Recovery of utilities and preferences from finite choice data

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Based on two papers:

- Recovering preferences from finite data (published).
- Recovering utility (available soon!)
Model

Alice (an experimenter)

Bob (a subject)
Alice presents Bob with choice problems:

“Hey Bob would you like \( x \) or \( y \)?”

\[ x \text{ vs. } y \]

Bob chooses one alternative.
Rinse and repeat $\rightarrow$ dataset of \( k \) choices.
An experimenter and a subject.

Subject makes choices according to some $\succeq^*$, or utility $u^*$, on set $X$.

Experimenter conducts a finite choice experiment of “size” $k$: $k$ questions, each one a binary choice problem.

Preference $\succeq_k$ or utility $u_k$ as rationalizations or estimates.

How are $\succeq_k$, $\succeq^*$, $u_k$ and $u^*$ related?
Subject chooses among alternatives: \( X = \mathbb{R}^n_+ \).

- Choices come from \( \succeq^* \), a continuous preference.
- \( \Sigma_i = \{x_i, y_i\} \).
- A finite experiment: choose an element from \( \Sigma_i, i = 1, \ldots, k \).
- Assumption: \( \Sigma_\infty = \bigcup_{k=1}^{\infty} \Sigma_k \) is dense.
Example 1

- $y$

- $x \succ^* y$

- $x$
Example 1

- $x \succ^* y$
- $U \succ^* V$
- $\exists x' \in U$ and $y' \in V$ s.t. $\forall k \exists$ rationalizing $\succeq_k$, with $y' \succ_k x'$
- But $x' \succ y'$. $\forall \succeq$ s.t. $\succeq$ is cont. and $\succeq|\Sigma_\infty = \succeq^*|\Sigma_\infty$. 

Chambers-Echenique-Lambert Recovery
Example 1: a “discontinuity.”

- Infinite data (\(\succeq^*\) on \(X\)): observe \(\succeq^*\); so \(x' \succ^* y'\)
- “Limiting” infinite data (\(\Sigma_\infty = \bigcup_{k=1}^{\infty} \Sigma_k\)):
  \[x' \succ y' \quad \forall \quad \text{s.t.} \quad \succeq \mid \Sigma_\infty = \succeq^* \mid \Sigma_\infty.\]
- Finite data: \((\Sigma_1 \ldots, \Sigma_k)\)
  can’t rule out \(y' \succ_k x'\), no matter how large \(k\).
Lesson 1

No amount of finite data may correct a mistaken inference.

Even when the (limiting) infinite data set leaves no room for error.
Example 2

Let $X = \mathbb{R}^n_+$. 

Fix a continuous preference $\succeq^*$ on $X$.

**Proposition (informal)**

There exists rationalizing $\succeq_k$ for each $k$ s.t.

$$\text{complete indifference} = \lim_{k \to \infty} \succeq_k$$
Example 2

Set of alternatives $X = [0, 1]$.

- Left: the subject prefers $x$ to $y$ iff $x \geq y$.
- Right: the subject is completely indifferent.
Example 2

Preference

\[ n=1 \]
Example 2

n=2
Example 2

\[ n=4 \]
Example 2

\[ n=6 \]
Example 2

Chambers-Echenique-Lambert

Recovery

$n=8$
Example 2

Preference

\[ n=10 \]
Example 2

$n=16$

Chambers-Echenique-Lambert  Recovery
Example 2

\[ n = 32 \]
Lesson 2

Discipline matters.

Empiricism is dangerous.

Inevitable role for theory (a Cartesian imperative).
Example 3

Choice under uncertainty:

- State space $S = \{s_1, s_2\}$.
- Choice among monetary acts: $x \in \mathbb{R}^S$.
- Bob is risk-neutral subjective exp. utility maximizer.
- So $x \succeq^* y$ iff $p \cdot x \geq p \cdot y$.
- Preferences described by a prior $p \in \Delta(S)$. 
Bob's preferences:
Suppose $y$ is chosen over $x$, and $x'$ over $y'$.
Suppose \( y \) is chosen over \( x \), and \( x' \) over \( y' \).
Example 3

Suppose $y$ is chosen over $x$, and $x'$ over $y'$.

