# Phillips Curve: Structural Estimation

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# Simple Theory

- Simple theory with Calvo pricing assumption implies:

$$\pi_t = \beta E_t \pi_{t+1} + \lambda mc_t$$

where

$$\lambda = \frac{(1-\theta)(1-\beta\theta)}{\theta}$$

and $1-\theta$ is frequency of price change, $\beta$ subjective discount factor

- Theory implies that $mc_t$ is the appropriate "forcing variable" in the Phillips curve

- Yet most empirical work uses simple measures of the output gap such as detrended output

# Gali-Gertler 99

Motivation:

- Is New Keynesian (Calvo) Phillips curve consistent with observed inflation persistence?
  - Implies disinflations can be costless
  - In practice, it seems disinflations are costly (Ball 94, 95)
    (Imperfect credibility could explain this)
- Do we need "sticky inflation" models or adaptive expectations?
- With quarterly data, hard to get statistically significant effect of real activity on inflation, when using output gap

# Simple Theory

- Under certain assumptions:

$$mc_t = \kappa x_t$$

  where $x_t = y_t - y_t^n$ denotes the output gap

- Maybe Phillips curve estimation doesn't work because:
  - These assumptions don't hold in reality
  - Output gap is mismeasured

# New Keynesian vs. Old Keynesian

- With rational expectations, NK Phillips curve can be written as

$$\pi_{t+1} - \pi_t = -\lambda\kappa x_t + \epsilon_{t+1}$$

  where $\epsilon_{t+1} = \pi_{t+1} - E_t\pi_{t+1}$, and assuming $\beta = 1$.

- Traditional Phillips curve with adaptive expectations:

$$\pi_t = E_{t-1}\pi_t + \lambda\kappa x_t$$

$$\pi_t - \pi_{t-1} = \lambda\kappa x_t$$

  where we are assuming $E_{t-1}\pi_t = \pi_{t-1}$

- Notice the difference in the sign on the output gap term!!
  (and difference in timing of inflation change)

# New Keynesian vs. Old Keynesian

$$\pi_{t+1} - \pi_t = -\lambda\kappa x_t + \epsilon_{t+1}$$

- NK Phillips curve implies tight labor market should lead inflation to fall!!
- Theoretical logic:
  - Inflation is a jump variable in this model
  - When output gaps are expected, inflation should jump up and start falling

$$\pi_t = \lambda\kappa \sum_{k=0}^{\infty} \beta^k E_t x_{t+k}$$

- I.e., inflation should lead output gap according to NK Phillips curve (Fuhrer-Moore 95)

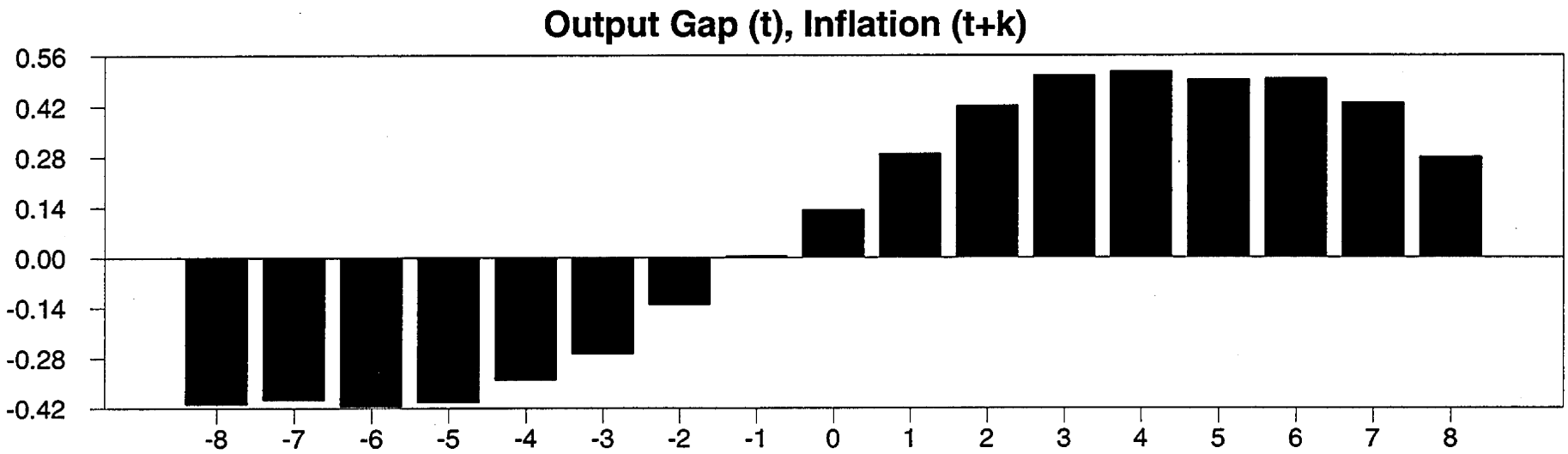
# New Keynesian vs. Old Keynesian

- Simple estimation using quadratically detrended log GDP yields:

$$\pi_{t+1} - \pi_t = 0.081 x_t + \epsilon_{t+1}$$

- Output gap term has "wrong sign" (from NK perspective)

# Output Gap Leads Inflation




Source: Gali and Gertler (1999) – Output gap measure as detrended output using HP-filter.
Sample period 1960Q1-1997Q4. Current output gap positively correlated with *future* inflation.

