

# Measuring Deviations from Theories of Choice Under Risk and Uncertainty

---

Federico Echenique – UC Berkeley

Taisuke Imai – LMU Munich

Kota Saito – Caltech

IECON – July 2023

Work in progress!!

Revealed preference theory asks

*When are agent's choices consistent with utility maximization?*

Revealed preference theory asks

*When are agent's choices consistent with utility maximization?*  
about general utility maximization

Revealed preference theory asks

*When are agent's choices consistent with utility maximization?*  
about general utility maximization

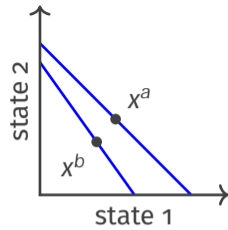
Recent theory is about specific functional forms

Revealed preference theory asks

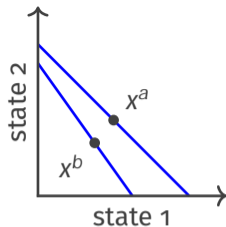
*When are agent's choices consistent with utility maximization?*  
about general utility maximization

Recent theory is about specific functional forms

This talk **expected utility**



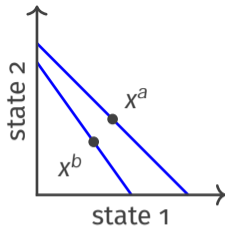
$$\begin{aligned} \max_{x^s} \quad & u(x^s) \\ \text{s.t.} \quad & p_s x_s \leq I \end{aligned}$$



When are **choices from budget sets**  $(x^k, p^k)$ ,  $k = a, b$ , consistent with EU?



$$\begin{aligned} \max_{x_s} & \quad u(x_s) \\ \text{s.t.} & \quad p_s x_s \leq I \end{aligned}$$

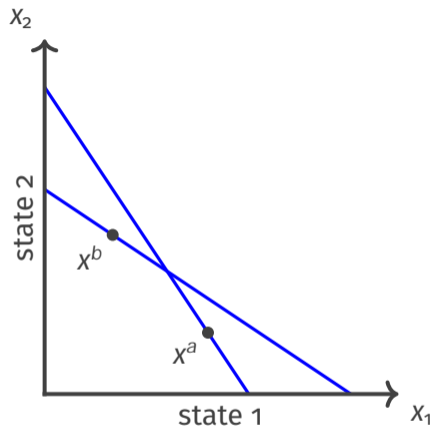


When are **choices from budget sets**  $(x^k, p^k)$ ,  $k = a, b$ , consistent with EU?

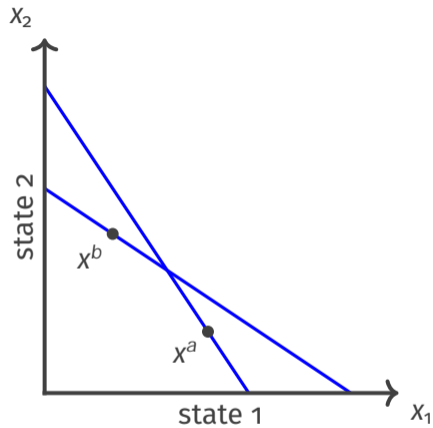
Can we find  $u$  (and  $\beta$ ) such that for each problem  $k \in \{a, b\}$

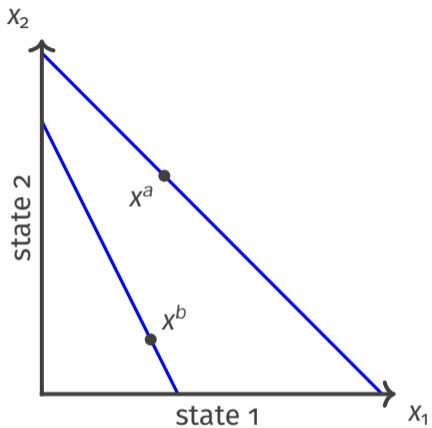
$$\begin{aligned} (x_1^k, x_2^k) & \geq \operatorname{argmax}_{(x_1, x_2)} \beta_1 u(x_1) + \beta_2 u(x_2) \\ \text{s.t.} & \quad p_1^k x_1 + p_2^k x_2 \leq p_1^k x_1^k + p_2^k x_2^k \end{aligned}$$

Green and Srivastava (1986), Kübler et al. (2014), Echenique and Saito (2015), Polisson et al. (2020)



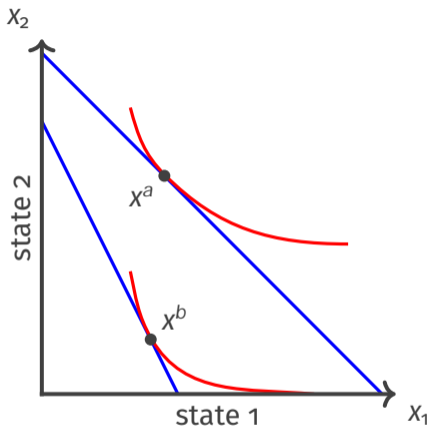
Violation of WARP





Risk-averse OEU agent solves

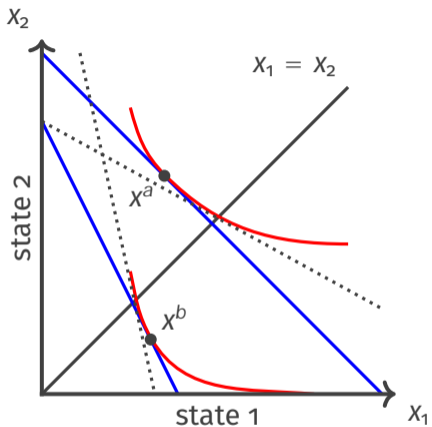
$$\begin{aligned} \max \quad & p_1 u(x_1) + p_2 u(x_2) \\ \text{s.t.} \quad & p_1 x_1 + p_2 x_2 \leq I \end{aligned}$$



Risk-averse OEU agent solves

$$\begin{aligned} \max \quad & {}_1u(x_1) + {}_2u(x_2) \\ \text{s: t: } & p_1x_1 + p_2x_2 \leq I \end{aligned}$$

$$\text{MRS} = \frac{{}_1u'(x_1)}{{}_2u'(x_2)}$$



Risk-averse OEU agent solves

$$\begin{aligned} \max \quad & p_1 u(x_1) + p_2 u(x_2) \\ \text{s.t.} \quad & p_1 x_1 + p_2 x_2 \leq I \end{aligned}$$

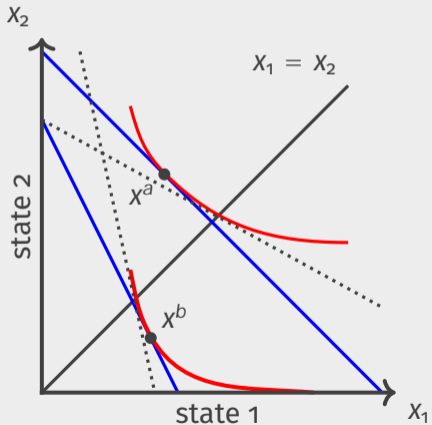
$$\text{MRS}_{j_{x_1=x_2}} = \frac{p_1 u'(x_1)}{p_2 u'(x_2)} = \frac{1}{2}$$

... but ..... have different slopes

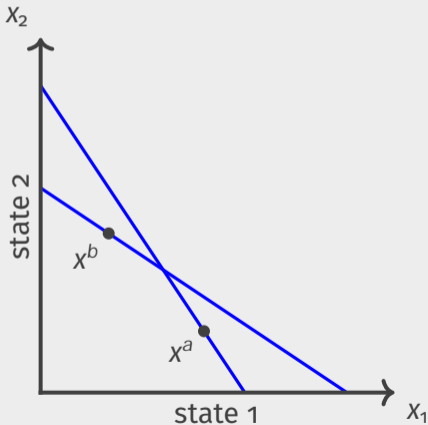
# When are choices from budget sets consistent with EU?

