,t+1})^\beta \left( \frac{C_{t+1}}{C_t} \right)^{-\gamma} \right] \]

Taking logs and assuming log-normality:

\[ 0 = E_t r_{i,t+1} + \log \beta - \gamma E_t \Delta \log C_{t+1} + \frac{1}{2} \left[ \sigma_i^2 + \gamma^2 \sigma_c^2 - 2\gamma \sigma_{ic} \right] \]

And for risk-free rate:

\[ 0 = r_{f,t} + \log \beta - \gamma E_t \Delta \log C_{t+1} + \frac{1}{2} \gamma^2 \sigma_c^2 \]

Combining these last two yields:

\[ E_t r_{i,t+1} - r_{f,t} + \frac{1}{2} \sigma_i^2 = \gamma \sigma_{ic} \]
Equity premium is risk aversion times covariance between consumption growth and return on equity

What is the intuition for this?
Complete markets

Representative agent with CRRA preferences:

\[ C_t^{-\gamma} = E_t[\beta C_{t+1}^{-\gamma} R_{i,t+1}] \]

Endowment economy ("Lucas tree"):

\[ \log C_{t+1} = \mu + \log C_t + \epsilon_{t+1} \]

\[ \epsilon_{t+1} \sim N(0, \sigma^2) \]

Equity modeled as a claim to the consumption process:

\[ R_{i,t+1} = R_{C,t+1} \]
In this case, equity premium:

\[ \log E_t R_{C,t+1} - \log R_{f,t} = \gamma \text{var}_t(\Delta \log C_{t+1}) \]

Does this model fit the data?

We need data on:
- Average stock returns
- Average returns on risk-free asset
- Variance of consumption growth

We need a view as to what values are “reasonable” for \( \gamma \)
- Mehra-Prescott: Values of \( \gamma < 10 \) “admissible”
Fig. 4. Set of admissible average equity risk premia and real returns.

Table 1

<table>
<thead>
<tr>
<th>Time periods</th>
<th>% growth rate of per capita real consumption</th>
<th>% real return on a relatively riskless security</th>
<th>% risk premium</th>
<th>% real return on S&amp;P 500</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>% growth rate of per capita real consumption</td>
<td>% real return on a relatively riskless security</td>
<td>% risk premium</td>
<td>% real return on S&amp;P 500</td>
</tr>
<tr>
<td></td>
<td>Mean (Std error)</td>
<td>Mean (Std error)</td>
<td>Mean (Std error)</td>
<td>Mean (Std error)</td>
</tr>
<tr>
<td>1889–1978</td>
<td>1.83 (0.38)</td>
<td>0.80 (0.60)</td>
<td>6.18 (1.76)</td>
<td>6.98 (1.74)</td>
</tr>
<tr>
<td>1889–1898</td>
<td>2.30 (4.90)</td>
<td>5.80 (3.23)</td>
<td>1.78 (11.57)</td>
<td>7.58 (10.02)</td>
</tr>
<tr>
<td>1899–1908</td>
<td>2.55 (5.31)</td>
<td>2.62 (2.59)</td>
<td>5.08 (16.86)</td>
<td>7.71 (17.21)</td>
</tr>
<tr>
<td>1909–1918</td>
<td>0.44 (3.07)</td>
<td>-1.63 (9.02)</td>
<td>1.49 (9.18)</td>
<td>-0.14 (12.81)</td>
</tr>
<tr>
<td>1919–1928</td>
<td>3.00 (3.97)</td>
<td>4.30 (6.61)</td>
<td>14.64 (15.94)</td>
<td>18.94 (16.18)</td>
</tr>
<tr>
<td>1929–1938</td>
<td>-0.25 (5.28)</td>
<td>2.39 (6.50)</td>
<td>0.18 (31.63)</td>
<td>2.56 (27.90)</td>
</tr>
<tr>
<td>1939–1948</td>
<td>2.19 (2.52)</td>
<td>-5.82 (4.05)</td>
<td>8.89 (14.23)</td>
<td>3.07 (14.67)</td>
</tr>
<tr>
<td>1949–1958</td>
<td>1.48 (1.00)</td>
<td>-0.81 (1.89)</td>
<td>18.30 (13.20)</td>
<td>17.49 (13.08)</td>
</tr>
<tr>
<td>1959–1968</td>
<td>2.37 (1.00)</td>
<td>1.07 (0.64)</td>
<td>4.50 (10.17)</td>
<td>5.58 (10.59)</td>
</tr>
<tr>
<td>1969–1978</td>
<td>2.41 (1.40)</td>
<td>-0.72 (2.06)</td>
<td>0.75 (11.64)</td>
<td>0.03 (13.11)</td>
</tr>
</tbody>
</table>