# Marginal Cost as Opposed to Output Gap

- One reaction: NK Phillips curve is empirically unrealistic.
  - Perhaps some sluggishness messes up this jump business
  - Perhaps information frictions play a role (yield $E_{t-1}\pi_t$)
- Gali-Gertler argue:
  - Use of output gap is the problem
  - Output gap measured with error
  - Marginal costs tends to lag output gap
- Gali-Gertler propose to estimate Phillips curve using marginal cost as forcing variable

# Measuring Marginal Cost

- But marginal costs are unobservable as well!!
- Gali-Gertler make following assumptions:
  - Production function: $Y_t = A_t K_t^{\alpha_k} N_t^{\alpha_n}$
  - Labor is hired on a spot market at constant wage
- Marginal cost:

$$MC_t = \frac{W_t/P_t}{\partial Y_t/\partial N_t} = \frac{W_t/P_t}{\alpha_n Y_t/N_t} = \frac{1}{\alpha_n}\frac{W_t N_t}{P_t Y_t} = \frac{S_t}{\alpha_n}$$

  proportional to labor share (average cost)

- In logs, we get:

$$mc_t = s_t$$

# Measuring Marginal Cost

- Assumptions that Gali-Gertler make to derive this are strong assumptions!!
- Worker-firm relationship often long-term relationship
  - Not clear that current wage is a good proxy for marginal cost
  - May just be an installment payment on a long-term contract
  - Suppose workers performs well at time $t$:
    - Wage may not reflect this at time $t$
    - Rather worker may expect a raise / promotion in the future
  - Firms may insure workers (labor hoarding)
- Wages at a given point in time complicated by overtime
  - Marginal wage may not be the same as average wage

# Supply Shocks

- Gali-Gertler estimate

$$\pi_t = \beta E_t \pi_{t+1} + \lambda s_t$$

- Advantage of using measure of marginal costs:
  - Supply shocks should be reflected in marginal costs

# Expectations of Inflations

- What do Gali-Gertler do about expectations of inflation?
- They assume rational expectations
- Under this assumption, Phillips curve can be written

$$\pi_t = \beta \pi_{t+1} + \lambda s_t + \epsilon_{t+1}$$

  where $\epsilon_{t+1} = \beta E_t \pi_{t+1} - \beta \pi_{t+1}$ (i.i.d.)
- They furthermore take structural model super seriously in assuming that there is **no other error term** than this expectations error
- This strong structural assumption allows them to use lagged variables as instruments (any variable dated at time $t$ or earlier)

# Empirical Specification

- Maintained assumptions:

$$\pi_t - \beta\pi_{t+1} - \lambda s_t = \epsilon_{t+1}$$

where $\epsilon_{t+1}$ is an i.i.d. expectations error and therefore uncorrelated with variables at time $t$ or earlier

- Implies:

$$E_t\{(\pi_t - \beta\pi_{t+1} - \lambda s_t)z_t\} = 0$$

where $z_t$ is in the time $t$ information set of agents

# Empirical Specification

- Gali-Gertler use GMM with these orthogonality conditions

$$E_t\{(\pi_t - \beta\pi_{t+1} - \lambda s_t)z_t\} = 0$$

- Sample period: 1960Q1-1997Q4
- Instruments: Four lags of inflation, labor income share, output gap, long-short interest rate spread, wage inflation, and commodity price inflation (24 instruments)

# Why IV and not OLS?

$$\pi_t = \beta \pi_{t+1} + \lambda s_t + \epsilon_{t+1}$$

- Under maintained assumption that error term is i.i.d. expectation error dated at time $t+1$, instrument only needed to estimate $\beta$
- More generally, other omitted variables (or cost push shocks) enter the equation and are dated at time $t$ (i.e., affect $\pi_t$):

$$\pi_t = \beta \pi_{t+1} + \lambda s_t + \eta_t$$

- In this case, both $\beta$ and $\lambda$ potentially biased

# Empirical Concerns

- Is $\epsilon_{t+1}$ really just an i.i.d. expectations error?
  - If assumptions needed for $mc_t = s_t$ don't hold, it's not
  - If expectations are not rational, it is not
- If it is not, then instruments may be invalid
  - Slow moving omitted variables correlated with past stuff
- 24 instruments raises concerns about many-weak instruments
  - Many/Weak instruments issue is an overfitting issue in small samples
  - Using 24 relatively weak instruments may lead to substantial overfitting

# Reduced Form Results

- Estimation with labor share:

