# Recovery of utilities and preferences from finite choice data

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Based on two papers:

- Recovering preferences from finite data (published).
- Recovering utility (available soon!)

# Model

Alice (an experimenter)

Bob (a subject)

# Model

- Alice presents Bob with choice problems:

"Hey Bob would you like $x$ or $y$?"

$x$ vs. $y$

- Bob chooses one alternative.
- Rinse and repeat $\rightarrow$ dataset of $k$ choices.

# Model

- An experimenter and a subject.
- Subject makes choices according to some $\succeq^*$, or utility $u^*$, on set $X$.
- Experimenter conducts a finite choice experiment of "size" $k$: $k$ questions, each one a binary choice problem.
- Preference $\succeq_k$ or utility $u_k$ as rationalizations or estimates.

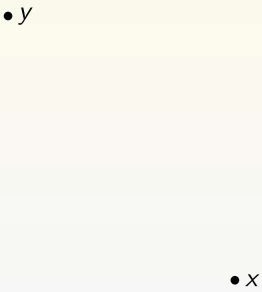
How are $\succeq_k$, $\succeq^*$, $u_k$ and $u^*$ related?

# Example 1

Subject chooses among alternatives: $X = \mathbf{R}^n_+$.

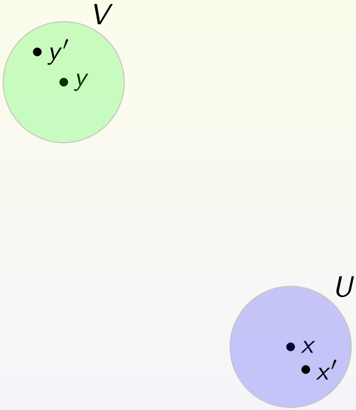
- Choices come from $\succeq^*$, a continuous preference.
- $\Sigma_i = \{x_i, y_i\}$.
- A *finite experiment:* choose an element from $\Sigma_i$, $i = 1, \ldots, k$.
- Assumption: $\Sigma_\infty = \cup_{k=1}^{\infty} \Sigma_k$ is dense.

# Example 1




- $x \succ^* y$

# Example 1




- $x \succ^* y$
- $U \succ^* V$
- $\exists x' \in U$ and $y' \in V$ s.t $\forall k$ $\exists$ rationalizing $\succeq_k$, with $y' \succ_k x'$
- But $x' \succ y'$. $\forall \succeq$ s.t. $\succeq$ is cont. and $\succeq|_{\Sigma_\infty} = \succeq^*|_{\Sigma_\infty}$.

# Example 1: a "discontinuity."

- Infinite data ($\succeq^*$ on $X$): observe $\succeq^*$; so $x' \succ^* y'$
- "Limiting" infinite data ($\Sigma_\infty = \cup_{k=1}^{\infty} \Sigma_k$):
  $x' \succ y' \; \forall \succeq$ s.t. $\succeq |_{\Sigma_\infty} = \succeq^* |_{\Sigma_\infty}$.
- Finite data: $(\Sigma_1 \ldots, \Sigma_k)$
  can't rule out $y' \succ_k x'$, no matter how large $k$.

No amount of finite data may correct a mistaken inference.

Even when the (limiting) infinite data set leaves no room for error.

# Example 2

Let $X = \mathbf{R}^n_+$.

Fix a continuous preference $\succeq^*$ on $X$.

**Proposition (informal)**

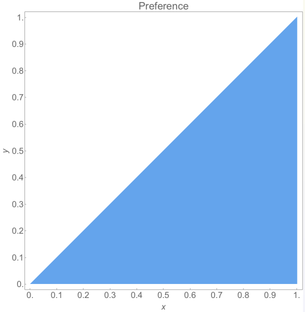
There exists rationalizing $\succeq_k$ for each $k$ s.t

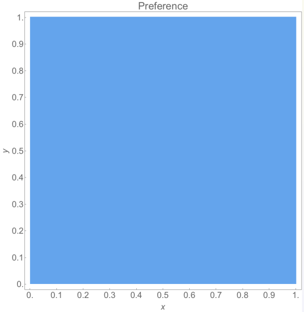
$$\text{complete indifference} = \lim_{k \to \infty} \succeq_k$$

# Example 2

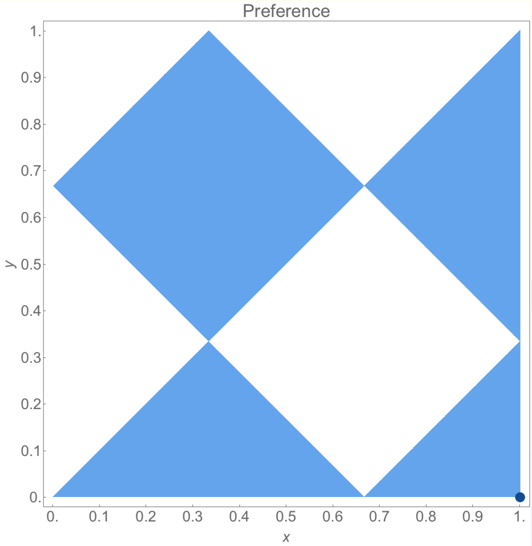
Set of alternatives $X = [0, 1]$.

- Left: the subject prefers $x$ to $y$ iff $x \geq y$.
- Right: the subject is completely indifferent.



