

# EXCHANGE RATE MODELS AND SPURIOUS REGRESSIONS

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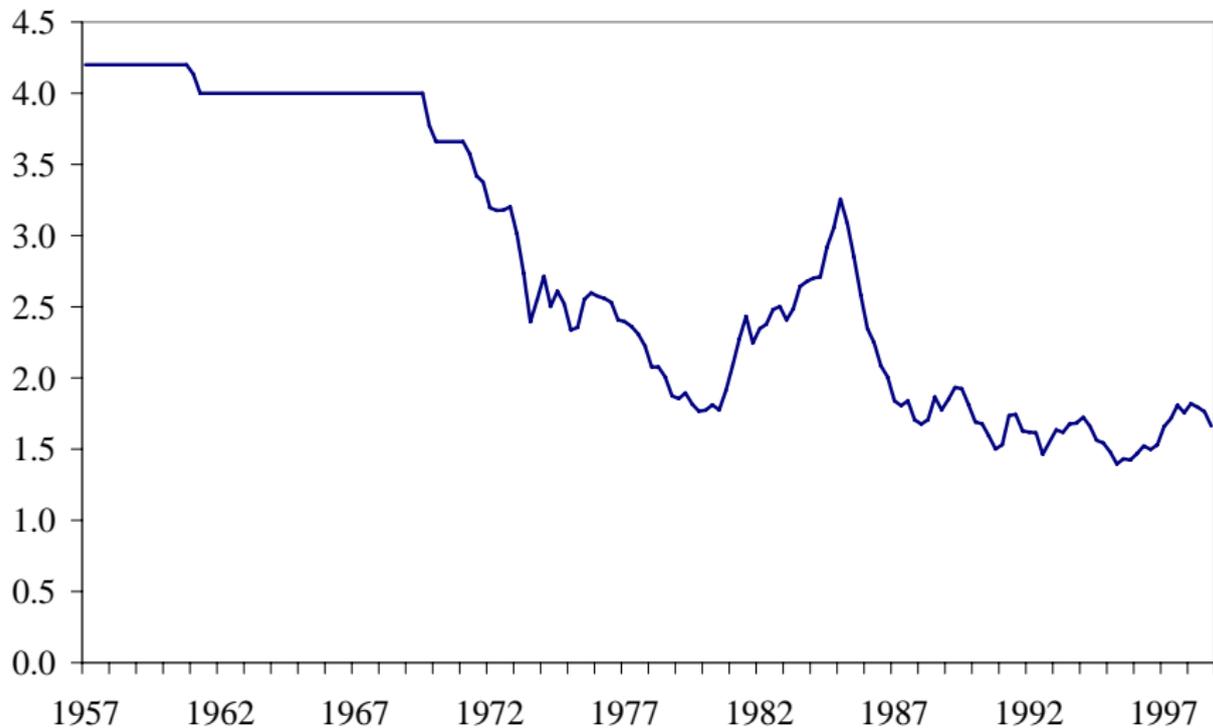
UC Berkeley

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“One of the most remarkable facts about G3 exchange rates is that they are so seemingly immune to systematic empirical explanation.”

– Kenneth Rogoff

## DEM/USD Exchange Rate



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- Exchange rate and “fundamentals”:

$$e_t = (m_t - m_t^*) - \phi_y (y_t - y_t^*) + \phi_i (i_t - i_t^*)$$

- Sample: German Mark, February 1920 - November 1923.
- Hyperinflation: Ignore a bunch of terms.

$$e_t = (m_t - m_t^*) - \phi_y(y_t - y_t^*) + \phi_i(i_t - i_t^*)$$

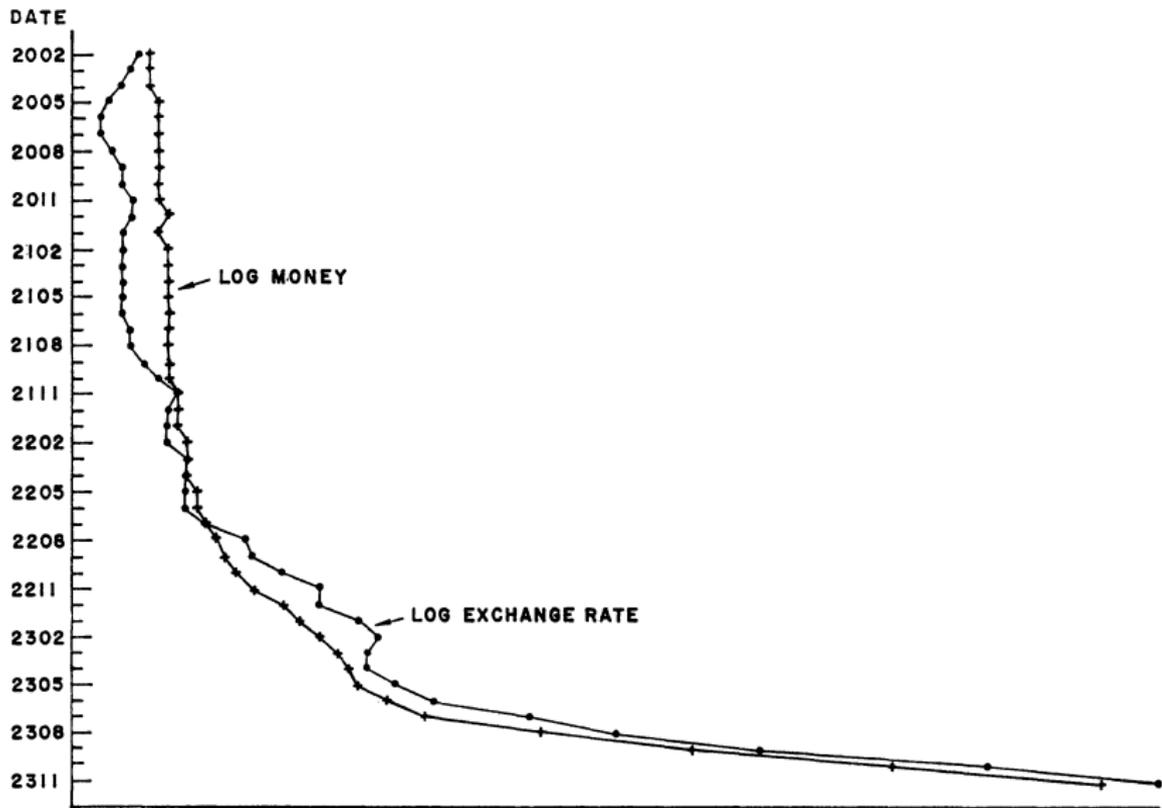
$$e_t = m_t - \phi_i(i_t - i_t^*)$$

$$\log S = -5.135 + 0.975 \log M + 0.591 \log \pi$$

**(0.731) (0.050) (0.073)**

$$R^2 = 0.994; \text{ s.e.} = 0.241; \text{ D.W.} = 1.91.$$

Source: Frenkel (1976).



*Fig. 1.*  
Source: Frenkel (1976).