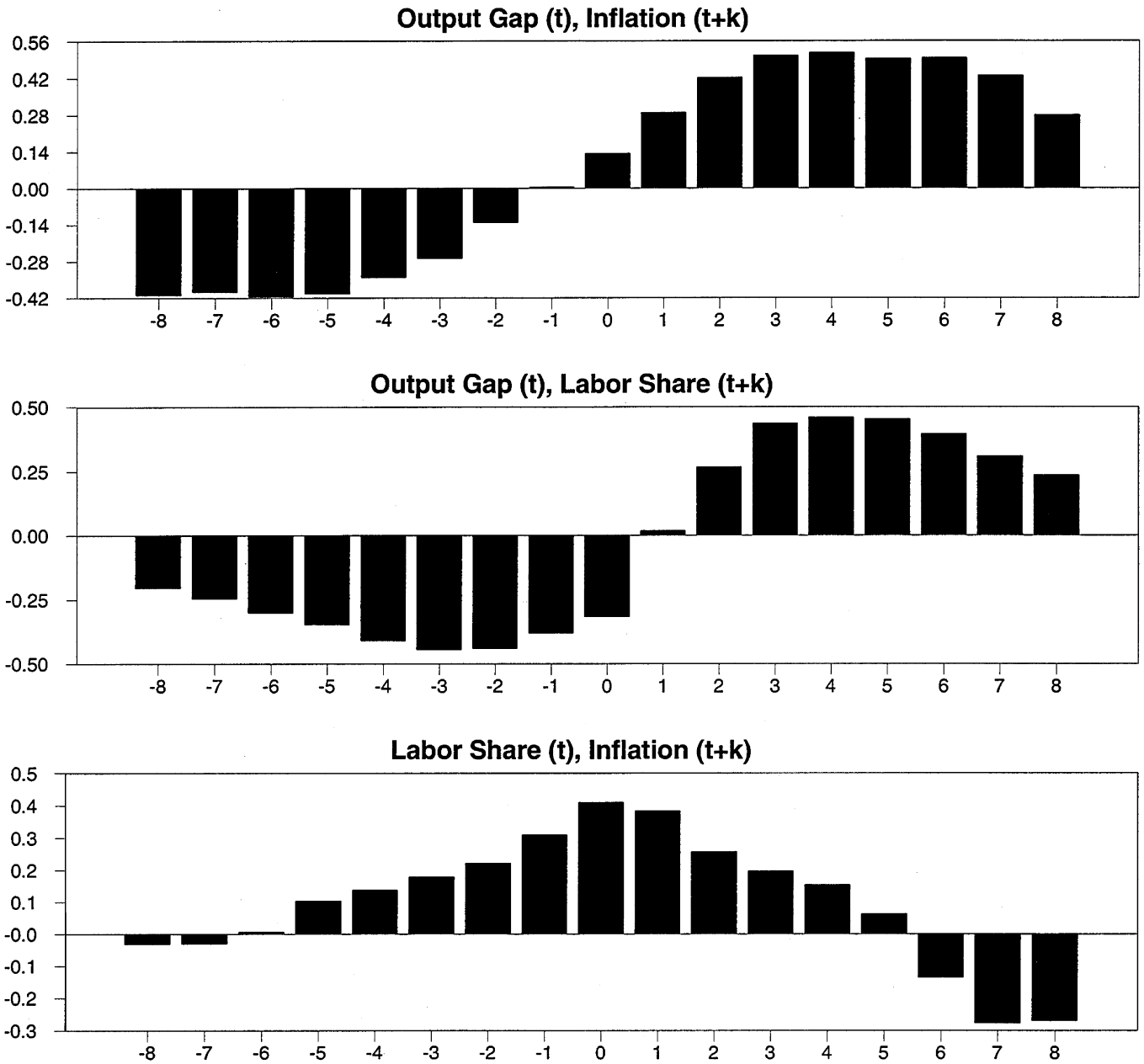
$$\pi_t = \underset{(0.012)}{0.023} s_t + \underset{(0.045)}{0.942} E_t \pi_{t+1}$$

  - Coefficients have "correct sign" and "sensible" magnitude

- Estimation with output gap (HP-filtered GDP):

$$\pi_t = \underset{(0.005)}{-0.016} s_t + \underset{(0.030)}{0.988} E_t \pi_{t+1}$$

  - Coefficient on output gap has "wrong sign"




Source: Gali and Gertler (1999) – Output gap measure as detrended output using HP-filter. Sample period 1960Q1-1997Q4.

# Output Gap vs. Labor Share

- Output gap leads inflation in contradiction to theory
- Labor share strongly correlated with inflation contemporaneously
- Labor share lags output gap
- Lag in response of labor share explains why it does better in Phillips curve estimation

Table 1
Estimates of the new Phillips curve

| | $\theta$ | $\beta$ | $\lambda$ |
|---|---|---|---|
| GDP deflator | | | |
| (1) | 0.829 | 0.926 | 0.047 |
| | (0.013) | (0.024) | (0.008) |
| (2) | 0.884 | 0.941 | 0.021 |
| | (0.020) | (0.018) | (0.007) |
| Restricted $\beta$ | | | |
| (1) | 0.829 | 1.000 | 0.035 |
| | (0.016) | | (0.007) |
| (2) | 0.915 | 1.000 | 0.007 |
| | (0.035) | | (0.006) |
| NFB deflator | | | |
| (1) | 0.836 | 0.957 | 0.038 |
| | (0.015) | (0.018) | (0.008) |
| (2) | 0.884 | 0.967 | 0.018 |
| | (0.023) | (0.016) | (0.008) |

*Notes*: This table reports GMM estimates of the structural parameters of Eq. (15). Rows (1) and (2) correspond to the two specifications of the orthogonality conditions found in Eqs. (18) and (19) in the text, respectively. Estimates are based on quarterly data and cover the sample period 1960:1–1997:4. Instruments used include four lags of inflation, labor income share, long-short interest rate spread, output gap, wage inflation, and commodity price inflation. A 12-lag Newey–West estimate of the covariance matrix was used. Standard errors are shown in brackets.

Source: Gali and Gertler (1999) – Two normalizations of moment conditions.