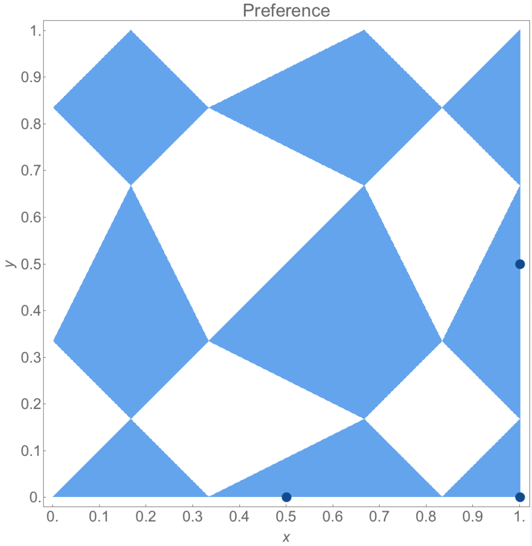



# Example 2


Preference
1.
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0.
y
0.
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1.
x

n=1

# Example 2



Preference
1.
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0.
y
0.
0.1
0.2
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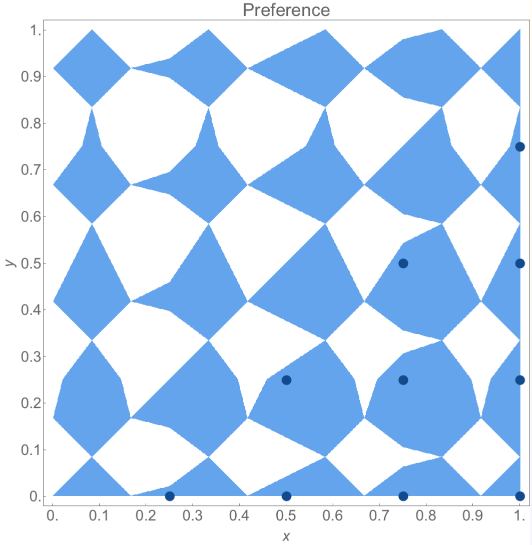
0.4
0.5
0.6
0.7
0.8
0.9
1.
x


n=2

# Example 2


Preference
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0.
y
0.
0.1
0.2
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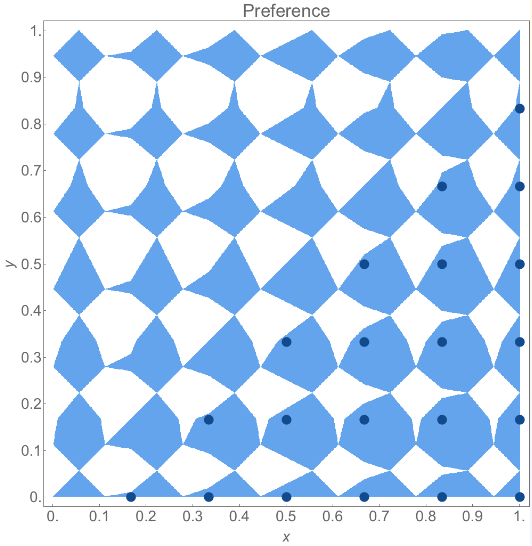
0.4
0.5
0.6
0.7
0.8
0.9
1.
x

n=4

# Example 2



Preference
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0.9
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0.7
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0.5
0.4
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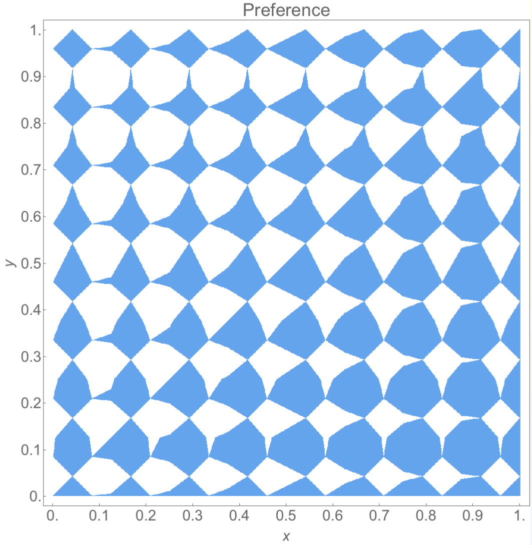
0.
y
0.
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1.
x

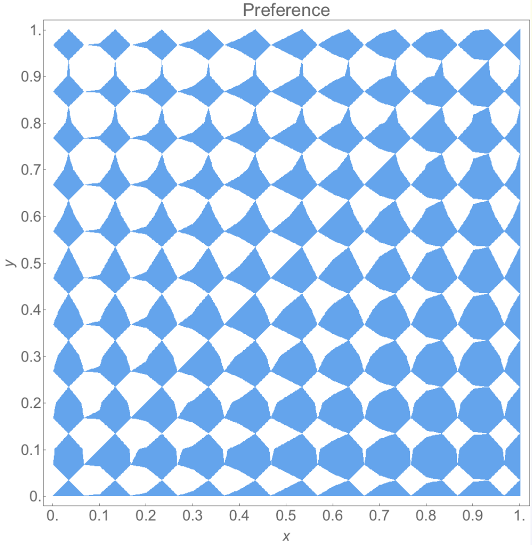

n=6

# Example 2


Preference
1.
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0.
y
0.
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1.
x

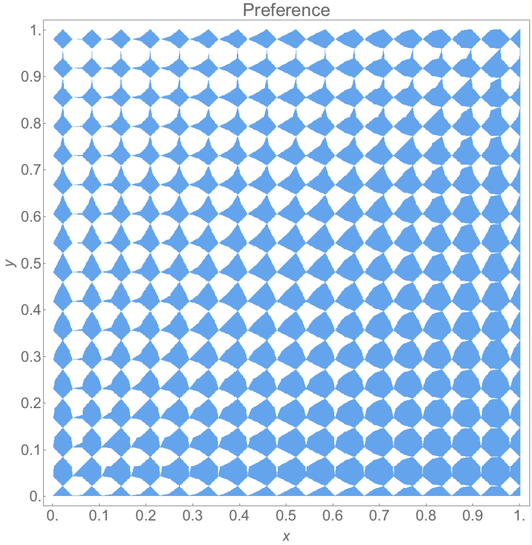
n=8

# Example 2



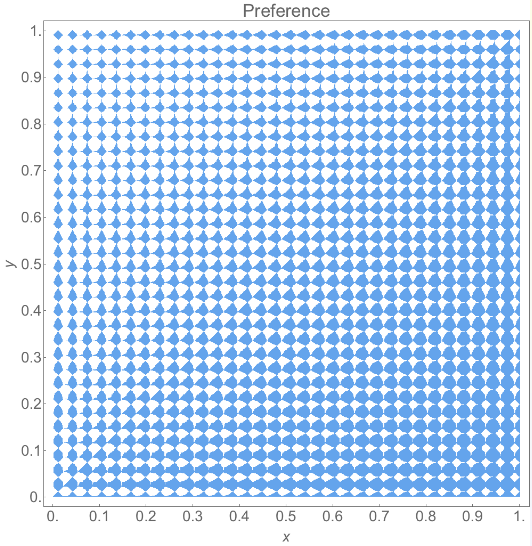

n=10

# Example 2




n=16

# Example 2




n=32

Discipline matters.