Source: Mehra and Prescott (1985)
Mehra-Prescott 85 made “extra” assumptions:
  - Endowment economy
  - Specific process for consumption growth
  - Equity is a consumption claim

Equity premium equation can be evaluated independent of these assumptions:

\[ E_t r_{i,t+1} - r_{f,t} + \frac{1}{2} \sigma_i^2 = \gamma \sigma_{ic} \]
<table>
<thead>
<tr>
<th>Country</th>
<th>Sample period</th>
<th>$\bar{\alpha}$</th>
<th>$\sigma(\alpha)$</th>
<th>$\sigma(m)$</th>
<th>$\sigma(\Delta c)$</th>
<th>$\rho(\alpha, \Delta c)$</th>
<th>Cov($\alpha, \Delta c$)</th>
<th>RRA(1)</th>
<th>RRA(2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>USA</td>
<td>1947.2–1996.3</td>
<td>7.852</td>
<td>15.218</td>
<td>51.597</td>
<td>1.084</td>
<td>0.193</td>
<td>3.185</td>
<td>246.556</td>
<td>47.600</td>
</tr>
<tr>
<td>AUL</td>
<td>1970.1–1996.2</td>
<td>3.531</td>
<td>23.194</td>
<td>15.221</td>
<td>2.142</td>
<td>0.156</td>
<td>7.725</td>
<td>45.704</td>
<td>7.107</td>
</tr>
<tr>
<td>CAN</td>
<td>1970.1–1996.2</td>
<td>3.040</td>
<td>16.673</td>
<td>18.233</td>
<td>2.034</td>
<td>0.159</td>
<td>5.387</td>
<td>56.434</td>
<td>8.965</td>
</tr>
<tr>
<td>GER</td>
<td>1978.4–1996.2</td>
<td>6.774</td>
<td>20.373</td>
<td>33.251</td>
<td>2.495</td>
<td>0.039</td>
<td>1.974</td>
<td>343.133</td>
<td>13.327</td>
</tr>
<tr>
<td>ITA</td>
<td>1971.2–1995.2</td>
<td>2.166</td>
<td>27.346</td>
<td>7.920</td>
<td>1.684</td>
<td>0.002</td>
<td>0.088</td>
<td>2465.323</td>
<td>4.703</td>
</tr>
<tr>
<td>NTH</td>
<td>1977.2–1996.1</td>
<td>9.943</td>
<td>15.632</td>
<td>63.607</td>
<td>2.654</td>
<td>0.023</td>
<td>0.946</td>
<td>1050.925</td>
<td>23.970</td>
</tr>
<tr>
<td>SWD</td>
<td>1970.1–1994.4</td>
<td>9.343</td>
<td>23.541</td>
<td>39.688</td>
<td>1.917</td>
<td>0.003</td>
<td>0.129</td>
<td>7215.176</td>
<td>20.705</td>
</tr>
<tr>
<td>SWT</td>
<td>1982.2–1996.2</td>
<td>12.393</td>
<td>20.466</td>
<td>60.553</td>
<td>2.261</td>
<td>-0.129</td>
<td>-5.978</td>
<td>&lt; 0</td>
<td>26.785</td>
</tr>
<tr>
<td>USA</td>
<td>1970.1–1996.3</td>
<td>5.817</td>
<td>16.995</td>
<td>34.228</td>
<td>0.919</td>
<td>0.248</td>
<td>3.875</td>
<td>150.136</td>
<td>37.255</td>
</tr>
<tr>
<td>SWD</td>
<td>1920–1993</td>
<td>6.000</td>
<td>18.906</td>
<td>31.737</td>
<td>2.862</td>
<td>0.169</td>
<td>9.141</td>
<td>65.642</td>
<td>11.091</td>
</tr>
</tbody>
</table>