Bob’s prior $p$ must be steeper than the blue line, and flatter than the green.
Suppose $y$ is chosen over $x$, and $x'$ over $y'$.

Bob’s prior $p$ must be steeper than the blue line, and flatter than the green.
Example 3

Suppose $y$ is chosen over $x$, and $x'$ over $y'$.

Narrows down unobserved comparison: $x'' \succ^* y''$. 
Example 3

Suppose Alice instead uses the max-min model for Bob:

$$u(x) = \min\{p \cdot x : p \in \Pi\}$$

With two states, $\Pi$ is described by four parameters. With more than two states, the model is non-parametric. Then from $y \succ x$ she learns something about the slope of the worst-case priors.
Example 3

$y$ is chosen over $x$, and $x'$ over $y'$. 
Example 3

$y$ is chosen over $x$, and $x'$ over $y'$.

No inference for $x''$ and $y''$. 
A more flexible theory may lead to overfitting.

In fact max-min with $|S| \geq 3$ is “hopeless.”

Any finite dataset will lead to poor out-of-sample predictions.
Example 4
Example 4
Model of preferences must be \textit{closed}.

Can’t allow for approximate behavior to “escape.”
Example 5

- Let $X = [0, 1]$, $\succeq^* = \succeq$ and $u^*(x) = x$.
- For each $k$, let $\succeq_k = \succeq$ and 
  \[
  u_k = \frac{x}{k}.
  \]
- Then $0 = \lim_k u_k$.
- But $\succeq_k = \succeq^*$ for all $k$!
Example 5

Let $X = [0, 1]$, $\geq^* \geq$ and $u^*(x) = x$.

For each $k$, let $\geq_k \geq$ and

$$u_k = \frac{x}{k}.$$

Then $0 = \lim_k u_k$.

But $\geq_k \geq^*$ for all $k$!

(For $\varepsilon > 0$, can choose $u_n$ with $\|u_n\|_\infty = 1$ or $\|u_n\|_1 = 1$ and $0 = \lim_n u_n(x)$ for all $x \in [0, 1 - \varepsilon]$.)
Utility estimates are more delicate than preferences.

Must choose the right utility representation.
Lessons for DT

Typical result in decision theory:

“Utility representation iff axioms. Moreover, utility is unique.”

Axioms $\Rightarrow$ testable implications. (But may require infinite data.)

Uniqueness $\Rightarrow$ identification. But more is needed to ensure utility recovery from finite data.
Alternatives: A topological space $X$.

Preference: A complete and continuous binary relation $\succeq$ over $X$

$\mathcal{P}$ a set of preferences.

A pair $(X, \mathcal{P})$ is a preference environment.
Example: Expected utility preferences

- There are $d$ prizes.
- $X$ is the set of lotteries over the prizes, $\Delta^{d-1} \subset \mathbb{R}^d$.
- An EU preference $\succeq$ is defined by $v \in \mathbb{R}^d$ such that $p \succeq p'$ iff $v \cdot p \geq v \cdot p'$.
- $\mathcal{P}$ is set of all the EU preferences.
Alice wants to recover Bob’s preference from his choices.

- Binary choice problem: \( \{x, y\} \subset X \).
- Bob is asked to choose \( x \) or \( y \).
  - Behavior encoded in a choice function \( c(\{x, y\}) \in \{x, y\} \).
- If Bob’s preference is \( \succeq \) then \( c(\{x, y\}) \succeq x \) and \( c(\{x, y\}) \succeq y \).
- Partial observability: indifference is not observable.
Alice gets finite dataset.