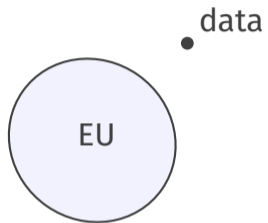
Kübler et al. (2014), Echenique and Saito (2015)

Not

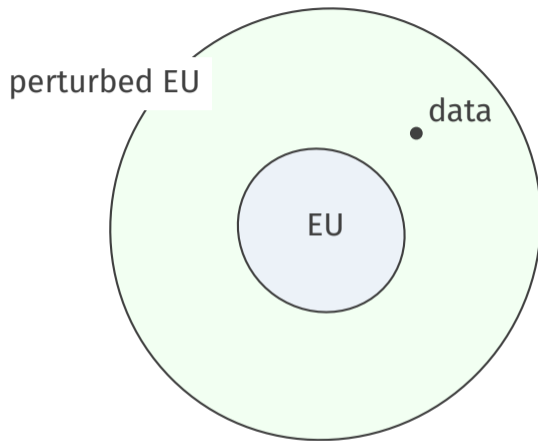


Not

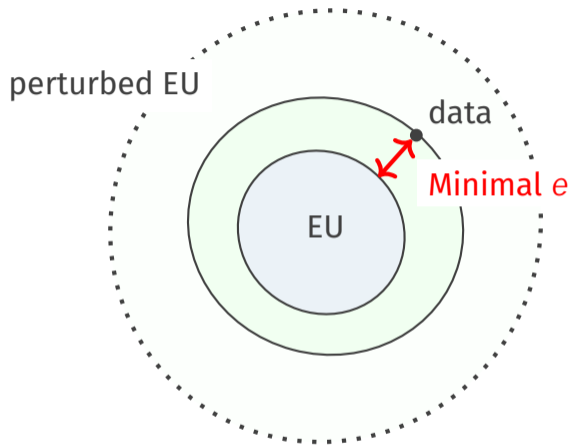








Echenique, Imai, Saito (202x) *JEEA*



Echenique, Imai, Saito (202x) *JEEA*

$$\begin{aligned} \max_{x^k} & \sum_{s \in S} u(x_s^k) \\ \text{s.t.} & \sum_{s \in S} p_s^k x_s^k \leq I, \quad k = 1, \dots, K \end{aligned}$$

$$\begin{aligned} \max_{x^k} \quad & \sum_{s \in S} u(x_s^k) \\ \text{s.t.} \quad & \sum_{s \in S} p_s^k x_s^k \leq I, \quad k = 1, \dots, K \end{aligned}$$

1. **Belief** perturbation:  $p_s \leftarrow e_s^k / \sum_{s \in S} e_s^k$

$$\begin{aligned} \max_{x^k} \quad & \sum_{s \in S} u(x_s^k) \\ \text{s.t.} \quad & \sum_{s \in S} p_s^k x_s^k \leq I, \quad k = 1, \dots, K \end{aligned}$$

1. Belief perturbation:  $p_s \rightarrow e_s^k / \sum_{s \in S} e_s^k$
2. Utility perturbation:  $u(x_s^k) \rightarrow u(x_s^k) + \epsilon_s^k$

$$\begin{aligned} \max_{x^k} \quad & \sum_{s \in S} u(x_s^k) \\ \text{s.t.} \quad & \sum_{s \in S} p_s^k x_s^k \leq I, \quad k = 1, \dots, K \end{aligned}$$

1. Belief perturbation:  $p_s \rightarrow e_s^k / \sum_{s \in S} e_s^k$
2. Utility perturbation:  $u(x_s^k) \rightarrow u(x_s^k) + \epsilon_s^k$
3. Price perturbation:  $p_s^k \rightarrow p_s^k + \epsilon_s^k$

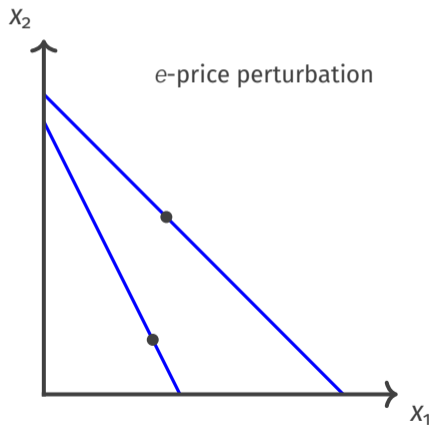
$$\begin{aligned} \max_{x^k} \quad & \sum_s u(x_s^k) \\ \text{s.t:} \quad & \sum_{s \in S} p_s^k x_s^k \leq I, \quad k = 1, \dots, K \end{aligned}$$

1. Belief perturbation:  $p_s \rightarrow e_s^k / \sum_{s \in S} e_s^k$
2. Utility perturbation:  $u(x_s^k) \rightarrow u(x_s^k)^k$
3. Price perturbation:  $p_s^k \rightarrow p_s^k$

Three ways of perturbation are equivalent

## Minimal $\epsilon$

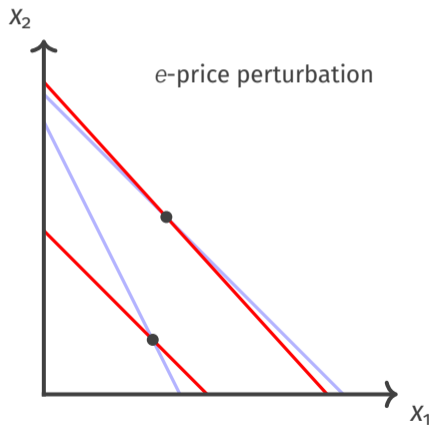
Echenique et al. (202x) *JEEA*





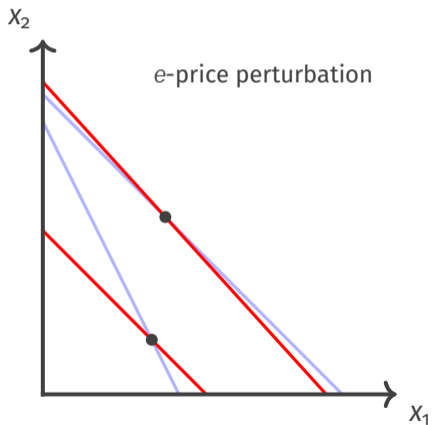
## Minimal $\epsilon$

Echenique et al. (202x) *JEEA*



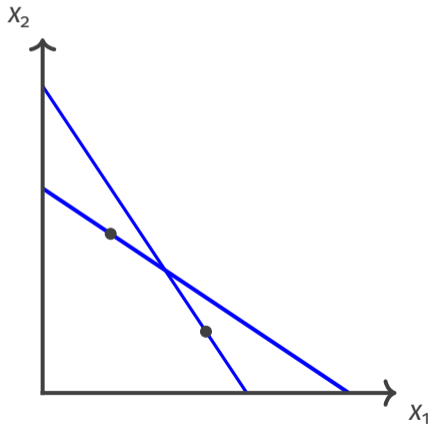
## Minimal $\epsilon$

Echenique et al. (202x) *JEEA*



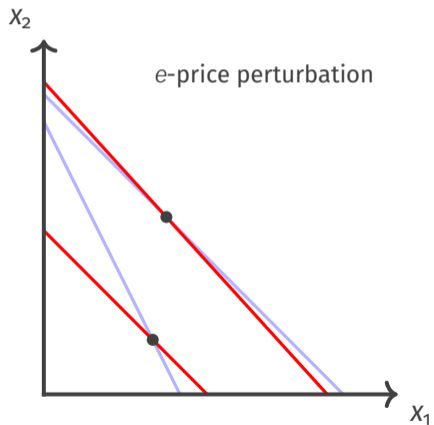
## Critical cost efficiency index

Afriat-Varian



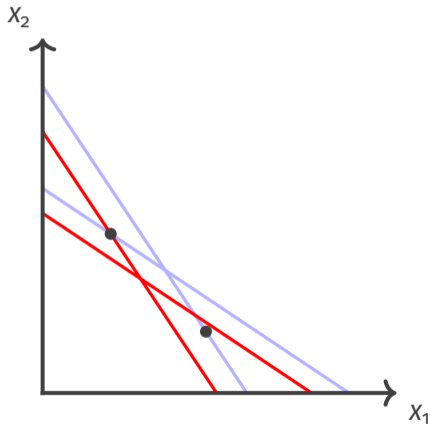
## Minimal $\epsilon$

Echenique et al. (202x) *JEEA*



## Critical cost efficiency index

Afriat-Varian



## Experiments    budgetary choice under risk

Choi et al. (2014)

