

Recovery of utilities and preferences from finite choice data

C. Chambers F. Echenique N. Lambert
Georgetown UC Berkeley USC

Renmin University of China — May 10th, 2023

Based on two papers:

- | Recovering preferences from finite data (published).
- | Recovering utility (available soon!)

Alice (an experimenter)



Bob (a subject)



- | Alice presents Bob with choice problems:

“Hey Bob would you like x or y ?”



x vs. y

- | Bob chooses one alternative.
- | Rinse and repeat ! dataset of k choices.

- | An experimenter and a subject.
- | Subject makes choices according to some \succsim , or utility u , on set X .
- | Experimenter conducts a finite choice experiment of “size” k : k questions, each one a binary choice problem.
- | Preference \succsim_k or utility u_k as rationalizations or estimates.

How are \succsim_k , \succsim , u_k and u related?

Example 1

Subject chooses among alternatives: $X = \mathbf{R}_+^n$.

- | Choices come from u , a continuous preference.
- | $\Sigma_i = \{x_i; y_i\}$.
- | A *nite experiment*: choose an element from Σ_i , $i = 1; \dots; k$.
- | Assumption: $\Sigma_1 = \bigcup_{k=1}^k \Sigma_k$ is dense.

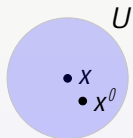
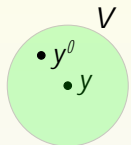
Example 1

• y

| x y

• x

Example 1



- | $x \succ y$
- | $U \succ V$
- | $\exists x^0 \in U$ and $y^0 \in V$ s.t. $\exists k > 0$
rationalizing \succ_k , with $y^0 \succ_k x^0$
- | But $x^0 \succ y^0$. \succ is cont. and
 $j_{\Sigma_1} = j_{\Sigma_1}$.

Example 1: a “discontinuity.”

- | Infinite data (on X): observe y^0 ; so $x^0 = y^0$
- | “Limiting” infinite data ($\Sigma_1 = \prod_{k=1}^{\infty} \Sigma_k$):
 $x^0 = y^0$ s.t. $j_{\Sigma_1} = j_{\Sigma_1}$.
- | Finite data: $(\Sigma_1, \dots, \Sigma_k)$
can't rule out $y^0 = k x^0$, no matter how large k .

No amount of finite data may correct
a mistaken inference.

Even when the (limiting) infinite data
set leaves no room for error.

Example 2

Let $X = \mathbf{R}_+^n$.

Fix a continuous preference \succsim on X .

Proposition (informal)

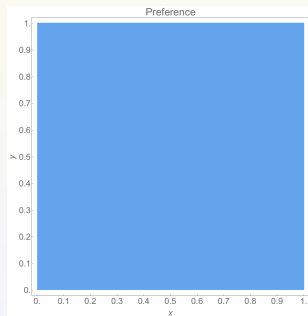
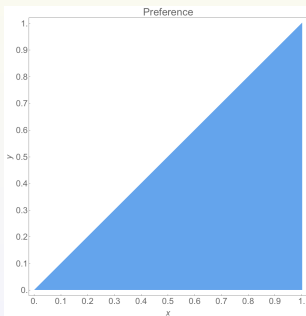
There exists rationalizing k for each k s.t

$$\text{complete indifference} = \lim_{k \rightarrow 1} k$$

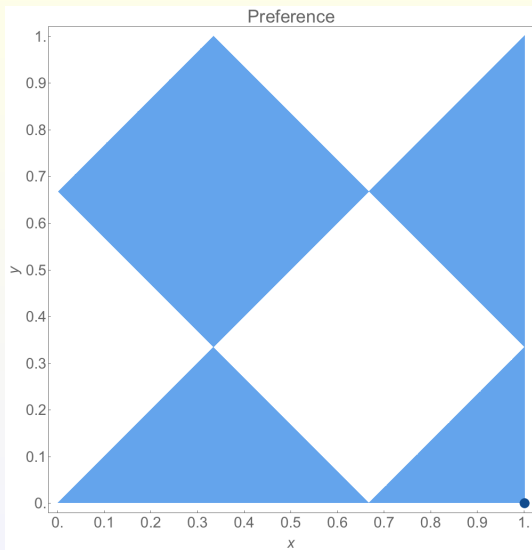
Example 2

Set of alternatives $X = [0; 1]$.

- | Left: the subject prefers x to y iff $x > y$.
- | Right: the subject is completely indifferent.

