Testable Implications of Models of Intertemporal Choice

Exponential Discounting and Its Generalizations

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Cowles F. conference, June 9 2015

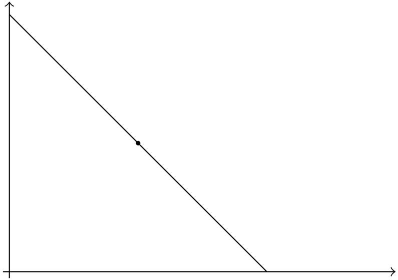
Utility and behavior

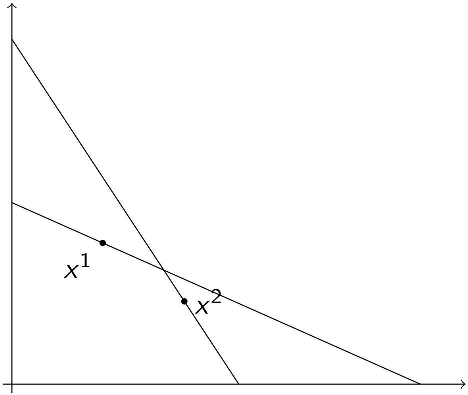
Model:

$$\max_{x \in \mathbf{R}_+^{\mathcal{T}}} \quad U(x)$$
$$\text{s.t } p \cdot x \leq I$$

Utility and behavior

(Market) behavior:




x^1
x^2

Utility and behavior

- Q: When is observable behavior consistent with utility max.?
- A: When SARP is satisfied.

This paper: $\max_{x \in \mathbf{R}_+^{\mathcal{T}}} U(x)$ s.t $p \cdot x \leq I$

- *Exponential discounting*:

$$U(x_0, \ldots, x_T) = \sum_{t \in \mathcal{T}} \delta^t u(x_t)$$

Importantly: u assumed to be st. increasing & concave.

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- *Quasi-hyperbolic discounting*:

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$$U(x_0, \ldots, x_T) = u(x_0) + \beta \sum_{t=1}^{T} \delta^t u(x_t)$$

- *Time-separable utility*:

$$U(x_0, \ldots, x_T) = \sum_{t \in \mathcal{T}} u_t(x_t)$$

Importantly: u assumed to be st. increasing & concave.

This paper.

- Q: When is observable behavior consistent with model M.?
- A: When SA-M is satisfied.

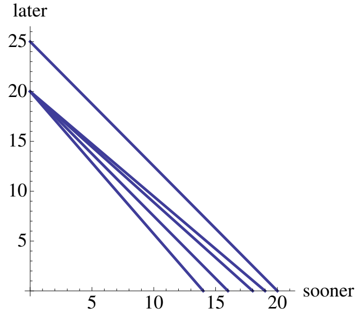
$M \in \{\text{TSU, QHD, EDU}\}$

Application to experimental data.

This paper.

Application to experimental data from Andreoni and Sprenger "Estimating Time Preferences from Convex Budgets" (AER 2012). Fits our framework:

- "Economic" budget sets;
- Planned (not realized) consumption.



Warmup

Warmup

The 2×2 case.

- 2 dates
- 2 observations
- Exp. discounting.

What is the meaning of this:

$$\max u(x_0) + \delta u(x_1)$$
$$p_0 x_0 + p_1 x_1 \leq I$$

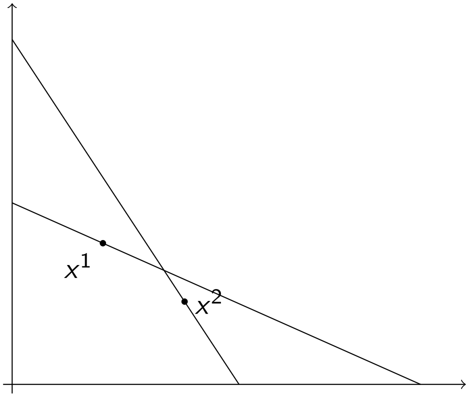
model for market behavior ?

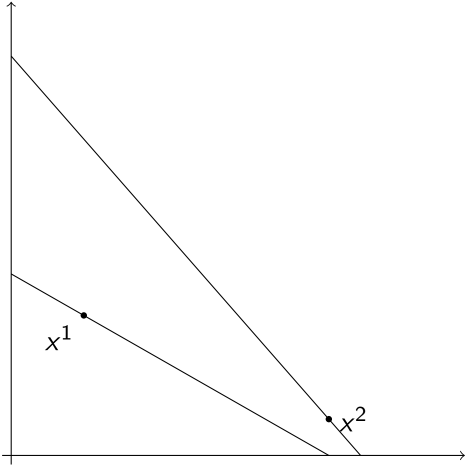
Unobservables:

- Utility $u : \mathbf{R}_+ \to \mathbf{R}$ st. inc. & conc.
- $\delta \in (0, 1]$

Observable:

- choices at different budgets


x^1
x^2



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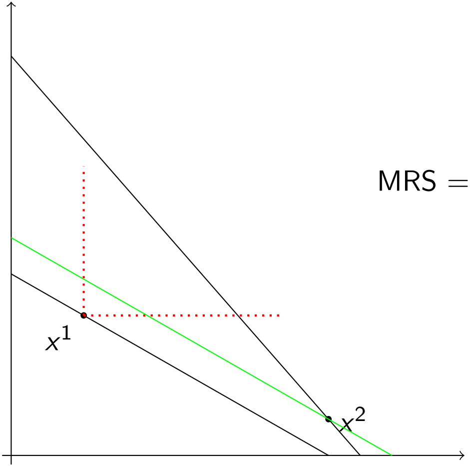
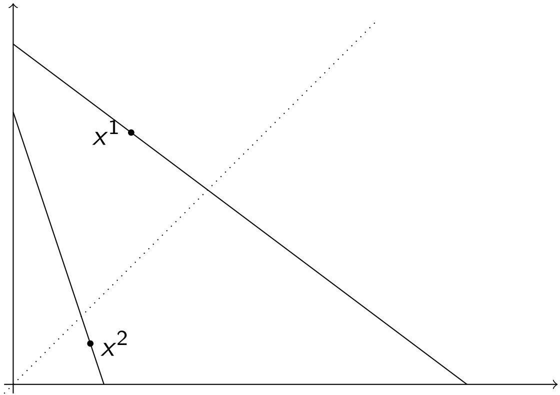