Carvalho et al. (2016)

Carvalho and Silverman (2019)

## Experiments    budgetary choice under risk

Choi et al. (2014)

Carvalho et al. (2016)

Carvalho and Silverman (2019)

Banks et al. (2019)

Castillo et al. (2017)

Cettolin et al. (2020)

Choi et al. (2007)

Choi et al. (2015)

Dembo et al. (2021)

Drichoutis and Nayga (2020)

Halevy et al. (2018)

Halevy and Mayraz (2022)

Kim et al. (2018)

Kurtz-David et al. (2019)

Loomes and Pogrebna (2014)

Stango and Zinman (2020a,b)

Zrill (2020)

...

## Experiments    budgetary choice under **risk**

Choi et al. (2014)

Carvalho et al. (2016)

Carvalho and Silverman (2019)

Banks et al. (2019)

Castillo et al. (2017)

Cettolin et al. (2020)

Choi et al. (2007)

Choi et al. (2015)

Dembo et al. (2021)

Drichoutis and Nayga (2020)

Halevy et al. (2018)

Halevy and Mayraz (2022)

Kim et al. (2018)

Kurtz-David et al. (2019)

Loomes and Pogrebna (2014)

Stango and Zinman (2020a,b)

Zrill (2020)

...

**Symmetric** (50-50) environment, **one-shot** experiment

1. Do choices individuals make satisfy **RP axioms**?
2. To what extent are choices **consistent** with theories of  $U$  max?

1. Do choices individuals make satisfy RP axioms?  
... and how are they influenced by the **environment**?
2. To what extent are choices consistent with theories of  $U$  max?  
... and how are they influenced by the **environment**?



1. Do choices individuals make satisfy RP axioms?  
... and how are they influenced by the environment?
2. To what extent are choices consistent with theories of  $U$  max?  
... and how are they influenced by the environment?
3. Are preferences **stable**? Are measures of consistency **reliable**?

Design

---

## Investment task

Investment task

Two **risky assets**: **Orange** and **Blue**

each asset has a price:  $p_O$  and  $p_B$

Investment task

Two risky assets: Orange and Blue

each asset has a price:  $p_O$  and  $p_B$

Endowment of 100 tokens    allocate between two assets  $(x_O, x_B)$

$$p_O x_O + p_B x_B = 100$$

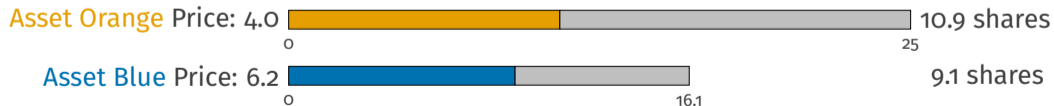
## Investment task

Two risky assets: **Orange** and **Blue**

each asset has a price:  $p_O$  and  $p_B$

Endowment of 100 tokens    allocate between two assets ( $x_O, x_B$ )

$$p_O x_O + p_B x_B = 100$$



## Investment task

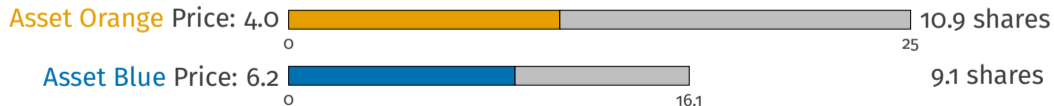
Two risky assets: **Orange** and **Blue**

each asset has a price:  $p_O$  and  $p_B$

Endowment of 100 tokens    allocate between two assets ( $x_O, x_B$ )

$$p_O x_O + p_B x_B = 100$$

**Payoff**  $x_s$  if state  $s \in \{O, B\}$  realizes



## Investment task

Two risky assets: **Orange** and **Blue**

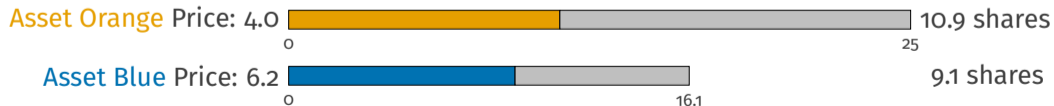
each asset has a price:  $p_O$  and  $p_B$

Endowment of 100 tokens    allocate between two assets ( $x_O, x_B$ )

$$p_O x_O + p_B x_B = 100$$

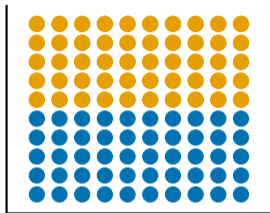
Payoff  $x_s$  if state  $s \in \{O, B\}$  realizes

$x_s = \text{Pr}(\text{state } s)$  or **info about**  $x_s$  varies by task

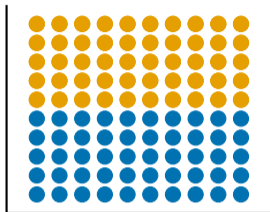




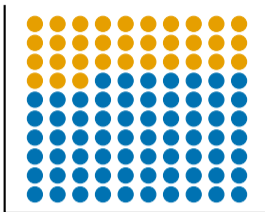
OBJSYM



OBJSYM



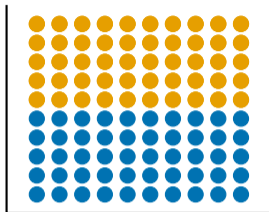
OBJASYMS



OBJASYML



OBJSYM



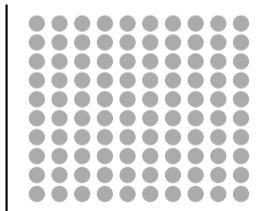
OBJASYMS



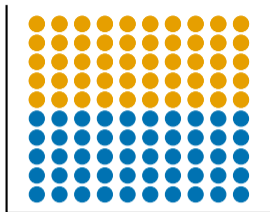
OBJASYML



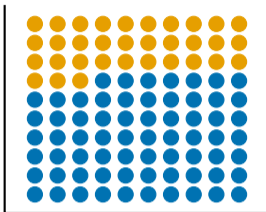
AMBFULL



OBJSYM



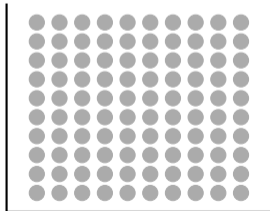
OBJASYMS



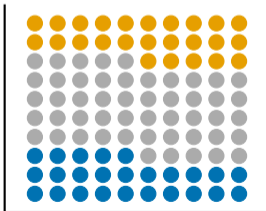
OBJASYML



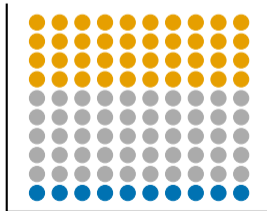
AMBFULL



AMBSYM



AMBASYM



20 questions in each task

20 questions in each task

3 sets of budget sets

equalize “risk-neutral” prices ( $p_s^{\text{rn}} = p_{s=}$ ) of 16 “core” budget sets