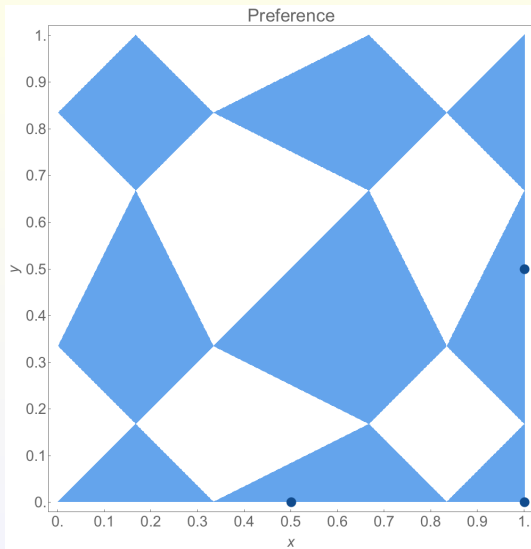


Example 2



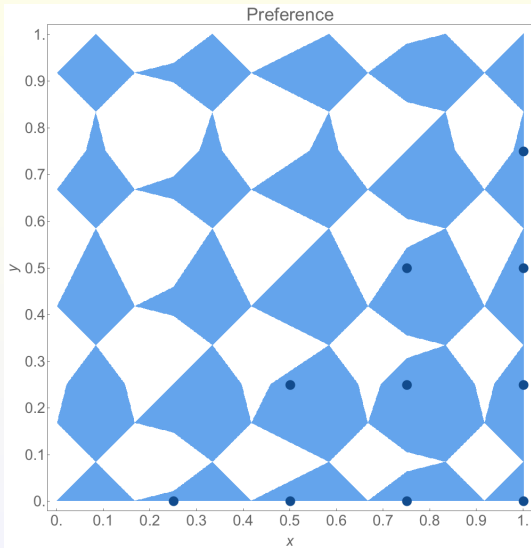
$n=1$

Example 2



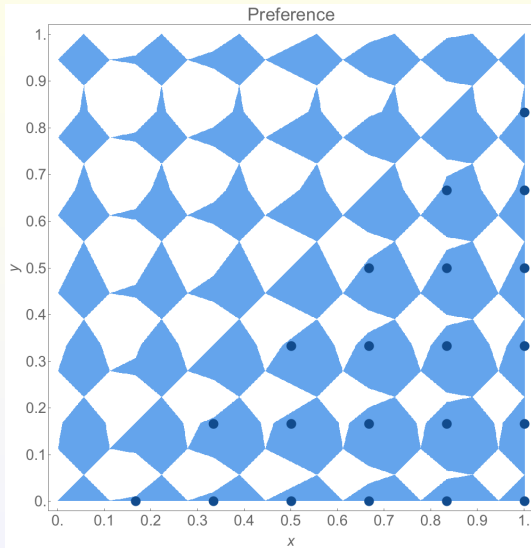
$n=2$

Example 2



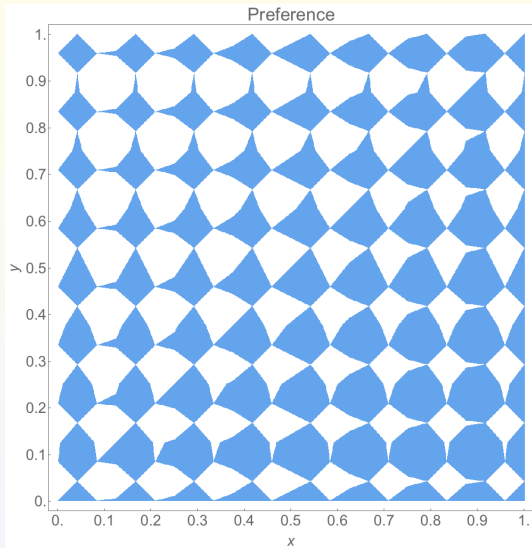
$n=4$

Example 2



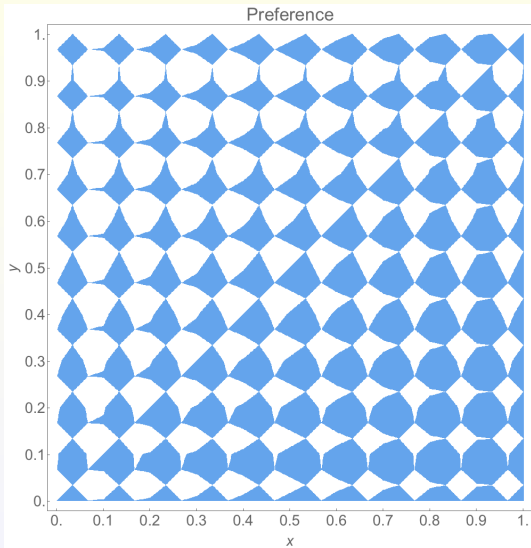
$n=6$

Example 2



$n=8$

Example 2



$n=10$

n=16

n=32

Discipline matters.

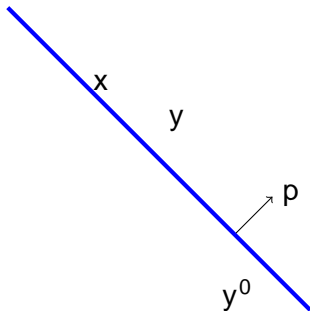
Empiricism is dangerous.

Inevitable role for theory (a Cartesian imperative).

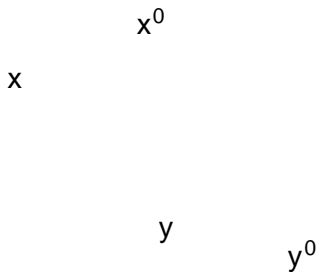
Choice under uncertainty:

- | State space $S = \{s_1, s_2\}$.
- | Choice among monetary acts $x \in \mathbb{R}^S$.
- | Bob is risk-neutral subjective exp. utility maximizer.
- | So $x \succ y \iff \sum_i p_i x_i > \sum_i p_i y_i$.
- | Preferences described by a prior $p \in \Delta(S)$.