- Sample: DEM/USD, July 1974 - February 1978.

$$\begin{aligned} e_t = & \phi_0 + \phi_m(m_t - m_t^*) - \phi_y(y_t - y_t^*) \\ & + \phi_i(i_t - i_t^*) + \phi_\pi(\pi_t^e - \pi_t^{e*}) + \epsilon_t \end{aligned}$$

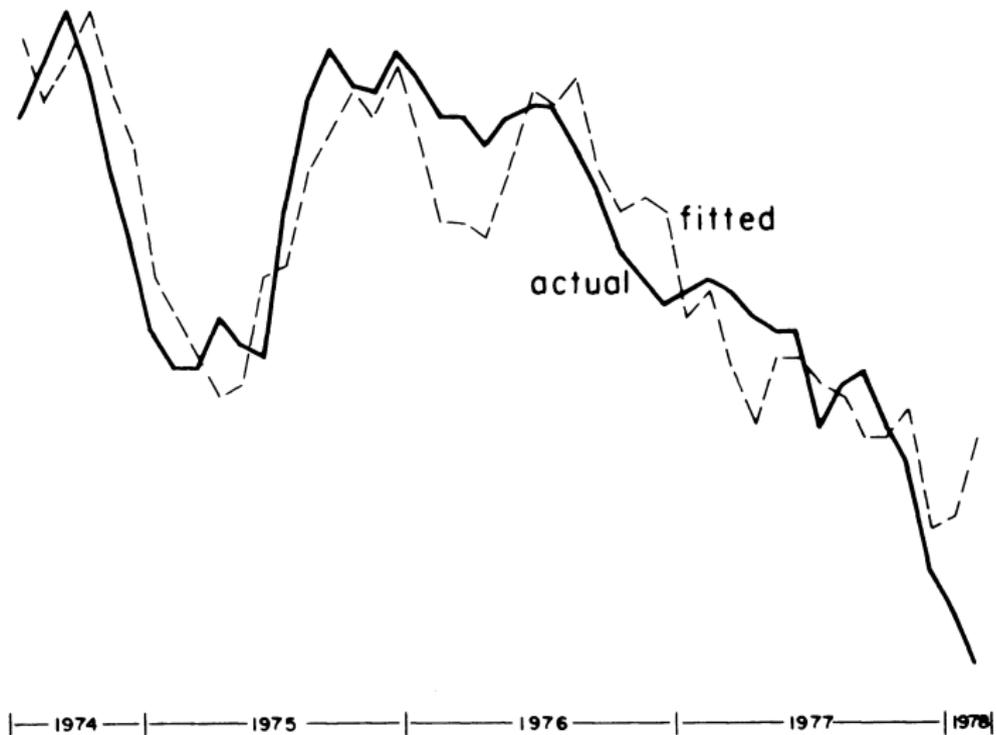


FIGURE 1. PLOT OF (*log OF*) MARK/DOLLAR RATE,  
*OLS* REGRESSION FROM TABLE 1

Source: Frankel (1979).

TABLE 1—TEST OF REAL INTEREST DIFFERENTIAL HYPOTHESIS  
(Sample: July 1974–February 1978)

Technique	Constant	$m - m_1^*$	$y - y^*$	$r - r^*$	$\pi - \pi^*$	$R^2$	$D.W.$	$\hat{\rho}$	Number of Observations
<i>OLS</i>	1.33 (.10)	.87 (.17)	-.72 (.22)	-1.55 (1.94)	28.65 (2.70)	.80	.76		44
<i>CORC</i>	.80 (.19)	.31 (.25)	-.33 (.20)	-.259 (1.96)	7.72 (4.47)	.91		.98	43
<i>INST</i>	1.39 (.08)	.96 (.14)	-.54 (.18)	-4.75 (1.69)	27.42 (2.26)		1.00		42
<i>FAIR</i>	1.39 (.12)	.97 (.21)	-.52 (.22)	-5.40 (2.04)	29.40 (3.33)			.46	41

Note: Standard errors are shown in parentheses.

Definitions: Dependent Variable (*log of*) Mark/Dollar Rate.

*CORC* = Iterated Cochrane-Orcutt.

*INST* = Instrumental variables for expected inflation differential are Consumer Price Index (*CPI*) inflation differential (average for past year), industrial Wholesale Price Index (*WPI*) inflation differential (average for past year), and long-term commercial bond rate differential.

*FAIR* = Instrumental variables are industrial *WPI* inflation differential and lagged values of the following: exchange rate, relative industrial production, short-term interest differential, and expected inflation differential. The method of including among the instruments lagged values of all endogenous and included exogenous variables, in order to insure consistency while correcting for first-order serial correlation, is attributed to Ray Fair.

$m - m^*$  = *log of* German  $M_1/U.S. M_1$

$y - y^*$  = *log of* German production/*U.S. production*

$r - r^*$  = Short-term German-*U.S.* interest differential

$(r - r^*)_{-1}$  = Short-term German-*U.S.* interest differential lagged

$\pi - \pi^*$  = Expected German-*U.S.* inflation differential, proxied by long-term government bond differential.

Source: Frankel (1979).

- Do the monetary models of exchange rates fit out of sample?

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- Generalized monetary model:

$$\begin{aligned} e_t = & \phi_0 + \phi_m(m_t - m_t^*) + \phi_y(y_t - y_t^*) + \phi_i(i_t - i_t^*) \\ & + \phi_\pi(\pi_t^e - \pi_t^{e*}) + \phi_{TB}TB_t + \phi_{TB^*}TB_t^* + \epsilon_t \end{aligned}$$

- Auto-regressive model

$$\mathbf{e}_t = \phi_0 + \sum_{j=1}^J \phi_j \mathbf{e}_{t-j} + \epsilon_t$$

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$$\mathbf{e}_t = \phi_0 + \sum_{j=1}^J \phi_j \mathbf{e}_{t-j} + \sum_{j=1}^J \Phi_j \mathbf{X}_{t-j} + \epsilon_t$$

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- Random Walk model

$$E_t \mathbf{e}_{t+j} = \mathbf{e}_t$$

# MEESE AND ROGOFF (1983)

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- Forecasts based on rolling regression starting November 1976
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- Forecasts based on rolling regression starting November 1976
- Forecast horizons: 1, 6 and 12 months
- Measure of out-of-sample accuracy: RMSE

$$\left\{ \sum_{s=0}^{N_k-1} [F(t+s+k) - A(t+s+k)]^2 / N_k \right\}^{1/2}$$

In structural models:

- Use actual realized future values of explanatory variables (as opposed to also forecasting explanatory variables)

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Two possible stories:

- Hard to predict exchange rate because it is hard to predict variables that it depends on

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Two possible stories:

- Hard to predict exchange rate because it is hard to predict variables that it depends on
- Hard to find any systematic relationship between exchange rates and other variables

Table 1  
Root mean square forecast errors.<sup>a</sup>

Model:	Random walk	Forward rate	Univariate autoregression	Vector autoregression	Frenkel–Bilson <sup>b</sup>	Dornbusch–Frankel <sup>b</sup>	Hooper–Morton <sup>b</sup>	
Exchange rate	Horizon							
\$/mark	1 month	3.72	3.20	3.51	5.40	3.17	3.65	3.50
	6 months	8.71	9.03	12.40	11.83	9.64	12.03	9.95
	12 months	12.98	12.60	22.53	15.06	16.12	18.87	15.69
\$/yen	1 month	3.68	3.72	4.46	7.76	4.11	4.40	4.20
	6 months	11.58	11.93	22.04	18.90	13.38	13.94	11.94
	12 months	18.31	18.95	52.18	22.98	18.55	20.41	19.20
\$/pound	1 month	2.56	2.67	2.79	5.56	2.82	2.90	3.03
	6 months	6.45	7.23	7.27	12.97	8.90	8.88	9.08
	12 months	9.96	11.62	13.35	21.28	14.62	13.66	14.57
Trade-weighted dollar	1 month	1.99	N.A.	2.72	4.10	2.40	2.50	2.74
	6 months	6.09	N.A.	6.82	8.91	7.07	6.49	7.11
	12 months	8.65	14.24	11.14	10.96	11.40	9.80	10.35

<sup>a</sup>Approximately in percentage terms.