across 3 OBJ tasks

Echenique and Saito (2015), Kübler et al. (2014)

same set used in OBJSYM and AMB\*

20 questions in each task

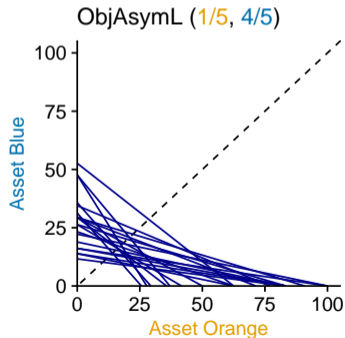
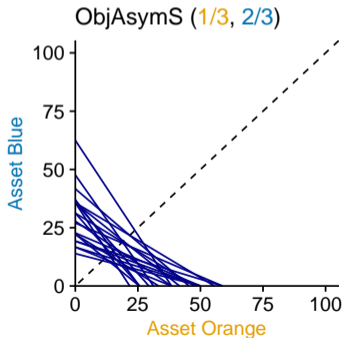
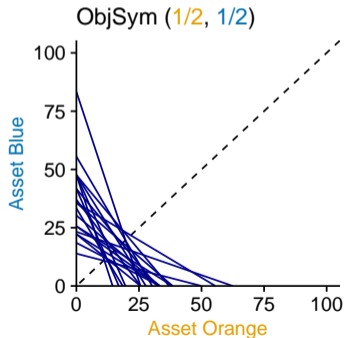
3 sets of budget sets

equalize “risk-neutral” prices ( $p_s^{rn} = p_{s= s}$ ) of 16 “core” budget sets

across 3 OBJ tasks

Echenique and Saito (2015), Kübler et al. (2014)

same set used in OBJSYM and AMB\*



## Study 1 effects of decision environment

Treatment		Task 1		Task 2
1	OBJSYM	(1/2, 1/2)	OBJASYMS	(1/3, 2/3)
2	OBJSYM	(1/2, 1/2)	OBJASYML	(1/5, 4/5)
3	OBJASYMS	(1/3, 2/3)	OBJASYML	(1/5, 4/5)
4	OBJSYM	(1/2, 1/2)	AMBFULL	(0, 1, 0)
5	AMBFULL	(0, 1, 0)	AMBSYM	(1/4, 1/2, 1/4)
6	AMBFULL	(0, 1, 0)	AMBASYM	(4/10, 5/10, 1/10)

Task order randomized



## Study 2 stability/reliability

Treatment		Task 1	Task 2
1	OBJSYM	(1/2, 1/2)	OBJSYM (1/2, 1/2)
2	OBJASYMS	(1/3, 2/3)	OBJASYMS (1/3, 2/3)
3	AMBFULL	(0, 1, 0)	AMBFULL (0, 1, 0)
4	AMBSYM	(1/4, 1/2, 1/4)	AMBSYM (1/4, 1/2, 1/4)

Prolific (US sample)

\$5 participation fee

1/5 “lucky” participants received additional bonus \$\$  
implemented 1 randomly-selected choice

3121 participants in 10 treatments

avg. 312 participants (min 268, max 352)

avg. 37 years old

53% female

# Results

---

## Study 1: Sensitivity to decision environment

OBJSYM



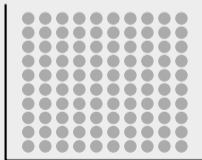
OBJASYMS



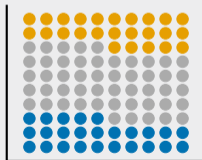
OBJASYML



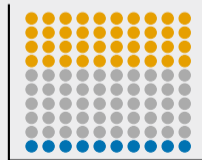
AMBFULL

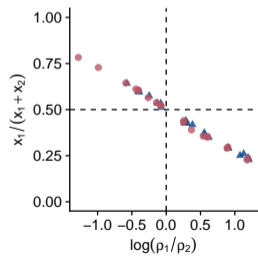
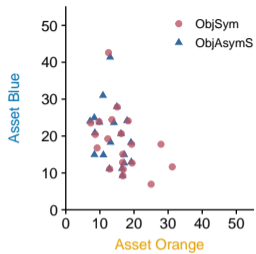


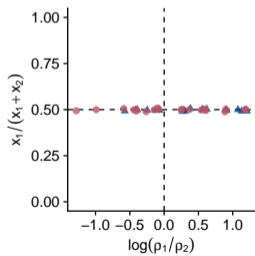
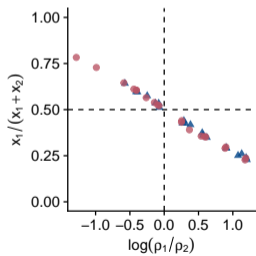
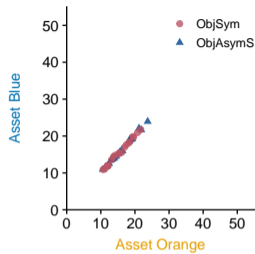
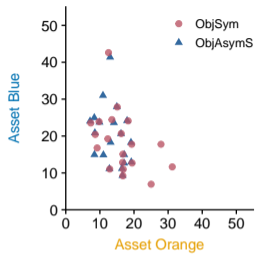
AMBSYM

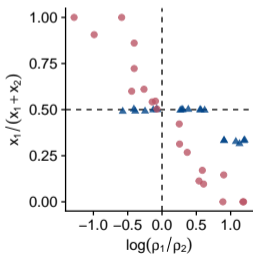
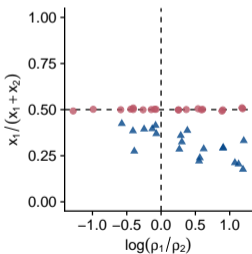
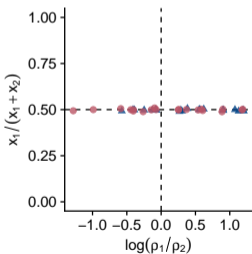
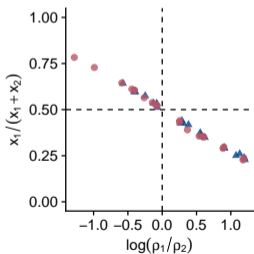
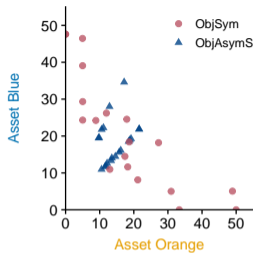
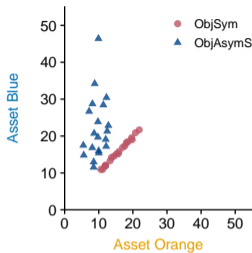
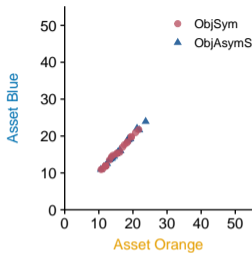
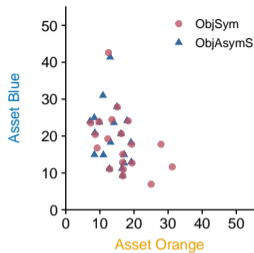