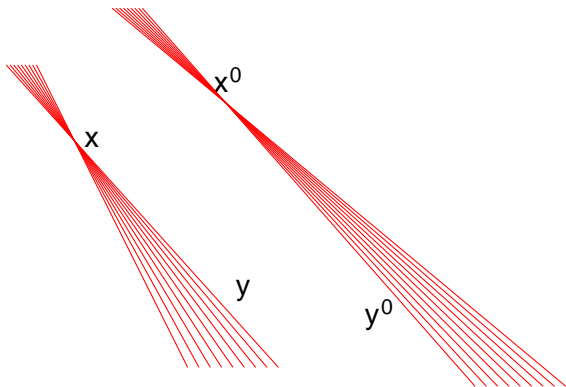
Bob's preferences:



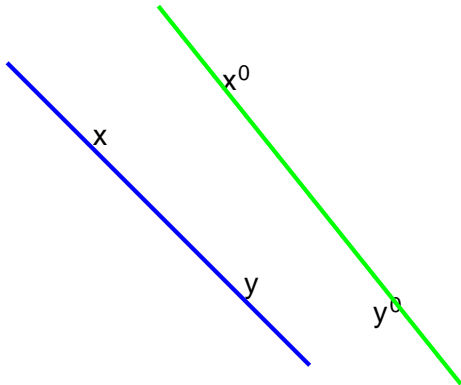
Suppose y is chosen over x , and x^0 over y^0 .



Suppose y is chosen over x , and x^0 over y^0 .

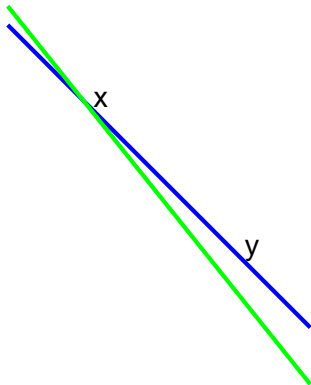


Suppose y is chosen over x , and x^0 over y^0 .



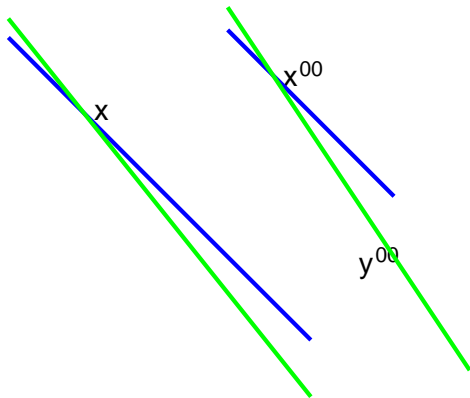
Bob's prior must be steeper than the blue line, and flatter than the green.

Suppose y is chosen over x , and x^0 over y^0 .



Bob's prior must be steeper than the blue line, and flatter than the green.

Suppose y is chosen over x , and x^0 over y^0 .



Narrows down unobserved comparison: x^0 y^0

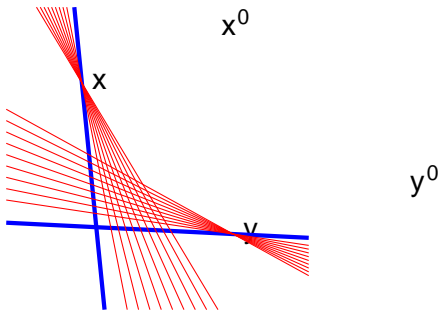
Suppose Alice instead uses the max-min model for Bob:

$$u(x) = \min_{p \in \mathcal{P}(x)} g$$

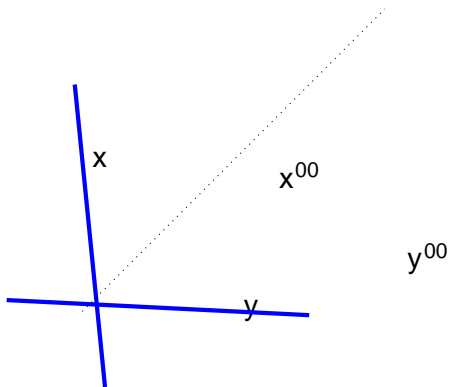
With two states, \mathcal{P} is described by four parameters. With more than two states, the model is non-parametric.

Then from $y = x$ she learns something about the slope of the worst-case priors.

y is chosen over x , and x^0 over y^0 .



y is chosen over x , and x^0 over y^0 .