<sup>b</sup>The three structural models are estimated using Fair's instrumental variable technique to correct for first-order serial correlation.

(\$/Mark 1-month number should be 3.17 not 3.72, see Table 3)

Source: Meese and Rogoff (1983).

- Nothing beats random walk out of sample

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- Stronger than just lack of predictability  
(since they use realized future values of explanatory variables)
- Nothing even explains exchange rates!!!

Rogoff (2001) recounts:

*For a long time, no one did believe us. The editor of the American Economic Review (Robert Clower) sent our manuscript back in return mail with a scathing letter saying that the results are obviously garbage and if we wish to remain in the economics profession, we had better develop a more positive attitude. ... One then young and now pre-eminent MIT macroeconomist, when told the findings, forcefully commented (with a French accent) "You just cannot possibly have done it right."*

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As of April 2019: 4776 Google scholar citations

## 1. Economics lesson:

- Exchange rate dominated by unpredictable shocks (unpredictable capital flows?)
- Exchange rate very forward looking variable

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- Exchange rate dominated by unpredictable shocks (unpredictable capital flows?)
- Exchange rate very forward looking variable

## 2. Econometric lesson:

- Beware regressing very persistent variable on another very persistent variable

- Uncovered interest rate parity:

$$i_t = i_t^* + E_t e_{t+1} - e_t$$

- Returns should be equalized across countries
- If interest rate is higher abroad, exchange rate should fall enough on average to equalize returns  
( $e_t$  is domestic currency price of foreign currency)

Rearranging and solving forward:

$$i_t = i_t^* + E_t e_{t+1} - e_t$$

$$e_t = (i_t^* - i_t) + E_t e_{t+1}$$

$$e_t = (i_t^* - i_t) + \sum_{j=1}^{\infty} E_t (i_{t+j}^* - i_{t+j}) + \lim_{j \rightarrow \infty} E_t e_{t+j}$$

What determines the change in the exchange rate:

$$e_{t+1} - e_t = -(i_t^* - i_t) + \sum_{j=1}^{\infty} \Delta E_{t+1}(i_{t+j}^* - i_{t+j}) + \lim_{j \rightarrow \infty} \Delta E_{t+1} e_{t+j}$$

where  $\Delta E_{t+1} x_{t+j} = E_{t+1} x_{t+j} - E_t x_{t+j}$  (time  $t + 1$  news about  $x_{t+j}$ )

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- Two components:
  - Current interest rate differential
  - News about all future interest rate differentials
- Not so implausible that the variance of the latter is huge compared to the former

# SIMPLIFIED ENGEL AND WEST (2005)

$$\mathbf{e}_{t+1} - \mathbf{e}_t = -(\mathbf{i}_t^* - \mathbf{i}_t) + \sum_{j=1}^{\infty} \Delta E_{t+1}(\mathbf{i}_{t+j}^* - \mathbf{i}_{t+j}) + \lim_{j \rightarrow \infty} \Delta E_{t+1} \mathbf{e}_{t+j}$$

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- But  $(\mathbf{i}_t^* - \mathbf{i}_t)$  not only thing observed
- Movements in longer-term bonds allow one to back out estimates of

$$\sum_{j=1}^{\infty} \Delta E_{t+1}(\mathbf{i}_{t+j}^* - \mathbf{i}_{t+j})$$

at least up to  $j = 40$  quarters (and assuming EHTS)

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- $\lim_{j \rightarrow \infty} \Delta E_{t+1} e_{t+j}$  still a potential problem
- But in real terms PPP should hold in the very long run (Clarida-Luo 14; Engel 15)

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- Why was Frankel's in-sample inference so much stronger than Meese-Rogoff's out-of-sample inference?
- Suggests that something is wrong with in-sample inference (This is a general concern)

- Monetary model of exchange rate:

$$e_t = \phi_0 + \phi_f f_t + \epsilon_t$$

- Both  $e_t$  and  $f_t$  have a unit-root.
- Granger and Newbold (1974):
  - Usual methods massively understate standard errors

As a preliminary, we looked at the regression

$$Y_t = \beta_0 + \beta_1 X_t,$$

where  $Y_t$  and  $X_t$  were, in fact, generated as *independent* random walks each of length 50. Table 1 shows values of

$$S = \frac{|\hat{\beta}_1|}{\widehat{S.E.}(\hat{\beta}_1)},$$

the customary statistic for testing the significance of  $\beta_1$ , for 100 simulations.

Table 1  
Regressing two independent random walks.

$S$ :	0-1	1-2	2-3	3-4	4-5	5-6	6-7	7-8
Frequency:	13	10	11	13	18	8	8	5
$S$ :	8-9	9-10	10-11	11-12	12-13	13-14	14-15	15-16
Frequency:	3	3	1	5	0	1	0	1

Source: Granger and Newbold (1974).

Table 2

Regressions of a series on  $m$  independent 'explanatory' series.

Series either all random walks or all A.R.I.M.A. (0, 1, 1) series, or changes in these.  $Y_0 = 100$ ,  $Y_t = Y_{t-1} + a_t$ ,  $Y_t' = Y_t + kb_t$ ;  $X_{j,0} = 100$ ,  $X_{j,t} = X_{j,t-1} + a_{j,t}$ ,  $X_{j,t}' = X_{j,t} + kb_{j,t}$ ;  $a_{j,t}, a_t, b_t, b_{j,t}$  sets of independent  $N(0, 1)$  white noises.  $k = 0$  gives random walks,  $k = 1$  gives A.R.I.M.A. (0, 1, 1) series.  $H_0 =$  no relationship, is true. Series length = 50, number of simulations = 100,  $\bar{R}^2 =$  corrected  $R^2$ .

		Per cent times $H_0$ rejected <sup>a</sup>	Average Durbin-Watson $d$	Average $\bar{R}^2$	Per cent $\bar{R}^2 > 0.7$
<i>Random walks</i>					
Levels	$m = 1$	76	0.32	0.26	5
	$m = 2$	78	0.46	0.34	8
	$m = 3$	93	0.55	0.46	25
	$m = 4$	95	0.74	0.55	34
	$m = 5$	96	0.88	0.59	37
Changes	$m = 1$	8	2.00	0.004	0
	$m = 2$	4	1.99	0.001	0
	$m = 3$	2	1.91	-0.007	0
	$m = 4$	10	2.01	0.006	0
	$m = 5$	6	1.99	0.012	0
<i>A.R.I.M.A. (0, 1, 1)</i>					
Levels	$m = 1$	64	0.73	0.20	3
	$m = 2$	81	0.96	0.30	7
	$m = 3$	82	1.09	0.37	11
	$m = 4$	90	1.14	0.44	9
	$m = 5$	90	1.26	0.45	19
Changes	$m = 1$	8	2.58	0.003	0
	$m = 2$	12	2.57	0.01	0
	$m = 3$	7	2.53	0.005	0
	$m = 4$	9	2.53	0.025	0
	$m = 5$	13	2.54	0.027	0

<sup>a</sup>Test at 5% level, using an overall test on  $\bar{R}^2$ .

Source: Granger and Newbold (1974).

- Two common responses:
  - Use HAC standard errors (e.g., Newey-West, 1987)
  - Series are persistent but don't have a unit root.