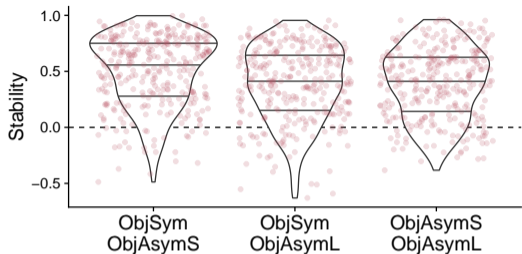
AMBASYM



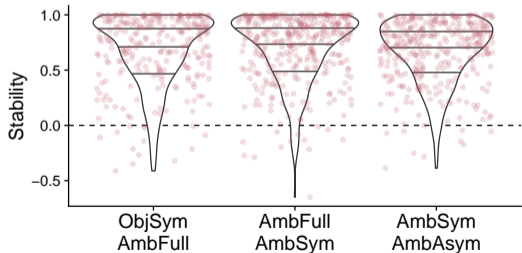






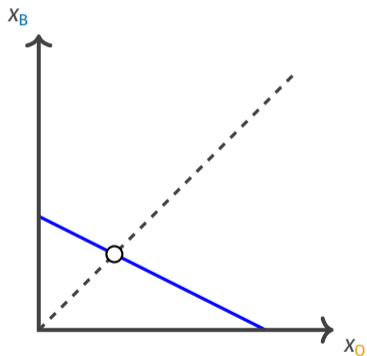


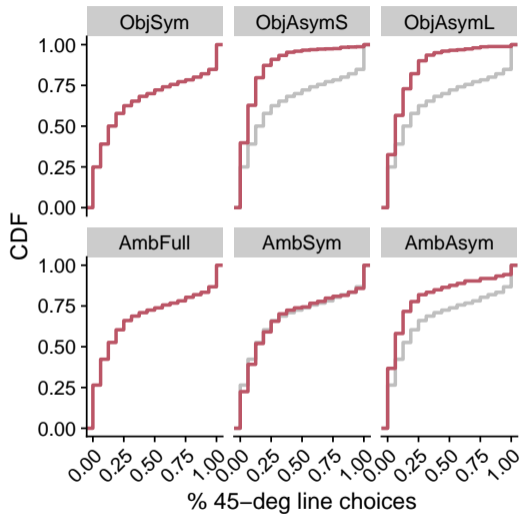
Correlation between choices made in two tasks





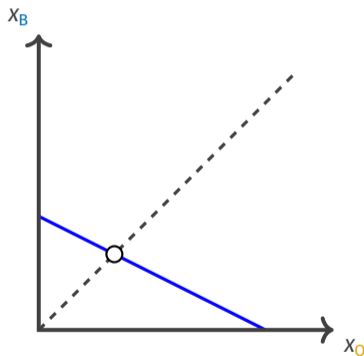
Choices on/close to the 45-degree line ( $x_O = x_B$ ) eliminate risk

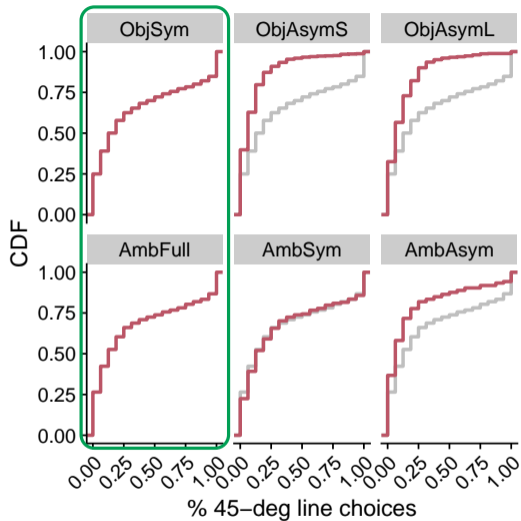




\* Gray lines represent CDF in ObjSym (top) or CDF in AmbFull (bottom)

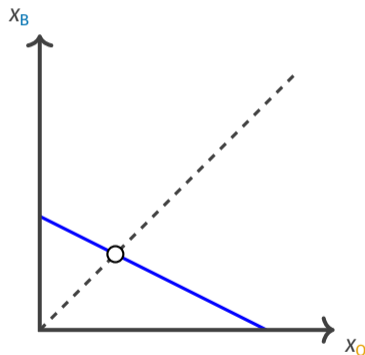
Choices on/close to the 45-deg line ( $x_O = x_B$ ) eliminate risk

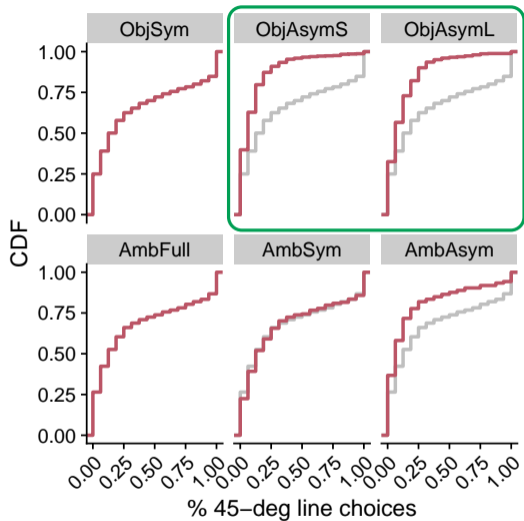




\* Gray lines represent CDF in ObjSym (top) or CDF in AmbFull (bottom)

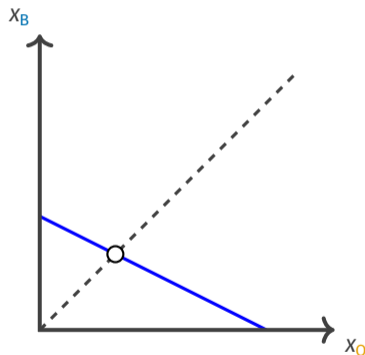
Choices on/close to the 45-deg line ( $x_O = x_B$ ) eliminate risk





\* Gray lines represent CDF in ObjSym (top) or CDF in AmbFull (bottom)

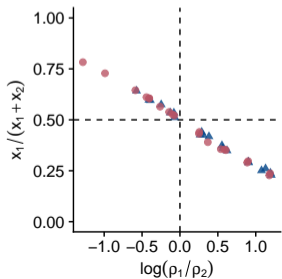
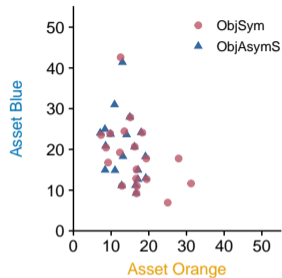
Choices on/close to the 45-deg line ( $x_O$   $x_B$ ) eliminate risk



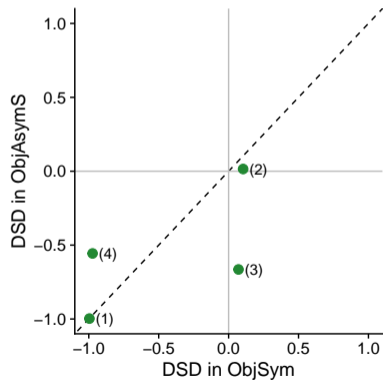
Downward-sloping demand  
sensitivity to price changes

$$\text{corr}(\log(x_O=x_B), \log(p_O=p_B))$$

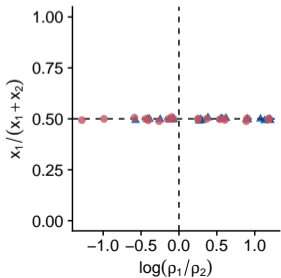
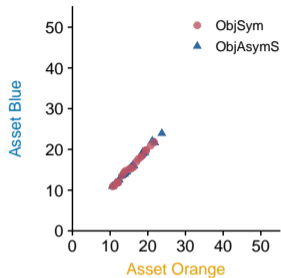
## Example 1