# Structural Estimates

Conparison vs. ex ante views:

- "Sensible" estimates for $\beta$
- Estimates of $\theta$ on the high end
  - Imply price rigidity of 5-6 quarters

# Inflation Inertia

- Does NK Phillips curve account for inflation inertia?
- Gali-Gertler estimate specification with fraction of rule-of-thumb agents
- Rule-of-thumb agents set

$$p_t^b = \bar{p}_{t-1}^* + \pi_{t-1}$$

- This yields

$$\pi_t = \lambda mc_t + \gamma_f E_t \pi_{t+1} + \gamma_b \pi_{t-1}$$

where

$$\lambda = \frac{(1-\omega)(1-\theta)(1-\beta\theta)}{\theta+\omega[1-\theta(1-\beta)]}$$

$$\gamma_f = \frac{\beta\theta}{\theta+\omega[1-\theta(1-\beta)]} \qquad \gamma_b = \frac{\omega}{\theta+\omega[1-\theta(1-\beta)]}$$

and $\omega$ denotes the fraction of rule-of-thump agents

Table 2
Estimates of the new hybrid Phillips curve

| | $\omega$ | $\theta$ | $\beta$ | $\gamma_b$ | $\gamma_f$ | $\lambda$ |
|---|---|---|---|---|---|---|
| GDP deflator | | | | | | |
| (1) | 0.265 | 0.808 | 0.885 | 0.252 | 0.682 | 0.037 |
| | (0.031) | (0.015) | (0.030) | (0.023) | (0.020) | (0.007) |
| (2) | 0.486 | 0.834 | 0.909 | 0.378 | 0.591 | 0.015 |
| | (0.040) | (0.020) | (0.031) | (0.020) | (0.016) | (0.004) |
| Restricted $\beta$ | | | | | | |
| (1) | 0.244 | 0.803 | 1.000 | 0.233 | 0.766 | 0.027 |
| | (0.030) | (0.017) | | (0.023) | (0.015) | (0.005) |
| (2) | 0.522 | 0.838 | 1.000 | 0.383 | 0.616 | 0.009 |
| | (0.043) | (0.027) | | (0.020) | (0.016) | (0.003) |
| NFB deflator | | | | | | |
| (1) | 0.077 | 0.830 | 0.949 | 0.085 | 0.871 | 0.036 |
| | (0.030) | (0.016) | (0.019) | (0.031) | (0.018) | (0.008) |
| (2) | 0.239 | 0.866 | 0.957 | 0.218 | 0.755 | 0.015 |
| | (0.043) | (0.025) | (0.021) | (0.031) | (0.016) | (0.006) |

*Notes*: This table reports GMM estimates of parameters of Eq. (26). Rows (1) and (2) correspond to the two specifications of the orthogonality conditions found in Eqs. (27) and (28) in the text, respectively. Estimates are based on quarterly data and cover the sample period 1960:1–1997:4. Instruments used include four lags of inflation, labor income share, long-short interest rate spread, output gap, wage inflation, and commodity price inflation. A 12-lag Newey–West estimate of the covariance matrix was used. Standard errors are shown in brackets.

Source: Gali and Gertler (1999) – Two normalizations of moment conditions.

# Estimation of Hybrid Phillips Curve

- Estimate of $\omega$ statistically significant
  - Normalization 1 yields: $\omega = 0.265(0.031)$
  - Normalization 2 yields: $\omega = 0.486(0.040)$
- A quarter to half of agents are rule-of-thumb
- Gali-Gertler conclusion:
  - Forward-looking behavior more important than backward-looking behavior
- Estimates of $\beta$ on the low side at around 0.9

# Critiques of Gali-Gertler

- Subsequent work has found Gali-Gertler's results to be highly sensitive to instruments used, vintage of data, model specification
- Mavroeidis-Plagborg-Moller-Stock 14 argue fundamental problem is weak instruments
  - Inflation is notoriously difficult to forecast
  - Lagged variables weak instruments for future inflation
- More recent literature has used many fewer instruments to avoid many-instruments problem