$$\text{MRS} = \frac{u'(x_0)}{\delta u'(x_1)}$$



x^1
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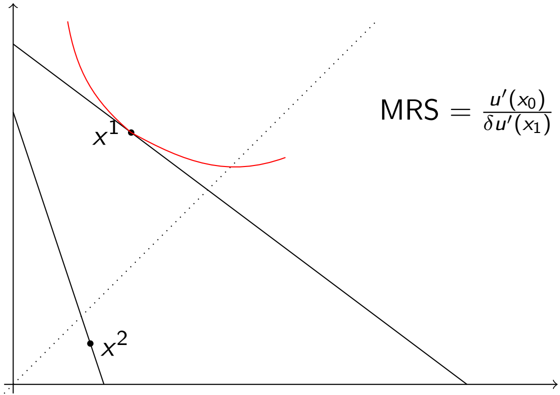

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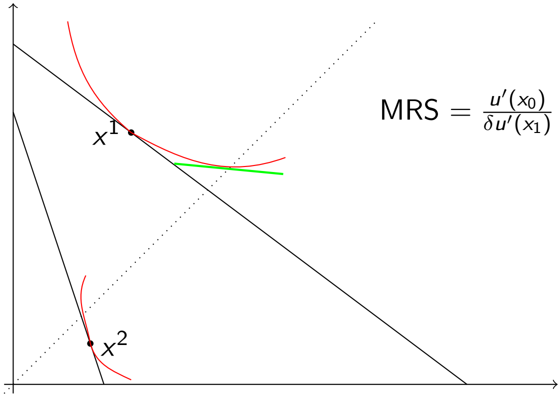
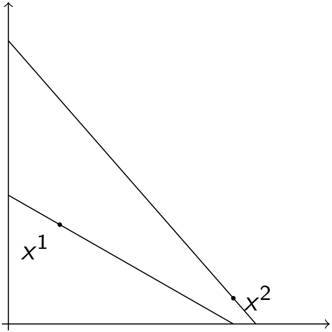
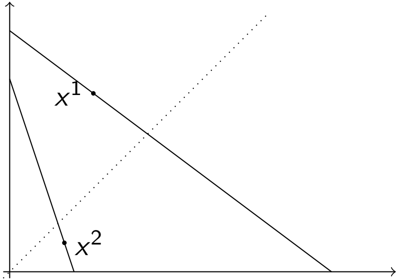


$$\text{MRS} = \frac{u'(x_0)}{\delta u'(x_1)}$$

Axiom 1
Not:



Axiom 2
Not:



END of Warmup

Main theorem(s):
A dataset is M-rationalizable iff it satisfies the "Strong Axiom of M" (SA-M).

M =

- TSU
- Q-Hyperbolic discounting (QHD)
- Exp. discounting (EDU)

Plug

"Savage in the market" Echenique - Saito (2014)

Subjective Expected Utility

$$U(x) = \sum_{s \in S} \mu_s u(x_s)$$

Data

- Time: $\mathcal{T} = \{0, 1, \ldots, T\}$; so $T + 1$ periods.
- Consumption path $x \in \mathbf{R}^{\mathcal{T}}_{+}$.
- Prices (interest rates): $p \in \mathbf{R}^{\mathcal{T}}_{++}$.

A *dataset* is a collection $\{(x^k, p^k)\}_{k=1}^{K}$, where $x^k \in \mathbf{R}^{\mathcal{T}}_{+}$ is a consumption path and and $p^k \in \mathbf{R}^{\mathcal{T}}_{++}$ a price vector.

Rationalizable data

- Let M be a set of functions $U : \mathbf{R}_{+}^{\mathcal{T}} \rightarrow \mathbf{R}$.

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- $B(p, I) = \{y \in \mathbf{R}^{\mathcal{T}}_{+} | p \cdot y \leq I\}$

Rationalizable data

- Let M be a set of functions $U : \mathbf{R}^{\mathcal{T}}_{+} \rightarrow \mathbf{R}$.
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Definition

Dataset $\{(x^k, p^k)\}_{k=1}^{K}$ is *M-rational* if $\exists$ U in the class M s.t.

$$y \in B(p^k, p^k \cdot x^k) \Rightarrow U(y) \leq U(x^k),$$

Notation

Let

- $\mathcal{C} = \{u : \mathbf{R}_+ \to \mathbf{R} | u \text{ is st. increasing and concave}\}$

Models: M $\in$ {EDU,QHD,TSU}

1. EDU: set of U s.t

$$U(x_0,\dots,x_T)=\sum_{t\in\mathcal{T}}\delta^t u(x_t),$$

   for some $u\in\mathcal{C}$, and $\delta\in(0,1]$.

2. QHD: set of U s.t

$$U(x_0,\dots,x_T)=u(x_0)+\beta\sum_{t=1}^{T}\delta^t u(x_t),$$

   for some $u\in\mathcal{C}$, $\beta>0$ and $\delta\in(0,1]$

3. TSU: set of U s.t

$$U(x_0,\dots,x_T)=\sum_{t\in\mathcal{T}}u_t(x_t),$$

   for some $u_t\in\mathcal{C}$, $t\in\mathcal{T}$.

