ESTIMATION OF THE INTERTEMPORAL ELASTICITY OF SUBSTITUTION

Emi Nakamura and Jon Steinsson

UC Berkeley

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1. Traditional estimation strategies for IES in macro (e.g., Hall, 1988; Campbell and Mankiw, 1989)
   - Very structural approach (although it doesn’t look it)
   - Example of a common type of reasoning in empirical macro

2. Critique of traditional strategy
   - Identification challenges are broader and more challenging than sometimes acknowledged

3. Example of different sort of structural approach (Best-Cloyne-Ilzetzki-Kleven 2019)
**Estimating the IES**

- Consumption Euler equation with power utility and log-normality:

\[ E_t \Delta \log C_{t+1} = \psi E_t r_{i,t+1} + \psi \log \beta + \frac{1}{2} [\psi \sigma_i^2 + \psi^{-1} \sigma_c^2 - 2 \sigma_{ic}] \]

- Can be rewritten as:

\[ \Delta \log C_{t+1} = \mu_i + \psi r_{i,t+1} + \epsilon_{i,t+1} \]

where

\[ \epsilon_{i,t+1} = \psi (E_t r_{i,t+1} - r_{i,t+1}) - (E_t \Delta \log C_{t+1} - \Delta \log C_{t+1}) \]
Estimating the IES

\[ \Delta \log C_{t+1} = \mu_i + \psi r_{i,t+1} + \epsilon_{i,t+1} \]

where

\[ \epsilon_{i,t+1} = \psi (E_t r_{i,t+1} - r_{i,t+1}) - (E_t \Delta \log C_{t+1} - \Delta \log C_{t+1}) \]

Can we estimate this using OLS?
Estimating the IES

\[ \Delta \log C_{t+1} = \mu_i + \psi r_{i,t+1} + \epsilon_{i,t+1} \]

where

\[ \epsilon_{i,t+1} = \psi (E_t r_{i,t+1} - r_{i,t+1}) - (E_t \Delta \log C_{t+1} - \Delta \log C_{t+1}) \]

Can we estimate this using OLS?

- Suppose there is a “good shock” that leads to a high realization of \( r_{i,t+1} \)
- This shock will be correlated with the error term
  (consumption (and return) will rise relative to expectation)
An IV Approach

\[ \Delta \log C_{t+1} = \mu_i + \psi r_{i,t+1} + \epsilon_{i,t+1} \]

where

\[ \epsilon_{i,t+1} = \psi(E_t r_{i,t+1} - r_{i,t+1}) - (E_t \Delta \log C_{t+1} - \Delta \log C_{t+1}) \]

Can we think of instruments that will work in this case?

(Hint: Error term is an expectation error)
An IV Approach

\[ \Delta \log C_{t+1} = \mu_i + \psi r_{i,t+1} + \epsilon_{i,t+1} \]

where

\[ \epsilon_{i,t+1} = \psi(E_t r_{i,t+1} - r_{i,t+1}) - (E_t \Delta \log C_{t+1} - \Delta \log C_{t+1}) \]

- Can we think of instruments that will work in this case? (Hint: Error term is an expectation error)
- Any variable known at time \( t \) works as an instrument
- Since \( \epsilon_{i,t+1} \) is an expectation error, it is orthogonal to all variables known at time \( t \) or earlier
- So, we can use lags of anything as instruments (Wow, lots of possible instruments)
If $r_{i,t+1}$ is the risk-free rate ($r_{f,t}$) it is known at time $t$.

Then we have:

$$\Delta \log C_{t+1} = \mu_i + \psi r_{f,t} + \epsilon_{i,t+1}$$

where

$$\epsilon_{i,t+1} = \Delta \log C_{t+1} - E_t \Delta \log C_{t+1}$$

In this case, OLS would work!

In practice, the real return on even Tbills is uncertain due to inflation.

Could estimate by OLS using TIPS (Treasury Inflation Protected Securities) although sample would be short (TIPS started trading in 1997).
Campbell and Mankiw (1989) estimate:

\[ \Delta \log C_{t+1} = \mu_i + \psi r_{i,t+1} + \epsilon_{i,t+1} \]

using lags of real rates, consumption growth, and nominal rates as instruments (see also Hall (1988))

Complication: \( C_t \) is a time average over a quarter

- Even if \( C_t \) were a random walk, time averaging would imply serial correlation of changes (Working, 1960)
- Campbell and Mankiw (1989) lag instruments by 2 periods to avoid this
Table 3  UNITED STATES, 1953-1986
\[ \Delta c_t = \mu + \sigma r_t \]