Empiricism is dangerous.

Inevitable role for theory (a Cartesian imperative).

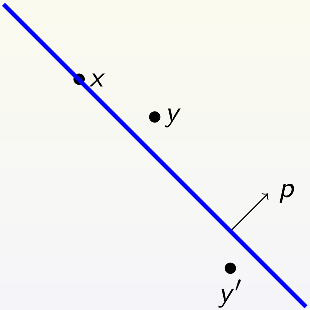
# Example 3

Choice under uncertainty:

- State space $S = \{s_1, s_2\}$.
- Choice among monetary acts: $x \in \mathbf{R}^S$.
- Bob is risk-neutral subjective exp. utility maximizer.
- So $x \succeq^* y$ iff $p \cdot x \geq p \cdot y$.
- Preferences described by a prior $p \in \Delta(S)$.

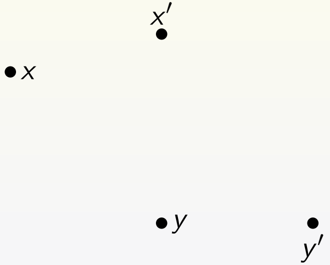
# Example 3

Bob's preferences:



# Example 3

Suppose $y$ is chosen over $x$, and $x'$ over $y'$.



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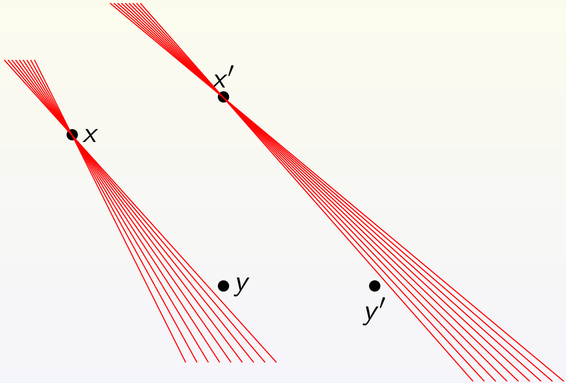

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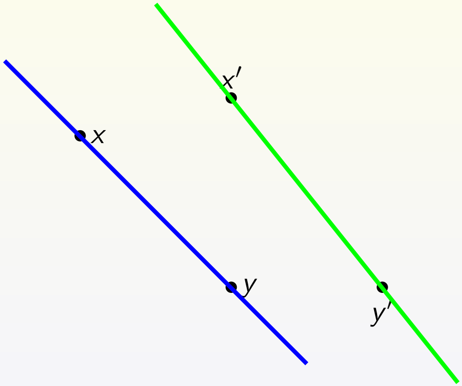


Bob's prior $p$ must be steeper than the blue line, and flatter than the green.

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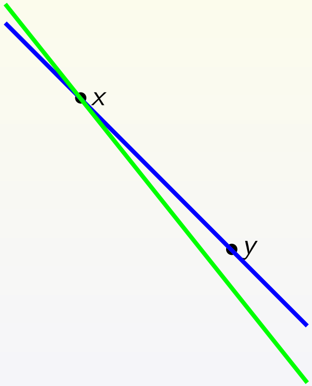
Suppose $y$ is chosen over $x$, and $x'$ over $y'$.

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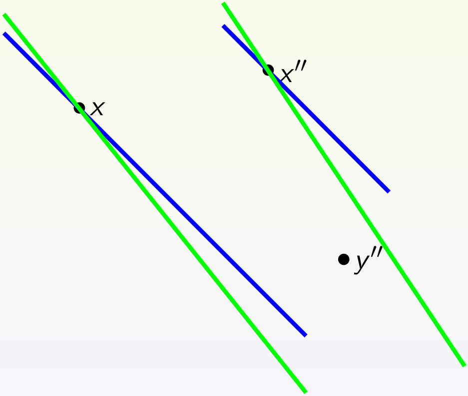



Narrows down unobserved comparison: $x'' \succ^* y''$.

# Example 3

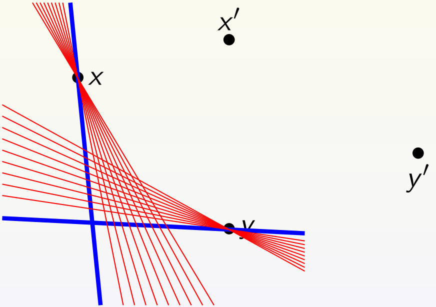
Suppose Alice instead uses the max-min model for Bob:

$$u(x) = \min\{p \cdot x : p \in \Pi\}$$

With two states, $\Pi$ is described by four parameters. With more than two states, the model is non-parametric.
Then from $y \succ x$ she learns something about the slope of the worst-case priors.

# Example 3

$y$ is chosen over $x$, and $x'$ over $y'$.