Main result: EDU

Derivation of SA-EDU.

$K = 1$ and $\delta = 1$.

Derivation of SA-EDU

- $K = 1$
- $\delta = 1$ (fixed and known)
- u differentiable.

$K = 1$ and $\delta = 1$.

$$\max_{x \in \mathbf{R}_+^{\mathcal{T}}} \sum_{t \in \mathcal{T}} u(x_t)$$
$$\sum_{t \in \mathcal{T}} p_t x_t \leq I$$

FOC:

$$u'(x_t) = \lambda p_t$$

$K = 1$ and $\delta = 1$.

$$u'(x_t) = \lambda p_t$$

So,

$$\frac{u'(x_t)}{u'(x_{t'})} = \frac{p_t}{p_{t'}}$$

$K = 1$ and $\delta = 1$.

$$u'(x_t) = \lambda p_t$$

So,

$$\frac{u'(x_t)}{u'(x_{t'})} = \frac{p_t}{p_{t'}}$$

Axiom (Downward sloping demand):

$$x_t > x_{t'} \Rightarrow \frac{p_t}{p_{t'}} \leq 1$$

$K = 1$ and $\delta = 1$.

Consider two pairs of observations:

$$x_{t_1} > x_{t_2} \Rightarrow \frac{p_{t_1}}{p_{t_2}} \leq 1$$

and

$$x_{t_3} > x_{t_4} \Rightarrow \frac{p_{t_3}}{p_{t_4}} \leq 1$$

Or, when $x_{t_1} > x_{t_2}$ and $x_{t_3} > x_{t_4}$ then

$$\frac{p_{t_1}}{p_{t_2}} \frac{p_{t_3}}{p_{t_4}} \leq 1.$$

$K = 1$ and $\delta = 1$.

A sequence $(x_{t_i}^{k_i}, x_{t_i'}^{k_i'})_{i=1}^n$ has the *downward sloping demand property* if

$$x_{t_i}^{k_i} > x_{t_i'}^{k_i'}, i = 1, \ldots, n \Rightarrow \prod_{i=1}^{n} \frac{p_{t_i}^{k_i}}{p_{t_i'}^{k_i'}} \leq 1.$$

Derive SA-EDU; $K > 1$ and $\delta = 1$.

Now: $K > 1$.

$$u'(x_t^k) = \lambda^k p_t^k$$

So,

$$\frac{u'(x_t^k)}{u'(x_{t'}^{k'})} = \frac{\lambda^k}{\lambda^{k'}} \frac{p_t^k}{p_{t'}^{k'}}$$

Derive SA-EDU; $K > 1$ and $\delta = 1$.

Now: $K > 1$.

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So,

$$\frac{u'(x_t^k)}{u'(x_{t'}^{k'})} = \frac{\cancel{\lambda^k}}{\cancel{\lambda^{k'}}} \frac{p_t^k}{p_{t'}^{k'}}$$

Axiom (Downward sloping demand):

$$x_t^k > x_{t'}^{k'} \text{ and } k = k' \Rightarrow \frac{p_t^k}{p_{t'}^{k'}} \leq 1$$

$$\frac{u'(x^k_{t_1})}{u'(x^{k'}_{t_2})}\frac{u'(x^{k'}_{t_3})}{u'(x^k_{t_4})} = \frac{\lambda^k}{\lambda^{k'}}\frac{\lambda^{k'}}{\lambda^k}\frac{p^k_{t_1}}{p^{k'}_{t_2}}\frac{p^{k'}_{t_3}}{p^{k'}_{t_4}}$$

$$\frac{u'(x_{t_1}^k)}{u'(x_{t_2}^{k'})}\frac{u'(x_{t_3}^{k'})}{u'(x_{t_4}^k)} = \frac{\cancel{\lambda^k}}{\cancel{\lambda^{k'}}}\frac{\cancel{\lambda^{k'}}}{\cancel{\lambda^k}}\frac{p_{t_1}^k}{p_{t_2}^{k'}}\frac{p_{t_3}^{k'}}{p_{t_4}^{k'}}$$

Hence

$$x_{t_1}^k > x_{t_2}^{k'} \text{ and } x_{t_3}^{k'} > x_{t_4}^k \Rightarrow \frac{p_{t_1}^k}{p_{t_2}^{k'}}\frac{p_{t_3}^{k'}}{p_{t_4}^{k'}} \leq 1$$

A sequence $(x_{t_i}^{k_i}, x_{t_i'}^{k_i'})_{i=1}^{n}$ is *balanced* if each k appears as k_i (on the left of the pair) the same number of times it appears as k_i' (on the right).

Axiom (for $\delta = 1$ and $K \geq 1$): Any balanced sequence has the downward sloping demand property.

Derive SA-EDU - general K and δ

Agent solves

$$\begin{array}{ll} \max_{x \in \mathbf{R}_+^{\mathcal{T}}} & \sum_{t \in \mathcal{T}} \delta^t u(x_t) \\ \text{s.t.} & \sum_{t \in \mathcal{T}} p_t x_t \leq I, \end{array}$$

FOC

$$\delta^t u'(x_t) = \lambda p_t,$$

So we need to find δ, u and λ^k s.t

$$\delta^t u'(x_t^k) = \lambda^k p_t^k,$$

for all k

$$\frac{u'(x_{t'}^{k'})}{u'(x_t^k)} = \frac{\delta^t}{\delta^{t'}} \frac{\lambda^{k'} p_{t'}^{k'}}{\lambda^k p_t^k}.$$

Suppose that $x_t^k > x_{t'}^{k'}$. Then:

$$\frac{\delta^t}{\delta^{t'}} \frac{\lambda^{k'} p_{t'}^{k'}}{\lambda^k p_t^k} \leq 1,$$