* $\bar{\alpha}$ is the average excess log return on stock over a money market instrument, plus one half the variance of this excess return: $\bar{\alpha} = r_e - r_f + \sigma^2(r_e - r_f)/2$. It is multiplied by 400 in quarterly data and 100 in annual data to express in annualized percentage points. $\sigma(\alpha)$ and $\sigma(\Delta c)$ are the standard deviations of the excess log return $\alpha = r_e - r_f$ and consumption growth $\Delta c$, respectively, multiplied by 200 in quarterly data and 100 in annual data to express in annualized percentage points. $\sigma(m) = 100\bar{\alpha} / \sigma(\alpha)$ is calculated from equation (12) as a lower bound on the standard deviation of the log stochastic discount factor, expressed in annualized percentage points. $\rho(\alpha, \Delta c)$ is the correlation of $\alpha$ and $\Delta c$. $\rho(\alpha, \Delta c)$ is the product $\sigma(\alpha) \sigma(\Delta c) \rho(\alpha, \Delta c)$. RRA(1) is $100\bar{\alpha} / \text{Cov}(\alpha, \Delta c)$, a measure of risk aversion calculated from equation (16) using the empirical covariance of excess stock returns with consumption growth. RRA(2) is $100\bar{\alpha} / \sigma(\alpha) \sigma(\Delta c)$, a measure of risk aversion calculated using the empirical standard deviations of excess stock returns and consumption growth, but assuming perfect correlation between these series.

Abbreviations: AUL, Australia; CAN, Canada; FR, France; GER, Germany; ITA, Italy; JPN, Japan; NTH, Netherlands; SWD, Sweden; SWT, Switzerland; UK, United Kingdom; USA, United States of America.