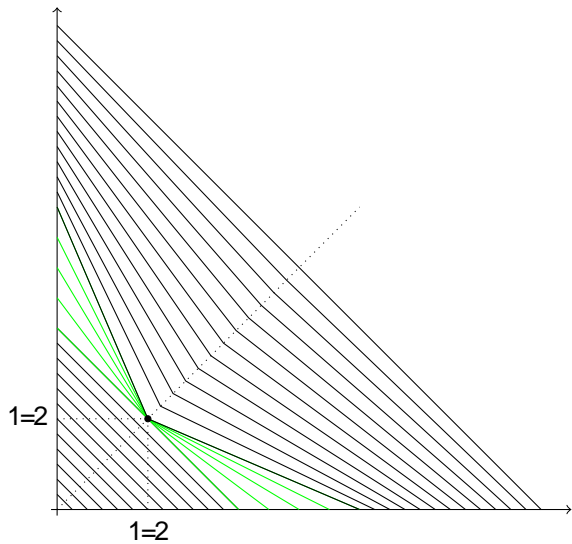


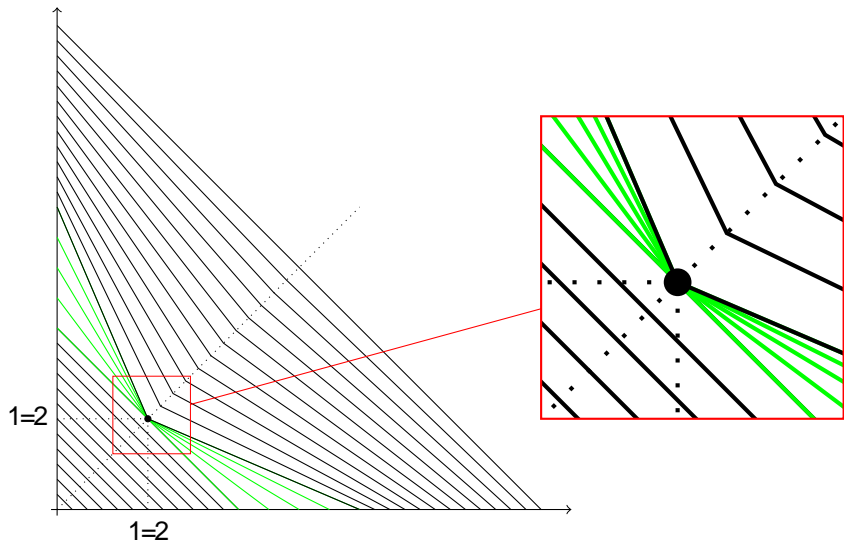
No inference for x^0 and y^0 .

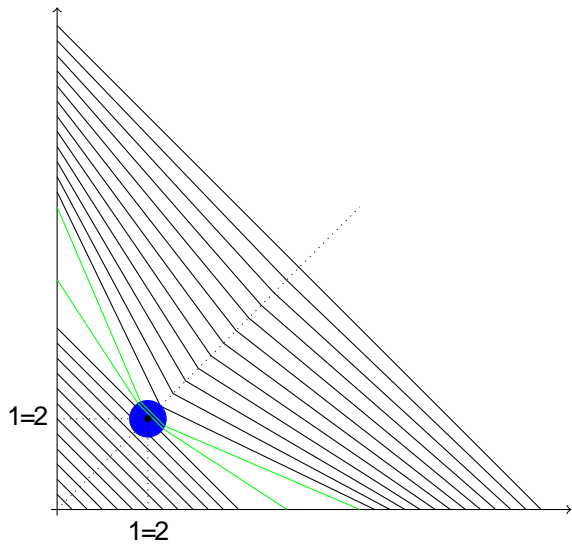
A more flexible theory may lead to
overfitting.

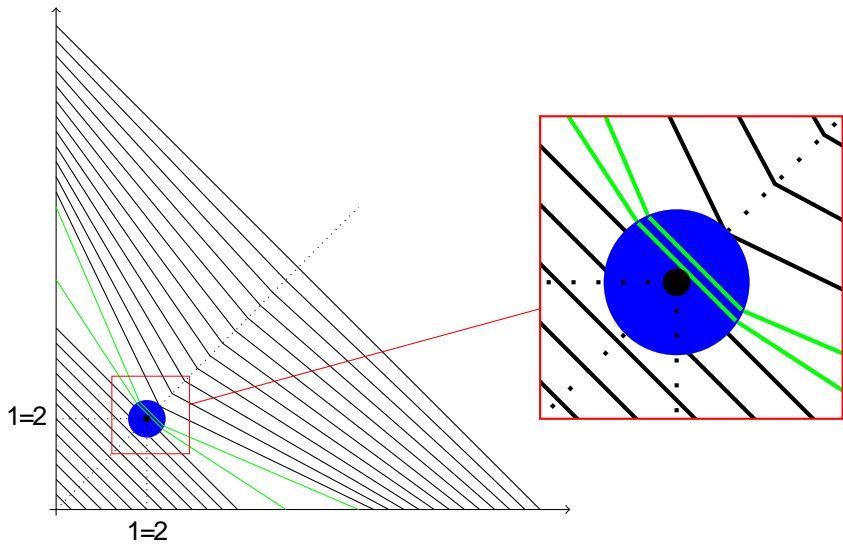
In fact max-min with $\frac{1}{n}$ is
"hopeless."

Any finite dataset will lead to poor
out-of-sample predictions.









Model of preferences must be closed

Can't allow for approximate behavior
to "escape."

| Let $X = [0; 1]$, $\mathcal{U} = \{u_k\}$ and $u(x) = x$.

| For each k , let $u_k(x) = \frac{x}{k}$ and

$$u_k = \frac{x}{k}:$$

| Then $0 = \lim_k u_k$.

| But $u_k(x) = \frac{x}{k}$ for all k !

| Let $X = [0; 1]$, $\epsilon = \frac{1}{k}$ and $u(x) = x$.

| For each k , let $u_k = \frac{x}{k}$ and

$$u_k = \frac{x}{k}:$$

| Then $0 = \lim_k u_k$.

| But $u_k = \frac{x}{k}$ for all k !