- Experiment of size $k$: $\Sigma^k = \{\Sigma_1, \ldots, \Sigma_k\}$ with $\Sigma_i = \{x_i, y_i\}$.
- Set of growing experiments: $\{\Sigma^k\} = \{\Sigma^1, \Sigma^2, \ldots\}$ with $\Sigma^k \subset \Sigma^{k+1}$. 
Afriat’s theorem and revealed preference tests: Afriat (1967); Diewert (1973); Varian (1982); Matzkin (1991); Chavas and Cox (1993); Brown and Matzkin (1996); Forges and Minelli (2009); Carvajal, Deb, Fenske, and Quah (2013); Reny (2015); Nishimura, Ok, and Quah (2017)


Consistency: Mas-Colell (1978); Forges and Minelli (2009); Kübler and Polemarchakis (2017); Polemarchakis, Selden, and Song (2017)

Identification: Matzkin (2006); Gorno (2019)

Econometric methods: Matzkin (2003); Blundell, Browning, and Crawford (2008); Blundell, Kristensen, and Matzkin (2010); Halevy, Persitz, and Zrill (2018)
OK, so far:

- $(X, \mathcal{P})$ preference env.
- $c$ encodes choice
- $\Sigma^k$ seq. of experiments
A preference $\succeq$ weakly rationalizes the observed choices on $\Sigma^k$ if $c(\{x, y\}) \succeq x$ and $c(\{x, y\}) \succeq y$ for all $\{x, y\} \in \Sigma^k$.

A preference $\succeq$ strongly rationalizes the observed choices on $\Sigma^k$ if $c(\{x, y\}) \succ z$ for $z \in \{x, y\}$, $z \neq c(\{x, y\})$, for all $\{x, y\} \in \Sigma^k$. 
Topology on preferences

Choice of topology: closed convergence topology.

- Standard topology on preferences (Kannai, 1970; Mertens (1970); Hildenbrand, 1970).

- \( \succeq_n \to \succeq \) when:
  1. For all \((x, y) \in \succeq\), there exists a seq. \((x_n, y_n) \in \succ_n\) that converges to 
    \((x, y)\).
  2. If a subsequence \((x_{n_k}, y_{n_k}) \in \succeq_{n_k}\) converges, the limit belongs to \(\succeq\).

- If \(X\) is compact and metrizable, same as convergence under the Hausdorff metric.

- \(X\) Euclidean and \(\mathcal{B}\) the strict parts of cont. weak orders. Then it’s the smallest topology for which the set

\[
\{(x, y, \succ) : x \in X, y \in X, \succ \in \mathcal{B} \text{ and } x \succ y\}
\]

is open.
Lemma

Let $X$ be a locally-compact Polish (separable and completely metrizable) space. Then the set of all continuous binary relations on $X$ is a compact metrizable space.
Topology of compact convergence

Let \( \{u_k\} \) be a sequence of functions,

\[ u_k : X \rightarrow \mathbb{R}. \]

The sequence \emph{converges compactly} to \( u : X \rightarrow \mathbb{R} \) if for every compact \( K \subseteq X \),

\[ u_k|_K \rightarrow u|_K \]

uniformly.

Turn out to be the right topology for utility functions when preferences are endowed with the closed convergence topology.
Results

Let $X$ be

- $X = \mathbb{R}^n$.
- or $X = \Delta([a, b])^\Omega$ (set of “monetary” Anscombe-Aumann acts) with finite $\Omega$.

Obs.

- Objective monotonicity.
- Connection between order and topology on $X$.
- Some of our results are more general.
A sequence of experiments \( \{ \Sigma^k \} \), with \( \Sigma^k = \{ \Sigma_1, \ldots, \Sigma_k \} \), is exhaustive when:

1. \( \bigcup_{i=1}^{\infty} \Sigma_i \) is dense in \( X \).
2. For all \( x, y \in \bigcup_{i=1}^{\infty} \Sigma_i \) with \( x \neq y \), there exists \( i \) s.t. \( \Sigma_i = \{ x, y \} \).
Theorem

Let

- \( \succeq^* \) be monotone and cont.;
- \( \succeq_k \) strongly rationalize the \( k \)-sized choice data generated by \( \succeq^* \).