Source: Campbell (1999)
### Table 5  Long-Period Averages of Rates of Return

<table>
<thead>
<tr>
<th>Country</th>
<th>Start</th>
<th>Stocks</th>
<th>Bills</th>
<th>Start</th>
<th>Bonds</th>
<th>Bills</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Part 1: OECD countries</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Australia</td>
<td>1876</td>
<td>0.1027 (0.1616)</td>
<td>0.0126 (0.0566)</td>
<td>1870</td>
<td>0.0352 (0.1157)</td>
<td>0.0125 (0.0569)</td>
</tr>
<tr>
<td>Belgium</td>
<td>--</td>
<td>--</td>
<td>--</td>
<td>1870</td>
<td>0.0291 (0.1584)**</td>
<td>0.0179 (0.1447)**</td>
</tr>
<tr>
<td>Canada</td>
<td>1916</td>
<td>0.0781 (0.1754)</td>
<td>--</td>
<td>1916</td>
<td>0.0392 (0.1199)</td>
<td>--</td>
</tr>
<tr>
<td>Denmark</td>
<td>1915</td>
<td>0.0750 (0.2300)</td>
<td>0.0265 (0.0652)</td>
<td>1870</td>
<td>0.0392 (0.1137)</td>
<td>0.0317 (0.0588)</td>
</tr>
<tr>
<td>Finland</td>
<td>1923</td>
<td>0.1268 (0.3155)</td>
<td>0.0128 (0.0935)</td>
<td>--</td>
<td>--</td>
<td>--</td>
</tr>
<tr>
<td>France</td>
<td>1870</td>
<td>0.0543 (0.2078)*</td>
<td>-0.0061 (0.0996)*</td>
<td>1870</td>
<td>0.0066 (0.1368)</td>
<td>-0.0079 (0.1000)</td>
</tr>
<tr>
<td>Germany</td>
<td>1870</td>
<td>0.0758 (0.2976)</td>
<td>-0.0153 (0.1788)</td>
<td>1924</td>
<td>0.0402 (0.1465)</td>
<td>0.0158 (0.1173)</td>
</tr>
<tr>
<td>Italy</td>
<td>1906</td>
<td>0.0510 (0.2760)</td>
<td>-0.0112 (0.1328)</td>
<td>1870</td>
<td>0.0173 (0.1879)</td>
<td>0.0046 (0.1191)</td>
</tr>
<tr>
<td>Japan</td>
<td>1894</td>
<td>0.0928 (0.3017)</td>
<td>-0.0052 (0.1370)</td>
<td>1883</td>
<td>0.0192 (0.1820)</td>
<td>0.0043 (0.1475)</td>
</tr>
<tr>
<td>Netherlands</td>
<td>1920</td>
<td>0.0901 (0.2116)**</td>
<td>0.0114 (0.0474)**</td>
<td>1881</td>
<td>0.0308 (0.1067)</td>
<td>0.0118 (0.0512)</td>
</tr>
<tr>
<td>New Zealand</td>
<td>1927</td>
<td>0.0762 (0.2226)</td>
<td>0.0234 (0.0529)</td>
<td>1926</td>
<td>0.0276 (0.1209)</td>
<td>0.0240 (0.0529)</td>
</tr>
<tr>
<td>Norway</td>
<td>1915</td>
<td>0.0716 (0.2842)</td>
<td>0.0098 (0.0782)</td>
<td>1877</td>
<td>0.0280 (0.1130)</td>
<td>0.0204 (0.0709)</td>
</tr>
<tr>
<td>Spain</td>
<td>1883</td>
<td>0.0610 (0.2075)†</td>
<td>0.0173 (0.0573)†</td>
<td>--</td>
<td>--</td>
<td>--</td>
</tr>
<tr>
<td>Sweden</td>
<td>1902</td>
<td>0.0923 (0.2347)</td>
<td>0.0180 (0.0719)</td>
<td>1922</td>
<td>0.0292 (0.0941)</td>
<td>0.0176 (0.0448)</td>
</tr>
<tr>
<td>Switzerland</td>
<td>1911</td>
<td>0.0726 (0.2107)††</td>
<td>0.0083 (0.0531)††</td>
<td>1916</td>
<td>0.0218 (0.0717)</td>
<td>0.0065 (0.0545)</td>
</tr>
<tr>
<td>U.K.</td>
<td>1870</td>
<td>0.0641 (0.1765)</td>
<td>0.0179 (0.0624)</td>
<td>1870</td>
<td>0.0280 (0.1049)</td>
<td>0.0179 (0.0624)</td>
</tr>
<tr>
<td>U.S.</td>
<td>1870</td>
<td>0.0827 (0.1866)</td>
<td>0.0199 (0.0482)</td>
<td>1870</td>
<td>0.0271 (0.0842)</td>
<td>0.0199 (0.0482)</td>
</tr>
<tr>
<td><strong>Part 2: Non-OECD countries</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Chile</td>
<td>1895</td>
<td>0.1430 (0.4049)</td>
<td>-0.0094 (0.1776)</td>
<td>--</td>
<td>--</td>
<td>--</td>
</tr>
<tr>
<td>India</td>
<td>1921</td>
<td>0.0514 (0.2341)***</td>
<td>0.0133 (0.0835)***</td>
<td>1874</td>
<td>0.0191 (0.1147)</td>
<td>0.0240 (0.0785)</td>
</tr>
<tr>
<td>South Africa</td>
<td>1911</td>
<td>0.0890 (0.206)</td>
<td>--</td>
<td>1911</td>
<td>0.0248 (0.1165)</td>
<td>--</td>
</tr>
<tr>
<td>Overall means†††</td>
<td>0.0814 (0.2449)</td>
<td>0.0085 (0.0880)</td>
<td>--</td>
<td>0.0266 (0.1234)</td>
<td>0.0147 (0.0805)</td>
<td></td>
</tr>
</tbody>
</table>