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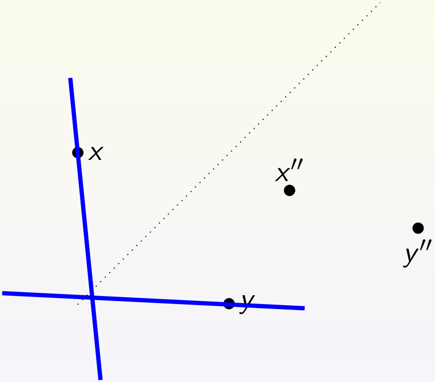


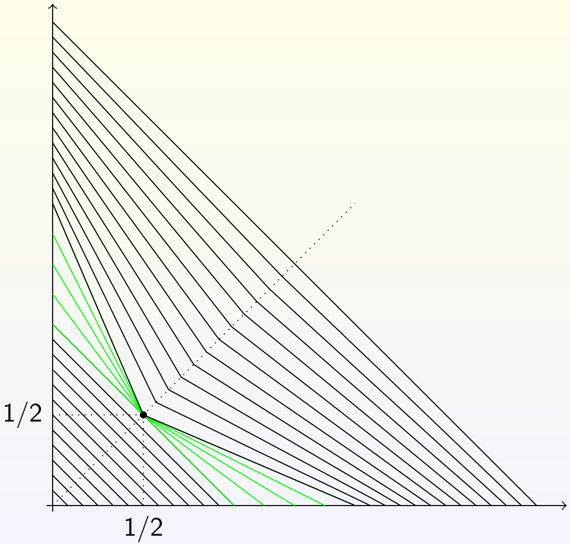

No inference for $x''$ and $y''$.

A more flexible theory may lead to *overfitting.*

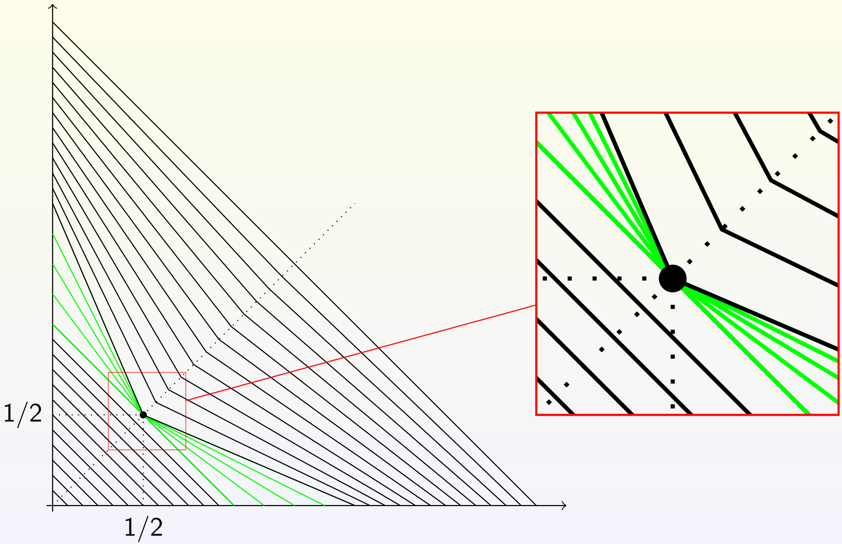
In fact max-min with $|S| \geq 3$ is "hopeless."

Any finite dataset will lead to poor out-of-sample predictions.

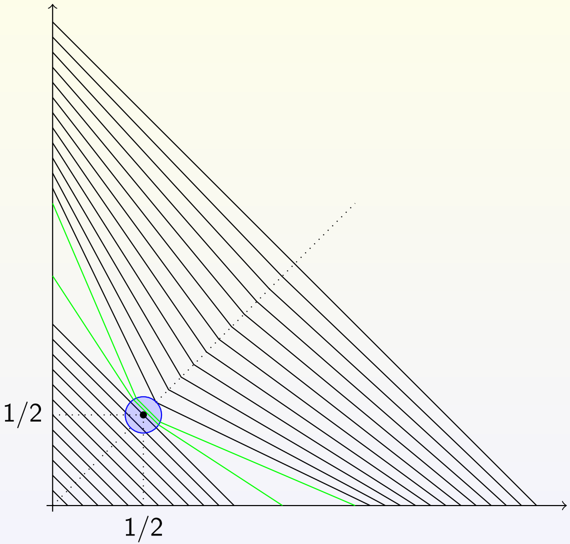
# Example 4



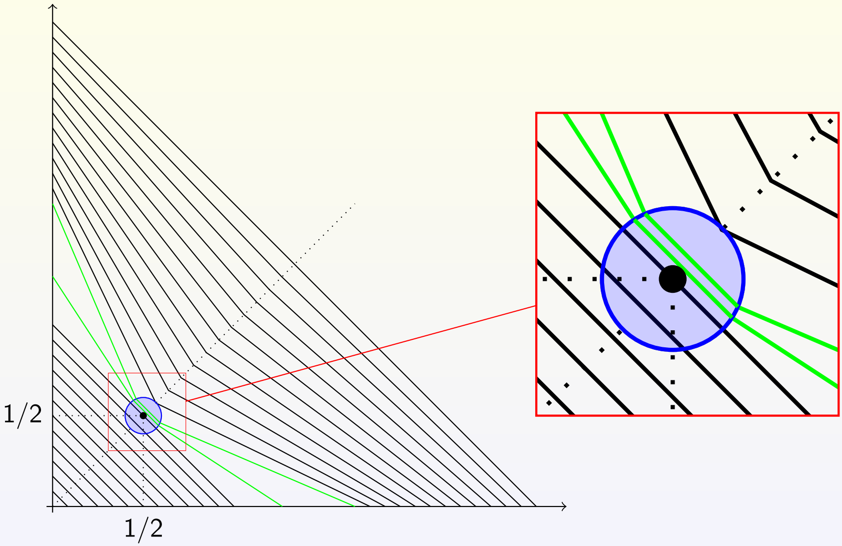
# Example 4



# Example 4



# Example 4



Model of preferences must be *closed*.

Can't allow for approximate behavior to "escape."