(For $\epsilon > 0$, can choose u_n with $\|u_n - u\|_1 = 1$ or $\|u_n - u\|_1 = 1$ and $0 = \lim_n u_n(x)$ for all $x \in [0; 1 - \epsilon]$.)

Utility estimates are more delicate
than preferences.

Must choose the right utility
representation.

Typical result in decision theory:

"Utility representation is **axioms**. Moreover, utility is **unique**."

Axioms) testable implications. (But may require infinite data.)

Uniqueness) identification. But more is needed to ensure utility recovery from finite data.

- | Alternatives: A topological space X .
- | Preference: A complete and continuous binary relation over X
- | \mathcal{P} a set of preferences.

A pair $(X; \mathcal{P})$ is a **preference environment**.

- | There are prizes.
- | X is the set of lotteries over the prizes, $d - 1 \quad \mathbb{R}^d$.
- | An EU preference is defined by $v \in \mathbb{R}^d$ such that $p \succsim p^0$ if $v \cdot p \geq v \cdot p^0$.
- | P is set of all the EU preferences.

Alice wants to recover Bob's preference from his choices.

- | Binary choice problem $f: X \times X \rightarrow \{0, 1\}$.
- | Bob is asked to choose x or y .
Behavior encoded in a choice function $c: X \times X \rightarrow \{0, 1\}$.
- | If Bob's preference is \succsim then $c(f(x, y)) = 1$ if and only if $x \succsim y$.
- | Partial observability: indifference is not observable.

Alice gets n i.i.d. dataset.

- | Experiment of size k : $\mathcal{D}^k = \{x_1, \dots, x_k\}$ with $y_i = f(x_i; \theta)$.
- | Set of growing experiments: $\mathcal{D}^k = \{x_1, x_2, \dots, x_k\}$ with $\mathcal{D}^k \subset \mathcal{D}^{k+1}$.

Afriat's theorem and revealed preference tests Afriat (1967);
Diewert (1973); Varian (1982); Matzkin (1991); Chavas and Cox (1993);
Brown and Matzkin (1996); Forges and Minelli (2009); Carvajal, Deb,
Fenske, and Quah (2013); Reny (2015); Nishimura, Ok, and Quah (2017)

Recoverability: Varian (1982); Cherchye, De Rock, and Vermeulen (2011);
Chambers, Echenique and Lambert (2021).

Consistency Mas-Colell (1978); Forges and Minelli (2009); Kubler and
Polemarchakis (2017); Polemarchakis, Selden, and Song (2017)

Identification: Matzkin (2006); Gorno (2019)

Econometric methods Matzkin (2003); Blundell, Browning, and Crawford
(2008); Blundell, Kristensen, and Matzkin (2010); Halevy, Persitz, and
Zrill (2018)

- | $(X; P)$ preference env.
- | c encodes choice
- | k seq. of experiments

- | A preference **weakly rationalizes the observed choices on** k if $c(f(x); y) = x$ and $c(f(x); y) = y$ for all $f(x); y \in 2^k$.
- | A preference **strongly rationalizes the observed choices on** k if $c(f(x); y) = z$ for $z \in f(x); y, z \notin c(f(x); y)$, for all $f(x); y \in 2^k$.

Choice of topology: **closed convergence topology**.

- | Standard topology on preferences (Kannai, 1970; Mertens (1970); Hildenbrand, 1970).

- | $\tau_n!$ when:

For all $(x; y) \in \tau_n$, there exists a seq. $(x_n; y_n) \in \tau_n$ that converges to $(x; y)$.

If a subsequence $(x_{n_k}; y_{n_k}) \in \tau_{n_k}$ converges, the limit belongs to τ_n .

- | If X is compact and metrizable, same as convergence under the Hausdorff metric.

- | X Euclidean and B the strict parts of cont. weak orders. Then it's the smallest topology for which the set

$$f(x; y; \tau) : x \in X; y \in X; \tau \in B \text{ and } x \succ y$$

is open.

Lemma

Let X be a locally-compact Polish (separable and completely metrizable) space. Then the set of all continuous binary relations $X \times X$ is a compact metrizable space.

Let $\{u_k\}$ be a sequence of functions,

$$u_k: X \rightarrow \mathbb{R}$$

The sequence **converges compactly** to $u: X \rightarrow \mathbb{R}$ if for every compact $K \subset X$,

$$u_{j_K} \rightarrow u|_K$$

uniformly.

Turn out to be the right topology for utility functions when preferences are endowed with the closed convergence topology.

Let X be

- | $X = \mathbb{R}^n$.
- | or $X = ([a; b])$ (set of "monetary" Anscombe-Aumann acts) with finite .

Obs.

- | Objective monotonicity.
- | Connection between order and topology $\times n$
- | Some of our results are more general.

A sequence of experiments $\{g^k\}$, with $g^k = f_{1, \dots, k} g$, is **exhaustive** when:

1. $\bigcup_{i=1}^{\infty} S_i$ is dense in X .
2. For all $x, y \in \bigcup_{i=1}^{\infty} S_i$ with $x \notin y$, there exists i s.t. $S_i = f(x; y)g$.