Then,

- \( \succeq_k \to \succeq^* \) (in the topology of closed convergence).
- For any utility \( u^* \) for \( \succeq^* \) \( \exists u_k \) for \( \succeq_k \) s.t \( u_k \to u^* \) (in the topology of compact convergence).
Discussion.

- Monotonicity.
- Convergence of any arbitrary preference rationalization.
- Utility can’t be arbitrary. Only get convergence of selected utility estimates. Require an identification theorem for each specific theory.
Why does monotonicity help?
Recall Example 1

\[ x \succ^* y \]

\[ U \succ^* V \]

\[ \exists x' \in U \text{ and } y' \in V \text{ s.t. } y' \succ_k x' \text{ for some rationalizing } \succeq_k \]

\[ \text{But } x' \succ y'. \forall \succeq \text{ s.t. } \succeq \text{ is cont. and } \succeq |_B = \preceq^*_B. \]
Monotone rationalizations.

\[ V \]
\[ y \]

\[ U \]
\[ x \]

- \[ x \succ^* y \]
- \[ U \succ^* V \]
Monotone rationalizations.

- $x \succ^* y$
- $U \succ^* V$
- Let $(x', y') \in U \times V$. 
Monotone rationalizations.

Let \((x', y') \in U \times V\).

\[ \exists x'', y'' \in B \]

\[ x'' \leq x' \]

\[ y' \leq y'' \]
Monotone rationalizations.

- $x \succ^* y$
- $U \succ^* V$
- Let $(x', y') \in U \times V$.
- $\implies \exists x'', y'' \in B$
- $x'' \leq x'$
- $y' \leq y''$
  $\implies x' \geq x'' \succ_k y'' \geq y'$

Chambers-Echenique-Lambert Recovery
Weak rationalizations

Let $X = \mathbb{R}^n$.

Let $\mathcal{P}^k(c)$ be the set of continuous and strictly monotone preferences that weakly rationalize the $k$ data.

For a set of binary relations $S$, define $\text{diam}(S) = \sup_{(\succeq, \succeq') \in S^2} \delta_C(\succeq, \succeq')$ to be the diameter of $S$ according to the metric $\delta_C$ which generates the topology on preferences.

**Theorem**

One of the following holds:

1. There is $k$ such that $\mathcal{P}^k(c) = \emptyset$.
2. $\lim_{k \to \infty} \text{diam}(\mathcal{P}^k(c)) \to 0$. 

Chambers-Echenique-Lambert Recovery
Weak rationalizations

A preference $\succeq$ is *locally strict* if

$$x \succeq y \implies \text{in every nbd. of } (x, y), \text{ there exists } (x', y') \text{ with } x' \succ y'$$

(Border and Segal, 1994).
Weak rationalizations

Let $X \subseteq \mathbb{R}^n$ and $\mathcal{P}$ be a closed set of locally strict preferences on $X$.

**Theorem**

Let $\succeq_k \in \mathcal{P}$ weakly rationalize the $k$-sized choice data.

- Then there is a preference $\succeq^* \in \mathcal{P}$ s.t $\succeq_k \rightarrow \succeq^*$.
- The limiting preference is unique: if, for every $k$, $\succeq'_k \in P$ rationalizes the $k$-data, then the same limit $\succeq'_k \rightarrow \succeq^*$ obtains.

Obs. that $\succeq^*$ generating the choice is not a hypothesis. May view this result as a definition of preference.

Obs. doesn’t require monotonicity.

(This result is in CEL (2021))
Utility functions
Finite state space: $S$.

Monetary consequences: $[a, b] \subseteq \mathbb{R}$

Anscombe-Aumann acts: $f : S \rightarrow \Delta([a, b])$

Preferences on $\Delta([a, b])^S$. 

Chambers-Echenique-Lambert Recovery
Let $U$ be the set of all continuous and monotone weakly increasing functions $u : [a, b] \to \mathbb{R}$ with $u(a) = 0$ and $u(b) = 1$.

A pair $(V, u)$ is a standard representation if $V : \Delta([a, b])^S \to \mathbb{R}$ and $u \in U$ are continuous functions such that $v(p, \ldots, p) = \int_{[a,b]} u \, dp$, for all constant acts $(p, \ldots, p)$.