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  - Use HAC standard errors (e.g., Newey-West, 1987)
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- Granger, Hyung, and Jeon (2001)

$$X_t = \alpha + \beta Y_t + u_t$$

$$X_t = \theta_x X_{t-1} + \epsilon_{x,t}$$

$$Y_t = \theta_y Y_{t-1} + \epsilon_{y,t}$$

Table 1. *Regressing between two independent AR series ( $\theta = \theta_x = \theta_y$ ), percentage of  $|t| > 1.96$*

Method	NOBS	$\theta = 0$	$\theta = 0.25$	$\theta = 0.5$	$\theta = 0.75$	$\theta = 0.9$	$\theta = 1.0$
OLS	100	5.3	7.6	13.3	29.1	51.5	77.0
	500	5.8	7.5	16.3	31.5	51.6	90.0
	2 000	5.8	7.1	13.5	29.4	52.5	94.5
	10 000	4.3	6.6	12.2	30.6	52.3	97.6
	$\infty$	5.0	7.0	13.0	30.0	53.0	100.0
BART	100	7.6	7.7	9.9	16.5	30.6	62.0
	500	6.4	6.8	9.0	14.1	23.9	79.6
	2 000	6.0	5.9	6.1	10.3	16.3	86.4
	10 000	4.6	5.2	5.5	7.7	12.8	92.5
	$\infty$	5.0	5.0	5.0	5.0	5.0	100.0

Notes: 1. The number of iteration = 1000.

2. % of rejection, i.e., absolute value of  $t$ -value  $> 1.96$ .

3.  $\infty$  means asymptotic case.

4. To avoid the problem of fixing  $X_0$  and  $Y_0$ , 100 pre-samples are generated and let  $X_{-100} = Y_{-100} = 0$ .

5. The number of rejections (BART) depends on the number of lags ( $l$ ) used to calculate  $\hat{v}$ .  $l = \text{integer } [4(T/100)^{1/4}]$  is set.

Source: Granger, Hyung, and Jeon (2001).

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- Big problem even if series are stationary if they are pretty persistent and sample is small
- Newey-West standard errors have very bad small sample properties
- Accurate standard errors require more sophisticated methods
- Lazarus-Lewis-Stock 18 suggest improvements
- Even this not so good. No really satisfactory methods exist

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- Intuitively, the key question is:
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(With unit root, all observations are correlated)
- Is higher frequency data useful?
  - It does increase the number of data points
  - But the correlation between data points goes up
  - Intuitively: No new information about low frequency stuff

- Whether a sample is “small” or “large” is not so simple a question
- Depends on how correlated observations are
- You can have hundreds of thousands of observations but a “small sample” problem if correlation between observations is very high
- Cross-sections correlation can also be a problem  
(hence importance of “clustering” in constructing standard errors)

1. “Revolution of identification”
  - More serious attention to credible identification of causal effects
2. Accurate standard errors
  - Clustering
  - Accounting for persistence

# HAS MEESE-ROGOFF 83 STOOD THE TEST OF TIME?

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- Mostly yes!
- Rossi 13 provides comprehensive survey
- Mark 95 long-run predictability results most serious challenge
- See also more recent work on Taylor rule fundamentals (Molodtsova-Papell JIE 09)

- Simple monetary model:

$$e_t = f_t + c$$

$$f_t = (m_t - m_t^*) - \lambda(y_t - y_t^*)$$

- Even if monetary model doesn't work in the short run, it may work in the long run
- Estimates partial adjustment model:

$$e_{t+k} - e_t = \alpha_k + \beta_k(f_t - e_t) + \nu_{t+k,t}$$

$$e_{t+k} - e_t = \alpha_k + \beta_k(f_t - e_t) + \nu_{t+k,t}$$

- Sample period: 1973:2 - 1991:4
- Pseudo-out-of-sample period: 1981:4 - 1991:4
- Currencies: Canada, Germany, Switzerland, Japan
- Horizons:  $k = 1, 4, 8, 12, 16$  (quarters)

$$\mathbf{e}_{t+k} - \mathbf{e}_t = \alpha_k + \beta_k(\mathbf{f}_t - \mathbf{e}_t) + \nu_{t+k,t}$$

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- Stambaugh 86/99 bias
  - $f_t - e_t$  predetermined but not strictly exogenous
  - Past values of  $e_{t+k} - e_t$  correlated with  $f_t - e_t$
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  - Causes finite sample bias in  $\beta_k$
- Standard errors produced using bootstrap that assumes  $f_t - e_t$  follows AR(p)
  - But  $e_t$  and  $f_t$  may not be cointegrated
  - Small sample bias in estimating AR(p)

- Why not use UK pound?

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$$f_t = (m_t - m_t^*) - (y_t - y_t^*)$$

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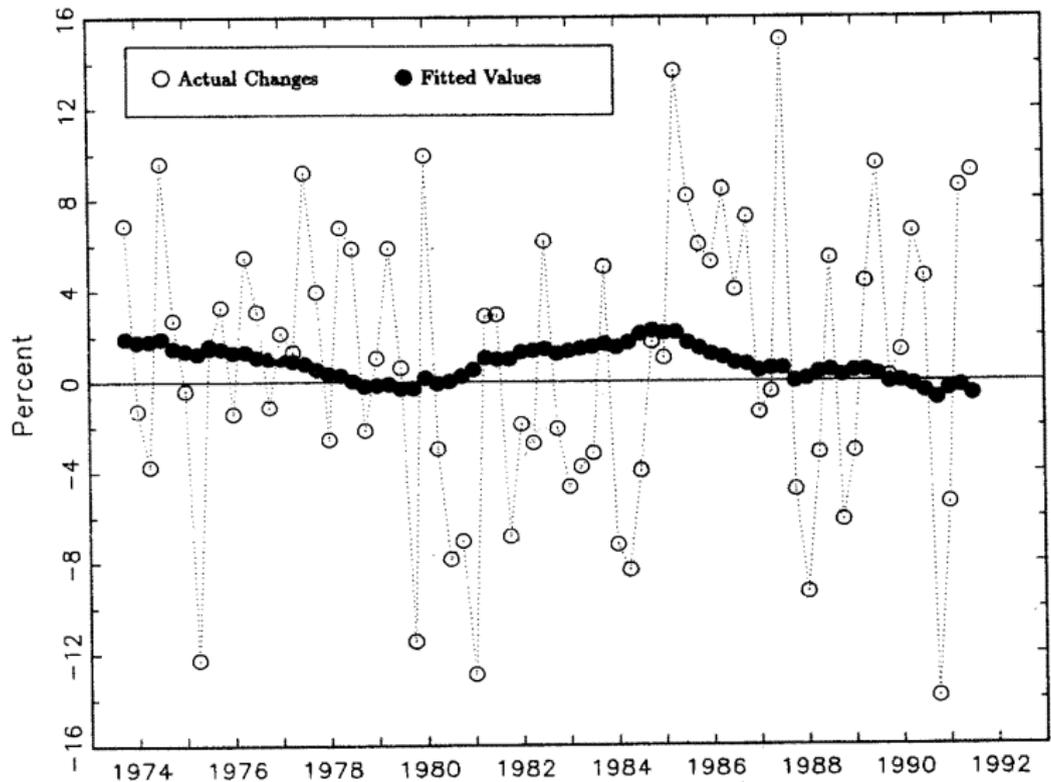
- GNP for US, GDP for all other countries. Why?
- M3 for Canada, M1 for all other countries. Why?

