CORRESPONDENCE PRINCIPLE

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The Correspondence Principle is the relation, which exists in certain economic models, between comparative statics of equilibria and the properties of out-of-equilibrium dynamics.

The Correspondence Principle (CP) implies that one obtains unambiguous comparative statics by selecting equilibria with desirable dynamic properties. Generally, the CP determines comparative statics in models with a one-dimensional endogenous variable, and in monotone multidimensional models. It does not determine comparative statics in general multidimensional models, such as Walrasian general equilibrium models with more than two goods.

One-dimensional models.

The CP holds quite generally in one-dimensional models. Consider, for example, a two-good economy with excess-demand function for good 1 given by z_1, shown in the figure below. We fix the price of good 2; by Walras's Law the equilibrium prices are the zeroes of z_1: there are three equilibria, p_1^1, p_1^2 and p_1^3.

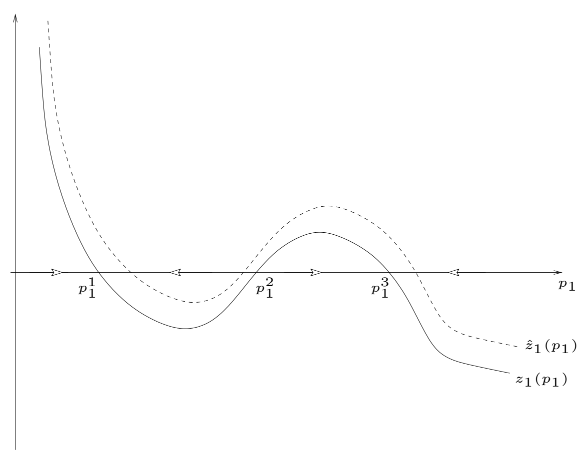

Figure 1. Two-good economy.

Date: October 2, 2006.

I thank Kim Border and Matias Iaryczower for comments.

Now consider the comparative-statics exercise of shifting excess demand up to $\hat{z}_1$. What is the effect on equilibrium price? Locally, the price increases if the equilibrium is p_1^1 or p_1^3, but it decreases if it is p_1^2. The different comparative statics at p_1^1 and p_1^2 corresponds exactly to the different behavior of tâtonnement dynamics after a small perturbation: p_1^1 is stable while p_1^2 is unstable.

The difference between comparative statics at p_1^1 and at p_1^2 is easy to explain. The comparative statics at p_1^1 says: slightly larger prices than p_1^1 are reached by increasing excess demand, and smaller prices are reached by decreasing excess demand. Since excess demand is zero at p_1^1, there must be *positive* excess demand at slightly larger prices and *negative* excess demand at slightly smaller prices. Hence, tâtonnement dynamics, which respond to the sign of excess demand, converges to p_1^1 after a small perturbation from p_1^1. On the other hand, at p_1^2, larger prices result from a decrease in excess demand; hence excess demand is positive at larger prices. Similarly, excess demand is negative at smaller prices. As a result, tâtonnement dynamics will not approach p_1^2 after a small perturbation from p_1^2.

If the economy is subject to sporadic shocks, one should not observe p_1^2, the unstable equilibrium. Hence, as a consequence of the correspondence between comparative statics and dynamics, one should expect an increase in excess demand to produce an increase in equilibrium price.

I shall give a general statement of the Correspondence Principle for the one-dimensional case. Consider a model where the endogenous variable takes values in $[0,1]$ and equilibria are determined as the fixed points of $f(\cdot,t):[0,1]\to[0,1]$; $t\in T\subseteq\mathbf{R}$ is an exogenous parameter. Assume that T is convex and that f is C^1.

A *selection of equilibria* is a function $e:T\to[0,1]$ such that $e(t)=f\left(e(t),t\right)$ for all $t\in T$. Say that a fixed point $x\in[0,1]$ is *stable* if there is a neighborhood V of x such that any sequence $\{x_n\}$ satisfying $x_0\in V$ and $x_{n+1}=f(x_n)$ for $n\geq 1$, converges to x. Say that $x\in[0,1]$ is *unstable* if, for any neighborhood V of x, there is a neighborhood W of x such that all sequences defined as above eventually lie in the complement of W.

Proposition 1. *Let f be monotone increasing in t. If e is a continuous selection of equilibria that is strictly decreasing over some interval $\left[\underline{t},\bar{t}\right]$, then for all $t\in(\underline{t},\bar{t})$, $e(t)$ is unstable.*

Multidimensional models.

The one-dimensional CP is a relation between the sign of the comparative-statics change in prices, and the sign of excess demand for smaller and larger prices. When more than one price is determined, this relation does not need to exist. Still, the CP holds for monotone models; models where the different dimensions of the endogenous variables are in some sense complements. Monotone economic models stem mainly from game theoretic models with strategic complementarities.

I proceed to give a statement of the CP. Consider a model where the endogenous variable takes values in a compact rectangle $X \subseteq \mathbf{R}^n$, and equilibria are determined as the fixed points of $f(\cdot, t) : X \to X$; $t \in T \subseteq \mathbf{R}$ is a parameter and T is convex.

Proposition 2. *Let f be monotone increasing in (x,t) and let e be a continuous selection of equilibria.*

- *If e is strictly decreasing over $\left[\underline{t}, \bar{t}\right] \subseteq T$, then for all $t \in (\underline{t}, \bar{t})$, $e(t)$ is unstable.*
- *If e is strictly increasing over $\left[\underline{t}, \bar{t}\right]$, then for all $t \in (\underline{t}, \bar{t})$, if $e(t)$ is locally isolated, it is stable.*

Literature

The CP was formulated by Paul Samuelson (1941; 1942; 1947), who also coined the term (though Hicks (1939) stated the CP informally). Samuelson formulated the one-dimensional CP. The version in Proposition 1 is taken from Echenique (2000). Bassett, Maybee, and Quirk (1968) study the scope of the CP. Arrow and Hahn (1971) present a critical discussion of the CP, and, because it fails in economies with more than two goods, conclude that "very few useful propositions are derivable from this principle." The monotone multidimensional CP is from Echenique (2002), who presents a general version of Proposition 2. Echenique (2004) presents a CP that does not rely on continuous selections of equilibria. The CP is also effective in dynamic optimization models (Brock, 1983; Burmeister and Long, 1977; Magill and Sheinkman, 1979) and in models of international trade (Bhagwati, Brecher, and Hatta, 1987).

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