$(V, u)$ is aggregative if there is an aggregator $H : [0, 1]^S \to \mathbb{R}$ with $V(f) = H((\int u \, df(s))_{s \in S})$ for $f \in \Delta([a, b])^S$.

An aggregative representation with aggregator $H$ is denoted by $(V, u, H)$. 

Chambers-Echenique-Lambert Recovery
A preference $\succeq$ on $\Delta([a, b])^S$ is *standard* if it is weakly monotone, and there is a standard representation $(V, u)$ in which $V$ represents $\succeq$. 
Example

Variational preferences (Maccheroni et al 2006) are standard and aggregative. Let

\[
V(f) = \inf \left\{ \int v(f(s))d\pi(s) + c(\pi) : \pi \in \Delta(S) \right\}
\]

where

1. \( v : \Delta([a, b]) \to \mathbb{R} \) is continuous and affine.
2. \( c : \Delta(S) \to [0, \infty] \) is lower semicontinuous, convex and grounded (meaning that \( \inf \{ c(\pi) : \pi \in \Delta(S) \} = 0 \)).

Let \( H : [0, 1]^S \to \mathbb{R} \) be \( H(x) = \inf \{ \sum_{s \in S} x(s)\pi(s) + c(\pi) : \pi \in \Delta(S) \} \)
Theorem

Let $\succeq$ be a standard preference with standard representation $(V, u)$, and \{\succeq^k\} a sequence of standard preferences, each with a standard representation $(V^k, u^k)$.

1. If $\succeq^k \rightarrow \succeq$, then $(V^k, u^k) \rightarrow (V, u)$.
2. If, in addition, these preferences are aggregative with representations $(V^k, u^k, H^k)$ and $(V, u, H)$, then $H^k \rightarrow H$. 
Statistical model

Given \((X, \mathcal{P})\). We change:

- How subjects make choices: they do not exactly follow a preference, but randomly deviate from it.
- How experiments are generated.
1. In a choice problem, alternatives drawn iid according to sampling distribution $\lambda$.

2. Subjects make “mistakes.”
   Upon deciding on $\{x, y\}$, a subject with preference $\succeq$ chooses $x$ over $y$ with probability $q(\succeq; x, y)$ (error probability function).

3. Only assumption: if $x \succ y$ then $q(\succeq; x, y) > 1/2$.

4. “Spatial” dependence of $q$ on $x$ and $y$ is arbitrary.
Kemeny-minimizing estimator: find a preference in $\mathcal{P}$ that minimizes the number of observations inconsistent with the preference.

- “Model free:” to compute estimator don’t need to assume a specific $q$ or $\lambda$.
- May be computationally challenging (depending on $\mathcal{P}$).
To sum up:

**Assumption 1**: $X$ is a locally compact, separable, and completely metrizable space.

**Assumption 2**: $\mathcal{P}$ is a closed set of locally strict preferences.

**Assumption 3’**: $\lambda$ has full support and for all $\succeq \in \mathcal{P}$, $\{(x, y) : x \sim y\}$ has $\lambda$-probability 0.
Theorem

Under Assumptions (1), (2), (3'), if the subject’s preference is $\succeq^* \in \mathcal{P}$ and $\succeq_n$ is the Kemeny-minimizing estimator for $\Sigma_n$, then, $\succeq_n \to \succeq^*$ in probability.
The VC dimension of $\mathcal{P}$ is the largest cardinality of an experiment that can always be rationalized by $\mathcal{P}$.

A measure of how flexible $\mathcal{P}$; how prone it is to overfitting.
Think of a game between Alicia and Roberto
Alicia defends $\mathcal{P}$; Roberto questions it.
Given is $k$
Alicia proposes a choice experiment of size $k$
Roberto fills in choices adversarily.
Alicia wins if she can rationalize the choices using $\mathcal{P}$.
The VC dimension of $\mathcal{P}$ is the largest $k$ for which Alicia always wins.
Convergence rates

- $\rho$ a metric on preferences.