<table>
<thead>
<tr>
<th>Row</th>
<th>Instruments</th>
<th>( \Delta c ) equation</th>
<th>( r ) equation</th>
<th>( \sigma ) estimate</th>
<th>Test of restrictions</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>None (OLS)</td>
<td>--</td>
<td>--</td>
<td>0.276</td>
<td>--</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>(0.079)</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>( r_{t-2}, \ldots, r_{t-4} )</td>
<td>0.063</td>
<td>0.431</td>
<td>0.270</td>
<td>0.031</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.009)</td>
<td>(0.000)</td>
<td>(0.118)</td>
<td>(0.029)</td>
</tr>
<tr>
<td>3</td>
<td>( r_{t-2}, \ldots, r_{t-6} )</td>
<td>0.067</td>
<td>0.426</td>
<td>0.281</td>
<td>0.034</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.014)</td>
<td>(0.000)</td>
<td>(0.118)</td>
<td>(0.050)</td>
</tr>
<tr>
<td>4</td>
<td>( \Delta c_{t-2}, \ldots, \Delta c_{t-4} )</td>
<td>0.024</td>
<td>-0.021</td>
<td>-0.707</td>
<td>0.000</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.101)</td>
<td>(0.966)</td>
<td>(2.586)</td>
<td>(0.215)</td>
</tr>
<tr>
<td>5</td>
<td>( \Delta c_{t-2}, \ldots, \Delta c_{t-6} )</td>
<td>0.018</td>
<td>0.007</td>
<td>0.992</td>
<td>0.008</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.007)</td>
<td>(0.316)</td>
<td>(0.478)</td>
<td>(0.189)</td>
</tr>
<tr>
<td>6</td>
<td>( \Delta i_{t-2}, \ldots, \Delta i_{t-4} )</td>
<td>0.061</td>
<td>0.024</td>
<td>1.263</td>
<td>-0.021</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.010)</td>
<td>(0.105)</td>
<td>(0.545)</td>
<td>(0.918)</td>
</tr>
<tr>
<td>7</td>
<td>( \Delta i_{t-2}, \ldots, \Delta i_{t-6} )</td>
<td>0.102</td>
<td>0.028</td>
<td>1.213</td>
<td>-0.022</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.002)</td>
<td>(0.119)</td>
<td>(0.445)</td>
<td>(0.700)</td>
</tr>
<tr>
<td>8</td>
<td>( r_{t-2}, \ldots, r_{t-4}, \Delta c_{t-2}, \ldots, \Delta c_{t+4} )</td>
<td>0.062</td>
<td>0.455</td>
<td>0.204</td>
<td>0.047</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.026)</td>
<td>(0.000)</td>
<td>(0.114)</td>
<td>(0.033)</td>
</tr>
<tr>
<td>9</td>
<td>( r_{t-2}, \ldots, r_{t-4}, \Delta c_{t-2}, \ldots, \Delta c_{t-4} )</td>
<td>0.103</td>
<td>0.476</td>
<td>0.150</td>
<td>0.100</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.006)</td>
<td>(0.000)</td>
<td>(0.111)</td>
<td>(0.005)</td>
</tr>
</tbody>
</table>

Note: The columns labeled “First-stage regressions” report the adjusted R² for the OLS regressions of the two variables on the instruments; in parentheses is the p-value for the null that all the coefficients except the constant are zero. The column labeled “\( \lambda \) estimate” reports the IV estimate of \( \lambda \) and, in parentheses, its standard error. The column labeled “Test of restrictions” reports the adjusted R² of the OLS regression of the residual on the instruments; in parenthesis is the p-value for the null that all the coefficients are zero.

Source: Campbell and Mankiw (1989)
Hall (1988) ran similar specifications. He favored estimates close to zero and interpreted them as estimates of the IES.

Campbell and Mankiw (1989) worry about misspecification due to hand-to-mouth consumers

1. Consumption growth predictable. Should not be true if $\psi = 0$
2. Over-identifying restrictions rejected
3. Estimates very unstable
4. Reverse regression not consistent with $\psi = 0$
Just as the consumption Euler equation implies that

\[ \Delta \log C_{t+1} = \mu_i + \psi r_{i,t+1} + \epsilon_{i,t+1} \]

it also implies that

\[ r_{i,t+1} = \alpha_i + \frac{1}{\psi} \Delta \log C_{t+1} + \eta_{i,t+1} \]

If \( \psi = 0 \), this “reverse regression” should yield a large estimate for \( 1/\psi \)

Under the maintained assumptions above, this “reverse regression” can be estimated using IV with the same set of instruments

This is the specification used by Hansen and Singleton (1983)
Table 4 UNITED STATES, 1953–1986

\[ r_t = \mu + 1/\sigma \Delta c_t \]

<table>
<thead>
<tr>
<th>Row</th>
<th>Instruments</th>
<th>First-stage regressions</th>
<th>1/\sigma estimate (s.e.)</th>
<th>Test of restrictions</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>( \Delta c ) equation</td>
<td>( r ) equation</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>None (OLS)</td>
<td>—</td>
<td>—</td>
<td>0.304 (0.087)</td>
</tr>
<tr>
<td>2</td>
<td>( r_{t-2}, \ldots, r_{t-4} )</td>
<td>0.063 (0.009)</td>
<td>0.431 (0.000)</td>
<td>1.581 (0.486) 0.086 (0.001)</td>
</tr>
<tr>
<td>3</td>
<td>( r_{t-2}, \ldots, r_{t-6} )</td>
<td>0.067 (0.014)</td>
<td>0.426 (0.000)</td>
<td>1.347 (0.390) 0.113 (0.001)</td>
</tr>
<tr>
<td>4</td>
<td>( \Delta c_{t-2}, \ldots, \Delta c_{t-4} )</td>
<td>0.024 (0.101)</td>
<td>-0.021 (0.966)</td>
<td>-0.342 (0.428) -0.021 (0.878)</td>
</tr>
<tr>
<td>5</td>
<td>( \Delta c_{t-2}, \ldots, \Delta c_{t-6} )</td>
<td>0.018 (0.007)</td>
<td>0.007 (0.316)</td>
<td>0.419 (0.258) -0.010 (0.440)</td>
</tr>
<tr>
<td>6</td>
<td>( \Delta i_{t-2}, \ldots, \Delta i_{t-4} )</td>
<td>0.061 (0.010)</td>
<td>0.024 (0.105)</td>
<td>0.768 (0.334) -0.021 (0.919)</td>
</tr>
<tr>
<td>7</td>
<td>( \Delta i_{t-2}, \ldots, \Delta i_{t-6} )</td>
<td>0.102 (0.002)</td>
<td>0.028 (0.119)</td>
<td>0.638 (0.249) -0.024 (0.747)</td>
</tr>
<tr>
<td>8</td>
<td>( r_{t-2}, \ldots, r_{t-4}, \Delta c_{t-2}, \ldots, \Delta c_{t-4} )</td>
<td>0.062 (0.026)</td>
<td>0.455 (0.000)</td>
<td>1.034 (0.333) 0.236 (0.000)</td>
</tr>
<tr>
<td>9</td>
<td>( r_{t-2}, \ldots, r_{t-4}, \Delta c_{t-2}, \ldots, \Delta c_{t-4}, \Delta i_{t-2}, \ldots, \Delta i_{t-4} )</td>
<td>0.103 (0.006)</td>
<td>0.476 (0.000)</td>
<td>0.521 (0.220) 0.455 (0.000)</td>
</tr>
</tbody>
</table>

