Decreasing Impatience

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What is discounting?

Value today of Fr. 1 in 4 years:

$$\delta^4 = \left(\frac{1}{1+r}\right)^4$$

Value today of x_t Francs or utils in t years: $x_t f(t)$.

$f : \mathrm{N} \to (0,1)$ is a *discount factor*.

Special case is *exponential discounting*, $f(t) = \delta^t$, $\delta \in (0,1)$.

Introduction

- Choice among *streams* $x \in \ell^\infty_+$.
- Each x_t expressed in money or utils.
- Preferences $\succeq$ for which there is a summable, nonincreasing, positive $f : \mathrm{N} \to [0,1]$ s.t

$$x \succeq y \Longleftrightarrow \sum_t f(t)x_t \geq \sum_t f(t)y_t.$$

- $f(t)$ is a *discount factor*.

Introduction

For example a *dated reward* (α, t) is a stream x with $x_s = 0$ for all $s \neq t$ and $x_t = \alpha$.

Willing to postpone α at time t for $\alpha' > \alpha$ at time $t+1$ if $\alpha' f(t+1) \geq \alpha f(t)$. That is, if

$$\frac{f(t+1)}{f(t)} \geq \frac{\alpha}{\alpha'}$$

Ratio $f(t+1)/f(t)$ is a measure of impatience.

Definition: f exhibits *decreasing impatience* if $f(t+1)/f(t)$ is (weakly) increasing.

For example

Exponential discounting: $f(t) = \delta^t$
(characterized by constant $f(t+1)/f(t)$).

Quasihyperbolic discounting:

$$f(t) = \beta^{\min\{t,1\}}\delta^t$$

Generalized quasihyperbolic

$$f(t) = \beta^{\min\{t^*,t\}}\delta^t$$

Motivation for decreasing impatience

Motivation 1: Behavioral. Decreasing impatience is at source of behavioral phenomena.

Motivation 2: Decreasing impatience is the outcome of aggregation : e.g Weitzman – Jackson/Yariv

Thought experiment

- Choose between $10 000 today and $ 10 500 next Thursday.
- Choose between $10 000 on Sept 23rd 2022 and $10 500 on Sept 30th 2022.

Thought experiment

- Choose between \$10 000 today and \$ 10 500 next Thursday.
- Choose between \$10 000 on Sept 23rd 2022 and \$10 500 on Sept 30th 2022.
- Decreasing impatience.
- This is incompatible with exponential discounting.

Very thirsty subjects (McLure et al 2007)

Choose between:

- Juice now vs. $2\times$ juice in 5 min
- Juice in 20min vs. $2\times$ juice in 25 min

Very thirsty subjects (McLure et al 2007)

Choose between:

- Juice now (60%)vs. 2× juice in 5 min (40%)
- Juice in 20min (30%) vs. 2× juice in 25 min (70%)

Decreasing impatience

Often captured by *quasi-hyperbolic discounting* Value of \$ 1 in t years:

$$f(t) = \beta\delta^t,$$

where $\beta, \delta \in (0, 1]$.

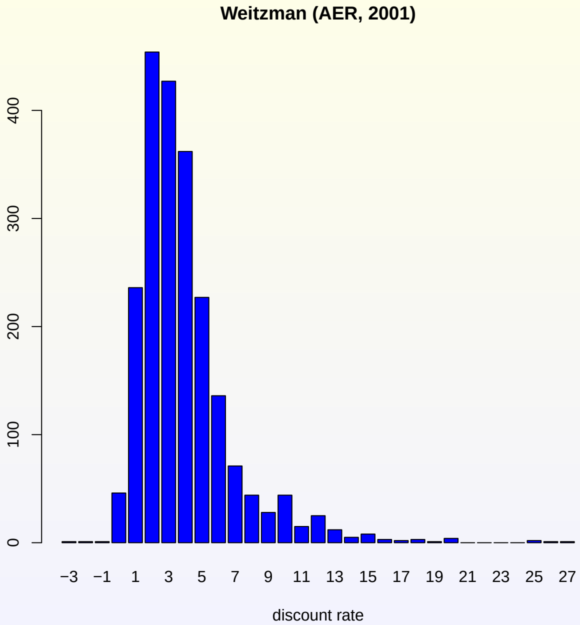
Motivation 2

Discounting for long-term projects.