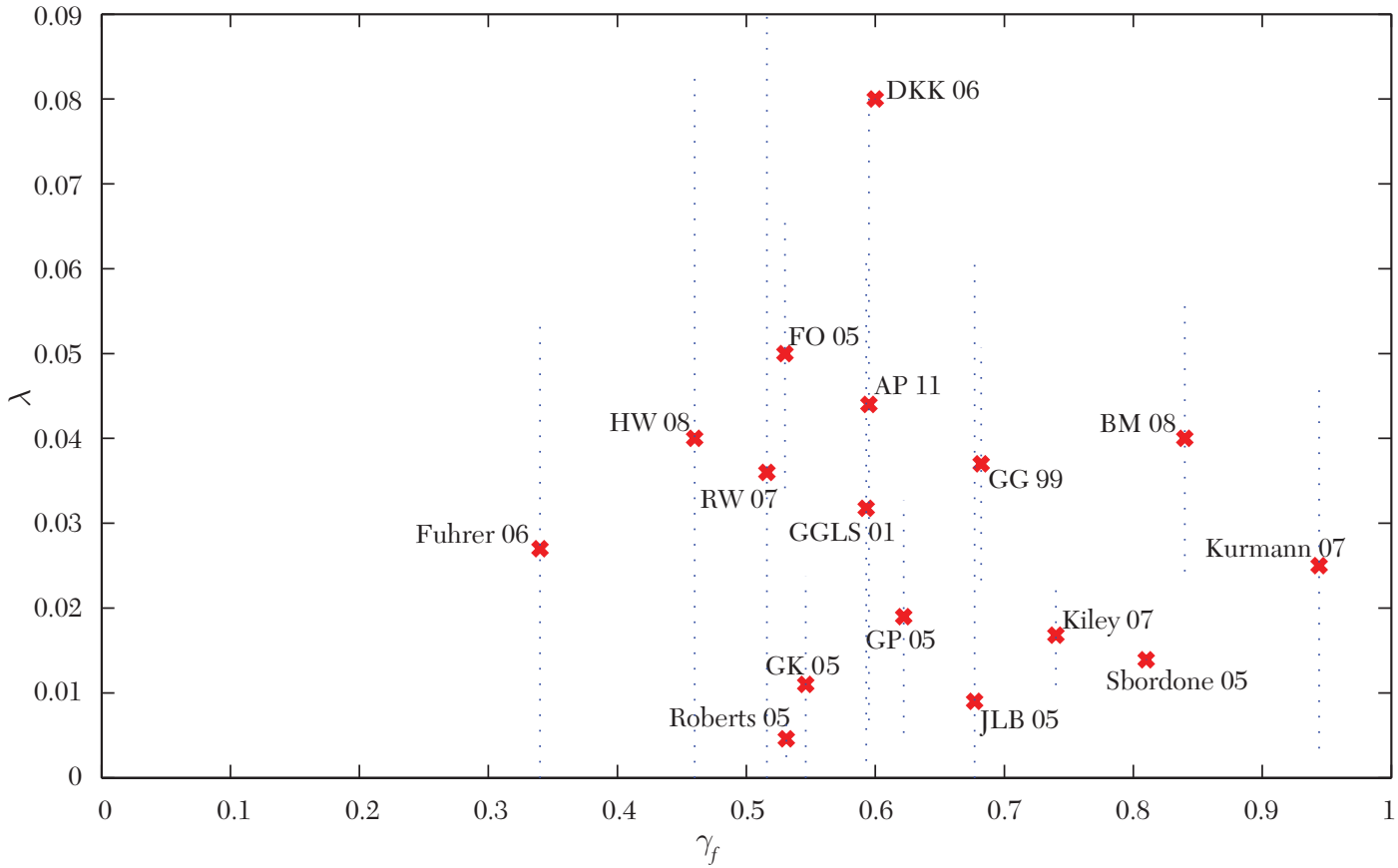



*Figure* 3. Point Estimates Reported in the Literature

*Notes:* Point estimates of $\lambda$ (vertical axis) and $\gamma_f$ (horizontal axis) reported in the literature. Only estimates that use U.S. data and the labor share as forcing variable are plotted. For some papers the semistructural point estimates have been imputed from point estimates of deeper parameters. The dotted blue lines indicate 95 percent confidence intervals for $\lambda$ where available. We include papers with readily available estimates and more than twenty-five Google Scholar citations as of mid-September 2012: Galí and Gertler (1999); Galí, Gertler, and López-Salido (2001); Fuhrer and Olivei (2005); Gagnon and Khan (2005); Guay and Pelgrin (2005); Henzel and Wollmershäuser (2008); Jondeau and Le Bihan (2005); Roberts (2005); Sbordone (2005); Dufour, Khalaf, and Kichian (2006); Fuhrer (2006); Kiley (2007); Kurmann (2007); Rudd and Whelan (2007); Brissimis and Magginas (2008); and Adam and Padula (2011).

Source: Mavroeidis, Plagborg-Moller, Stock (2014)

# Sensitivity to Data Vintage

- Rudd-Whelan 07 emphasize sensitivity to data vintage
- Mavroeidis-Plagborg-Moller-Stock 14 run Gali-Gertler 99 hybrid specification with Gali-Gertler-Lopez-Salido 01 instruments on Gali-Gertler 99 sample period for two data vintages
  - Roughly replicate Gali-Gertler 99 results for 2008 data vintage
  - With 2012 data vintage, slope of Phillips curve 30% smaller and insignificant

TABLE 3
BASELINE GIV ESTIMATES USING DIFFERENT DATA VINTAGES

| Data vintage | Const. | $\lambda$ | $\gamma_f$ | $\gamma_b$ | Hansen test |
|---|---|---|---|---|---|
| 1998 | 0.041 | 0.026 | 0.615 | 0.340 | 5.263 |
| | (0.030) | (0.013) | (0.057) | (0.058) | [0.628] |
| 2012 | –0.049 | 0.018 | 0.719 | 0.240 | 9.816 |
| | (0.040) | (0.012) | (0.099) | (0.095) | [0.199] |

*Notes:* Comparison of GIV estimates of the hybrid NKPC based on 1998 and 2012 vintages of data. The estimation sample is 1970q1 to 1998q1. Inflation: GDP deflator. Labor share: NFB. Instruments: four lags of inflation and two lags of the labor share, wage inflation, and quadratically-detrended output. Estimation method: CUE GMM. Weight matrix: Newey and West (1987) with automatic lag truncation (4 lags). Standard errors in parentheses and $p$-values in square brackets.