*missing 1940-41, **missing 1945-46, †missing 1936-40, ††missing 1914-16, ***missing 1926-27
†††Averages of means and standard deviations for 17 countries with stock and bill data and 15 countries with bond and bill data

Source: Barro and Ursua (2008)
Volatility of consumption seems to be relatively modest

World seems to be a relatively safe place

People must be very risk averse to not want to bid up prices of stocks

High equity premium implies that stocks are cheap!!
Suppose we invest $1 in:

- Equity with 8% real return
- Tbills with 1% real return

<table>
<thead>
<tr>
<th>Horizon</th>
<th>Equity</th>
<th>Tbills</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1.08</td>
<td>1.01</td>
</tr>
<tr>
<td>5</td>
<td>1.50</td>
<td>1.05</td>
</tr>
<tr>
<td>10</td>
<td>2.15</td>
<td>1.10</td>
</tr>
<tr>
<td>25</td>
<td>6.85</td>
<td>1.28</td>
</tr>
<tr>
<td>50</td>
<td>46.90</td>
<td>1.64</td>
</tr>
<tr>
<td>100</td>
<td>2199.76</td>
<td>2.70</td>
</tr>
</tbody>
</table>

Dutch (supposedly) bought Manhattan from natives for $24 in 1626

Suppose the invested this in the stock market:

\[ \$24 \times 1.08^{(2019-1626)} = \$1.36 \times 10^{13} = \$13.6 \text{ Trillion} \]
Mean equity premium: ≈ 6.5%

Standard deviation of equity premium: ≈ 18%

Standard error on equity premium: \( \frac{\sigma}{\sqrt{T}} = 2.1\% \) (post-WWII)
\[ \frac{\sigma}{\sqrt{T}} = 1.5\% \] (post-1870)

Using post-WWII standard error:

95% confidence interval for equity premium: [2.3%, 10.7%]

Perhaps last 100 years have been unusually good
Equity Premium is VERY BIG ... Or Is It?

- Relative to prior history, 20th century was good for growth and stocks
- Simple Gordon growth model:
  \[ \frac{P}{D} = \frac{1}{r - g} \]
- Maybe expectations about future growth have risen
- Maybe equity premium has fallen
- Would yield an unusually high return not to be repeated in the future
Hansen-Jaganathan Bound

\[ E_t r_{i,t+1} - r_{f,t} + \frac{1}{2} \sigma_i^2 = -\sigma_{im} \]

Definition of correlation coefficient:

\[ \rho_{im} = \frac{\sigma_{im}}{\sigma_i \sigma_m} \]

\[ -1 \leq \rho_{im} \]

\[ \sigma_m \geq \frac{-\sigma_{im}}{\sigma_i} \]

\[ \sigma_m \geq \frac{E_t r_{i,t+1} - r_{f,t} + \frac{1}{2} \sigma_i^2}{\sigma_i} \]

Ratio on right-hand-side called “Sharpe ratio”
\[ \sigma_m \geq \frac{E_t r_{i,t+1} - r_{f,t} + \frac{1}{2} \sigma_i^2}{\sigma_i} \]

- Sharp ratio for stocks: 0.4
- Sharp ratio for other assets: >1
- Hansen-Jaganathan bound implies that volatility of stochastic discount factor is enormous
- Seems implausible
**Risk-Free Rate Puzzle**

\[ r_{f,t} = -\log \beta + \gamma E_t \Delta C_{t+1} - \frac{1}{2} \gamma^2 \sigma_c^2 \]