Weitzman (2001)

Survey of 2,160 Ph.D-level economists.

- "what real interest rate should be used to discount over time the benefits and costs of projects being proposed to mitigate the possible effects of global climate change."
- use "professionally considered gut feeling"


Weitzman (AER, 2001)
0
100
200
300
400
−3
−1
1
3
5
7
9
11
13
15
17
19
21
23
25
27
discount rate

Weitzman (2001)

Survey of 2,160 Ph.D-level economists:

- Mean rate : 3.96 %
- StdDev: 2.94 %

Smaller survey: 50 leading economists *(incl. G. Akerlof, K. Arrow, G. Becker, P. Krugman, D. McFadden, R. Lucas, R. Solow, J. Stiglitz, J. Tobin . . .)*

- Mean rate : 4.09 %
- StdDev: 3.07 %

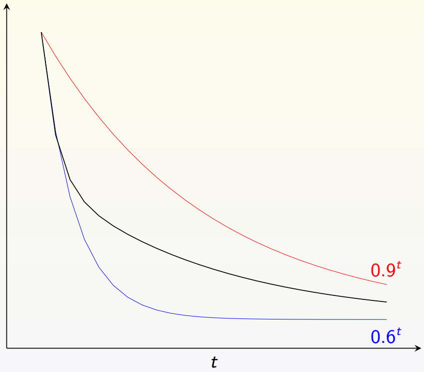
Gamma Discounting

- Natural to use average of discount factors.
- Utilitarian aggregation.
- Ex:

$$\frac{1}{2}0.6^t + \frac{1}{2}0.9^t$$

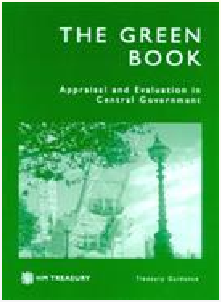
- The average will mimic the largest factor for large t.

Example



Bottom line: Utilitarian aggregation leads to decreasing impatience.

Project evaluation



H.M. Treasury's "Green book" recommends a discount rate that decreases from 3.50% to 2.50% as the time horizon increases from 30 to 75 years.

Main result (informally)

Decreasing impatience equivalent to :

- One axiom ("compound interest convexity").
- "Dynamically consistent" aggregation of generalized quasihyperbolic discounting
- Another aggregation/multiple-selves representation.

One axiom

Compound interest convexity: For all $k > 0$, all $t \geq 1$ and all $r > 0$,

$$\left(\frac{k}{2}(1+r)^{t-1}, t-1\right) + \left(\frac{k}{2}(1+r)^{t+1}, t+1\right) \succeq (k(1+r)^t, t).$$

Requires only one observation to falsify.

Main result: 3/4

Theorem

Let $\succeq$ be a preference over streams, with associated discount factor f. Assume $\frac{f(t+1)}{f(t)}$ bded away 1. The following statements are equivalent:

1. $\succeq$ satisfies decreasing impatience.
2. $\succeq$ satisfies compound-interest convexity.
3. f is the (finite or countable) geometric mean of gen. quasihyperbolic discount factors.

Main result: 3/4

Theorem

Let $\succeq$ be a preference over streams, with associated discount factor f. Assume $\frac{f(t+1)}{f(t)}$ bded away 1. The following statements are equivalent:

1. $\succeq$ satisfies decreasing impatience.
2. $\succeq$ satisfies compound-interest convexity.
3. f is the (finite or countable) geometric mean of gen. quasihyperbolic discount factors. So $\exists$ β-δ discount factors f_s, and $\eta_s > 0$ with $\sum_s \eta_s = 1$, such that $f(t) \propto \prod_s f_s(t)^{\eta_s}$.