Note: The columns labeled “First-stage regressions” report the adjusted \( R^2 \) for the OLS regressions of the two variables on the instruments; in parentheses is the p-value for the null that all the coefficients except the constant are zero. The column labeled “\( \lambda \) estimate” reports the IV estimate of \( \lambda \) and, in parentheses, its standard error. The column labeled “Test of restrictions” reports the adjusted \( R^2 \) of the OLS regression of the residual on the instruments; in parenthesis is the p-value for the null that all the coefficients are zero.

Source: Campbell and Mankiw (1989)
What is going on!!

It can’t both be true that:

- $\psi$ is close to zero
- $1/\psi$ is relatively small

Yogo (2004): Puzzle due to weak instruments
Consumption growth notoriously hard to predict!!
Employs first-stage F-stat for weak instruments developed by Stock and Yogo (2003)
Concludes that reverse regression is unreliable due to weak instruments, but regression with real Tbill rate as regressor is reliable
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- Concludes that reverse regression is unreliable due to weak instruments, but regression with real Tbill rate as regressor is reliable
### Table 1.—Test for Weak Instruments

<table>
<thead>
<tr>
<th>Country</th>
<th>Sample Period</th>
<th>Variable</th>
<th>( F )</th>
<th>TSLS Bias</th>
<th>TSLS Size</th>
<th>Fuller-( k )</th>
<th>LIML</th>
</tr>
</thead>
<tbody>
<tr>
<td>USA</td>
<td>1947.3–1998.4</td>
<td>( \Delta c )</td>
<td>2.93</td>
<td>0.93</td>
<td>1.00</td>
<td>0.53</td>
<td>0.37</td>
</tr>
<tr>
<td></td>
<td></td>
<td>( r_f )</td>
<td>15.53</td>
<td>0.00</td>
<td>0.66</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td></td>
<td></td>
<td>( r_e )</td>
<td>2.88</td>
<td>0.93</td>
<td>1.00</td>
<td>0.54</td>
<td>0.39</td>
</tr>
<tr>
<td>AUL</td>
<td>1970.3–1998.4</td>
<td>( \Delta c )</td>
<td>1.79</td>
<td>0.99</td>
<td>1.00</td>
<td>0.81</td>
<td>0.69</td>
</tr>
<tr>
<td></td>
<td></td>
<td>( r_f )</td>
<td>21.81</td>
<td>0.00</td>
<td>0.14</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td></td>
<td></td>
<td>( r_e )</td>
<td>1.82</td>
<td>0.99</td>
<td>1.00</td>
<td>0.80</td>
<td>0.68</td>
</tr>
<tr>
<td>CAN</td>
<td>1970.3–1999.1</td>
<td>( \Delta c )</td>
<td>3.03</td>
<td>0.92</td>
<td>1.00</td>
<td>0.50</td>
<td>0.35</td>
</tr>
<tr>
<td></td>
<td></td>
<td>( r_f )</td>
<td>15.37</td>
<td>0.00</td>
<td>0.67</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td></td>
<td></td>
<td>( r_e )</td>
<td>2.51</td>
<td>0.96</td>
<td>1.00</td>
<td>0.64</td>
<td>0.48</td>
</tr>
<tr>
<td>FR</td>
<td>1970.3–1998.3</td>
<td>( \Delta c )</td>
<td>0.17</td>
<td>1.00</td>
<td>1.00</td>
<td>1.00</td>
<td>1.00</td>
</tr>
<tr>
<td></td>
<td></td>
<td>( r_f )</td>
<td>38.43</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
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<tr>
<td></td>
<td></td>
<td>( r_e )</td>
<td>3.09</td>
<td>0.91</td>
<td>1.00</td>
<td>0.49</td>
<td>0.34</td>
</tr>
<tr>
<td>GER</td>
<td>1979.1–1998.3</td>
<td>( \Delta c )</td>
<td>0.83</td>
<td>1.00</td>
<td>1.00</td>
<td>0.97</td>
<td>0.93</td>
</tr>
<tr>
<td></td>
<td></td>
<td>( r_f )</td>
<td>17.66</td>
<td>0.00</td>
<td>0.45</td>
<td>0.00</td>
<td>0.00</td>
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<td></td>
<td></td>
<td>( r_e )</td>
<td>0.69</td>
<td>1.00</td>
<td>1.00</td>
<td>0.98</td>
<td>0.95</td>
</tr>
<tr>
<td>USA</td>
<td>1970.3–1998.4</td>
<td>( \Delta c )</td>
<td>3.53</td>
<td>0.86</td>
<td>1.00</td>
<td>0.39</td>
<td>0.25</td>
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<td></td>
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<td>( r_f )</td>
<td>11.92</td>
<td>0.02</td>
<td>0.92</td>
<td>0.00</td>
<td>0.00</td>
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<tr>
<td></td>
<td></td>
<td>( r_e )</td>
<td>2.16</td>
<td>0.97</td>
<td>1.00</td>
<td>0.72</td>
<td>0.58</td>
</tr>
<tr>
<td>SWD</td>
<td>1921–1994</td>
<td>( \Delta c )</td>
<td>1.02</td>
<td>1.00</td>
<td>1.00</td>
<td>0.95</td>
<td>0.89</td>
</tr>
<tr>
<td></td>
<td></td>
<td>( r_f )</td>
<td>5.50</td>
<td>0.55</td>
<td>1.00</td>
<td>0.10</td>
<td>0.05</td>
</tr>
<tr>
<td></td>
<td></td>
<td>( r_e )</td>
<td>1.67</td>
<td>0.99</td>
<td>1.00</td>
<td>0.84</td>
<td>0.72</td>
</tr>
<tr>
<td>UK</td>
<td>1921–1994</td>
<td>( \Delta c )</td>
<td>1.93</td>
<td>0.98</td>
<td>1.00</td>
<td>0.78</td>
<td>0.65</td>
</tr>
<tr>
<td></td>
<td></td>
<td>( r_f )</td>
<td>4.87</td>
<td>0.66</td>
<td>1.00</td>
<td>0.16</td>
<td>0.08</td>
</tr>
<tr>
<td></td>
<td></td>
<td>( r_e )</td>
<td>4.18</td>
<td>0.77</td>
<td>1.00</td>
<td>0.26</td>
<td>0.15</td>
</tr>
<tr>
<td>USA</td>
<td>1891–1995</td>
<td>( \Delta c )</td>
<td>1.55</td>
<td>0.99</td>
<td>1.00</td>
<td>0.86</td>
<td>0.76</td>
</tr>
<tr>
<td></td>
<td></td>
<td>( r_f )</td>
<td>2.87</td>
<td>0.93</td>
<td>1.00</td>
<td>0.54</td>
<td>0.39</td>
</tr>
<tr>
<td></td>
<td></td>
<td>( r_e )</td>
<td>1.00</td>
<td>1.00</td>
<td>1.00</td>
<td>0.95</td>
<td>0.90</td>
</tr>
</tbody>
</table>