# Example 5

- Let $X = [0,1]$, $\succeq^* = \geq$ and $u^*(x) = x$.
- For each $k$, let $\succeq_k = \geq$ and
$$u_k = \frac{x}{k}.$$
- Then $0 = \lim_k u_k$.
- But $\succeq_k = \succeq^*$ for all $k$!

# Example 5

- Let $X = [0,1]$, $\succeq^* = \geq$ and $u^*(x) = x$.
- For each $k$, let $\succeq_k = \geq$ and

$$u_k = \frac{x}{k}.$$

- Then $0 = \lim_k u_k$.
- But $\succeq_k = \succeq^*$ for all $k$!

(For $\varepsilon > 0$, can choose $u_n$ with $\|u_n\|_\infty = 1$ or $\|u_n\|_1 = 1$ and $0 = \lim_n u_n(x)$ for all $x \in [0, 1-\varepsilon]$.)

Utility estimates are more delicate than preferences.

Must choose the right utility representation.

# Lessons for DT

Typical result in decision theory:

"Utility representation iff axioms. Moreover, utility is unique."

Axioms $\Rightarrow$ testable implications. (But may require infinite data.)

Uniqueness $\Rightarrow$ identification. But more is needed to ensure utility recovery from finite data.

# Model

- Alternatives: A topological space $X$.
- Preference: A complete and continuous binary relation $\succeq$ over $X$
- $\mathcal{P}$ a set of preferences.

A pair $(X, \mathcal{P})$ is a preference environment.

# Example: Expected utility preferences

- There are $d$ prizes.
- $X$ is the set of lotteries over the prizes, $\Delta^{d-1} \subset \mathbf{R}^d$.
- An EU preference $\succeq$ is defined by $v \in \mathbf{R}^d$ such that $p \succeq p'$ iff $v \cdot p \geq v \cdot p'$.
- $\mathcal{P}$ is set of all the EU preferences.

# Experiment

Alice wants to recover Bob's preference from his choices.

- Binary choice problem : $\{x, y\} \subset X$.
- Bob is asked to choose $x$ or $y$.
  Behavior encoded in a choice function $c(\{x, y\}) \in \{x, y\}$.
- If Bob's preference is $\succeq$ then $c(\{x, y\}) \succeq x$ and $c(\{x, y\}) \succeq y$.
- Partial observability: indifference is not observable.

# Experiment

Alice gets finite dataset.

- Experiment of size $k$ : $\Sigma^k = \{\Sigma_1, \ldots, \Sigma_k\}$ with $\Sigma_i = \{x_i, y_i\}$.
- Set of growing experiments: $\{\Sigma^k\} = \{\Sigma^1, \Sigma^2, \ldots\}$ with $\Sigma^k \subset \Sigma^{k+1}$.

# Literature

Afriat's theorem and revealed preference tests: Afriat (1967); Diewert (1973); Varian (1982); Matzkin (1991); Chavas and Cox (1993); Brown and Matzkin (1996); Forges and Minelli (2009); Carvajal, Deb, Fenske, and Quah (2013); Reny (2015); Nishimura, Ok, and Quah (2017)

Recoverability: Varian (1982); Cherchye, De Rock, and Vermeulen (2011); Chambers, Echenique and Lambert (2021).

Consistency: Mas-Colell (1978); Forges and Minelli (2009); Kübler and Polemarchakis (2017); Polemarchakis, Selden, and Song (2017)

Identification: Matzkin (2006); Gorno (2019)

Econometric methods: Matzkin (2003); Blundell, Browning, and Crawford (2008); Blundell, Kristensen, and Matzkin (2010); Halevy, Persitz, and Zrill (2018)

# OK, so far:

- $(X, \mathcal{P})$ preference env.
- $c$ encodes choice
- $\Sigma^k$ seq. of experiments

# Rationalization

- A preference $\succeq$ weakly rationalizes the observed choices on $\Sigma^k$ if $c(\{x,y\}) \succeq x$ and $c(\{x,y\}) \succeq y$ for all $\{x,y\} \in \Sigma^k$.
- A preference $\succeq$ strongly rationalizes the observed choices on $\Sigma^k$ if $c(\{x,y\}) \succ z$ for $z \in \{x,y\}$, $z \neq c(\{x,y\})$, for all $\{x,y\} \in \Sigma^k$.

# Topology on preferences

Choice of topology: closed convergence topology.

- Standard topology on preferences (Kannai, 1970; Mertens (1970); Hildenbrand, 1970).
- $\succeq_n \to \succeq$ when:
    1. For all $(x, y) \in \succeq$, there exists a seq. $(x_n, y_n) \in \succ_n$ that converges to $(x, y)$.
    2. If a subsequence $(x_{n_k}, y_{n_k}) \in \succeq_{n_k}$ converges, the limit belongs to $\succeq$.
- If $X$ is compact and metrizable, same as convergence under the Hausdorff metric.
- $X$ Euclidean and $\mathcal{B}$ the strict parts of cont. weak orders. Then it's the smallest topology for which the set

$$\{(x, y, \succ) : x \in X, y \in X, \succ \in \mathcal{B} \text{ and } x \succ y\}$$

is open.

# Topology on preferences

**Lemma**

Let $X$ be a locally-compact Polish (separable and completely metrizable) space. Then the set of all continuous binary relations on $X$ is a compact metrizable space.

# Topology of compact convergence

Let $\{u_k\}$ be a sequence of functions,

$$u_k \colon X \to \mathbf{R}.$$

The sequence *convergences compactly* to $u \colon X \to \mathbf{R}$ if for every compact $K \subseteq X$,

$$u_k|_K \to u|_K$$

uniformly.

Turn out to be the right topology for utility functions when preferences are endowed with the closed convergence topology.

# Results

Let $X$ be

- $X = \mathbf{R}^n$.
- or $X = \Delta([a,b])^\Omega$ (set of "monetary" Anscombe-Aumann acts) with finite $\Omega$.

Obs.

- Objective monotonicity.
- Connection between order and topology on $X$.
- Some of our results are more general.