But δ, $\lambda^{k'}$ and λ^k are unobservable.

$$x_{t_1}^{k_1} > x_{t_2}^{k_2} \text{ and } x_{t_3}^{k_2} > x_{t_4}^{k_1}.$$

such that

$$t_1 + t_3 \geq t_2 + t_4.$$

$$\frac{u'(x_{t_1}^{k_1})}{u'(x_{t_2}^{k_2})} \cdot \frac{u'(x_{t_3}^{k_2})}{u'(x_{t_4}^{k_1})} = \left(\frac{\delta^{t_2}}{\delta^{t_1}} \frac{\lambda^{k_1} p_{t_1}^{k_1}}{\lambda^{k_2} p_{t_2}^{k_2}} \right) \cdot \left(\frac{\delta^{t_4}}{\delta^{t_3}} \frac{\lambda^{k_2} p_{t_3}^{k_2}}{\lambda^{k_1} p_{t_4}^{k_1}} \right)$$
$$= \delta^{(t_2+t_4)-(t_1+t_3)} \frac{p_{t_1}^{k_1}}{p_{t_2}^{k_2}} \frac{p_{t_3}^{k_2}}{p_{t_4}^{k_1}}$$

$$x_{t_1}^{k_1} > x_{t_2}^{k_2} \text{ and } x_{t_3}^{k_2} > x_{t_4}^{k_1}.$$

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$$\frac{u'(x_{t_1}^{k_1})}{u'(x_{t_2}^{k_2})} \cdot \frac{u'(x_{t_3}^{k_2})}{u'(x_{t_4}^{k_1})} = \left(\frac{\delta^{t_2}}{\delta^{t_1}} \frac{\cancel{\lambda^{k_1}} p_{t_1}^{k_1}}{\lambda^{k_2} p_{t_2}^{k_2}}\right) \cdot \left(\frac{\delta^{t_4}}{\delta^{t_3}} \frac{\lambda^{k_2} p_{t_3}^{k_2}}{\cancel{\lambda^{k_1}} p_{t_4}^{k_1}}\right)$$

$$= \delta^{(t_2+t_4)-(t_1+t_3)} \frac{p_{t_1}^{k_1}}{p_{t_2}^{k_2}} \frac{p_{t_3}^{k_2}}{p_{t_4}^{k_1}}$$

$$x_{t_1}^{k_1} > x_{t_2}^{k_2} \text{ and } x_{t_3}^{k_2} > x_{t_4}^{k_1}.$$

such that

$$t_1 + t_3 \geq t_2 + t_4.$$

$$\frac{u'(x_{t_1}^{k_1})}{u'(x_{t_2}^{k_2})} \cdot \frac{u'(x_{t_3}^{k_2})}{u'(x_{t_4}^{k_1})} = \left(\frac{\delta^{t_2}}{\delta^{t_1}} \frac{\cancel{\lambda^{k_1}} p_{t_1}^{k_1}}{\cancel{\lambda^{k_2}} p_{t_2}^{k_2}} \right) \cdot \left(\frac{\delta^{t_4}}{\delta^{t_3}} \frac{\cancel{\lambda^{k_2}} p_{t_3}^{k_2}}{\cancel{\lambda^{k_1}} p_{t_4}^{k_1}} \right)$$

$$= \delta^{(t_2+t_4)-(t_1+t_3)} \frac{p_{t_1}^{k_1}}{p_{t_2}^{k_2}} \frac{p_{t_3}^{k_2}}{p_{t_4}^{k_1}}$$

$$\frac{u'(x^{k_1}_{t_1})}{u'(x^{k_2}_{t_2})} \cdot \frac{u'(x^{k_2}_{t_3})}{u'(x^{k_1}_{t_4})} = \left(\delta^{(t_2+t_4)-(t_1+t_3)} \right) \cdot \left(\frac{p^{k_1}_{t_1}}{p^{k_2}_{t_2}} \cdot \frac{p^{k_2}_{t_3}}{p^{k_1}_{t_4}} \right)$$

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- ≤ 1 by concavity

$$\frac{u'(x_{t_1}^{k_1})}{u'(x_{t_2}^{k_2})} \cdot \frac{u'(x_{t_3}^{k_2})}{u'(x_{t_4}^{k_1})} = \left(\delta^{(t_2+t_4)-(t_1+t_3)}\right) \cdot \left(\frac{p_{t_1}^{k_1}}{p_{t_2}^{k_2}} \cdot \frac{p_{t_3}^{k_2}}{p_{t_4}^{k_1}}\right)$$

- ≤ 1 by concavity
- ≥ 1 as $\delta \in (0, 1]$ and $t_1 + t_3 \geq t_2 + t_4$.

$$\frac{u'(x_{t_1}^{k_1})}{u'(x_{t_2}^{k_2})} \cdot \frac{u'(x_{t_3}^{k_2})}{u'(x_{t_4}^{k_1})} = \left(\delta^{(t_2+t_4)-(t_1+t_3)}\right) \cdot \left(\frac{p_{t_1}^{k_1}}{p_{t_2}^{k_2}} \cdot \frac{p_{t_3}^{k_2}}{p_{t_4}^{k_1}}\right)$$

- ≤ 1 by concavity
- ≥ 1 as $\delta \in (0, 1]$ and $t_1 + t_3 \geq t_2 + t_4$.
- $\Rightarrow \leq 1$

Recall

$(x_{t_i}^{k_i}, x_{t_i'}^{k_i'})_{i=1}^n$ is *balanced* if each k appears as k_i (on the left of the pair) the same number of times it appears as k_i' (on the right).