The table reports the first-stage \( F \)-statistic from a regression of the endogenous variable onto the instruments. The endogenous variables are consumption growth (\( \Delta c \)), real interest rate (\( r_f \)), and real stock return (\( r_e \)). The instruments are twice lagged nominal interest rate, inflation, consumption growth, and log dividend-price ratio. The table also reports the \( p \)-value of the test for weak instruments. The null hypotheses are: (1) the TSLS relative bias is greater than 10%, (2) the size of the 5% TSLS \( t \)-test can be greater than 10%, (3) the Fuller-\( k \) relative bias is greater than 10%, and (4) the size of 5% LIML \( t \)-test can be greater than 10%.

Table 2.—Estimates of the EIS Using the Interest Rate

<table>
<thead>
<tr>
<th>Country</th>
<th>Sample Period</th>
<th>(1/\psi)</th>
<th>(\psi)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>TSLS</td>
<td>Fuller-(k)</td>
</tr>
<tr>
<td>USA</td>
<td>1947.3–1998.4</td>
<td>0.68 (0.48)</td>
<td>3.30 (2.00)</td>
</tr>
<tr>
<td>AUL</td>
<td>1970.3–1998.4</td>
<td>0.50 (0.48)</td>
<td>2.37 (2.45)</td>
</tr>
<tr>
<td>CAN</td>
<td>1970.3–1999.1</td>
<td>-1.04 (0.39)</td>
<td>-2.40 (1.13)</td>
</tr>
<tr>
<td>FR</td>
<td>1970.3–1998.3</td>
<td>-3.12 (3.75)</td>
<td>-1.83 (1.72)</td>
</tr>
<tr>
<td>GER</td>
<td>1979.1–1998.3</td>
<td>-1.05 (0.62)</td>
<td>-1.38 (0.90)</td>
</tr>
<tr>
<td>ITA</td>
<td>1971.4–1998.1</td>
<td>-3.34 (1.98)</td>
<td>-5.82 (4.47)</td>
</tr>
<tr>
<td>JAP</td>
<td>1970.3–1998.4</td>
<td>-0.18 (0.43)</td>
<td>-0.86 (1.23)</td>
</tr>
<tr>
<td>NTH</td>
<td>1977.3–1998.4</td>
<td>-0.53 (0.41)</td>
<td>-1.41 (1.33)</td>
</tr>
<tr>
<td>SWD</td>
<td>1970.3–1999.2</td>
<td>-0.10 (1.10)</td>
<td>-0.21 (1.54)</td>
</tr>
<tr>
<td>SWT</td>
<td>1976.2–1998.4</td>
<td>-1.56 (0.83)</td>
<td>-1.51 (0.79)</td>
</tr>
<tr>
<td>UK</td>
<td>1970.3–1999.1</td>
<td>1.06 (0.45)</td>
<td>3.76 (2.42)</td>
</tr>
<tr>
<td>USA</td>
<td>1970.3–1998.4</td>
<td>0.53 (0.50)</td>
<td>2.19 (2.60)</td>
</tr>
<tr>
<td>SWD</td>
<td>1921–1994</td>
<td>1.17 (1.13)</td>
<td>3.30 (3.34)</td>
</tr>
<tr>
<td>UK</td>
<td>1921–1994</td>
<td>2.40 (1.01)</td>
<td>2.99 (1.33)</td>
</tr>
<tr>
<td>USA</td>
<td>1891–1995</td>
<td>-0.38 (1.12)</td>
<td>-1.17 (2.90)</td>
</tr>
</tbody>
</table>

The reciprocal of the EIS is estimated from \( r_{i,t+1} = \mu + (1/\psi) \Delta c_{i,t+1} + \eta_{i,t+1} \), and the EIS is estimated from \( \Delta c_{i,t+1} = \tau + \psi r_{i,t+1} + \xi_{i,t+1} \). The instruments are the twice lagged nominal interest rate, inflation, consumption growth, and log dividend-price ratio. Standard errors in parentheses.