- \( \sigma_c^2 \ll E_t \Delta C_{t+1} \)
- High value of \( \gamma \) therefore implies high risk free rate
- What is the intuition for this?
Risk-Free Rate Puzzle

\[ r_{f,t} = -\log \beta + \gamma E_t \Delta C_{t+1} - \frac{1}{2} \gamma^2 \sigma_c^2 \]

- \( \sigma_c^2 \ll E_t \Delta C_{t+1} \)
- High value of \( \gamma \) therefore implies high risk free rate
- What is the intuition for this?
  - Consumers must be compensated a lot to allow their consumption profile to be upward sloping
  - This is \( \gamma \) acting in its incarnation as 1/IES
$r_{f,t} = -\log \beta + \gamma E_t \Delta C_{t+1} - \frac{1}{2} \gamma^2 \sigma_c^2$

- $\sigma_c^2 << E_t \Delta C_{t+1}$
- High value of $\gamma$ therefore implies high risk free rate
- What is the intuition for this?
  - Consumers must be compensated a lot to allow their consumption profile to be upward sloping
  - This is $\gamma$ acting in its incarnation as $1/\text{IES}$
- To get a low risk-free rate, $\beta > 1$
Table 6  
The riskfree rate puzzle$^a$

<table>
<thead>
<tr>
<th>Country</th>
<th>Sample period</th>
<th>$\bar{r}_f$</th>
<th>$\Delta c$</th>
<th>$\sigma(\Delta c)$</th>
<th>RRA(1)</th>
<th>TPR(1)</th>
<th>RRA(2)</th>
<th>TPR(2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>USA</td>
<td>1947.2–1996.3</td>
<td>0.794</td>
<td>1.908</td>
<td>1.084</td>
<td>246.556</td>
<td>-112.474</td>
<td>47.600</td>
<td>-76.710</td>
</tr>
<tr>
<td>AUL</td>
<td>1970.1–1996.2</td>
<td>1.820</td>
<td>1.854</td>
<td>2.142</td>
<td>45.704</td>
<td>-34.995</td>
<td>7.107</td>
<td>-10.196</td>
</tr>
<tr>
<td>FR</td>
<td>1973.2–1996.2</td>
<td>2.736</td>
<td>1.581</td>
<td>2.130</td>
<td>&lt; 0</td>
<td>N/A</td>
<td>14.634</td>
<td>-15.536</td>
</tr>
<tr>
<td>SWD</td>
<td>1970.1–1994.4</td>
<td>1.520</td>
<td>0.750</td>
<td>1.917</td>
<td>&gt;1000</td>
<td>&gt;1000</td>
<td>20.705</td>
<td>-6.126</td>
</tr>
<tr>
<td>SWT</td>
<td>1982.2–1996.2</td>
<td>1.466</td>
<td>0.414</td>
<td>2.261</td>
<td>&lt; 0</td>
<td>N/A</td>
<td>26.785</td>
<td>8.698</td>
</tr>
<tr>
<td>USA</td>
<td>1970.1–1996.3</td>
<td>1.350</td>
<td>1.710</td>
<td>0.919</td>
<td>150.136</td>
<td>-160.275</td>
<td>37.255</td>
<td>-56.505</td>
</tr>
<tr>
<td>SWD</td>
<td>1920–1993</td>
<td>2.073</td>
<td>1.748</td>
<td>2.862</td>
<td>65.642</td>
<td>63.778</td>
<td>11.091</td>
<td>-12.274</td>
</tr>
</tbody>
</table>