$(x_{t_i}^{k_i}, x_{t_i'}^{k_i'})_{i=1}^n$ has the *downward sloping demand property* if:

$$x_{t_i}^{k_i} > x_{t_i'}^{k_i'}, i = 1, \ldots, n$$

$$\Rightarrow \prod_{i=1}^{n} \frac{p_{t_i}^{k_i}}{p_{t_i'}^{k_i'}} \leq 1.$$

Exp. discounted utility

St. Axiom of Revealed Exp. Discounted Utility (SA-EDU)

Any balanced sequence $(x_{t_i}^{k_i}, x_{t'_i}^{k'_i})_{i=1}^n$ s.t. $\sum_{i=1}^n t_i \geq \sum_{i=1}^n t'_i$; has the downward sloping demand property.

Theorem

A dataset is EDU rational iff it satisfies SA-EDU.

More general models.

- TSU
- QHD

Recall the 2×2 case

Two possibilities:

$$\frac{u'(x_{t_1}^{k_1})}{u'(x_{t_1}^{k_2})}\frac{u'(x_{t_2}^{k_2})}{u'(x_{t_2}^{k_1})} \leq 1$$

$$\frac{u'(x_{t_1}^{k_1})}{u'(x_{t_2}^{k_1})}\frac{u'(x_{t_2}^{k_2})}{u'(x_{t_1}^{k_2})} \leq 1$$

Recall the 2×2 case

$$\frac{u'(x_{t_1}^{k_1})}{u'(x_{t_1}^{k_2})} \cdot \frac{u'(x_{t_2}^{k_2})}{u'(x_{t_2}^{k_1})} \leq 1$$

Then,

$$\frac{p_{t_1}^{k_1}}{p_{t_2}^{k_2}} \frac{p_{t_3}^{k_2}}{p_{t_4}^{k_1}} \leq 1$$

Recall the 2×2 case

$$\frac{u'_{t_1}(x_{t_1}^{k_1})}{u'_{t_1}(x_{t_1}^{k_2})} \cdot \frac{u'_{t_2}(x_{t_2}^{k_2})}{u'_{t_2}(x_{t_2}^{k_1})} \leq 1$$

Then,

$$\frac{p_{t_1}^{k_1}}{p_{t_2}^{k_2}} \frac{p_{t_3}^{k_2}}{p_{t_4}^{k_1}} \leq 1$$

Time-separable utility

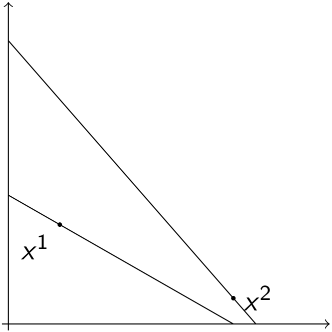
SA-TSU

Any balanced sequence $(x_{t_i}^{k_i}, x_{t_i'}^{k_i'})_{i=1}^n$ *in which each* $t_i = t_i'$ *for all* i *has the downward sloping demand property.*

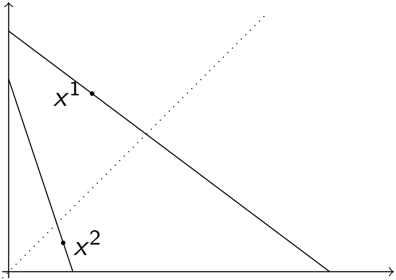
Theorem

A dataset is TSU rational iff it satisfies SA-TSU.

Axiom 1
TSU



Axiom 2



SA-TSU

Any balanced sequence $(x_{t_i}^{k_i}, x_{t'_i}^{k'_i})_{i=1}^n$ *in which each* $t_i = t'_i$ *for all* i *has the downward sloping demand property.*

SA-EDU

Any balanced sequence $(x_{t_i}^{k_i}, x_{t'_i}^{k'_i})_{i=1}^n$ *in which* $\sum_{i=1}^n t_i \geq \sum_{i=1}^n t'_i$ *has the downward sloping demand property.*

SA-TSU

Any balanced sequence $(x_{t_i}^{k_i}, x_{t_i'}^{k_i'})_{i=1}^n$ *in which each* $t_i = t_i'$ *for all* i *has the downward sloping demand property.*

SA-EDU

Any balanced sequence $(x_{t_i}^{k_i}, x_{t_i'}^{k_i'})_{i=1}^n$ *in which* $\sum_{i=1}^n t_i \geq \sum_{i=1}^n t_i'$ *has the downward sloping demand property.*

SA-QHD

Any balanced sequence $(x_{t_i}^{k_i}, x_{t_i'}^{k_i'})_{i=1}^n$ *in which*

1. $\sum_{i=1}^n t_i \geq \sum_{i=1}^n t_i'$;
2.

has the downward sloping demand property.

SA-TSU

Any balanced sequence $(x_{t_i}^{k_i}, x_{t'_i}^{k'_i})_{i=1}^n$ *in which each* $t_i = t'_i$ *for all* i *has the downward sloping demand property.*

SA-EDU

Any balanced sequence $(x_{t_i}^{k_i}, x_{t'_i}^{k'_i})_{i=1}^n$ *in which* $\sum_{i=1}^n t_i \geq \sum_{i=1}^n t'_i$ *has the downward sloping demand property.*

SA-QHD

Any balanced sequence $(x_{t_i}^{k_i}, x_{t'_i}^{k'_i})_{i=1}^n$ *in which*

1. $\sum_{i=1}^n t_i \geq \sum_{i=1}^n t'_i$;
2. $\#\{i : t_i > 0\} = \#\{i : t'_i > 0\}$;

has the downward sloping demand property.