Weak instruments is not the only empirical challenge!

Above approach relies **heavily** on

$$\Delta \log C_{t+1} = \mu_i + \psi r_{i,t+1} + \epsilon_{i,t+1} \tag{1}$$

being a structural equation

In particular, heavy reliance on $\epsilon_{i,t+1}$ being **only** an expectation error

But what if equation (1) is misspecified?
Suppose $U'(C_t, \eta_t)$ and $\eta_t$ is persistent

$\eta_t$ can be:
- Preference shocks (e.g., sentiment, preference for borrowing)
- Labor supply (i.e., non-separable utility)

If $\eta_t$ is persistent, then $\eta_{t-j}$ will affect both
- Lagged variables being used as instruments
- Current $\eta_t$

This will lead IV with lagged variables to be biased
Suppose we are estimating

$$\Delta \log C_{t+1} = \mu_i + \psi r_{i,t+1} + \epsilon_{i,t+1}$$

by OLS in the presence of preference shocks

- Increased desire to save drives down interest rates, and raises consumption growth
- Implies current interest rates negatively correlated with $\epsilon_{i,t+1}$
- Downward bias in OLS estimate of $\psi$
Suppose we are estimating

$$\Delta \log C_{t+1} = \mu_i + \psi r_{i,t+1} + \epsilon_{i,t+1}$$

by IV with lagged instruments and persistent preference shocks

Increase in desired savings in period $t - j$ will affect instruments at time $t - j$ and also increase desire to save in period $t + 1$

Part of correlation between instruments and $r_{i,t+1}$ due to lagged preference shock

Lagged preference shock lowers $r_{i,t+1}$ and raises $\Delta \log C_{t+1}$ due to affect on current preference shock

Same downward bias as OLS
1. Hand-to-mouth consumers (more generally: liquidity constraints) (see Werning (2015))

2. Time-varying volatility:

$$E_t \Delta \log C_{t+1} = \psi E_t r_{i,t+1} + \psi \log \beta + \frac{1}{2} [\psi \sigma_i^2 + \psi^{-1} \sigma_c^2 - 2\sigma_{ic}]$$

- We have been assuming that all the $\sigma$ terms are constant
- What if they are not?
- Bansal and Yaron (2004): $\psi$ will be downward biased
- Persistent increase in $\sigma_c$ lowers $E_t r_{i,t+1}$ and is part of error term
  (see also Carrol, 1997; Blundell et al., 1994; Guvenen, 2000)

3. Consumption commitments (housing, cars) lead to more complicated consumption Euler equation (Chetty and Szeidl, 2016)
This lagged instrument strategy has been common in macro
  - E.g., Phillips curve estimation

Stems from taking simple structural model extremely literally

Applied micro approach very different:
  - Error term contains all sorts of things
  - We don’t know the true model
  - Want conclusions to be robust to many structural stories

Lagged instrument strategy becoming less common in macro
Most work estimates IES using time-series variation, but hard to find exogenous variation in the time series

Exploits variation in rates of return in the cross-section to identify the IES

After-tax rates of return are influenced by capital tax rates

Exploits exogenous variation in capital tax rates
Most variation in tax rates potentially endogenous (e.g., due to variation in income)

Constructs “simulated” tax rates based on predicted income from exogenous characteristics (e.g., education, age, and sex)

Controls flexibly for these characteristics

Identification comes only from changes in the tax system over the sample period
Estimating the IES

- Specification:

\[ GC_{i,t+1} = \alpha + \beta ATRATE_{it} + X_{it}\delta + \Delta Z_{it,t+1}\eta + \epsilon \]

- GC: Non-durable consumption growth for household \( i \) (from CEX)
- ATRATE: Income specific after tax rate of return for household \( i \) (SCF portfolio shares and NBER TAXSIM tax rates)
- \( X \): vector of baseline demographic characteristics
- \( \Delta Z \): vector of demographic changes
- Includes time and state fixed effects

- IV: Instrument for ATRATE using tax rate based on predicted income for a given demographic group
consumption growth by lowering the degree of consumption uncertainty. As a result, there will be an upward bias to my estimate of the EIS: it will incorporate both the effect of taxes on the rate of return and the effect of taxes on the variance of consumption growth. With only one instrument, the tax rate, I cannot identify both effects. I suggest an approach to addressing this issue below.

Table 1 presents the means of the data. The mean level of non-durable consumption in the data is $25,268, and consumption growth over a nine-month period averages 1.1%.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Mean</th>
<th>Standard Deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Non-Durable Consumption</td>
<td>25,268</td>
<td>15,306</td>
</tr>
<tr>
<td>Consumption Growth</td>
<td>0.011</td>
<td>0.394</td>
</tr>
<tr>
<td>After-Tax Interest Rate</td>
<td>0.039</td>
<td>0.049</td>
</tr>
<tr>
<td>Capital Income Marginal Tax Rate</td>
<td>0.236</td>
<td>0.094</td>
</tr>
</tbody>
</table>

4. Results

4.1. Basic results

Table 2 presents the results of estimating Eq. (1). The regression includes all the control variables described above, but I show here only the coefficient of interest, that on the interest rate.