**Theorem**

Under the same conditions as in Part A,

$$N(\eta, \delta) \leq \frac{2}{r(\eta)^2} \left( \sqrt{\frac{2}{\delta}} + C \sqrt{VC(P)} \right)^2$$

with $C$ a universal constant.
Convergence rates

- $\rho$ a metric on preferences.
- $N(\eta, \delta)$: smallest value of $N$ such that for all $n \geq N$, and all subject preferences $\succeq^* \in \mathcal{P}$,

$$\Pr(\rho(\succeq_n, \succeq^*) < \eta) \geq 1 - \delta.$$ 

Theorem

Under the same conditions as in Part A,

$$N(\eta, \delta) \leq \frac{2}{r(\eta)^2} \left( \sqrt{\frac{2}{\delta}} + C \sqrt{\text{VC}(\mathcal{P})} \right)^2$$

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Convergence rates

- $\rho$ a metric on preferences.
- $N(\eta, \delta)$: smallest value of $N$ such that for all $n \geq N$, and all subject preferences $\succeq^* \in \mathcal{P}$,

$$\Pr(\rho(\succeq_n, \succeq^*) < \eta) \geq 1 - \delta.$$ 

- $\mu(\succeq'; \succeq)$: prob. choice of preference $\succeq$ is consistent with $\succeq'$.

$$r(\eta) = \inf \left\{ \mu(\succeq; \succeq) - \mu(\succeq'; \succeq) : \succeq, \succeq' \in \mathcal{P}, \rho(\succeq, \succeq') \geq \eta \right\}.$$ 

Theorem

Under the same conditions as in Part A,

$$N(\eta, \delta) \leq \frac{2}{r(\eta)^2} \left( \sqrt{2/\delta} + C \sqrt{\text{VC}(\mathcal{P})} \right)^2$$

with $C$ a universal constant.
Convergence rates

- \( \rho \) a metric on preferences.
- \( N(\eta, \delta) \) : smallest value of \( N \) such that for all \( n \geq N \), and all subject preferences \( \succeq^* \in \mathcal{P} \),

\[
\Pr(\rho(\succeq_n, \succeq^*) < \eta) \geq 1 - \delta.
\]

- \( \mu(\succeq'; \succeq) \) : prob. choice of preference \( \succeq \) is consistent with \( \succeq' \).

\[
r(\eta) = \inf \{ \mu(\succeq; \succeq) - \mu(\succeq'; \succeq) : \succeq, \succeq' \in \mathcal{P}, \rho(\succeq, \succeq') \geq \eta \}.
\]

- \( \text{VC}(\mathcal{P}) \) the VC dimension of the class \( \mathcal{P} \).

**Theorem**

Under the same conditions as in Part A,

\[
N(\eta, \delta) \leq \frac{2}{r(\eta)^2} \left( \sqrt{\frac{2}{\delta}} + C \sqrt{\text{VC}(\mathcal{P})} \right)^2
\]

with \( C \) a universal constant.
Expected utility

1. \( X \) is the set of lotteries over \( d \) prizes.
2. \( \mathcal{P} \) is the set of nonconstant EU preferences: there are always lotteries \( p, p' \) such as \( p \) is strictly preferred to \( p' \).

This preference environment satisfies Assumptions 1 and 2.

Suppose: there is \( C > 0 \) and \( k > 0 \) s.t

\[
q(x, y; \succeq) \geq \frac{1}{2} + C(v \cdot x - v \cdot y)^k,
\]

when \( x \succeq y \) and \( v \) represents \( \succeq \).
Under these assumptions, we can bound $r(\eta)$ and $VC(\mathcal{P})$, which implies

$$N(\eta, \delta) = O \left( \frac{1}{\delta \eta^{4d-2}} \right).$$

Other examples: Cobb-Douglas, CES, and CARA subjective EU preferences, and intertemporal choice with discounted, Lipschitz-bounded utilities.
Monotone preferences

- $K$ be a compact set in $X \equiv \mathbb{R}^d_{++}$, and fix $\theta > 0$.
- $\mathcal{P}$ has finite VC-dimension and is identified on $K$.
- $\lambda$ is the uniform probability measure on $K^{\theta/2}$.
- $q$ satisfies: probability of choosing $y$ instead of $x$ when $x \succ y$ is a function of $\|x - y\|$.