Corners

Proposition

Suppose that a dataset $(x^k, p^k)_{k=1}^K$ *satisfies that* $x_0^k = 0$ *for all* $k \in K$. *Then* $(x^k, p^k)_{k=1}^K$ *is QHD rational iff it is EDU rational.*

Strategy in the proof: need to find δ, u and λ^k s.t

$$\delta^t u'(x_t^k) = \lambda^k p_t^k,$$

linearized:

$$t \log \delta + v_t^k = \log \lambda^k + \log p_t^k,$$

but need $\log p_t^k \in \mathbf{Q}$;
so... approximation result (complicated by lack of compactness).

Discussion

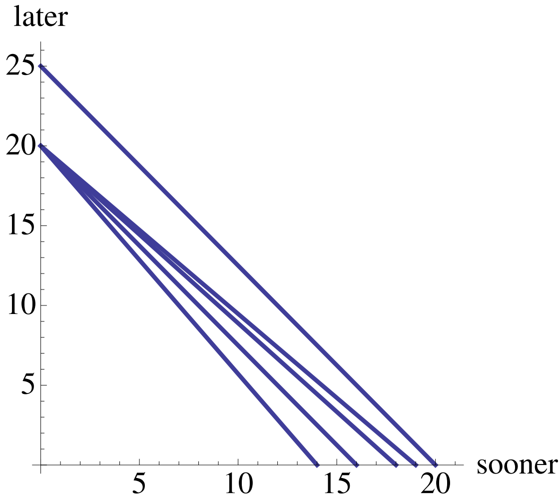
- Contrast Koopmans' axioms and other axiomatizations.
- Previous revealed preference results.
- Experimental literature : Andreoni & Sprenger

Andreoni and Sprenger

Andreoni and Sprenger "Estimating Time Preferences from Convex Budgets" (AER 2012).
Fits our framework:

- "Economic" budget sets;
- Planned (not realized) consumption.
- A-S minimize role of uncertainty.

Experimental design



Andreoni and Sprenger

- Parametric estimation of CRRA demand functions.
- Find no "hyperbolic" discounting.

$$u(x_0) + \beta \sum_{t=1}^{T} \delta^t u(x_t)$$

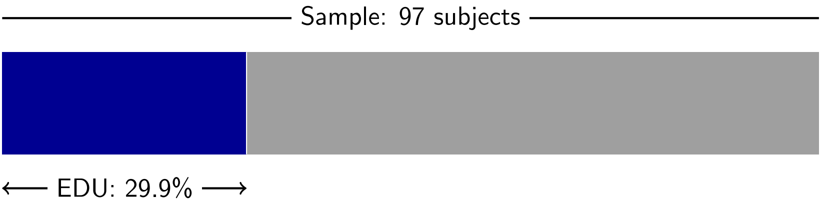
A-S find $\beta \sim 1$.

CTB Experiment

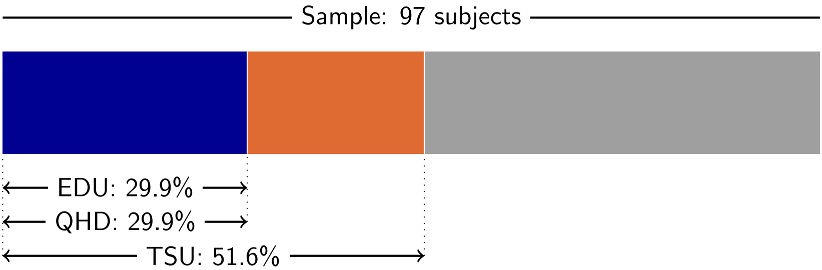
- Details:
  - 97 participants
  - Time (payment dates)
    - "Sooner" $\in \{0, 7, 35\}$ days
    - Delay $\in \{35, 70, 98\}$ days
  - 45 choices (9 pairs of "sooner" and "later" $\times$ 5 budgets)
- Took great care of uncertainty in future payments
  - E.g., participants received authors' business card and are told to call or email if check does not arrive
  - 97% of participants believed that they would get paid

More details

Sample: 97 subjects


Sample: 97 subjects
EDU: 29.9%


Sample: 97 subjects
EDU: 29.9%
QHD: 29.9%


Sample: 97 subjects
EDU: 29.9%
QHD: 29.9%
TSU: 51.6%

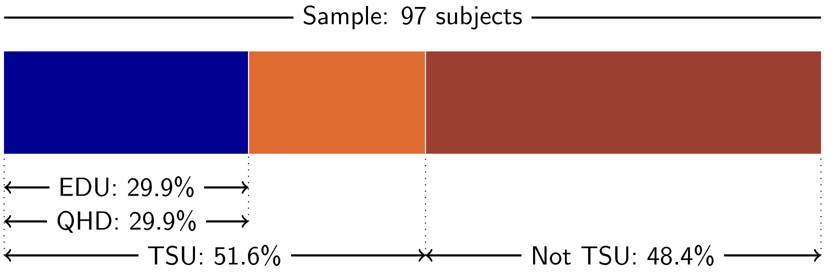
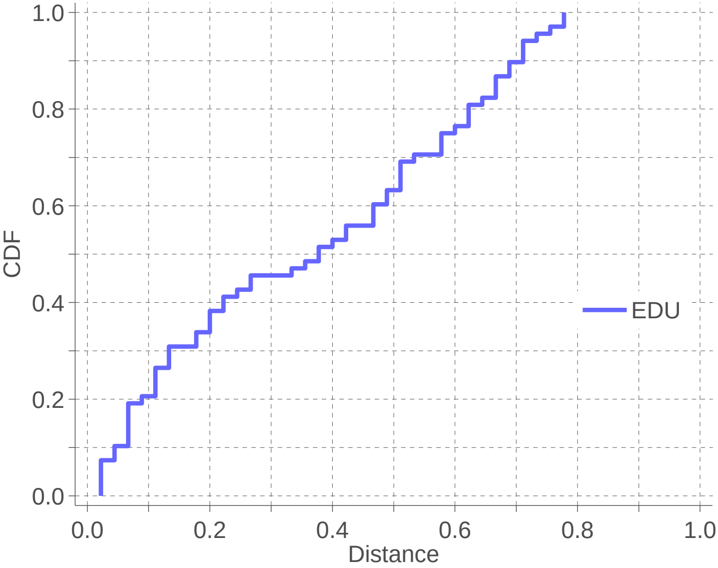

Sample: 97 subjects
EDU: 29.9%
QHD: 29.9%
TSU: 51.6%
Not TSU: 48.4%

Table : Pass rates.