TABLE 2—REGRESSION ESTIMATES AND BOOTSTRAP DISTRIBUTIONS

(i)	(ii)	(iii)	(iv)	(v)	(vi)	(vii)	(viii)	(ix)	(x)	(xi)	(xii)	(xiii)	(xiv)
$k$	$\hat{\beta}_k$	Adj-p	Adj-n	$R^2$	Adj-p	Adj-n	$t_k(20)$	MSL-p	MSL-n	$t_k(A)$	$A$	MSL-p	MSL-n
Canadian dollar:													
1	0.040	0.027	0.028	0.059	0.050	0.051	3.051	0.076	0.077	2.172	1	0.065	0.066
4	0.155	0.109	0.114	0.179	0.144	0.146	2.389	0.186	0.195	2.168	12	0.172	0.175
8	0.349	0.264	0.264	0.351	0.287	0.285	2.539	0.222	0.230	2.527	19	0.201	0.216
12	0.438	0.320	0.315	0.336	0.251	0.235	1.961	0.317	0.343	1.936	29	0.323	0.343
16	0.450	0.295	0.287	0.254	0.146	0.121	1.542	0.420	0.447	1.512	33	0.436	0.456
Deutsche mark:													
1	0.035	0.012	0.016	0.015	0.005	0.006	1.836	0.280	0.252	0.929	2	0.408	0.391
4	0.205	0.114	0.126	0.104	0.065	0.068	2.902	0.169	0.157	2.290	15	0.206	0.193
8	0.554	0.380	0.410	0.265	0.196	0.190	3.487	0.174	0.159	3.558	26	0.147	0.143
12	0.966	0.733	0.759	0.527	0.432	0.410	6.329	0.059	0.057	6.510	29	0.047	0.048
16	1.324	1.015	1.046	0.762	0.638	0.603	9.256	0.027	0.033	9.124	23	0.024	0.025
Swiss franc:													
1	0.074	0.046	0.046	0.051	0.042	0.042	2.681	0.109	0.119	2.073	2	0.084	0.086
4	0.285	0.171	0.171	0.180	0.147	0.145	3.248	0.121	0.126	3.196	14	0.096	0.102
8	0.568	0.356	0.350	0.336	0.278	0.276	4.770	0.080	0.085	4.696	21	0.078	0.073
12	0.837	0.527	0.519	0.538	0.458	0.452	8.013	0.026	0.024	8.013	20	0.026	0.021
16	1.086	0.706	0.671	0.771	0.673	0.655	17.406	0.001	0.002	12.665	14	0.005	0.006
Yen:													
1	0.047	0.014	0.016	0.020	0.010	0.011	1.396	0.388	0.365	1.331	3	0.285	0.259
4	0.263	0.136	0.138	0.125	0.088	0.090	2.254	0.271	0.262	2.153	14	0.247	0.231
8	0.575	0.328	0.329	0.301	0.233	0.232	3.516	0.199	0.189	3.496	19	0.188	0.177
12	0.945	0.592	0.579	0.532	0.432	0.427	4.889	0.129	0.143	4.735	17	0.153	0.156
16	1.273	0.819	0.802	0.694	0.565	0.548	4.919	0.154	0.156	4.901	16	0.174	0.177

Notes: The table presents OLS estimates of the regression  $e_{t+k} - e_t = \alpha_k + \beta_k(f_t - e_t) + \nu_{t+k,k}$ , where  $f_t \equiv (m_t - m_t^*) - (y_t - y_t^*)$ . The (Gaussian) parametric and nonparametric bootstrap distributions are generated under the null hypothesis that the regressor follows an AR(4) for the Canadian dollar, the Swiss franc, and the yen, and an AR(5) for the deutsche mark. Exchange rates are dollars per unit of foreign currency. Adj-p and Adj-n are bias-adjusted values obtained by subtracting median values generated by the parametric and nonparametric bootstrap distributions, respectively, from the estimates. MSL-p and MSL-n are, respectively, the parametric and nonparametric bootstrap marginal significance levels for a one-tail test.  $A$  is the truncation lag determined by Andrews's (1991) univariate AR(1) rule used for constructing the  $t$  ratios with the data.

Source: Mark (1995). Note: Big  $\beta$ , big  $R^2$ , large  $t_k(20)$  for DM, CHF, JPY



**FIGURE 1. ONE-QUARTER CHANGES IN THE LOG DOLLAR/DEUTSCHE-MARK EXCHANGE RATE**

Source: Mark (1995)

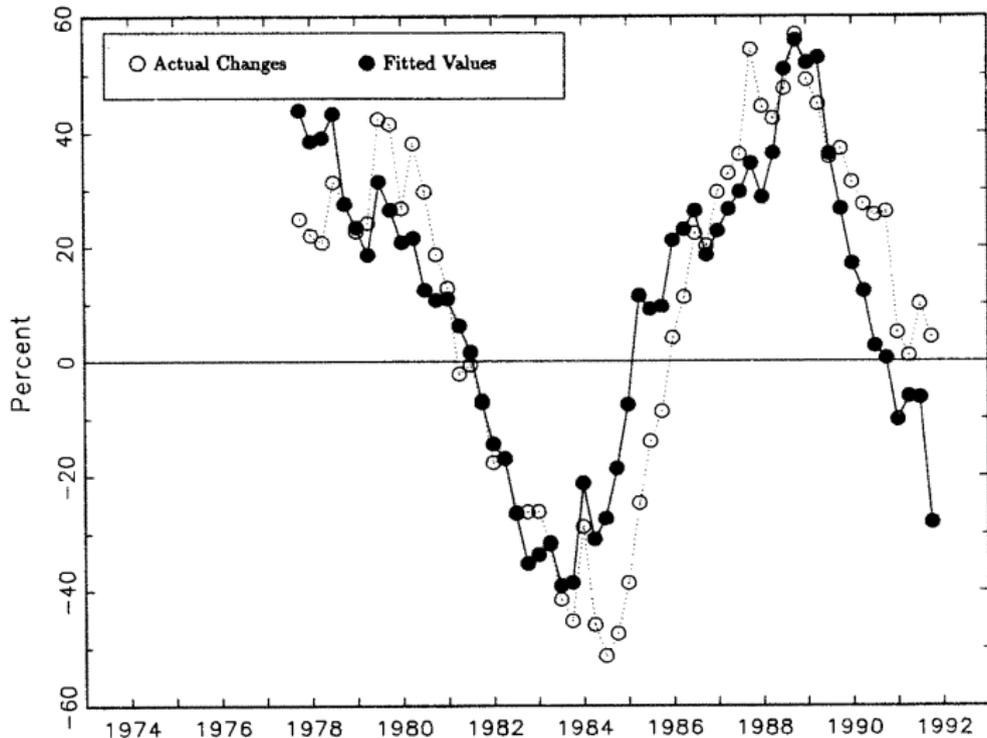


FIGURE 5. SIXTEEN-QUARTER CHANGES IN THE LOG DOLLAR/DEUTSCHE-MARK EXCHANGE RATE

Source: Mark (1995)