$^a$ $\bar{r}_f$ is the mean money market return from Table 2, in annualized percentage points. $\Delta c$ and $\sigma(\Delta c)$ are the mean and standard deviation of consumption growth from Table 3, in annualized percentage points. RRA(1) and RRA(2) are the risk aversion coefficients from Table 5. $TPR(1) = \bar{r}_f - RRA(1)\Delta c + RRA(1)^2 \sigma^2(\Delta c)/200$, and $TPR(2) = \bar{r}_f - RRA(2)\Delta c + RRA(2)^2 \sigma^2(\Delta c)/200$. From Equation (17), these time preference rates give the real interest rate, in annualized percentage points, that would prevail if consumption growth had zero mean and zero standard deviation and risk aversion were RRA(1) or RRA(2), respectively.
Perhaps low return on short term bonds is a liquidity premium for “moneylike” features

Campbell argues against this based on the term premium:
  - Long-term bonds don’t have this type of liquidity premium
  - Yet their returns are only slightly higher than those of short-term bonds
Table 7
International yield spreads and bond excess returns

<table>
<thead>
<tr>
<th>Country</th>
<th>Sample period</th>
<th>$\bar{s}$</th>
<th>$\sigma(s)$</th>
<th>$\rho(s)$</th>
<th>$\bar{e}r_b$</th>
<th>$\sigma(er_b)$</th>
<th>$\rho(er_b)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>USA</td>
<td>1947.2–1996.4</td>
<td>1.199</td>
<td>0.999</td>
<td>0.783</td>
<td>0.011</td>
<td>8.923</td>
<td>0.070</td>
</tr>
<tr>
<td>AUL</td>
<td>1970.1–1996.3</td>
<td>0.938</td>
<td>1.669</td>
<td>0.750</td>
<td>0.156</td>
<td>8.602</td>
<td>0.162</td>
</tr>
<tr>
<td>CAN</td>
<td>1970.1–1996.3</td>
<td>1.057</td>
<td>1.651</td>
<td>0.819</td>
<td>0.950</td>
<td>9.334</td>
<td>-0.009</td>
</tr>
<tr>
<td>FR</td>
<td>1973.2–1996.3</td>
<td>0.917</td>
<td>1.547</td>
<td>0.733</td>
<td>1.440</td>
<td>8.158</td>
<td>0.298</td>
</tr>
<tr>
<td>GER</td>
<td>1978.4–1996.3</td>
<td>0.991</td>
<td>1.502</td>
<td>0.869</td>
<td>0.899</td>
<td>7.434</td>
<td>0.117</td>
</tr>
<tr>
<td>ITA</td>
<td>1971.2–1995.3</td>
<td>-0.200</td>
<td>2.025</td>
<td>0.759</td>
<td>-1.386</td>
<td>9.493</td>
<td>0.335</td>
</tr>
<tr>
<td>JPN</td>
<td>1970.2–1996.3</td>
<td>0.593</td>
<td>1.488</td>
<td>0.843</td>
<td>1.687</td>
<td>9.165</td>
<td>-0.058</td>
</tr>
<tr>
<td>NTH</td>
<td>1977.2–1996.2</td>
<td>1.212</td>
<td>1.789</td>
<td>0.574</td>
<td>1.549</td>
<td>7.996</td>
<td>0.032</td>
</tr>
<tr>
<td>SWD</td>
<td>1970.1–1995.1</td>
<td>0.930</td>
<td>2.046</td>
<td>0.724</td>
<td>-0.212</td>
<td>7.575</td>
<td>0.244</td>
</tr>
<tr>
<td>SWT</td>
<td>1982.2–1996.3</td>
<td>0.471</td>
<td>1.655</td>
<td>0.755</td>
<td>1.071</td>
<td>6.572</td>
<td>0.268</td>
</tr>
<tr>
<td>UK</td>
<td>1970.1–1996.3</td>
<td>1.202</td>
<td>2.106</td>
<td>0.893</td>
<td>0.959</td>
<td>11.611</td>
<td>-0.057</td>
</tr>
<tr>
<td>USA</td>
<td>1970.1–1996.4</td>
<td>1.562</td>
<td>1.190</td>
<td>0.737</td>
<td>1.504</td>
<td>10.703</td>
<td>0.033</td>
</tr>
<tr>
<td>SWD</td>
<td>1920–1994</td>
<td>0.284</td>
<td>1.140</td>
<td>0.280</td>
<td>-0.075</td>
<td>6.974</td>
<td>-0.185</td>
</tr>
<tr>
<td>UK</td>
<td>1919–1994</td>
<td>1.272</td>
<td>1.505</td>
<td>0.694</td>
<td>0.318</td>
<td>8.812</td>
<td>-0.098</td>
</tr>
<tr>
<td>USA</td>
<td>1891–1995</td>
<td>0.720</td>
<td>1.550</td>
<td>0.592</td>
<td>0.172</td>
<td>6.499</td>
<td>0.153</td>
</tr>
</tbody>
</table>