Main result

Note that utilitarian aggregation cannot account for all dec. impatience discount factors.

Indeed quasi-hyperbolic discount factors cannot be obtained in this fashion. In a sense, this is the point of Weitzman and Jackson-Yariv.

There would be discount factors δ_i, with weights a_i such that $\beta\delta^t = \sum_i a_i \delta_i^t$ for all $t > 1$.

So $\beta = \sum_i a_i (\delta_i/\delta)^t$. The RHS is constant in t, which is only possible if $\delta_i = \delta$ for all i.

Geometric mean

Why the geometric mean?

Let's think of a preference aggregation rule $\phi : \mathcal{NI}^M \rightarrow \mathcal{NI}$.

Axioms

Pareto: If for all $i \in M$, $f_i(t)x \geq f_i(s)y$, then $\varphi(f_1, \ldots, f_n)(t)x \geq \varphi(f_1, \ldots, f_n)(s)y$, with a strict inequality if any individual inequality is strict.

Independence of Irrelevant Alternatives: For any $t, s > 0$. For all $f, f' \in \mathcal{NI}^M$, if for all $i \in M$ and all $x, y \in \mathrm{R}_{++}$: $f_i(t)x \geq f_i(s)y$ iff $f'_i(t)x \geq f'_i(s)y$, then for all $x, y \in \mathrm{R}_{++}$, $\varphi(f)(t)x \geq \varphi(f)(s)y$ iff $\varphi(f')(t)x \geq \varphi(f')(s)y$.

Dynamic consistency

- Suppose we ask agents to rank dated rewards (α, s) and (α', s') at time t.
- Suppose φ chooses (α, s).
- Dynamic consistency requires then that φ prefer $(\alpha, s + t)$ to $(\alpha', s' + t)$ at time 0.
- (Otherwise the plan would be reversed once time t arrives.)

Dynamic consistency

Notation: f^t is the discount factor starting at time t: $f^t(s) = f(t+s)$.

Dynamic Consistency: For all $t \geq 0$, $\varphi(f_1^t, \ldots, f_n^t) \propto \varphi(f_1, \ldots, f_n)^t$.

Either can aggregate and wait, or wait and aggregate. Assuming individual preferences are dynamically consistent, axiom states it doesn't matter.

Analogue of commuting with respect to Bayesian updating for prob. aggregation literature.

An aggregator φ is a *geometric mean* if there exists $\eta_i > 0$ for each $i \in M$ such that $\sum_i \eta_i = 1$ and

$$\varphi(f_1, \ldots, f_m) \propto \phi_\alpha(f_1, \ldots, f_m)(t) = \prod_{i \in M} f_i(t)^{\eta_i},$$

for all $(f_1, \ldots, f_m) \in \mathcal{N}\mathcal{I}^M$.

Theorem

An aggregator satisfies Pareto, IIA, and Dynamic Consistency iff it is a geometric mean.

Seemingly unrelated

Parimutuel betting: Eisenberg-Gale (1959)

- Bettors on race track
- Each have beliefs over horse that win the race.
- Place bets; payouts are made according to the parimutuel system.
- Can be formulated as a market in which securities are traded: one for each horse.
- Price reflects an aggregate of each bettor's beliefs.

Parimutuel aggregation

A *parimutuel economy* is a collection $(\succeq_i, w_i)_{i \in I}$, where I is finite or countable, each $\succeq_i$ being an exponential preference (on ℓ^∞_+) and $w_i > 0$ satisfying $\sum_i w_i = 1$.