Theorem

Let

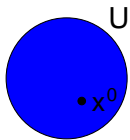
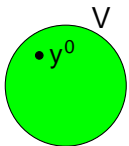
- | be monotone and cont.;
- | k strongly rationalize the k -sized choice data generated by .

Then,

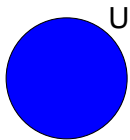
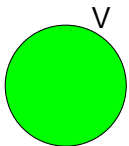
- | k ! (in the topology of closed convergence).
- | For any utility u for $\exists u_k$ for k s.t $u_k \rightarrow u$ (in the topology of compact convergence).

- | Monotonicity.
- | Convergence of **any arbitrary** preference rationalization.
- | Utility **can't be arbitrary**. Only get convergence of selected utility estimates. Require an identification theorem for each specific theory.

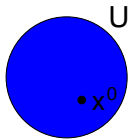
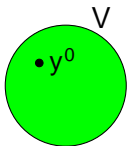
Why does monotonicity help?



- | $x \succ y$
- | $U \succ V$
- | $\exists x^0 \in U$ and $y^0 \in V$ s.t. $y^0 \succ_k x^0$ for some rationalizing k
- | But $x^0 \succ y^0$ s.t. is cont. and $j_B = j_B$.



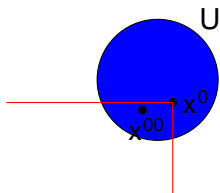
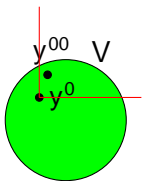
| x y
| U V



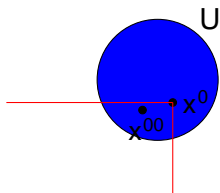
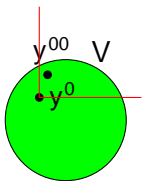
| $x \in y$

| $U \cap V$

| Let $(x^0, y^0) \in U \cap V$.



- | $x \quad y$
- | $U \quad V$
- | Let $(x^0, y^0) \in U \times V$.
- | $\Rightarrow \exists x^{00}, y^{00} \in B$
- | $x^{00} \sim x^0$
- | $y^0 \sim y^{00}$



- | $x \quad y$
- | $U \quad V$
- | Let $(x^0, y^0) \in U \cap V$.
- | $\Rightarrow \exists x^{00}, y^{00} \in B$
- | $x^{00} \in x^0$
- | $y^0 \in y^{00}$
- | $\Rightarrow x^0 = x^{00} + k(y^{00} - y^0)$

Let $X = \mathbb{R}^n$.

Let $P^k(c)$ be the set of continuous and strictly monotone preferences that weakly rationalize the data.

For a set of binary relations \mathcal{S} , define $\text{diam}(\mathcal{S}) = \sup_{(c; \theta) \in \mathcal{S}} \text{diam}(c; \theta)$ to be the diameter of \mathcal{S} according to the metric c which generates the topology on preferences.

Theorem

One of the following holds:

1. There is k such that $P^k(c) = \emptyset$.
2. $\lim_{k \rightarrow \infty} \text{diam}(P^k(c)) = 0$.

A preference is **locally strict** if

$x \succ y \Rightarrow$ in every nbd. of $(x; y)$, there exists (x^0, y^0) with $x^0 \succ y^0$

(Border and Segal, 1994).

Let $X \subseteq \mathbb{R}^n$ and P be a closed set of locally strict preferences on X .

Theorem

Let $\{C_k\}_{k \in \mathbb{N}}$ weakly rationalize the k -sized choice data.

- | Then there is a preference $P^0 \in P$ s.t. $C_k \subseteq C_{P^0}$.
- | The limiting preference is unique: if, for every $P^0 \in P$ rationalizes the k -data, then the same limit P^0 obtains.

Obs. that generating the choice is not a hypothesis. May view this result as a definition of preference.

Obs. doesn't require monotonicity.

(This result is in CEL (2021))

Utility functions

Finite state space S .

Monetary consequences $[a; b] \in \mathbb{R}$

Anscombe-Aumann acts $f : S \rightarrow ([a; b])$

Preferences on $([a; b])^S$.

Let U be the set of all continuous and monotone weakly increasing functions $u : [a; b] \rightarrow \mathbb{R}$ with $u(a) = 0$ and $u(b) = 1$.

A pair $(V; u)$ is a **standard representation** if $V : (\mathbb{R} \times [a; b])^S \rightarrow \mathbb{R}$ and $u \in U$ are continuous functions such that $V(p; \cdot; \cdot; p) = \int_{[a; b]} u dp$, for all constant acts $(p; \cdot; \cdot; p)$.

$(V; u)$ is **aggregative** if there is an **aggregator** $H : [0; 1]^S \rightarrow \mathbb{R}$ with $V(f) = H((\int u df(s))_{s \in S})$ for $f \in ([a; b])^S$.

An aggregative representation with aggregator H is denoted by $(V; u; H)$.

A preference \succsim on $([a; b])^S$ is **standard** if it is weakly monotone, and there is a standard representation $V(\cdot)$ in which V represents \succsim .