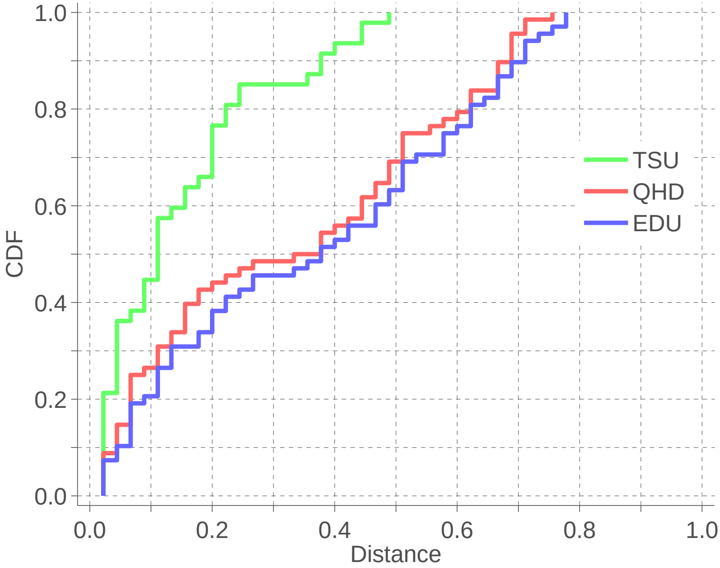
	EDU	QHD	PQHD	MTD	GTD	TSU
Pass rates	29.9%	29.9%	29.9%	39.2%	42.3%	51.6%

Distance to rationalizability.

Distance



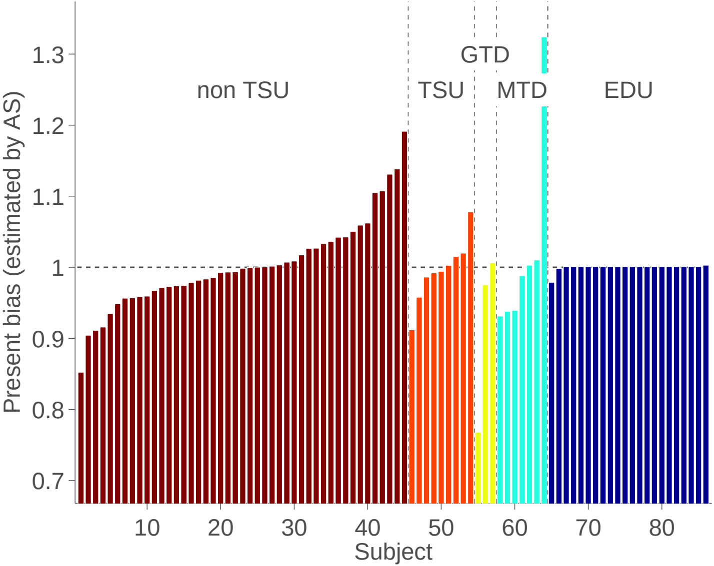
Distance



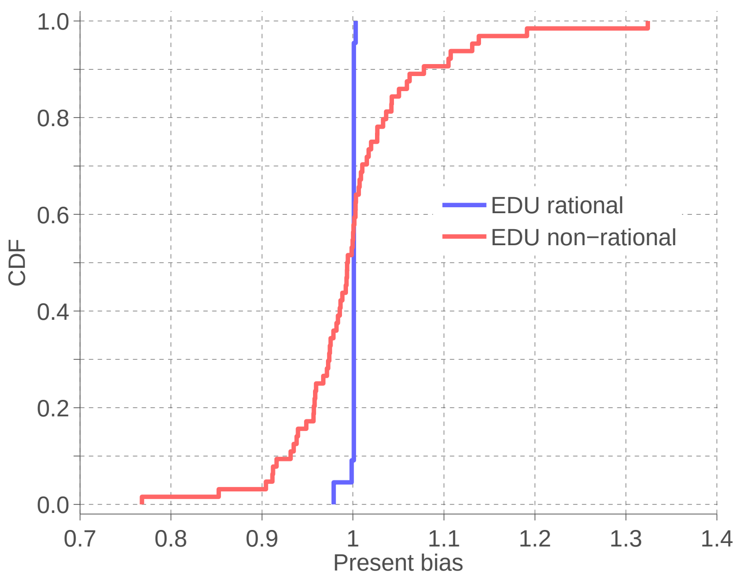
Comparison with A-S

$$u(x_0) + \beta \sum_{t=1}^{T} \delta^t u(x_t)$$

Recall that $\beta \sim 1$.