# Results

A sequence of experiments $\{\Sigma^k\}$, with $\Sigma^k = \{\Sigma_1, \ldots, \Sigma_k\}$, is exhaustive when:

1. $\bigcup_{i=1}^{\infty} \Sigma_i$ is dense in $X$.
2. For all $x, y \in \bigcup_{i=1}^{\infty} \Sigma_i$ with $x \neq y$, there exists $i$ s.t $\Sigma_i = \{x, y\}$.

# Results

**Theorem**

Let

- $\succeq^*$ be monotone and cont.;
- $\succeq_k$ strongly rationalize the $k$-sized choice data generated by $\succeq^*$.

Then,

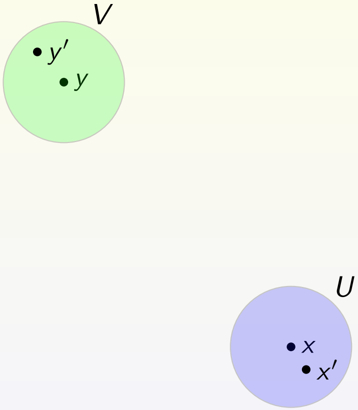
- $\succeq_k \to \succeq^*$ (in the topology of closed convergence).
- For any utility $u^*$ for $\succeq^*$ $\exists$ $u_k$ for $\succeq_k$ s.t $u_k \to u^*$ (in the topology of compact convergence).

# Discussion.

- Monotonicity.
- Convergence of *any arbitrary* preference rationalization.
- Utility *can't be arbitrary*. Only get convergence of selected utility estimates. Require an identification theorem for each specific theory.

Why does monotonicity help?

# Recall Example 1




- $x \succ^* y$
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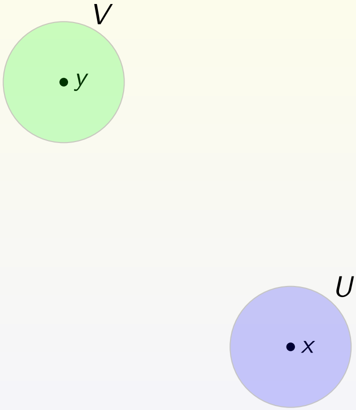
- $\exists x' \in U$ and $y' \in V$ s.t $y' \succ_k x'$ for some rationalizing $\succeq_k$
- But $x' \succ y'$. $\forall \succeq$ s.t. $\succeq$ is cont. and $\succeq|_B = \succeq^*|_B$.

# Monotone rationalizations.




- $x \succ^* y$
- $U \succ^* V$

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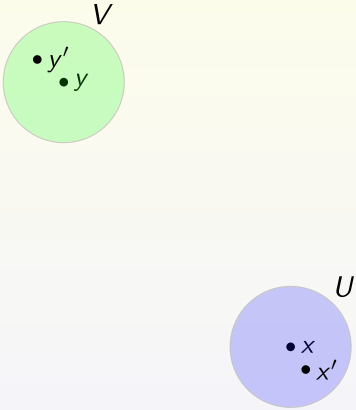
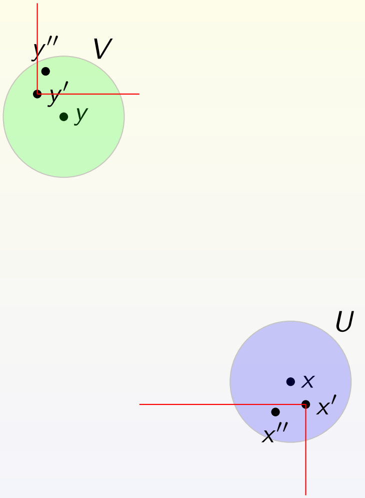


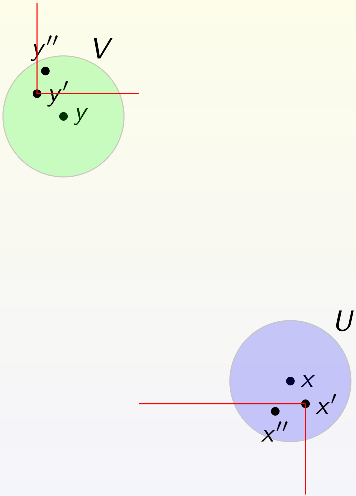

- $x \succ^* y$
- $U \succ^* V$
- Let $(x', y') \in U \times V$.

# Monotone rationalizations.




- $x \succ^* y$
- $U \succ^* V$
- Let $(x', y') \in U \times V$.
- $\implies \exists x'', y'' \in B$
- $x'' \leq x'$
- $y' \leq y''$

# Monotone rationalizations.




- $x \succ^* y$
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- Let $(x', y') \in U \times V$.
- $\implies \exists x'', y'' \in B$
- $x'' \leq x'$
- $y' \leq y''$
  $\implies x' \geq x'' \succ_k y'' \geq y'$

# Weak rationalizations

Let $X = \mathbf{R}^n$.

Let $\mathcal{P}^k(c)$ be the set of continuous and strictly monotone preferences that weakly rationalize the $k$ data.

For a set of binary relations $S$, define $\operatorname{diam}(S) = \sup_{(\succeq,\succeq')\in S^2} \delta_C(\succeq,\succeq')$ to be the diameter of $S$ according to the metric $\delta_C$ which generates the topology on preferences.

**Theorem**

One of the following holds:

1. There is $k$ such that $\mathcal{P}^k(c) = \varnothing$.
2. $\lim_{k\to\infty} \operatorname{diam}(\mathcal{P}^k(c)) \to 0$.

# Weak rationalizations

A preference $\succeq$ is *locally strict* if

$x \succeq y \implies$ in every nbd. of $(x, y)$, there exists $(x', y')$ with $x' \succ y'$

(Border and Segal, 1994).