Variational preferences (Maccheroni et al 2006) are standard and aggregative. Let

$$V(f) = \inf_{g \in \mathcal{Z}} \int v(f(s)) d\mu(s) + c(g) : g \in \mathcal{Z}(\mathcal{S})$$

where

1. $v : ([a; b]) \rightarrow \mathbb{R}$ is continuous and $a > 0$.
2. $c : \mathcal{Z}(\mathcal{S}) \rightarrow [0; 1]$ is lower semicontinuous, convex and grounded (meaning that $\inf_{g \in \mathcal{Z}(\mathcal{S})} c(g) = 0$).

Let $H : [0; 1]^{\mathcal{S}} \rightarrow \mathbb{R}$ be $H(x) = \inf_{g \in \mathcal{P}} \int x(s) d\mu(s) + c(g) : g \in \mathcal{Z}(\mathcal{S})$

Theorem

Let \succsim be a standard preference with standard representation (V, u) , and $\{ \succsim^k \}$ a sequence of standard preferences, each with a standard representation $(V^k; u^k)$.

1. If $\succsim^k \succsim$, then $(V^k; u^k) \succsim (V; u)$.
2. If, in addition, these preferences are aggregative with representation $(V^k; u^k; H^k)$ and $(V; u; H)$, then $H^k \succsim H$.

Given $(X; P)$. We change:

- | How subjects make choices: they do not exactly follow a preference but randomly deviate from it.
- | How experiments are generated.

1. In a choice problem, alternatives drawn iid according to **sampling distribution** .
2. Subjects make "mistakes."
Upon deciding on $x; y$, a subject with preference chooses x over y with probability $q(x; y)$ (**error probability function**).
3. Only assumption: if $x \succ y$ then $q(x; y) > 1/2$.
4. "Spatial" dependence of q on x and y is arbitrary.

Kemeny-minimizing estimator: find a preference P that minimizes the number of observations inconsistent with the preference.

- | "Model free:" to compute estimator don't need to assume a specific form.
- | May be computationally challenging (depending on n).

Assumption 1 : X is a locally compact, separable, and completely metrizable space.

Assumption 2 : P is a closed set of locally strict preferences.

Assumption 3' : f has full support and for all $x, y \in X$, $f(x; y) > 0$.

Theorem

Under Assumptions (1), (2), (3'), if the subject's preference is $2 P$ and \hat{p}_n is the Kemeny-minimizing estimator for p_n , then, $\hat{p}_n \rightarrow p_n$ in probability.

The **VC dimension** of \mathcal{P} is the largest cardinality of an experiment that can always be rationalized by \mathcal{P} .

A measure of how flexible \mathcal{P} ; how prone it is to overfitting.

- | Think of a game between Alicia and Roberto
- | Alicia defends \mathcal{P} ; Roberto questions it.
- | Given \mathcal{P}
- | Alicia proposes a choice experiment of size k
- | Roberto chooses adversarially.
- | Alicia wins if she can rationalize the choices using \mathcal{P}
- | The VC dimension of \mathcal{P} is the largest k for which Alicia always wins.

Convergence rates

- | a metric on preferences.

Theorem

Under the same conditions as in Part A,

$$N(\epsilon; \rho) \leq \frac{2}{r(\epsilon)^2} \rho^{\frac{1}{2}} + C \rho^{\frac{1}{2}} \sqrt{VC(P)}$$

with C a universal constant.

Convergence rates

- | a metric on preferences.
- | $N(\epsilon; P)$: smallest value of N such that for all $n \geq N$, and all subject preferences $\succsim \in P$,

$$\Pr\left(\left| \bar{v}_n - v \right| < \epsilon\right) \geq 1 - \epsilon$$

Theorem

Under the same conditions as in Part A,

$$N(\epsilon; P) \leq \frac{2}{\epsilon^2} \left(\frac{1}{2\epsilon} + C \sqrt{\text{VC}(P)} \right)^2$$

with C a universal constant.

Convergence rates

- | a metric on preferences.
- | $N(\epsilon; \mathcal{P})$: smallest value of N such that for all $n \geq N$, and all subject preferences $\rho \in \mathcal{P}$,

$$\Pr(\| \hat{\rho}_n - \rho \| < \epsilon) \geq 1 - \epsilon :$$

- | $r(\epsilon; \mathcal{P})$: prob. choice of preference ρ is consistent with ϵ .

$$r(\epsilon) = \inf_{\mathcal{P}} \left(\epsilon; \mathcal{P} \right) \quad \left(\epsilon; \mathcal{P} \right) : \epsilon \in \mathcal{P}; \left(\epsilon; \epsilon \right) :$$

Theorem

Under the same conditions as in Part A,

$$N(\epsilon; \mathcal{P}) \leq \frac{2}{r(\epsilon)^2} \log \frac{1}{2\epsilon} + C \log \frac{1}{VC(\mathcal{P})} \log \frac{1}{\epsilon^2}$$

with C a universal constant.

Convergence rates

- | a metric on preferences.
- | $N(\epsilon; \mathcal{P})$: smallest value of N such that for all $n \geq N$, and all subject preferences $\succ \in \mathcal{P}$,

$$\Pr(\max_{i \leq n} (r_i - r) < \epsilon) \geq 1 - \epsilon :$$

- | $(\theta; \mathcal{P})$: prob. choice of preference \succ is consistent with θ .

$$r(\theta) = \inf_{\succ \in \mathcal{P}} (r(\succ) - \theta) : \theta \in \mathcal{P}; (r(\succ) - \theta) \geq 0 :$$

- | $VC(\mathcal{P})$ the VC dimension of the class \mathcal{P} .