Source: Mavroeidis, Plagborg-Moller, Stock (2014)

# Mavroeidis, Plagborg-Moller, Stock 14

- Run huge number of different a priori reasonable specifications with a common dataset
- Main findings:
  - Large amount of specification uncertainty
  - Large amount of sampling uncertainty
- Both conclusions due to weakness of identification

TABLE 4
NKPC Specification Combinations

| Specification settings | Options |
|---|---|
| Inflation ($\pi_t$) | GDP deflator, CPI, chained GDP def., GNP def., chained GNP def., NFB GDP def., PCE, core PCE, core CPI, filtered GDP def. gap, smoothed GDP def. gap, filt. CPI gap, sm. CPI gap, SPF-based CPI gap, filt. core CPI gap, sm. core CPI gap, filt. PCE gap, sm. PCE gap, filt. core PCE gap, sm. core PCE gap |
| Labor share (ls) | NFB, NFB coint. relation, HP filtered NFB gap, Baxter-King filt. NFB gap, linearly detrended NFB gap, quadratically detrended NFB gap, real-time NFB HP gap, real-time NFB BK gap, real-time NFB lin. detr. gap, real-time NFB quadr. detr. gap |
| Output gap (ygap) | CBO, HP filt., BK filt., lin. detr., quadr. detr., real-time HP filt., real-time BK filt., real-time lin. detr., real-time quadr. detr. |
| Reduced form | Unrestricted, VAR |
| Survey forecasts ($\pi^s_{t\|\tau}$) | SPF CPI, SPF GDP def., GB GDP def. |
| Expectations | $\pi_{t+1}$ (endogenous), $\pi^s_{t+1\|t}$ (endog.), $\pi^s_{t+1\|t}$ (exogenous)<br>$\pi^s_{t+1\|t-1}$ (endog.), $\pi^s_{t+1\|t-1}$ (exog.) |
| Instruments | GG: 4 lags of $\pi_t$, ls, ygap, 10y–90d yield spread, wage infl., commodity price infl.<br>GGLS: 4 lags of $\pi_t$ and 2 lags of ls, ygap, wage infl.<br>small: 4 lags of $\pi_t$ and 3 lags of forcing variable<br>exact: 1 extra lag of each endog. regr. (just-identified)<br>RT: 2 real-time lags of GDP def. inflation, $\Delta$ls, ygap<br>survey: 2 lags of 1-quarter SPF/GB forecasts, forcing variable<br>Extra regressors (e.g., oil) added to instruments (if endog., use 2 lags) |
| Inflation lags | 0 lags (pure NKPC), 1 lag, 4 lags |
| Parameter restrictions | No restrictions, $\gamma(1) = \gamma_f$ (inflation coefficients sum to 1)<br>With $\gamma(1) = \gamma_f$, use lags of $\Delta\pi_t$ instead of $\pi_t$ as instruments |
| Oil shocks | None, log change of WTI spot price divided by GDP def. |
| Interest rate | None, 90-day Treasury rate |
| Sample | Full available, 1960–1997, 1968–2005, 1968–2008, 1971–2008, 1981–2008, 1984–end of sample |
| GMM estimator | 2-step, CUE |

*Notes:* List of the specification options that we consider when estimating the NKPC (9). The efficient GMM weight matrix is computed using the Newey and West (1987) heteroskedasticity and autocorrelation consistent estimator with automatic lag truncation, except for VAR specifications, which use the White (1980) heteroskedasticity consistent estimator.

Source: Mavroeidis, Plagborg-Moller, Stock (2014)