# Weak rationalizations

Let $X \subseteq \mathbf{R}^n$. and $\mathcal{P}$ be a closed set of locally strict preferences on $X$.

**Theorem**

Let $\succeq_k \in \mathcal{P}$ weakly rationalize the $k$-sized choice data.

- Then there is a preference $\succeq^* \in \mathcal{P}$ s.t $\succeq_k \to \succeq^*$.
- The limiting preference is unique: if, for every $k$, $\succeq'_k \in P$ rationalizes the $k$-data, then the same limit $\succeq'_k \to \succeq^*$ obtains.

Obs. that $\succeq^*$ generating the choice is not a hypothesis. May view this result as a definition of preference.

Obs. doesn't require monotonicity.

(This result is in CEL (2021))

# Utility functions

# Standard representation

Finite state space: $S$.

Monetary consequences: $[a, b] \subseteq \mathbf{R}$

Anscombe-Aumann acts: $f : S \rightarrow \Delta([a, b])$

Preferences on $\Delta([a, b])^S$.

# Standard representation

Let $U$ be the set of all continuous and monotone weakly increasing functions $u : [a, b] \to \mathbf{R}$ with $u(a) = 0$ and $u(b) = 1$.

A pair $(V, u)$ is a *standard representation* if $V : \Delta([a, b])^S \to \mathbf{R}$ and $u \in U$ are continuous functions such that $v(p, \dots, p) = \int_{[a,b]} u \,\mathrm{d}p$, for all constant acts $(p, \dots, p)$.

$(V, u)$ is *aggregative* if there is an *aggregator* $H : [0, 1]^S \to \mathbf{R}$ with $V(f) = H((\int u \,\mathrm{d}f(s))_{s \in S})$ for $f \in \Delta([a, b])^S$.

An aggregative representation with aggregator $H$ is denoted by $(V, u, H)$.

# Standard representation

A preference $\succeq$ on $\Delta([a,b])^S$ is *standard* if it is weakly monotone, and there is a standard representation $(V, u)$ in which $V$ represents $\succeq$.

# Example

Variational preferences (Maccheroni et al 2006) are standard and aggregative. Let

$$V(f) = \inf\{\int v(f(s))d\pi(s) + c(\pi) : \pi \in \Delta(S)\}$$

where

1. $v : \Delta([a, b]) \to \mathbf{R}$ is continuous and affine.
2. $c : \Delta(S) \to [0, \infty]$ is lower semicontinuous, convex and grounded (meaning that $\inf\{c(\pi) : \pi \in \Delta(S)\} = 0$).

Let $H : [0, 1]^S \to \mathbf{R}$ be $H(x) = \inf\{\sum_{s\in S} x(s)\pi(s) + c(\pi) : \pi \in \Delta(S)\}$

# Standard representation

**Theorem**

Let $\succeq$ be a standard preference with standard representation $(V, u)$, and $\{\succeq^k\}$ a sequence of standard preferences, each with a standard representation $(V^k, u^k)$.

1. If $\succeq^k \to \succeq$, then $(V^k, u^k) \to (V, u)$.
2. If, in addition, these preferences are aggregative with representations $(V^k, u^k, H^k)$ and $(V, u, H)$, then $H^k \to H$.

# Statistical model

Given $(X, \mathcal{P})$. We change:

- How subjects make choices: they do not exactly follow a preference, but randomly deviate from it.
- How experiments are generated.

# Statistical model

1. In a choice problem, alternatives drawn iid according to sampling distribution $\lambda$.
2. Subjects make "mistakes."
   Upon deciding on $\{x, y\}$, a subject with preference $\succeq$ chooses $x$ over $y$ with probability $q(\succeq; x, y)$ (error probability function).
3. Only assumption: if $x \succ y$ then $q(\succeq; x, y) > 1/2$.
4. "Spatial" dependence of $q$ on $x$ and $y$ is arbitrary.

# Estimator

Kemeny-minimizing estimator: find a preference in $\mathcal{P}$ that minimizes the number of observations inconsistent with the preference.

- "Model free:" to compute estimator don't need to assume a specific $q$ or $\lambda$.
- May be computationally challenging (depending on $\mathcal{P}$).

# To sum up:

ASSUMPTION 1 : $X$ is a locally compact, separable, and completely metrizable space.

ASSUMPTION 2 : $\mathcal{P}$ is a closed set of locally strict preferences.

ASSUMPTION 3' : $\lambda$ has full support and for all $\succeq \in \mathcal{P}$, $\{(x, y) : x \sim y\}$ has $\lambda$-probability 0.

# Consistency

**Theorem**

Under Assumptions (1), (2), (3'), if the subject's preference is $\succeq^* \in \mathcal{P}$ and $\succeq_n$ is the Kemeny-minimizing estimator for $\Sigma_n$, then, $\succeq_n \to \succeq^*$ in probability.

# Convergence rates: Digression

The VC dimension of $\mathcal{P}$ is the largest cardinality of an experiment that can always be rationalized by $\mathcal{P}$.

A measure of how flexible $\mathcal{P}$; how prone it is to overfitting.

# Convergence rates: Digression

- Think of a game between Alicia and Roberto
- Alicia defends $\mathcal{P}$; Roberto questions it.
- Given is $k$
- Alicia proposes a choice experiment of size $k$
- Roberto fills in choices adversarily.
- Alicia wins if she can rationalize the choices using $\mathcal{P}$.
- The VC dimension of $\mathcal{P}$ is the largest $k$ for which Alicia always wins.

# Convergence rates

- $\rho$ a metric on preferences.

**Theorem**

Under the same conditions as in Part A,

$$N(\eta, \delta) \le \frac{2}{r(\eta)^2}\left(\sqrt{2/\delta} + C\sqrt{\mathrm{VC}(\mathcal{P})}\right)^2$$

with $C$ a universal constant.