An *allocation* is a collection $x = (x_i)_{i \in I}$ of sequences in ℓ^∞_+ s.t

$$\sum_{i \in I} x_i(t) = 1$$

A *parimutuel equilibrium* in $(\succeq_i, w_i)$ is a pair (x^*, p^*) in which x^* is an allocation and $p^* \in \ell^1_+$ is a sequence of prices, for which x^*_i is maximal for $\succeq_i$ in the budget set

$$\{x \in \ell^\infty_+ : \sum_t p(t)x(t) \leq w_i\}.$$

Main result

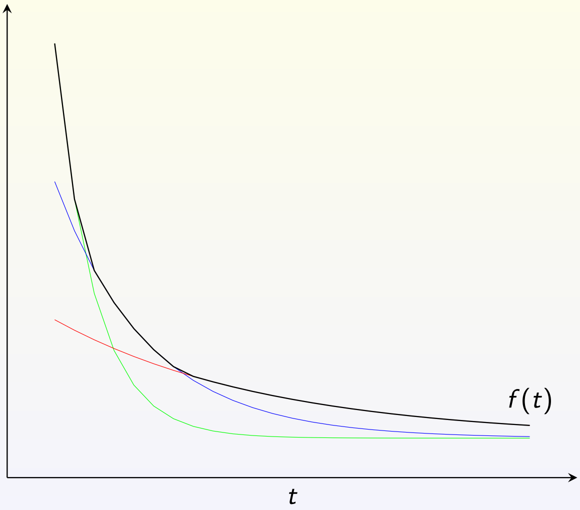
Theorem

Let $\succeq$ be a preference over streams, with associated discount factor f. Assume $\frac{f(t+1)}{f(t)}$ bded away 1. The following statements are equivalent:

1. $\succeq$ satisfies decreasing impatience.
2. $\succeq$ satisfies compound-interest convexity.
3. f is proportional to the (finite or countable) geometric mean of gen. quasihyperbolic discount factors. So $\exists$ β-δ discount factors f_s, and $\eta_s > 0$ with $\sum_s \eta_s = 1$, such that $f(t) \propto \prod_s f_s(t)^{\eta_s}$.
4. There is a parimutuel economy, and a parimutuel equilibrium (x^*, p^*) s.t $p^*_t = f(t)$ for all t.

Remark: In fact eqm. prices are unique.

Example



Ideas in the proof

Prelec (2004): Decreasing impatience equivalent to log convexity.

Ideas in the proof

Consider first compound interest convexity.

A known result in mathematics (Montel):

Theorem

f is log convex iff for all $\beta > 1$, $\beta^t f(t)$ is convex.

Equivalent to compound interest convexity in our framework.

Consider geometric mean property.

Roughly: a decreasing convex function is in the linear span of decreasing, convex piecewise linear in two pieces.

So a decreasing log-convex function is in the “geometric” span of decreasing, log-convex, piecewise exponential in two pieces.

Consider parimutuel property.

Remember: every decreasing convex function is a pointwise sup of affine functions.

So: every decreasing log-convex function is a pointwise sup of functions like: $\alpha\beta^t$, where $\alpha > 0$ and $\beta < 1$.

Now, for any linear economy, equilibrium prices are a pointwise sup of scales of utilities—this is a general result; does not depend on exponential or even a countable-dimensional vector space.

Conversely (and roughly), given any pointwise sup of scales of utilities, there are individual wealth levels for which this constitutes an equilibrium price.

So: characterize those functions which can be pointwise sups of scales of exponentials, but we already claimed these are the log-convex functions.

Related Literature

- Loewenstein and Prelec (1992), Prelec (2004), Laibson (1997)
- Halevy (2015)
- Chakraborty (2021)
- DeJarnette, Dillenberger, Gottlieb, and Ortoleva (2020)
- Weitzman (2001) Zuber (2011) Jackson and Yariv (2015) Feng and Ke (2018) Chambers and Echenique (2018)

Conclusions

- Decreasing impatience: behavioral phenomenon, and a property of discounting in project evaluation.
- Main result contains several different characterizations: One behavioral axiom. Two "multiple selves" representations.
- Decreasing impatience is dynamically consistent aggregation of generalized hyperbolic discounting.
- Connection to parimutuel aggregation of beliefs.

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