**Proposition**

The Kemeny-minimizing estimator is consistent and, as $\eta \to 0$ and $\delta \to 0$,

$$N(\eta, \delta) = O \left( \frac{1}{\eta^{2d+2} \ln \frac{1}{\delta}} \right).$$
Applications: preferences from utilities

A set $\mathcal{P}$ is defined from utilities when there is a class $\mathcal{U}$ of utility functions such that for all $\succeq \in \mathcal{P}$

$$x \succeq y \iff U(x) \geq U(y)$$

for some $U \in \mathcal{U}$.

**Proposition 1**

Under Assumption 1, if $\mathcal{U}$ is compact and represents locally strict preferences, then Assumption 2 is met.

Implied by the continuity theorem of Border and Segal (1994).
Revisit the case of expected utility preferences:

1. \( \mathcal{X} \) is the set of lotteries over \( d \) prizes.
2. \( \mathcal{P} \) is the set of nonconstant EU preferences: there are always lotteries \( p, p' \) such as \( p \) is strictly preferred to \( p' \).

This preference environment satisfies Assumptions 1 and 2. When the probability of error of choosing \( y \) instead of \( x \) when \( x \succ y \) is a function of \( \|x - y\| \), we can bound \( r(\eta) \) and \( VC(\mathcal{P}) \), which implies

\[
N(\eta, \delta) = O \left( \frac{1}{\delta \eta^{4d-2}} \right).
\]
Literature

Afriat’s theorem and revealed preference tests: Afriat (1967); Diewert (1973); Varian (1982); Matzkin (1991); Chavas and Cox (1993); Brown and Matzkin (1996); Forges and Minelli (2009); Carvajal, Deb, Fenske, and Quah (2013); Reny (2015); Nishimura, Ok, and Quah (2017)

Recoverability: Varian (1982); Cherchye, De Rock, and Vermeulen (2011)

Approximation: Mas-Colell (1978); Forges and Minelli (2009); Kübler and Polemarchakis (2017); Polemarchakis, Selden, and Song (2017)

Identification: Matzkin (2006); Gorno (2019)

Econometric methods: Matzkin (2003); Blundell, Browning, and Crawford (2008); Blundell, Kristensen, and Matzkin (2010); Halevy, Persitz, and Zrill (2018)
Call a dominance relation any binary relation on \( X \) that is not reflexive.

Say that \( \succeq \) is strictly monotone wrt \( \succ \) if \( x \succ y \) implies \( x \succ y \).

Say that \( \succeq \) is Grodal-transitive if \( x \succeq y \succ z \succeq w \) implies \( x \succeq w \).

Proposition 2

Take a set of alternatives \( X \) that meets Assumption 1, and suppose:

1. \( \succ \) is a dominance relation that is open,
2. for each \( x \), there are \( y, z \) arbitrarily close to \( x \) such that \( y \succ x \) and \( x \succ z \).

Then the class of preferences that are Grodal-transitive and strictly monotone wrt \( \succ \) meets Assumption 2.
Example: back to preferences over commodity bundles.

- There are $d$ commodities.
- $X \equiv \mathbb{R}^d_{++}$, where for $(x_1, \ldots, x_d) \in X$, $x_i$ is quantity of good $i$ consumed.
- $x \gg y$ iff $x_i > y_i$ for all $i = 1, \ldots, d$.

The set of all preferences that are Grodal-transitive and strictly monotone wrt $\gg$ meets Assumption 2.
Conclusion

- Binary choice
- Finite data
- “Consistency” – Large sample theory
- Unified framework: RP and econometrics.

Applicable to:

- Large-scale (online) experiments/surveys.
- Voting (roll-call data).