# Convergence rates

- $\rho$ a metric on preferences.
- $N(\eta, \delta)$ : smallest value of $N$ such that for all $n \geq N$, and all subject preferences $\succeq^* \in \mathcal{P}$,

$$\mathbf{Pr}(\rho(\succeq_n, \succeq^*) < \eta) \geq 1 - \delta.$$

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- $\mu(\succeq'; \succeq)$ : prob. choice of preference $\succeq$ is consistent with $\succeq'$.

$$r(\eta) = \inf \left\{ \mu(\succeq;\succeq) - \mu(\succeq';\succeq) : \succeq, \succeq' \in \mathcal{P}, \rho(\succeq, \succeq') \geq \eta \right\}.$$

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## Convergence rates

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- $\mathrm{VC}(\mathcal{P})$ the VC dimension of the class $\mathcal{P}$.

**Theorem**

Under the same conditions as in Part A,

$$N(\eta, \delta) \leq \frac{2}{r(\eta)^2} \left( \sqrt{2/\delta} + C \sqrt{\mathrm{VC}(\mathcal{P})} \right)^2$$

with $C$ a universal constant.

1. $X$ is the set of lotteries over $d$ prizes.
2. $\mathcal{P}$ is the set of nonconstant EU preferences: there are always lotteries $p, p'$ such as $p$ is strictly preferred to $p'$.

This preference environment satisfies Assumptions 1 and 2.

Suppose: there is $C > 0$ and $k > 0$ s.t

$$q(x, y; \succeq) \geq \frac{1}{2} + C(v \cdot x - v \cdot y)^k,$$

when $x \succeq y$ and $v$ represents $\succeq$.

Under these assumptions, we can bound $r(\eta)$ and $\mathrm{VC}(\mathcal{P})$, which implies

$$N(\eta, \delta) = O\left(\frac{1}{\delta \eta^{4d-2}}\right).$$

Other examples: Cobb-Douglas, CES, and CARA subjective EU preferences, and intertemporal choice with discounted, Lipschitz-bounded utilities.

# Monotone preferences

- $K$ be a compact set in $X \equiv \mathbf{R}^d_{++}$, and fix $\theta > 0$.
- $\mathcal{P}$ has finite VC-dimension and is identified on $K$
- $\lambda$ is the uniform probability measure on $K^{\theta/2}$,
- $q$ satisfies: probability of choosing $y$ instead of $x$ when $x \succ y$ is a function of $\|x - y\|$,

**Proposition**

The Kemeny-minimizing estimator is consistent and, as $\eta \to 0$ and $\delta \to 0$,

$$N(\eta, \delta) = O\left(\frac{1}{\eta^{2d+2}} \ln \frac{1}{\delta}\right).$$

# Applications: preferences from utilities

A set $\mathcal{P}$ is defined fom utilities when there is a class $\mathcal{U}$ of utility functions such that for all $\succeq \in \mathcal{P}$

$$x \succeq y \qquad \Leftrightarrow \qquad U(x) \geq U(y)$$

for some $U \in \mathcal{U}$.

**Proposition 1**

Under Assumption 1, if $\mathcal{U}$ is compact and represents locally strict preferences, then Assumption 2 is met.

Implied by the continuity theorem of Border and Segal (1994).

Revisit the case of expected utility preferences:

1. $X$ is the set of lotteries over $d$ prizes.
2. $\mathcal{P}$ is the set of nonconstant EU preferences: there are always lotteries $p, p'$ such as $p$ is strictly preferred to $p'$.

This preference environment satisfies Assumptions 1 and 2. When the probability of error of choosing $y$ instead of $x$ when $x \succ y$ is a function of $\|x - y\|$, we can bound $r(\eta)$ and $\mathrm{VC}(\mathcal{P})$, which implies

$$N(\eta, \delta) = O\left(\frac{1}{\delta \eta^{4d-2}}\right).$$

# Literature

Afriat's theorem and revealed preference tests: Afriat (1967); Diewert (1973); Varian (1982); Matzkin (1991); Chavas and Cox (1993); Brown and Matzkin (1996); Forges and Minelli (2009); Carvajal, Deb, Fenske, and Quah (2013); Reny (2015); Nishimura, Ok, and Quah (2017)

Recoverability: Varian (1982); Cherchye, De Rock, and Vermeulen (2011)

Approximation: Mas-Colell (1978); Forges and Minelli (2009); Kübler and Polemarchakis (2017); Polemarchakis, Selden, and Song (2017)

Identification: Matzkin (2006); Gorno (2019)

Econometric methods: Matzkin (2003); Blundell, Browning, and Crawford (2008); Blundell, Kristensen, and Matzkin (2010); Halevy, Persitz, and Zrill (2018)

# Applications: monotone preferences

- Call a dominance relation any binary relation on $X$ that is not reflexive.
- Say that $\succeq$ is strictly monotone wrt $\rhd$ if $x \rhd y$ implies $x \succ y$.
- Say that $\succeq$ is Grodal-transitive if $x \succeq y \succ z \succeq w$ implies $x \succeq w$.

**Proposition 2**

Take a set of alternatives $X$ that meets Assumption 1, and suppose:

1. $\rhd$ is a dominance relation that is open,
2. for each $x$, there are $y, z$ arbitrarily close to $x$ such that $y \rhd x$ and $x \rhd z$.

Then the class of preferences that are Grodal-transitive and strictly monotone wrt $\rhd$ meets Assumption 2.

Example: back to preferences over commodity bundles.

- There are $d$ commodities.
- $X \equiv \mathbf{R}^d_{++}$, where for $(x_1, \ldots, x_d) \in X$, $x_i$ is quantity of good $i$ consumed.
- $x \gg y$ iff $x_i > y_i$ for all $i = 1, \ldots, d$.

The set of all preferences that are Grodal-transitive and strictly monotone wrt $\gg$ meets Assumption 2.

# Conclusion

- Binary choice
- Finite data
- "Consistency" – Large sample theory
- Unified framework: RP and econometrics.

Applicable to:

Large-scale (online) experiments/surveys.

Voting (roll-call data).