# Specification Uncertainty

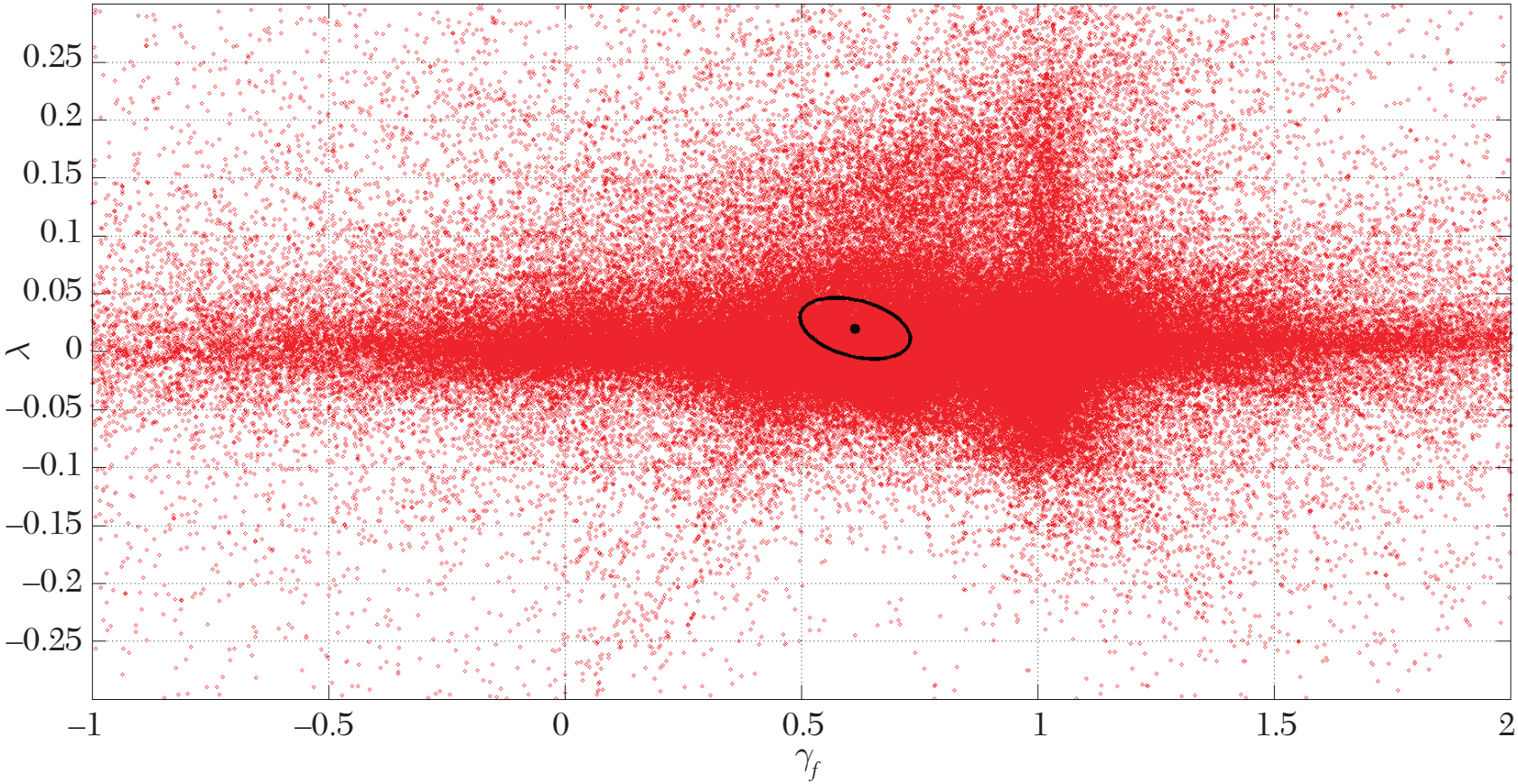



*Figure* 4. Point Estimates: Labor Share Specifications

*Notes:* Point estimates of $\lambda$, $\gamma_f$ from the various specifications listed in table 4 that use the labor share as forcing variable, excluding real-time and survey instrument sets. The black dot and ellipse represent the point estimate and 90 percent joint Wald condence set from the 1998 vintage results in table 3.

Source: Mavroeidis, Plagborg-Moller, Stock (2014)

# Specification Uncertainty

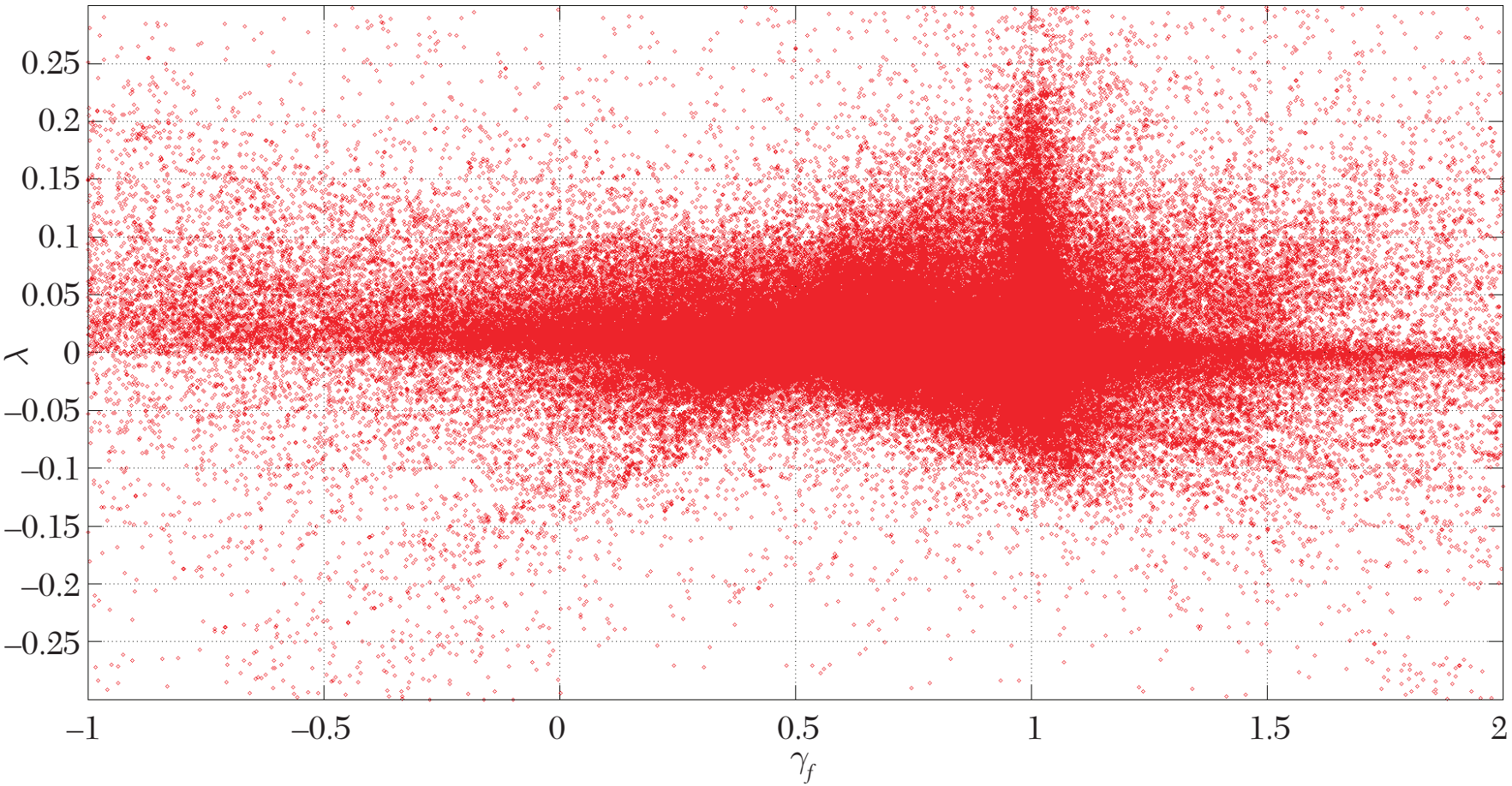



*Figure* 5. Point Estimates: Output Gap Specifications

*Notes:* Point estimates of $\lambda$, $\gamma_f$ from the various specifications listed in table 4 that use the output gap as forcing variable, excluding real-time and survey instrument sets.