Theorem

Under the same conditions as in Part A,

$$N(\epsilon; \mathcal{P}) \leq \frac{2}{\epsilon^2} \left(\frac{VC(\mathcal{P})}{2} + C \right) \frac{1}{\epsilon^2}$$

with C a universal constant.

Expected utility

1. X is the set of lotteries over d prizes.
2. \mathcal{P} is the set of **nonconstant** EU preferences: there are always lotteries $p; p^\ell$ such as p is strictly preferred to p^ℓ .

This preference environment satisfies Assumptions 1 and 2.

Suppose: there is $C > 0$ and $k > 0$ s.t

$$q(x; y; \cdot) = \frac{1}{2} + C(v(x) - v(y))^k;$$

when $x \succ y$ and v represents \cdot .

Under these assumptions, we can bound $r(\cdot)$ and $VC(P)$, which implies

$$N(\epsilon; \delta) = O\left(\frac{1}{4d\epsilon^2}\right) :$$

Other examples: Cobb-Douglas, CES, and CARA subjective EU preferences, and intertemporal choice with discounted, Lipschitz-bounded utilities.

Monotone preferences

- | K be a compact set in $X \subset \mathbf{R}_{++}^d$, and fix $\epsilon > 0$.
- | P has finite VC-dimension and is identified on K
- | μ is the uniform probability measure on K ≈ 2 ,
- | q satisfies: probability of choosing y instead of x when $x \succ y$ is a function of $k(x - y)_k$,

Proposition

The Kemeny-minimizing estimator is consistent and, as $\epsilon \rightarrow 0$ and $\mu \rightarrow 0$,

$$N(\epsilon; \mu) = O\left(\frac{1}{\epsilon^{2d+2}} \ln \frac{1}{\epsilon}\right);$$

Applications: preferences from utilities

A set P is defined from utilities when there is a class \mathcal{U} of utility functions such that for all $x, y \in X$

$$x \succ y \iff \exists U \in \mathcal{U} \text{ such that } U(x) > U(y)$$

for some $U \in \mathcal{U}$.

Proposition 1

Under Assumption 1, if U is compact and represents locally strict preferences, then Assumption 2 is met.

Implied by the continuity theorem of Border and Segal (1994).

Revisit the case of expected utility preferences:

1. X is the set of lotteries over d prizes.
2. \mathcal{P} is the set of **nonconstant** EU preferences: there are always lotteries $p; p^\theta$ such as p is strictly preferred to p^θ .

This preference environment satisfies Assumptions 1 and 2. When the probability of error of choosing y instead of x when $x \succ y$ is a function of $kx \succ yk$, we can bound $r(\epsilon)$ and $VC(\mathcal{P})$, which implies

$$N(\epsilon; \mathcal{P}) = O\left(\frac{1}{4d^2}\right) \epsilon^{-2}$$

Afriat's theorem and revealed preference tests: Afriat (1967); Diewert (1973); Varian (1982); Matzkin (1991); Chavas and Cox (1993); Brown and Matzkin (1996); Forges and Minelli (2009); Carvajal, Deb, Fenske, and Quah (2013); Reny (2015); Nishimura, Ok, and Quah (2017)

Recoverability: Varian (1982); Cherchye, De Rock, and Vermeulen (2011)

Approximation: Mas-Colell (1978); Forges and Minelli (2009); Kübler and Polemarchakis (2017); Polemarchakis, Selden, and Song (2017)

Identification: Matzkin (2006); Gorno (2019)

Econometric methods: Matzkin (2003); Blundell, Browning, and Crawford (2008); Blundell, Kristensen, and Matzkin (2010); Halevy, Persitz, and Zrill (2018)

Applications: monotone preferences

- | Call a **dominance relation** any binary relation on X that is not reflexive.
- | Say that \succsim is **strictly monotone** wrt \triangleright if $x \triangleright y$ implies $x \succ y$.
- | Say that \succsim is **Grodal-transitive** if $x \succ y \succ z \succ w$ implies $x \succ w$.

Proposition 2

Take a set of alternatives X that meets Assumption 1, and suppose:

1. \triangleright is a dominance relation that is open,
2. for each x , there are $y; z$ arbitrarily close to x such that $y \triangleright x$ and $x \triangleright z$.

Then the class of preferences that are Grodal-transitive and strictly monotone wrt \triangleright meets Assumption 2.

Example: back to preferences over commodity bundles.

- | There are d commodities.
- | $X \subseteq \mathbf{R}_{++}^d$, where for $(x_1; \dots; x_d) \in X$, x_i is quantity of good i consumed.
- | $x \succ y$ iff $x_i > y_i$ for all $i = 1; \dots; d$.

The set of all preferences that are Grodal-transitive and strictly monotone wrt \succ meets Assumption 2.

Conclusion

- | Binary choice
- | Finite data
- | “Consistency” – Large sample theory
- | Unified framework: RP and econometrics.

Applicable to:

Large-scale (online) experiments/surveys.

Voting (roll-call data).