TABLE 4—OUT-OF-SAMPLE FORECAST EVALUATION

(i) <i>k</i>	(ii) IN/RW	(iii) OUT/IN	(iv) OUT/RW	(v) MSL-p	(vi) MSL-n	(vii) $\mathcal{DM}(20)$	(viii) MSL-p	(ix) MSL-n	(x) $\mathcal{DM}(A)$	(xi) <i>A</i>	(xii) MSL-p	(xiii) MSL-n
Canadian dollar:												
1	0.960	1.040	0.998	0.209	0.194	0.061	0.215	0.202	0.036	1	0.218	0.201
4	0.889	1.258	1.119	0.571	0.538	-1.270	0.526	0.487	-0.925	8	0.494	0.468
8	0.675	1.695	1.145	0.447	0.397	-1.036	0.427	0.377	-0.890	17	0.420	0.390
12	0.654	2.197	1.436	0.613	0.578	-1.916	0.574	0.556	-1.661	18	0.587	0.579
16	0.799	2.128	1.699	0.654	0.636	-2.596	0.578	0.542	-1.857	15	0.567	0.555
Deutsche mark:												
1	0.988	1.027	1.015	0.397	0.339	-0.932	0.458	0.393	-0.846	4	0.536	0.493
4	0.927	1.120	1.037	0.345	0.288	-1.345	0.563	0.511	-0.852	9	0.478	0.427
8	0.833	1.203	1.002	0.268	0.217	-0.027	0.270	0.220	-0.020	18	0.270	0.221
12	0.670	1.188	0.796	0.127	0.092	4.246	0.068	0.059	0.094	16	0.151	0.136
16	0.431	1.216	0.524	0.040	0.025	8.719 <sup>a</sup>	0.061	0.047	8.719	18	0.021	0.011
Swiss franc:												
1	0.972	1.026	0.997	0.305	0.266	0.066	0.320	0.278	0.064	3	0.315	0.271
4	0.886	1.108	0.981	0.291	0.263	0.218	0.304	0.272	0.162	12	0.298	0.274
8	0.780	1.176	0.917	0.256	0.219	0.703	0.260	0.236	0.560	17	0.253	0.227
12	0.625	1.181	0.738	0.152	0.132	2.933	0.161	0.137	0.938	13	0.255	0.211
16	0.335	1.229	0.411	0.033	0.023	9.650 <sup>b</sup>	0.080	0.058	1.996	8	0.192	0.159
Yen:												
1	0.962	1.027	0.988	0.304	0.257	1.571	0.168	0.132	0.836	3	0.177	0.134
4	0.822	1.129	0.928	0.257	0.207	2.302	0.151	0.118	1.487	10	0.134	0.105
8	0.688	1.191	0.819	0.214	0.162	3.096	0.142	0.117	1.803	13	0.152	0.117
12	0.536	1.329	0.712	0.196	0.148	3.319	0.174	0.148	1.147	17	0.164	0.135
16	0.363	1.579	0.574	0.152	0.119	5.126	0.178	0.160	3.096	16	0.151	0.131

Notes: The table presents ratios of root-mean-squared errors for the regression's out-of-sample forecasts (OUT), the driftless random walk (RW), and the in-sample regression residual during the forecast period (IN). The first forecast is made on 1981:4.  $\mathcal{DM}(20)$  and  $\mathcal{DM}(A)$  are the Diebold-Mariano statistics constructed using the method of Newey and West (1987) with the truncation lag of the Bartlett window set to 20 and set by Andrews's (1991) AR(1) rule, respectively. In instances where the estimated spectral density at frequency zero of the squared error differential is nonpositive (see footnote 8), the Bartlett-window truncation lag is decreased by 1. MSL-p and MSL-n are marginal significance levels, generated by the parametric and nonparametric bootstrap distributions, respectively, for one-tail tests.

Source: Mark (1995). Note: OUT/RW much smaller than 1.

# TRUE OUT-OF-SAMPLE TEST

- Jon wrote a class paper on this for Jim Stock's Time Series class in 2003
- True out-of-sample period: 1992:1-2000:4

# TRUE OUT-OF-SAMPLE TEST

- Jon wrote a class paper on this for Jim Stock's Time Series class in 2003
- True out-of-sample period: 1992:1-2000:4
- Used slightly different data:
  - M2 as opposed to M3 for Canada
  - GDP as opposed to GNP for US
  - Results sensitive to this (not comforting)
- Main results do not survive in 1990s

**Table 1 -- Replication, Extensions and Out of Sample Performance**

	Mark's published results		Mark's data -- My replication		Current Data -- Mark's sample period		Mark's RMSE results	RMSE ratios for 1990's
	beta	R2	beta	R2	beta	R2	OUT/RW	OUT/RW
	(ii)	(iii)	(iv)	(v)	(vi)	(vii)	(viii)	(ix)
<b>Canadian dollar vs. U.S. dollar</b>								
1	0.040	0.059	0.041	0.061	-0.005	0.002	0.998	0.963
4	0.155	0.179	0.157	0.182	-0.041	0.022	1.119	0.933
8	0.349	0.351	0.350	0.354	-0.073	0.020	1.145	0.932
12	0.438	0.336	0.442	0.342	-0.187	0.061	1.436	1.037
16	0.450	0.254	0.456	0.262	-0.305	0.107	1.699	1.208
<b>Deutsche mark vs. U.S. dollar</b>								
1	0.035	0.015	0.037	0.016	0.036	0.016	1.015	1.029
4	0.205	0.104	0.204	0.104	0.181	0.087	1.037	0.987
8	0.554	0.265	0.552	0.264	0.503	0.231	1.002	0.992
12	0.966	0.527	0.961	0.524	0.911	0.485	0.796	1.511
16	1.324	0.762	1.318	0.758	1.274	0.715	0.524	1.957
<b>Swiss franc vs. U.S. dollar</b>								
1	0.074	0.051	0.073	0.050	0.087	0.073	0.997	0.984
4	0.285	0.180	0.284	0.178	0.314	0.227	0.981	0.937
8	0.568	0.336	0.566	0.335	0.571	0.351	0.917	0.786
12	0.837	0.538	0.834	0.536	0.804	0.511	0.738	0.763
16	1.086	0.771	1.085	0.770	1.064	0.751	0.411	0.980
<b>Japanese Yen vs. U.S. dollar</b>								
1	0.047	0.020	0.047	0.020	0.030	0.011	0.988	1.014
4	0.263	0.125	0.264	0.126	0.195	0.085	0.928	1.117
8	0.575	0.301	0.576	0.302	0.452	0.227	0.819	1.177
12	0.945	0.532	0.948	0.534	0.769	0.421	0.712	1.139
16	1.273	0.694	1.274	0.696	1.063	0.557	0.574	1.185

Source: Steinsson (2003)

## FOLLOW UP ON MARK (1995)

- Killian 99 makes same point as I did. Also critiques bootstrap.
- Faust-Rogers-Wright 03: Doesn't work with other vintages of data