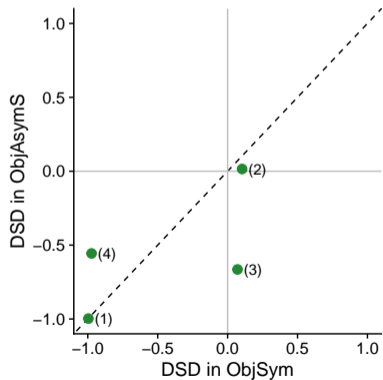
Downward-sloping demand  
sensitivity to price changes  
 $\text{corr}(\log(x_O=x_B), \log(p_O=p_B))$



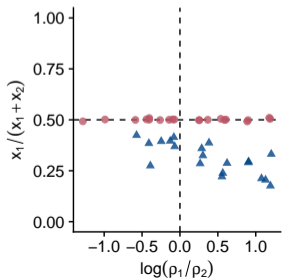
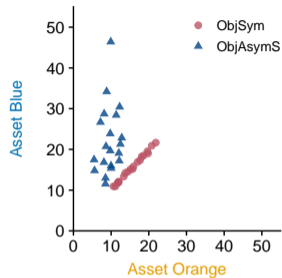
## Example 2



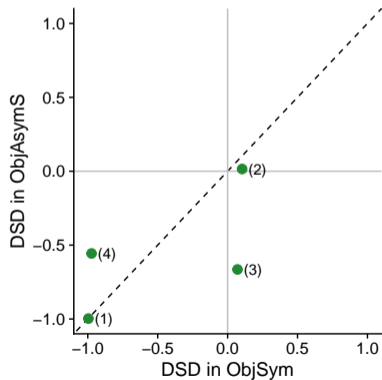
Downward-sloping demand  
sensitivity to price changes  
 $\text{corr}(\log(x_O=x_B), \log(p_O=p_B))$



### Example 3

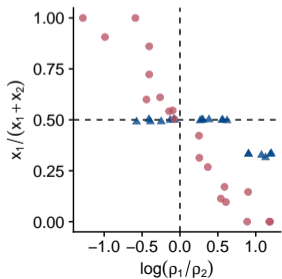
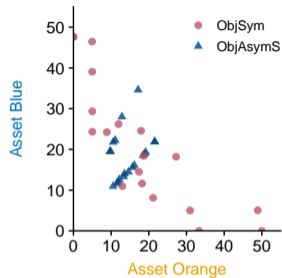


Downward-sloping demand  
sensitivity to price changes  
 $\text{corr}(\log(x_O=x_B), \log(p_O=p_B))$

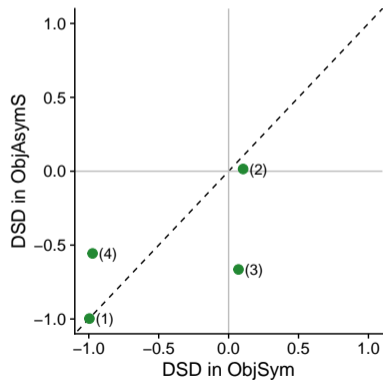




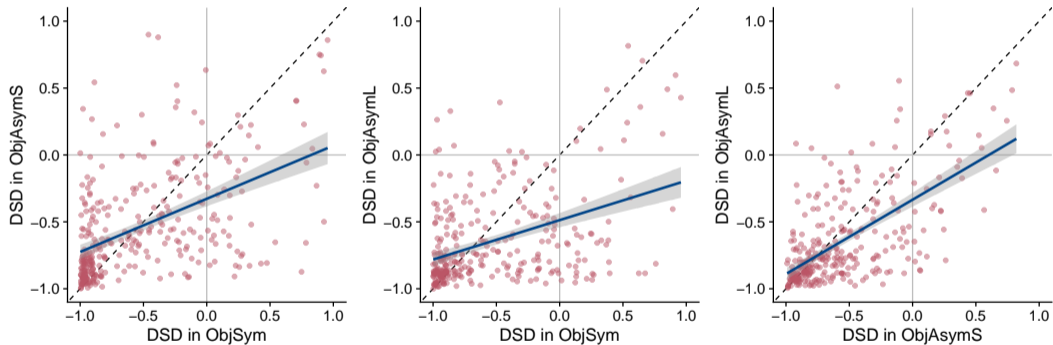
## Example 4



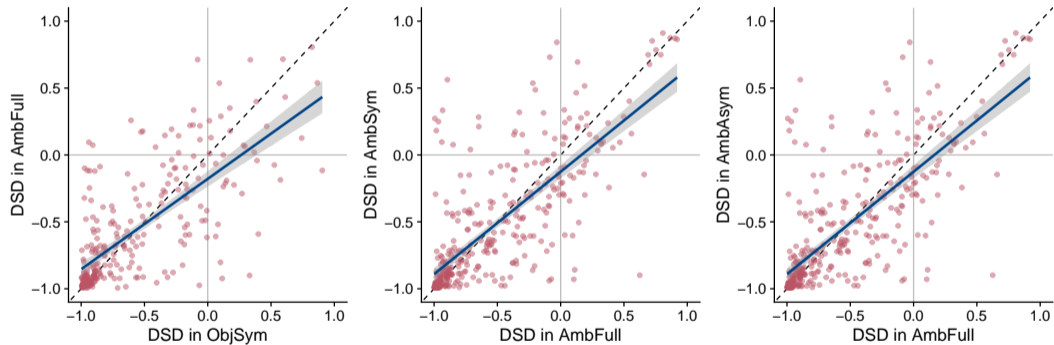
Downward-sloping demand  
sensitivity to price changes  
 $\text{corr}(\log(x_O=x_B), \log(p_O=p_B))$



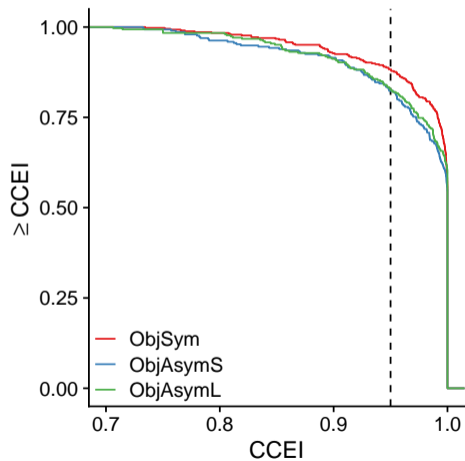
## Objective probabilities: symmetric vs. asymmetric



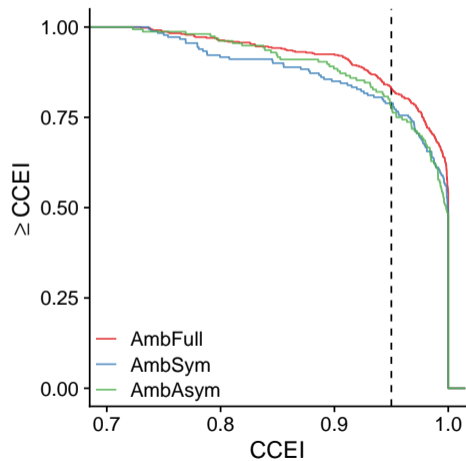
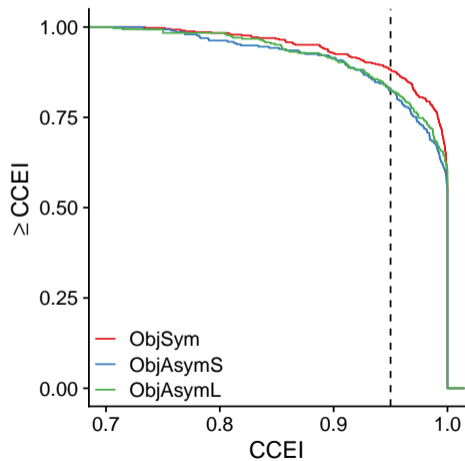
## Full ambiguity vs. partial ambiguity