<table>
<thead>
<tr>
<th></th>
<th>After-Tax T-Bill Rate</th>
<th>After-Tax Rate of Return</th>
</tr>
</thead>
<tbody>
<tr>
<td>OLS, no year dummies</td>
<td>-0.551</td>
<td>0.105</td>
</tr>
<tr>
<td></td>
<td>(0.116)</td>
<td>(0.032)</td>
</tr>
<tr>
<td>Lag IV, no year dummies</td>
<td>2.616</td>
<td>0.328</td>
</tr>
<tr>
<td></td>
<td>(0.490)</td>
<td>(0.130)</td>
</tr>
<tr>
<td>Tax IV</td>
<td>2.032</td>
<td>2.239</td>
</tr>
<tr>
<td></td>
<td>(0.796)</td>
<td>(0.894)</td>
</tr>
<tr>
<td>Number Obs.</td>
<td>66,314</td>
<td>66,208</td>
</tr>
</tbody>
</table>

Notes: Estimates from models such as Eq. (1) in text. Each cell represents the estimated EIS from a separate model: first column uses after-tax T-bill rate, while second column uses weighted average after-tax rate of return. Standard errors in parentheses.

Source: Gruber (2013)
No consensus in the literature!!

Macro people often use IES < 1, influenced by Hall (1988)

Asset pricing people often use IES > 1 because values < 1 yield counter-intuitive responses of asset prices to shocks

- With IES < 1, bad news about future growth increases stock prices because of strong desire to save
Simplest form of inference:

- Run regression in which one of the coefficients may be interpreted as direct causal evidence of parameter in question (e.g., IES)

- Often what we can measure is not directly what we are interested in estimating

- Such statistics can, however, often yield powerful inference about what we are interested in if viewed through the lens of a structural model (provide tell-tale signs about parameter of interest)

(See Nakamura-Steinsson (2018) for more discussion of this idea.)
Use “mortgage notches” in the UK to shed light on IES

Statistics they calculate do not provide direct estimates of IES

But viewed through sensible structural models, they (arguably) provide powerful inference about IES

How to make convincing inference through the lens of a structural model is a complicated art
  - Which model to use?
  - How robust are the conclusions?

Best et al. paper is a good example of this
UK mortgages have low fixed rates for 2, 3, or 5 years then much higher flexible rates.

High penalty for refinancing early.

Most people refinance at the time of interest rate reset.

Authors focus on refinancers so as to abstract from the effect of mortgage size on home size.
**Figure A.1: Refinancing Happens When the Reset Rate Kicks In**

Notes: The figure shows the distribution of the time to refinance, excluding individuals where the date on which the reset rate kicks in is unobserved. The figure shows individuals who refinance more than 6 months after their reset rate kicks in in black, individuals who refinance more than 2 months before their reset rate kicks in in white, and the remainder who refinance around their reset date in gray.

Source: Best-Cloyne-Ilzetzki-Kleven (2019)
UK Mortgage Market

- Interest rate jumps by discrete amounts (features notches) at certain loan-to-value (LTV) thresholds
- Very salient: daily menu in newspapers, on bank websites, etc.
- Estimate rate function:
  \[ r_i = f(LTV_i) + \beta_1 \text{bank}_i + \beta_2 \text{variability}_i \otimes \text{duration}_i \otimes \text{month}_i + \beta_3 \text{repayment}_i + \beta_4 \text{term}_i + \nu_i \]
- No individual characteristics because UK mortgage market is like a supermarket (no individual negotiation)
- But adding age, income and family status has no effect on results
Figure 3
Interest rate jumps at notches

Notes: This figure shows the conditional interest rate as a function of the LTV ratio from the non-parametric regression (2.1). In each LTV bin, we plot the coefficient on the LTV bin dummy plus a constant given by the mean predicted value $E[\hat{r}_i]$ from all the other covariates (i.e. omitting the contribution of the LTV bin dummies). The figure shows that the mortgage interest rate evolves as a step function with sharp notches at LTV ratios of 60%, 70%, 75%, 80%, and 85%.

Source: Best-Cloyne-Ilzetzki-Kleven (2019)
A fundamental difficulty in addressing this question is how to find exogenous variation in interest rates. Most studies rely on time series movements in interest rates, which are gradual and almost certainly endogenous to unobserved factors that affect consumption. Our starting point is a novel source of quasi-experimental variation in interest rates arising from the fact that U.K. banks offer notched mortgage interest schedules. That is, the mortgage interest rate features discrete jumps at critical thresholds for the loan-to-value (LTV) ratio. For example, the interest rate increases by almost 0.5pp on the entire loan when crossing the 80% LTV threshold. This creates very strong incentives to reduce borrowing to a level below the notch, thereby giving up consumption today in order to get a lower interest rate and more consumption in the future. Intuitively, the magnitude of such borrowing and consumption responses to interest rate notches is governed by the value of the EIS.

Our study is based on administrative mortgage data from the Financial Conduct Authority (FCA). The data cover the universe of household mortgages in the U.K. between 2008 and 2014, including rich information on mortgage contracts and borrower characteristics. The majority of U.K. mortgage products carry a relatively low interest rate for a period of 2–5 years after which a much higher reset rate kicks in, creating strong incentives to refinance at the time the reset rate starts to apply. This makes refinancing a common occurrence in the U.K. We focus on the population of refinancers, because they allow for a clean assessment of borrowing and intertemporal consumption choices. Specifically, because housing choices are pre-determined for refinancers, estimating LTV responses in this sample allows us to isolate borrowing choices from housing choices.

