Preference and Choice

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We'll introduce some of the themes of this class through a general, abstract, model of choice.

The model describes an individual agent that makes choices from different sets of available options.

The purpose of the model is to be able to explain and predict individual choice. The basic hypothesis (our theory of individual choice) is that choice is **rational**.

Preference and Choice

A triple $(X, \mathcal{B}, c)$ is a *choice structure* when

- X is a nonempty set.
- $\mathcal{B}$ is a family of nonempty subsets of X.
- $c : \mathcal{B} \to 2^X$ is a function with $c(B) \subseteq B$ being nonempty for all $B \in \mathcal{B}$.

The elements of $\mathcal{B}$ are the possible choice problems facing the agent, and $c(B) \subseteq B$ the chosen elements from $B \in \mathcal{B}$.

Preference and Choice

Think of a choice structure as describing choices of dinner.
$X = \{\text{fish}, \text{chicken}, \text{steak}, \text{salad}\}$

There are two restaurants:

$$\mathcal{B} = \{\{\text{fish}, \text{salad}\}, \{\text{chicken}, \text{steak}, \text{salad}\}\}$$

$$c(\{\text{fish}, \text{salad}\}) = \{\text{fish}\}$$

$$c(\{\text{chicken}, \text{steak}, \text{salad}\}) = \{\text{chicken}, \text{salad}\}$$

Preference and Choice

Consider a choice structure where $\mathcal{B}$ consists of all subsets of X with cardinality 2.

All "doubletons."

Now write $x \succeq y$ when $x \in c(\{x, y\})$.

This defines a binary relation.

Digression: binary relations

Let X be a set. A *binary relation* on X is a subset of $X \times X$.

If $B \subseteq X \times X$ is a binary relation, write $x \, B \, y$ when $(x, y) \in B$.

Examples:

- If $f : X \to X$ is a function then its *graph* $\{(x, f(x)) : x \in X\}$ is a binary relation on X.
- If $X = \mathbf{R}$ then $\geq$ is a binary relation on X.
- If $X = \mathbf{N}$ is the set of natural numbers then "is divisible by" is a binary relation on X.

Digression: binary relations

A binary relation B is:

- *complete* when $x \, B \, y$ or $y \, B \, x$ (or both) for all $x, y \in X$.
- *transitive* when $x \, B \, y$ and $y \, B \, z$ implies that $x \, B \, z$

A binary relation that is complete and transitive is called a *weak order*.

Digression: binary relations

A function $u : X \to \mathbf{R}$ *represents* (or *is a representation of*) the binary relation B if

$$x \, B \, y \text{ iff } u(x) \geq u(y).$$

Proposition

If u represents B, then B is a weak order.

Preference and Choice

A weak order is called a *(rational) preference relation.*

Two ways you can think of preferences:

1. As shorthand for a choice defined on all doubletons.
2. As a ranking in an agent's mind.

The preferred interpretation in neoclassical economics is 1.

In the context of rational preference relations, when a function represents $\succeq$ we say it is a *utility function* for $\succeq$.

Strict Preference and Indifference

When $\succeq$ is a rational preference and it is false that $y \succeq x$ then we write $x \succ y$. (*Strict* preference.)

When $x \succeq y$ and $y \succeq x$ we write $x \sim y$. (*Indifference*)

Obs. that if u represents $\succeq$, then $x \succ y$ iff $u(x) > u(y)$; and $x \sim y$ iff $u(x) = u(y)$.

Preference and Choice

Given $B \subset X$ and a (rational) preference $\succeq$,

$$c^*_{\succeq}(B) = \{x \in B : x \succeq y \text{ for all } y \in B\}$$

is the choices that are *maximal* for the preference $\succeq$.

So: given X and $\mathcal{B}$, a preference $\succeq$ defines (or generates) a choice structure $(X, \mathcal{B}, c^*_{\succeq})$.

Preference and Choice

Let $\succeq$ be a rational preference and $(X, \mathcal{B}, c)$ a choice structure.

Say that $\succeq$ *(strongly) rationalizes* the choice structure if

$$(X, \mathcal{B}, c^*_{\succeq}) = (X, \mathcal{B}, c)$$

When there is a preference that rationalizes a choice structure, we say that the structure is rationalizable.

Preference and Choice

Let $(X, \mathcal{B}, c)$ a choice structure.

The *revealed preference* defined from $(X, \mathcal{B}, c)$ is the binary relation $\succeq^*$ defined by $x \succeq^* y$ whenever $x \in c(B)$ and $y \in B$, for some $B \in \mathcal{B}$.