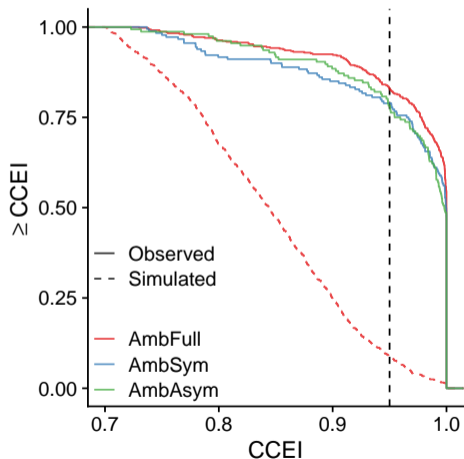
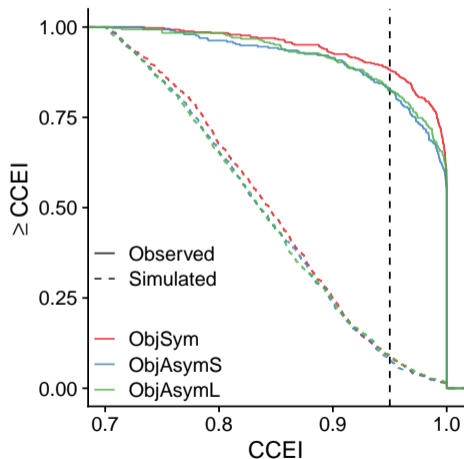
## Measure of consistency: CCEI



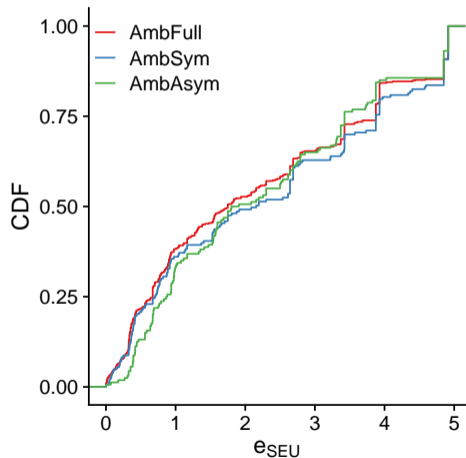
## Measure of consistency: CCEI



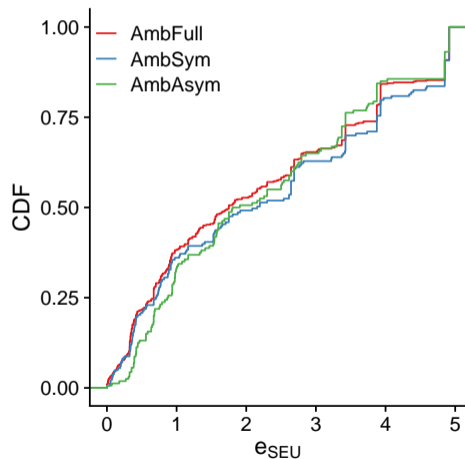
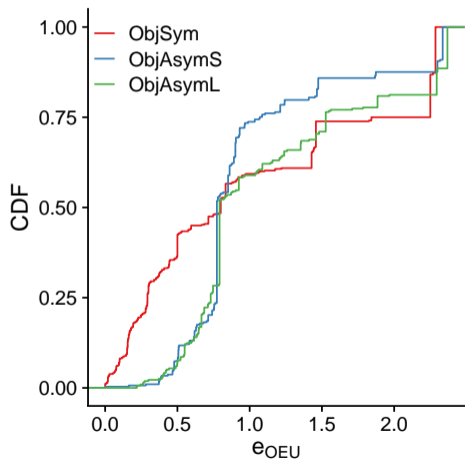
## Measure of consistency: CCEI



# Measure of consistency: Minimal $\epsilon$

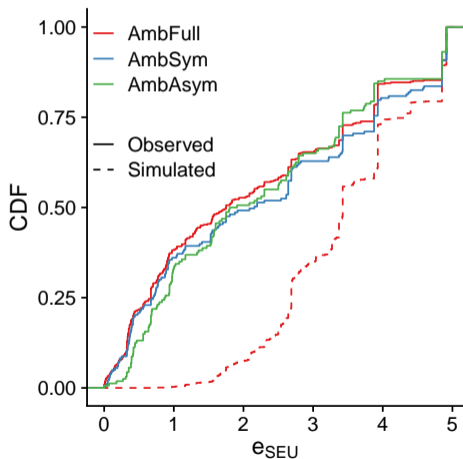
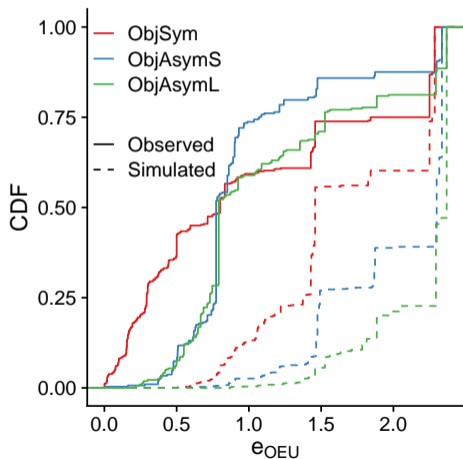


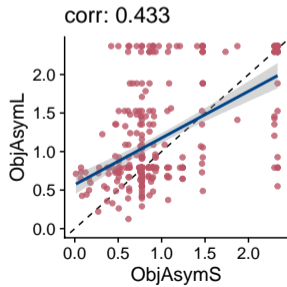
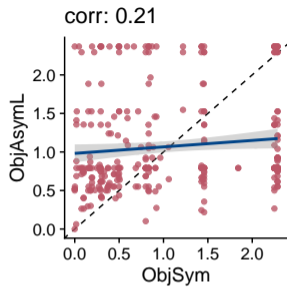
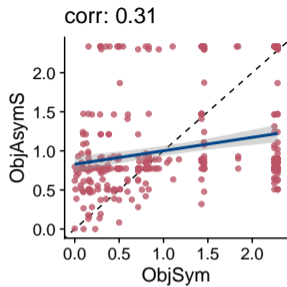
# Measure of consistency: Minimal $\epsilon$

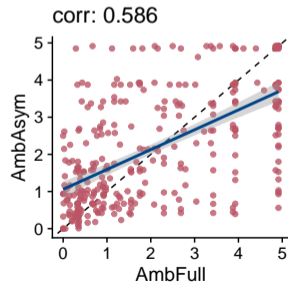
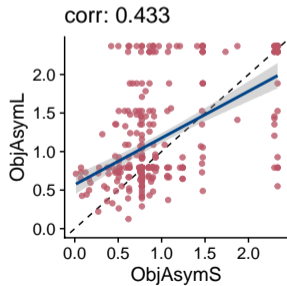
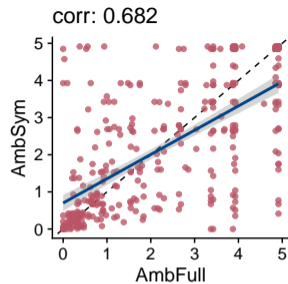
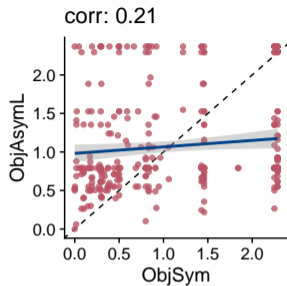
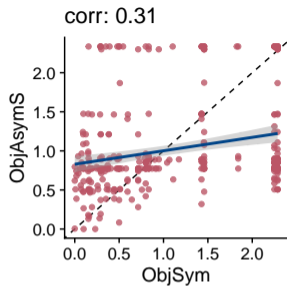