Figure 1 plots the LTV distribution for U.K. home refinancers around the different interest rate notches, depicted by vertical lines. There is large and sharp bunching below every notch along with missing mass above every notch, which provides direct evidence that borrowers respond to interest rates. A recent literature in public economics has developed approaches to translate such bunching moments into reduced-form price elasticities, mostly focusing on behavioural responses.

**Figure 1**

**Observed LTV distribution among U.K. refinancers**

*Notes:* This figure shows the observed distribution of LTV ratios among refinancers in the U.K. between 2008–14. There are interest rate notches at LTV ratios of 60%, 70%, 75%, 80%, 85%, and 90% (depicted by vertical lines).

Bunching and the IES

- Large amount of bunching below interest rate notches
- Intuitively, this is informative about IES:
  - Households must cut consumption to get below notch
  - How willing are households to cut consumption now to raise life-time consumption?
Two Challenges:

1. Need to translate bunching into IES estimate
   - What is counterfactual?
   - Is observed bunching a lot or a little?

2. Many other features of reality affect bunching
   - Patience
   - Demand for buffer stock savings (i.e., income risk and risk aversion)
   - Frictions to household optimization
Bunching and the IES

- Authors write down a structural model
- Ask for what parameter values model can match bunching in the data
- Bunching highly sensitive to the IES
- Relatively insensitive to reasonable variation in other parameters
This is a good example of the use of structural estimation

The moments being used are shown to be highly informative about something specific in the model and are used to estimate that thing

Often structural estimation is a big black box with lots of moments estimating lots of parameters without a clear sense of what identifies what
Notes: This figure shows the two steps in the construction of the counterfactual LTV distribution among refinancers. Each panel shows the actual LTV distribution with dots (as in Figure 1). Panel A shows the distribution of passive LTVs with crosses, calculated based on the LTV of the previous mortgage, amortization, and the house value at the time of refinancing. Panel B shows the distribution of counterfactual LTVs with crosses, which adjusts passive LTVs for the average equity extraction of non-bunchers in the actual distribution.

Source: Best-Cloyne-Ilzetzki-Kleven (2019)
Simple Structural Model

- Two periods (0 and 1)
- Households have decided to stay in current home
- Face a refinancing decision
- Utility:
  \[
  \frac{\sigma}{\sigma - 1} \left( \frac{c_0^{(\sigma-1)/\sigma}}{\sigma} + \delta c_1^{(\sigma-1)/\sigma} \right)
  \]
  where \(\delta\) is discount factor and \(\sigma\) is IES
- Budget constraints:
  \[
  c_0 = y_0 + W_0 - (1 - \lambda)P_0H
  \]
  \[
  c_1 = y_1 + R\lambda P_0H + (1 - d)P_1H
  \]
Authors simulate model for different values of IES

Distribution of $W_0$ is calibrated to replicate counterfactual LTV distribution when $R$ is constant

Other parameters are calibrated to “reasonable” values:

\[
\delta = 0.96
\]

\[
d = 0.025
\]

\[
\frac{P_1}{P_0} = 1.026
\]

\[
y_1 = y_0
\]
Figure 6 compares the observed LTV distribution to simulated LTV distributions under four different EIS scenarios. The other parameters of the model are assigned reasonable values that do not vary across the different EIS scenarios. The distribution of initial wealth $W_0$ is calibrated using equation (3.5) in order to replicate the counterfactual LTV distribution shown in Figure 4. In this counterfactual scenario, we assume that each borrower faces a flat interest rate $R$ given by

\[ R = \text{constant} \]

Specifically, the discount factor is set at an annual rate of $\delta = 0.96$ (a common value in the literature), real house price growth is set at an annual rate of $P_1/P_0 = 1.026$ (the historical average in the U.K.), the depreciation rate is set at $d = 0.025$ (taken from the literature), while for simplicity real income is assumed to be constant over time $y_1 = y_0$. 

Source: Best-Cloyne-Ilzetzki-Kleven (2019)
But could it be that other values of the other parameters could justify a large IES?

Authors set IES = 1 and then vary other parameters to maximize fit at notches

Varying other parameters cannot give good fit with IES = 1 even allowing for very unreasonable values for other parameters
Panel B: $\sigma = 1$; Calibrated $\delta, y, P$

Source: Best-Cloyne-Ilzetzki-Kleven (2019). Parameter values: $\delta = 0.24$, $P_1/P_0 = 0.88$, $y_1 = 0.58y_0$
Precautionary Savings

- Adjustment to interest rate notches affected by demand for buffer stock savings

- Simple structural model doesn’t capture this (no risk)

- But simple structural model provides an upper bound:
  - Makes extreme assumption of no liquid wealth
  - All adjustment borne by consumption

- Addition of liquid wealth and precautionary savings would add an adjustment margin

- Even lower IES needed to justify small amount of bunching
Most important “treat to identification” is frictions to household optimization (inattention, inertia, myopia)

Estimate fraction of non-optimizers as those in dominated region (right above notch) relative to counterfactual

Redoes model simulations assuming this fraction of non-optimizers
Notes: This figure shows the two steps in the construction of the counterfactual LTV distribution among refinancers. Each panel shows the actual LTV distribution with dots (as in Figure 1). Panel A shows the distribution of passive LTVs with crosses, calculated based on the LTV of the previous mortgage, amortization, and the house value at the time of refinancing. Panel B shows the distribution of counterfactual LTVs with crosses, which adjusts passive LTVs for the average equity extraction of non-bunchers in the actual distribution.