Weak axiom of revealed preference

A choice structure $(X, \mathcal{B}, c)$ satisfies the *weak axiom of revealed preference (WARP)* if whenever there is $B \in \mathcal{B}$ and $y \in B$ and $x \in c(B)$ then for any $B' \in \mathcal{B}$ with $x \in B'$

$$y \in c(B') \Longrightarrow x \in c(B').$$

Exercise

Show that any rationalizable choice structure must satisfy WARP.

Weak axiom of revealed preference

Theorem

Let $(X, \mathcal{B}, c)$ be a choice structure in which $\mathcal{B}$ contains all subsets of cardinality two and three. Then $(X, \mathcal{B}, c)$ is rationalizable iff it satisfies WARP.

Proof

Before we do the proof. Please press pause and try to prove this on your own.

Hint: think of the revealed preference relation. First show that this is complete and transitive.

Proof

($\Longrightarrow$) Let $\succeq^*$ be the revealed preference relation defined from $(X, \mathcal{B}, c)$. Since $\mathcal{B}$ contains all doubletons, and choice is nonempty, $\succeq^*$ is complete.

Now suppose that $x \succeq^* y$ and $y \succeq^* z$, meaning that (1) $c(\{x, y\}) \ni x$ and (2) $c(\{y, z\}) \ni y$.

Consider $c(\{x, y, z\})$. If this choice does not contain x then it has to contain y or z. If it contains y then we have a violation of WARP with (1). If it contains neither y or x then it must have z and we have a violation of WARP with (2).

So $\succeq^*$ is a preference.

Proof

Now to show that $c_{\succeq^*}(B) = c(B)$.

If $x \in c_{\succeq^*}(B)$ and $y \in c(B)$ then WARP requires that $x \in c(B)$ as $x \succeq^* y$ implies $x \in c(B')$ and $y \in B'$ for some B'.

Conversely, $x \in c(B)$ and $y \in c_{\succeq^*}(B)$ implies $y \in B$ and thus $x \succeq^* y$. So $x \in c_{\succeq^*}(B)$.

Proof

($\Longleftarrow$) Let $(X, \mathcal{B}, c)$ be rationalized by the preference $\succeq$.

If $x \in c(B)$ and $y \in B$ then $x \succeq y$.

So, for any $B' \ni x, y$, if $y \in c(B')$ then $x \succeq y$ implies $x \in c(B')$.

Violations of WARP

Category: 35mm camera		Share (%)	
Brand	Price ($)	Set 1 (n = 106)	Set 2 (n = 115)
x Minolta X-370	169.99	50	22
y Minolta Maxxum 3000i	239.99	50	57
z Minolta Maxxum 7000i	469.99	—	21

From Simonson and Tversky (1989)

Violations of WARP

Other examples:

- Just-noticeable differences.
- Collective (household) choice.
- Framing.
- Unstable preferences (changing tastes).

Walrasian budgets and choice.

Let $X \subseteq \mathbf{R}^n_+$, and define

$$B(p, I) = \{x \in X : p \cdot x \leq I\}.$$

where $p \cdot x = \sum_l p_l x_l$.

$B(p, I)$ is a *Walrasian budget set* for prices p and income I.

Let $\mathcal{B}$ be the set of Walrasian budget sets for some prices and incomes.

$$\mathcal{B} = \{B(p, I) : p \in \mathbf{R}^n_+, I > 0\}$$

Obs: No $B(p, I)$ is finite. Result assuming doubletons and tripletons do not apply.

Walrasian demand.

In this context, a choice structure $(X, \mathcal{B}, c)$ can be identified with a demand function (or correspondence)

$$x^*(p, I) = c(B(p, I)),$$

which depends on prices and income.

Note that, for any $\lambda > 0$, $B(\lambda p, \lambda I) = B(p, I)$. Hence $x^*(\lambda p, \lambda I) = x^*(p, I)$: demand is *homogeneous of degree zero*.

There is no "monetary illusion" in demand.

Walrasian demand.

In general, $x^*(p, I)$ may contain more than one element. It is called a demand correspondence.

When $x^*(p, I)$ is a singleton for all (p, I) we call it a demand function.

Walrasian demand.

A demand correspondence x^* satisfies the *weak axiom of revealed preference (WARP)* if, for any $(p, I), (p', I') \in \mathbb{R}^n_+ \times \mathbb{R}_+$, $x \in x^*(p, I)$, and $x' \in x^*(p', I')$:

$$x \neq x' \text{ and } p \cdot x' \leq I \Longrightarrow x \in x^*(p', I') \text{ or } p' \cdot x > I'$$

A demand function x^* satisfies the *weak axiom of revealed preference (WARP)* if, for any $(p, I), (p', I') \in \mathbb{R}^n_+ \times \mathbb{R}_+$:

$$x^*(p, I) \neq x^*(p', I') \text{ and } p \cdot x^*(p', I') \leq I \Longrightarrow p' \cdot x^*(p, I) > I'.$$