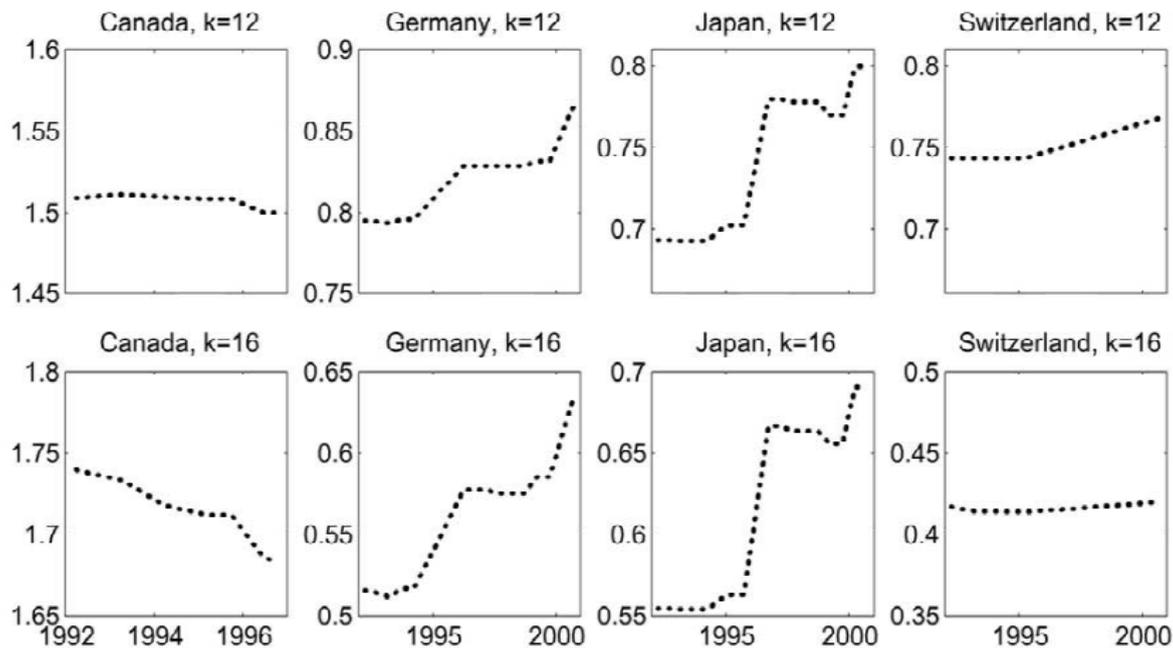


Fig. 6. Out-of-sample relative RMSE using different data vintages (Mark's sample period).

Source: Faust-Rogers-Wright (2003)

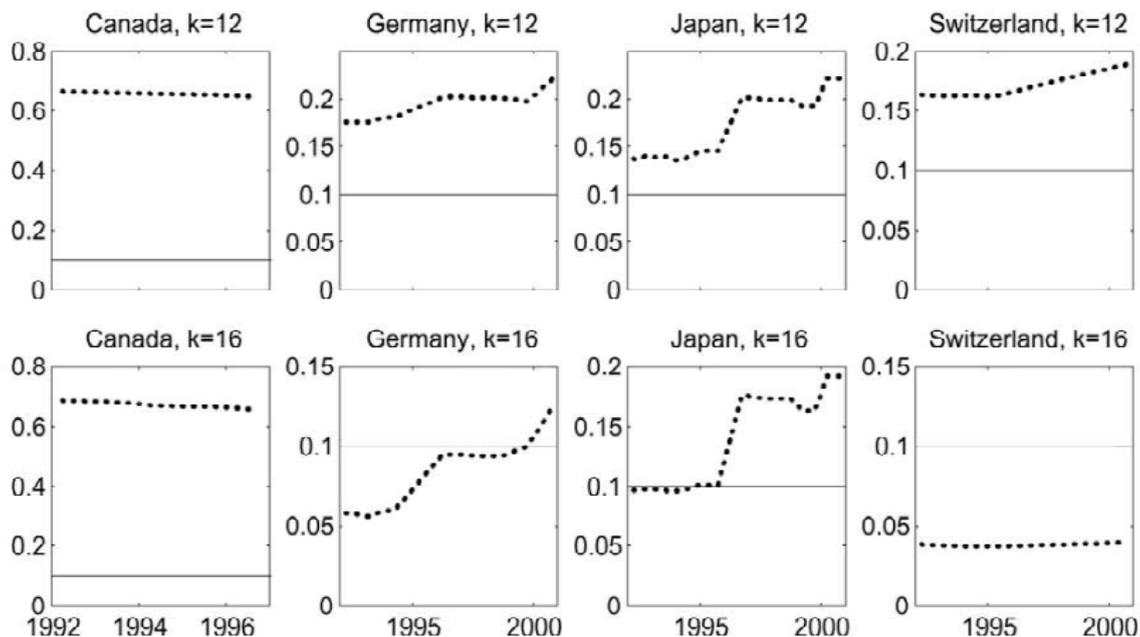
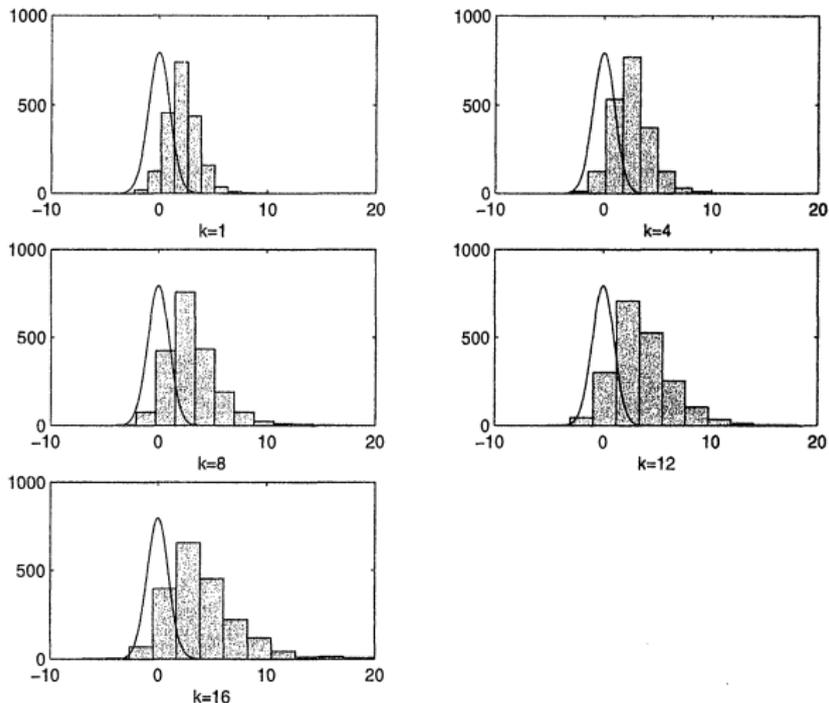


Fig. 7. Bootstrap  $P$ -values for out-of-sample relative RMSE using different data vintages (Mark's sample period).

Source: Faust-Rogers-Wright (2003)

- Killian 99 makes same point as I did. Also critiques bootstrap.
- Faust-Rogers-Wright 03: Doesn't work with other vintages of data
- Berkowitz-Giorgianni 01:
  - Mark's bootstrap assumes  $e_t$  and  $f_t$  are cointegrated  
(It assume AR(p) for  $z_t = f_t - e_t$ )
  - Standard errors much larger under null of no-cointegration  
(also standard diagnostic test stats fail in this case)

FIGURE 1.—MONTE CARLO DISTRIBUTIONS OF NEWEY-WEST  $t$ -STATISTICS  
UNDER  $H_0$ : INDEPENDENCE



For each horizon of interest,  $k = 1, 4, 8, 12,$  and  $16$ , we plot the histogram of the Newey-West  $t$ -statistic with a bandwidth of 20. The solid line corresponds to the density of a  $t$ -distributed random variable with degrees of freedom equal to  $85 - (k + 1)$ .