Source: Mavroeidis, Plagborg-Moller, Stock (2014)

# Mavroeidis, Plagborg-Moller, Stock 14

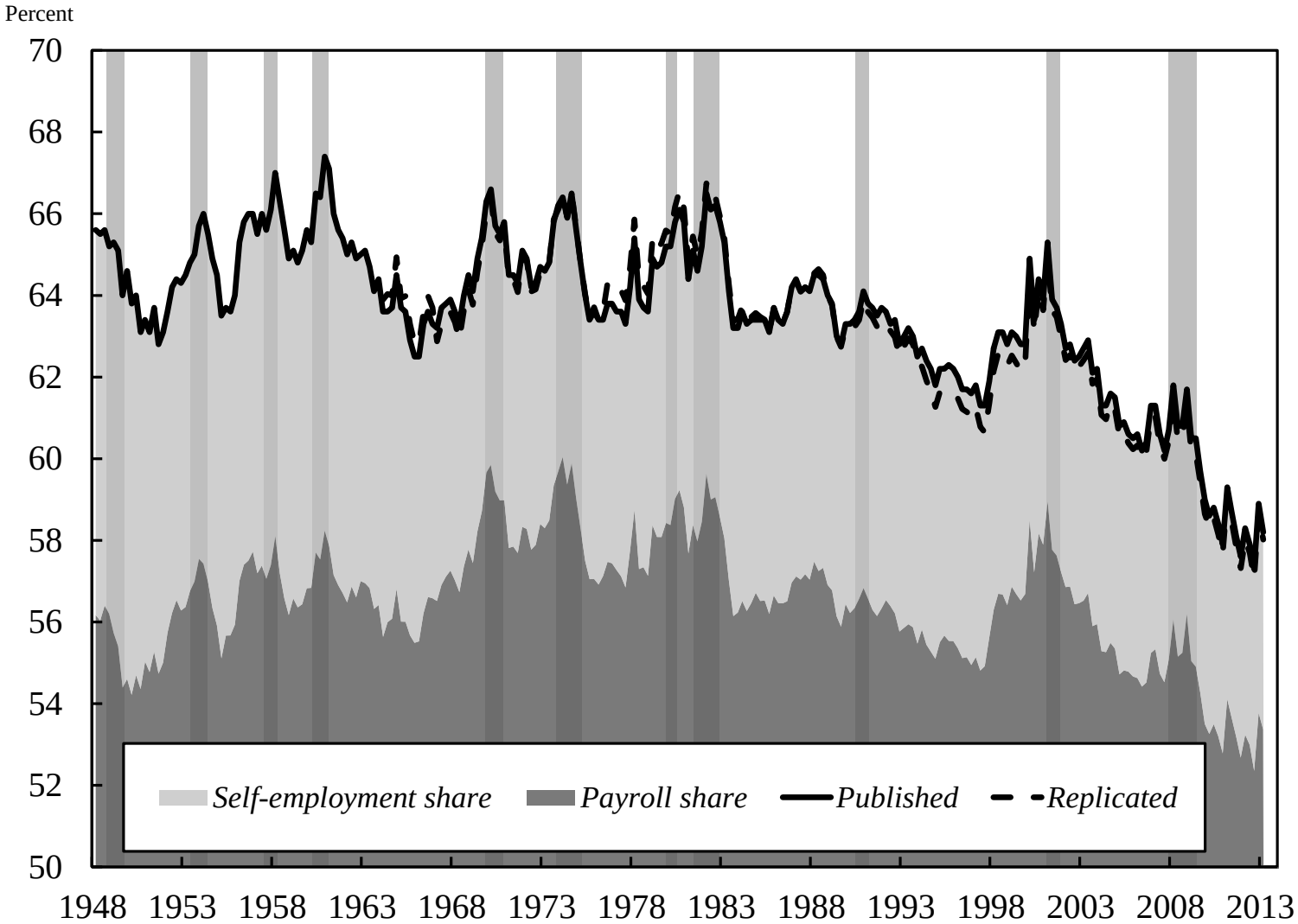
Overall conclusion:

> *"Literature has reached a limit on how much can be learned about the New Keynesian Phillips curve from aggregate macroeconomic time series."*
>
> *"New identification approaches and new datasets are needed to reach an empirical consensus."*

# Recent Behavior of the Labor Share

- Since about 2000, labor share has been trending downward
- If labor share is a good measure of marginal costs, downward trend should create massive deflationary pressure
  (Coibion-Gorodichenko 15)
- Doesn't seem to line up with the evolution of inflation

**Figure 1.** Labor share, payroll share, and replicated labor share in U.S. nonfarm business sector.




*Source: Bureau of Labor Statistics, Bureau of Economic Analysis, and authors' calculations*

Source: Elsby, Hobijn, Sahin (2014)