$\bar{s}$ is the mean of the log yield spread, the difference between the log yield on long-term bonds and the log 3-month money market return, expressed in annualized percentage points. $\sigma(s)$ is the standard deviation of the log yield spread and $\rho(s)$ is its first-order autocorrelation. $\bar{e}r_b$, $\sigma(er_b)$, and $\rho(er_b)$ are defined in the same way for the excess 3-month return on long-term bonds over money market instruments, where the bond return is calculated from the bond yield using the par-bond approximation given in Campbell, Lo and MacKinlay (1997), Chapter 10, equation (10.1.19). Full details of this calculation are given in the Data Appendix.

Abbreviations: AUL, Australia; CAN, Canada; FR, France; GER, Germany; ITA, Italy; JPN, Japan; NTH, Netherlands; SWD, Sweden; SWT, Switzerland; UK, United Kingdom; USA, United States of America.
Restatement of Problem:

- To fit equity premium evidence, need high risk aversion

- High risk aversion implies low IES (with CRRA utility)

- Low IES implies high risk-free interest rate
“Obvious” solution:

- Consider preferences where IES may differ from 1/CRRRA
- Make IES and CRRA high

- Epstein-Zin-Weil preferences deliver this
Epstein-Zin (1989, 1991) and Weil (1989) propose:

\[ U_t = \left\{ (1 - \delta)C_t^{1-\gamma} + \delta \left( E_t U_{t+1}^{1-\gamma} \right)^{\frac{1}{\theta}} \right\}^{\frac{\theta}{1-\gamma}} \]

- Parameters:
  \[ \theta = \frac{1 - \gamma}{1 - 1/\psi} \]

  - \( \gamma \): Coefficient of relative risk aversion
  - \( \psi \): Intertemporal elasticity of substitution

- Falls outside expected utility framework
- Large literature about “weird” properties
Consumption Euler equation with Epstein-Zin-Weil preferences:

$$1 = E_t \left[ \beta^\theta \left( \frac{C_{t+1}}{C_t} \right)^{-\theta/\psi} (1 + R_{W,t+1})^{-(1-\theta)} (1 + R_{i,t+1}) \right]$$

- $R_{W,t+1}$ return on wealth
With power utility case, it is not clear whether $\gamma$ appears in a particular equation because it is the CRRA or because it is $1/\text{IES}$.

This is clarified in EZW case:

$$E_t r_{i,t+1} - r_{f,t} + \frac{1}{2} \sigma_i^2 = \theta \frac{\sigma_{ic}}{\psi} + (1 - \theta) \sigma_{iw}$$

$$r_{f,t} = -\log \beta + \frac{1}{\psi} E_t \Delta \log C_{t+1} + \frac{1}{2} (\theta - 1) \sigma_w^2 - \frac{1}{2} \frac{\theta}{\psi^2} \sigma_c^2$$

Since both $\gamma$ and $\psi$ can be big at the same time, EP and RF puzzles can be resolved.

But are large values of $\gamma$ and $\psi$ "reasonable"