Source: Berkowitz-Giorgianni (2001)

TABLE 1.—LONG-HORIZON MONTE CARLO ESTIMATES NULL HYPOTHESIS: INDEPENDENCE  
(SPOT RATES FOLLOW A RANDOM WALK AND FUNDAMENTALS AN AR(2) PROCESS)

$k$	%-ile	$\hat{\beta}_k$	$t(\text{LS})$	$t(20)$	$t(\text{A})$	$R_k^2$	OUT/RW	DM(20)	DM(A)
Panel A: sample size = 85									
1	50	0.046	1.479	2.028	1.525	0.026	1.008	-0.377	-0.300
	90	0.112	2.517	3.941	2.740	0.072	0.969	2.222	1.341
	95	0.143	2.827	4.502	3.132	0.088	0.957	3.197	1.758
4	50	0.183	2.982	2.359	2.092	0.101	1.032	-0.441	-0.372
	90	0.403	5.220	4.823	4.180	0.256	0.881	2.458	1.816
	95	0.495	6.012	5.666	4.947	0.314	0.840	3.430	2.400
8	50	0.354	4.270	2.769	2.635	0.195	1.050	-0.420	-0.366
	90	0.711	7.741	6.079	5.689	0.444	0.768	2.752	2.230
	95	0.839	9.004	7.442	6.813	0.519	0.699	4.058	3.015
12	50	0.507	5.333	3.200	3.203	0.286	1.045	-0.320	-0.301
	90	0.958	9.798	7.173	6.839	0.574	0.678	3.108	2.574
	95	1.067	11.267	8.928	8.112	0.641	0.605	4.552	3.524
16	50	0.635	6.132	3.465	3.585	0.359	1.029	-0.223	-0.194
	90	1.122	11.505	8.292	7.976	0.663	0.601	3.796	2.956
	95	1.244	13.360	9.790	9.488	0.727	0.516	5.089	4.122
Panel B: sample size = 1085									
1	50	0.004	1.559	1.593	1.572	0.002	0.999	0.198	0.134
	90	0.011	2.565	2.734	2.595	0.006	0.991	2.495	1.559
	95	0.013	2.834	2.991	2.854	0.007	0.987	3.107	1.943
4	50	0.016	3.102	1.651	1.646	0.009	0.997	0.212	0.150
	90	0.041	5.145	2.805	2.853	0.024	0.962	2.562	2.014
	95	0.050	5.683	3.142	3.160	0.029	0.946	3.423	2.571
8	50	0.031	4.433	1.728	1.723	0.017	0.995	0.181	0.164
	90	0.082	7.385	2.976	3.053	0.048	0.920	2.908	2.452
	95	0.100	8.088	3.321	3.391	0.057	0.890	3.915	3.157
12	50	0.046	5.449	1.815	1.791	0.026	0.992	0.208	0.189
	90	0.121	9.098	3.146	3.191	0.072	0.874	3.509	3.007
	95	0.149	9.982	3.520	3.614	0.085	0.831	4.884	3.951
16	50	0.062	6.260	1.911	1.835	0.035	0.992	0.173	0.149
	90	0.158	10.505	3.292	3.374	0.093	0.826	4.433	3.807
	95	0.198	11.504	3.715	3.815	0.110	0.773	6.308	5.278

The table presents estimated slope coefficients,  $\hat{\beta}_k$ , for equation (2) with the LS  $t$ -statistics, heteroskedasticity, and autocorrelation-corrected  $t$ -statistics using a Bartlett kernel and a truncation lag of 20 and Andrews' (1991) rule; respectively,  $t(\text{LS})$ ,  $t(20)$ , and  $t(\text{A})$ . OUT/RW denotes the ratio of regression mean-squared out-of-sample forecast error to the random-walk, mean-squared, out-of-sample forecast error. DM(20) and DM(A) denote the Diebold-Mariano statistics with a Bartlett kernel and truncation lags of 20 and via Andrews' (1991) rule, respectively.

Source: Berkowitz-Giorgianni (2001)

TABLE 4.—LONG-HORIZON REGRESSION ESTIMATES NULL HYPOTHESIS: NO COINTEGRATION (1973:2–1991:4)

	<i>k</i>	$\hat{\beta}_k$	<i>t</i> (20)	<i>p</i> -val	<i>t</i> (A)	<i>p</i> -val	$R_k^2$	OUT/RW	<i>p</i> -val	DM(20)	<i>p</i> -val	DM(A)	<i>p</i> -val
<i>Canadian Dollar</i>	1	0.040	3.051	0.095	2.172	0.086	0.059	0.998	0.405	0.061	0.441	0.036	0.443
	4	0.155	2.398	0.217	2.168	0.190	0.179	1.119	0.412	-1.270	0.849	-0.925	0.975
	8	0.349	2.539	0.225	2.527	0.212	0.351	1.145	0.712	-1.036	0.958	-0.890	0.901
	12	0.438	1.961	0.352	1.936	0.350	0.336	1.436	0.317	-1.916	0.592	-1.661	0.695
	16	0.450	1.542	0.458	1.512	0.465	0.254	1.699	0.196	-2.596	0.466	-1.857	0.620
<i>German Mark</i>	1	0.035	1.836	0.510	0.929	0.668	0.015	1.015	0.969	-0.932	0.724	-0.846	0.669
	4	0.205	2.902	0.354	2.290	0.406	0.104	1.037	0.914	-1.345	0.522	-0.852	0.780
	8	0.554	3.487	0.354	3.558	0.313	0.265	1.002	0.809	-0.027	0.814	-0.020	0.814
	12	0.966	6.329	*0.165	6.510	*0.135	0.527	0.796	*0.406	4.246	0.093	0.094	0.563
	16	1.324	9.256	0.096	9.124	0.082	0.762	0.524	*0.113	8.719	0.030	8.719	0.016
<i>Japanese Yen</i>	1	0.047	1.396	0.516	1.331	0.423	0.020	0.988	0.477	1.571	0.286	0.836	0.322
	4	0.263	2.254	0.353	2.153	0.341	0.125	0.928	0.396	2.302	0.215	1.487	0.310
	8	0.575	3.516	0.228	3.496	0.227	0.301	0.819	0.304	3.096	0.172	1.803	0.309
	12	0.945	4.889	0.166	4.735	0.190	0.532	0.712	0.233	3.319	0.173	1.147	0.276
	16	1.273	4.919	0.216	4.901	0.199	0.694	0.574	0.142	5.126	0.109	3.096	0.241
<i>Swiss Franc</i>	1	0.074	2.681	0.210	2.073	*0.166	0.051	0.997	0.642	0.066	0.704	0.064	0.693
	4	0.285	3.248	0.181	3.196	*0.131	0.180	0.981	0.596	0.218	0.676	0.162	0.686
	8	0.568	4.770	0.095	4.696	0.081	0.336	0.917	0.458	0.703	0.621	0.560	0.635
	12	0.837	8.013	0.032	8.013	0.023	0.538	0.738	0.214	2.933	0.203	0.938	0.624
	16	1.086	17.41	0.006	12.66	0.010	0.771	0.411	0.026	9.650	#0.019	1.996	0.411
			$\rho_1$		$\rho_2$		$\rho_3$		$\rho_4$		$\sigma^2$		
	Canadian Dollar		1.227		0.028		-0.045		-0.233		0.011		
	German Mark		1.253		-0.305						0.016		
	Japanese Yen		1.267		-0.062		-0.204				0.013		
	Swiss Franc		1.916		-1.154		0.236				0.014		

Data were kindly supplied by Nelson Mark (originally taken from the OECD *Main Economic Indicators*). For detailed descriptions of the statistics, see the notes to table 1.

\* indicates *p*-values that are no longer significant at a 90% level, but were under the null of cointegration.

# indicates the reverse.

The table presents least-squares estimates of equation (2) over horizons of  $k = 1, 4, 8, 12$ , and 16 quarters and Monte Carlo *p*-values, tabulated under the null hypothesis that  $s_t$  is a random walk, independent of  $f_t$ . The  $f_t$  are generated using the following AR models with lag order selected by the BIC criterion.

Source: Berkowitz-Giorgianni (2001)