Source: Best-Cloyne-Ilzetzki-Kleven (2019)
Figure A.7: Observed vs Simulated LTV Distributions With Friction Adjustment

Panel A: $\sigma = 0.12$

Panel B: $\sigma = 0.5$

Panel C: $\sigma = 1$

Panel D: $\sigma = 2$

Notes: The figure shows simulations of a model introduced in Section 3 for a range of EIS values. The simulations include a friction adjustment so that a fraction $\alpha$ of non-bunching households are assumed to be “non-optimizers”, who behave as though they face the counterfactual interest rate schedule (and thus choose the corresponding counterfactual LTV). The blue lines show the predicted LTV distribution from the model. The black lines show the empirical LTV distribution. The upper left hand corner has $\sigma = 0.12$, which is the EIS that minimizes the MSE of the predicted bunching masses. Higher EIS values predict far greater bunching masses than found in the data, with a large share of households jumping more than one notch in the LTV distribution to exploit lower interest charges. The distribution largely hollows out between notches, in contrast to the data.

Source: Best-Cloyne-Ilzetzki-Kleven (2019)
TABLE 3

Bounding optimization frictions and the EIS

<table>
<thead>
<tr>
<th>Panel A: Adjustment factor $a$</th>
<th>70</th>
<th>75</th>
<th>80</th>
<th>85</th>
<th>Pooled</th>
</tr>
</thead>
<tbody>
<tr>
<td>(1) Notched banks only</td>
<td>0.11</td>
<td>0.15</td>
<td>0.15</td>
<td>0.03</td>
<td>0.12</td>
</tr>
<tr>
<td></td>
<td>(0.02)</td>
<td>(0.02)</td>
<td>(0.03)</td>
<td>(0.01)</td>
<td>(0.01)</td>
</tr>
<tr>
<td>(2) Dominated region</td>
<td>0.21</td>
<td>0.30</td>
<td>0.15</td>
<td>0.08</td>
<td>0.22</td>
</tr>
<tr>
<td></td>
<td>(0.02)</td>
<td>(0.03)</td>
<td>(0.02)</td>
<td>(0.03)</td>
<td>(0.01)</td>
</tr>
<tr>
<td>(3) Entire hole</td>
<td>0.67</td>
<td>0.60</td>
<td>0.57</td>
<td>0.40</td>
<td>0.61</td>
</tr>
<tr>
<td></td>
<td>(0.05)</td>
<td>(0.02)</td>
<td>(0.04)</td>
<td>(0.09)</td>
<td>(0.02)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Panel B: Elasticity of intertemporal substitution $\sigma$</th>
<th>70</th>
<th>75</th>
<th>80</th>
<th>85</th>
<th>Pooled</th>
</tr>
</thead>
<tbody>
<tr>
<td>(4) Unadjusted</td>
<td>0.02</td>
<td>0.08</td>
<td>0.06</td>
<td>0.11</td>
<td>0.05</td>
</tr>
<tr>
<td></td>
<td>(0.00)</td>
<td>(0.01)</td>
<td>(0.01)</td>
<td>(0.04)</td>
<td>(0.01)</td>
</tr>
<tr>
<td>(5) Dominated region: notched banks only</td>
<td>0.02</td>
<td>0.11</td>
<td>0.08</td>
<td>0.11</td>
<td>0.07</td>
</tr>
<tr>
<td></td>
<td>(0.00)</td>
<td>(0.01)</td>
<td>(0.02)</td>
<td>(0.04)</td>
<td>(0.01)</td>
</tr>
<tr>
<td>(6) Dominated region: all banks</td>
<td>0.03</td>
<td>0.17</td>
<td>0.08</td>
<td>0.13</td>
<td>0.09</td>
</tr>
<tr>
<td></td>
<td>(0.00)</td>
<td>(0.02)</td>
<td>(0.02)</td>
<td>(0.05)</td>
<td>(0.01)</td>
</tr>
<tr>
<td>(7) All mass in the hole is friction</td>
<td>0.16</td>
<td>0.50</td>
<td>0.31</td>
<td>0.30</td>
<td>0.30</td>
</tr>
<tr>
<td></td>
<td>(0.05)</td>
<td>(0.07)</td>
<td>(0.08)</td>
<td>(8.53)</td>
<td>(0.03)</td>
</tr>
</tbody>
</table>

Notes: The table shows how the estimated EIS is affected by assumptions on optimization frictions. The top panel of the table shows the friction adjustment factor $a$ estimated in three different cases. Row (1) shows the friction adjustment based on mass in the dominated region using only notched banks, row (2) shows the friction adjustment based on mass in the dominated region using all banks (our baseline estimates), while row (3) shows the friction adjustment assuming that all mass in the hole is due to friction. The bottom panel of the table shows the estimated EIS when not adjusting for optimization friction (in row (4)), and when adjusting for friction using each of the three measures provided in the top panel (in rows (5)–(7)). As explained in the main text of the article, the EIS estimates provided in rows (4) or (5) are in general lower bounds, whereas the EIS estimate provided in row (7) is an upper bound. The upper bound is based on the extreme assumption that all density mass in the hole—not just the mass in the much narrower dominated region—can be explained by friction rather than by heterogeneity in true preferences (i.e. true preferences are assumed to be homogeneous in the population).

Source: Best-Cloyne-Ilzetzki-Kleven (2019)