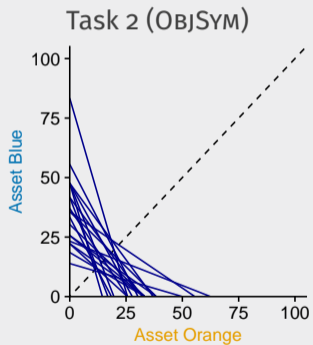
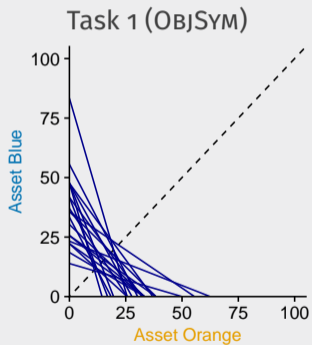
## Measure of consistency: Minimal e

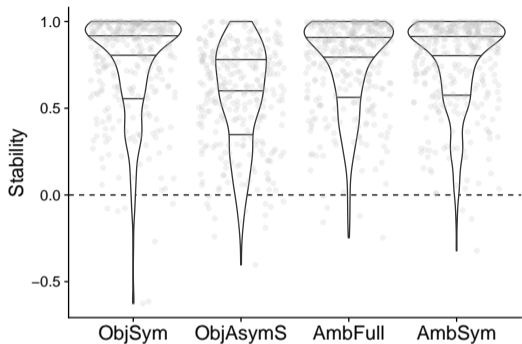




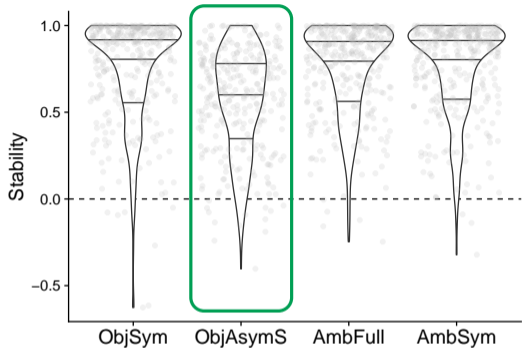


## Study 2: Stability of preferences

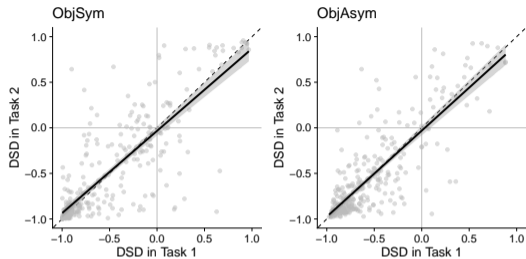
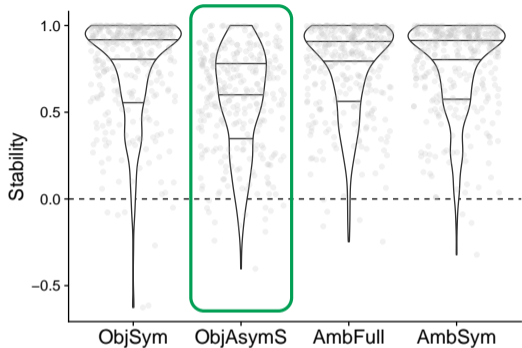




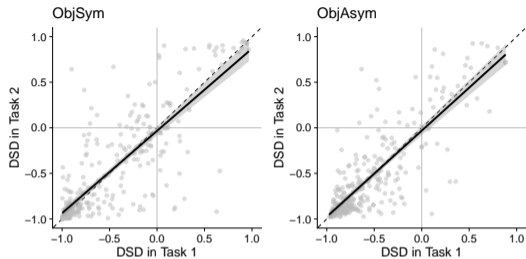
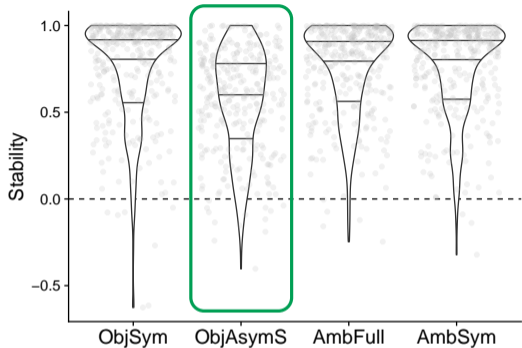
High correlation between  
choices made in two tasks



High correlation between  
choices made in two tasks  
... except for ObjAsym

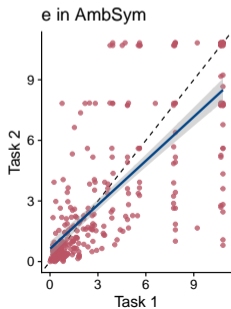
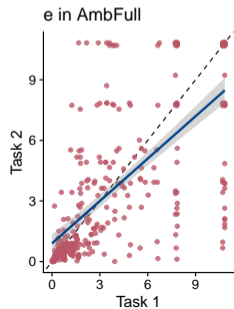
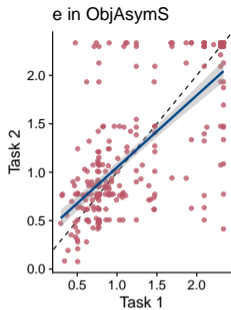
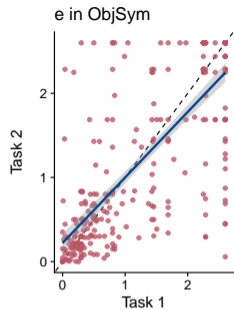


High correlation between  
choices made in two tasks  
... except for ObjAsym



High correlation between  
choices made in two tasks  
... except for ObjAsym





High correlation between e from two tasks

1. Do choices individuals make satisfy RP axioms?  
... and how are they influenced by the environment?
2. To what extent are choices consistent with theories of  $U$  max?  
... and how are they influenced by the environment?
3. Are preferences stable? Are measures of consistency reliable?

1. Do choices individuals make satisfy RP axioms?  
... and how are they influenced by the environment?
2. To what extent are choices consistent with theories of  $U$  max?  
... and how are they influenced by the environment?
3. Are preferences stable? Are measures of consistency reliable?

### Across tasks

participants responded to asymmetry

OBJSYM and AMBFULL were treated similarly

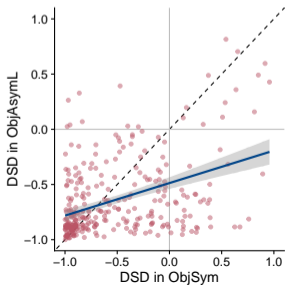
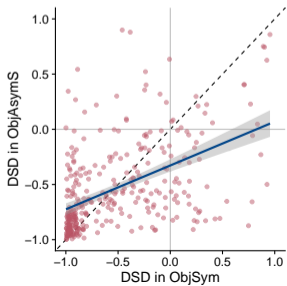
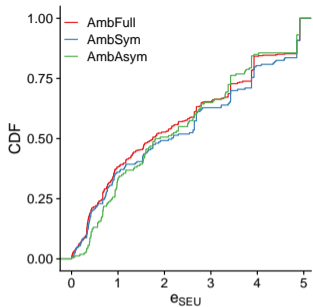
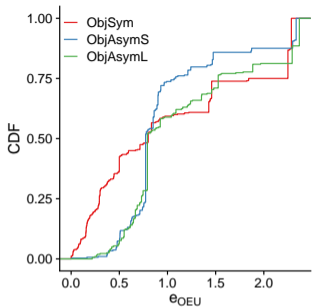
1. Do choices individuals make satisfy RP axioms?  
... and how are they influenced by the environment?
2. To what extent are choices consistent with theories of  $U$  max?  
... and how are they influenced by the environment?
3. Are preferences stable? Are measures of consistency reliable?

### Across tasks

participants responded to asymmetry  
OBJSYM and AMBFULL were treated similarly

### Within task

stable choice patterns



Comments / Questions  
 fede@econ.berkeley.edu