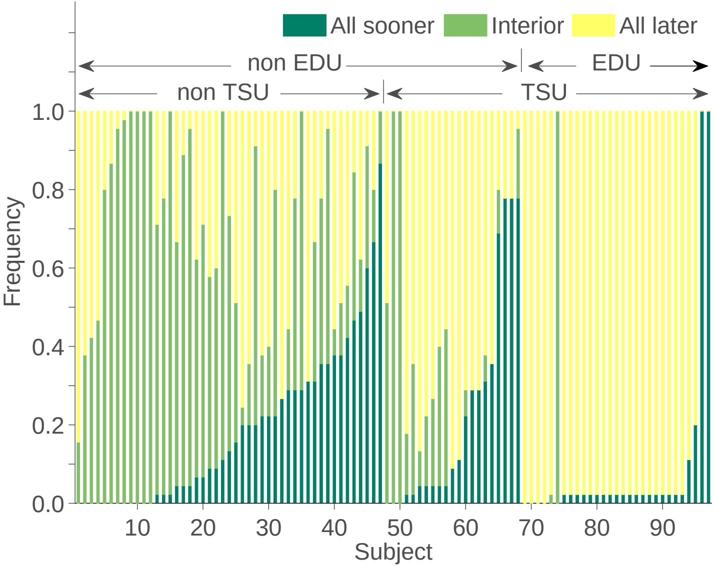

GTD
non TSU
TSU
MTD
EDU
Present bias (estimated by AS)
1.3
1.2
1.1
1
0.9
0.8
0.7
10
20
30
40
50
60
70
80
Subject

Estimated β



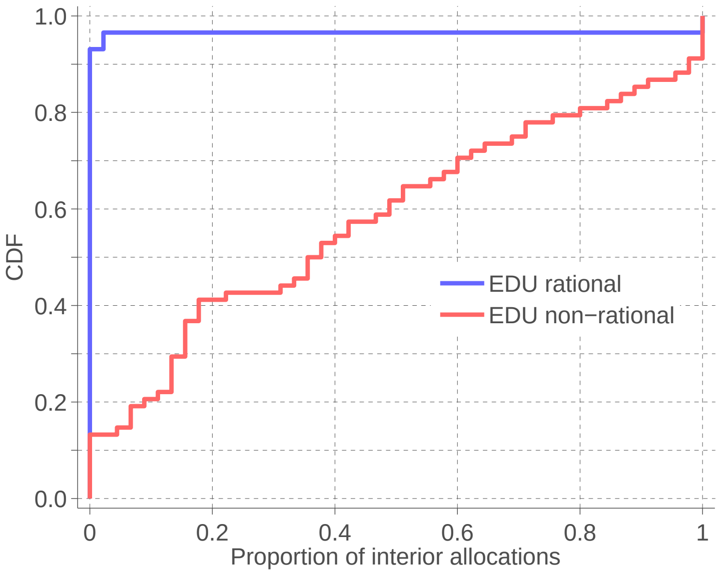
Interior/corner solutions

- All but two of EDU-rational subjects make only corner choices.
- Typically "all later."
- Proposition applies to over 80% of EDU rational subjects.

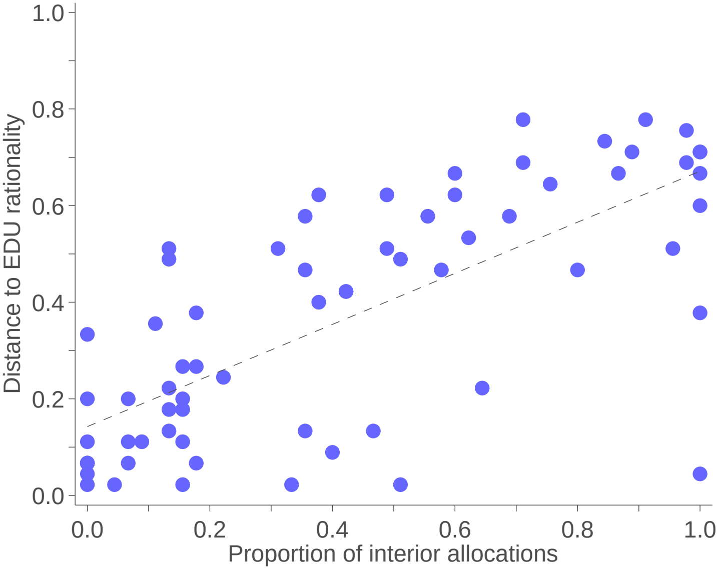


All sooner
Interior
All later
non EDU
EDU
non TSU
TSU
Frequency
0.0
0.2
0.4
0.6
0.8
1.0
10
20
30
40
50
60
70
80
90
Subject

Interior choices



Interior choices



Power against QHD w/linear utility

We generated choices from a fictitious QHD agent.

- with linear utility u;
- either small or large β (severely present or future biased).

Almost all agents always pass the EDU test (only very large or small β trigger a violation of EDU).

TSU $\Rightarrow$ Normal Demand

- 43 out of 47 non-TSU subjects violate normality of demand.

Power

Sampling	EDU	QHD	TSU
Uniform random	0.00	0.00	0.00
Simple Bootstrap	0.00	0.00	0.00

Jittering

Starting from AS estimated

$$U(x_0, \ldots, x_T) = \frac{1}{\alpha}x_0^\alpha + \beta \sum_{t=1}^{T} \delta^t \frac{1}{\alpha}x_t^\alpha \tag{1}$$

with $(\alpha, \delta, \beta) = (0.897, 0.999, 1.007)$,
and estimated std. devs. we:

- simulate 1,000 "jittered" versions of parameters;
- perform our QHD test;
- observe 100% pass rate.

Conclusions

- First "revealed preference axiomatization" of
  - EDU
  - QHD
  - TSU
- Application to A-S data:
  - 30% of subjects are EDU
  - scope of QHD is not more than EDU

Putting AS Data into Our Framework

- In AS, every choice is about "sooner" and "later"
- $T = 133 = 35$ (latest "sooner") $+ 98$ (largest delay)
- $K = 45$
- For each k, we observe
  - (p_τ, x_τ): price and consumption for "sooner"
  - $(p_{\tau+d}, x_{\tau+d})$: price and consumption for "later"
- For each k,

$$(p(k,t), x(k,t)) = \begin{cases} (p_\tau, x_\tau) & \text{if } t = \tau \\ (p_{\tau+d}, x_{\tau+d}) & \text{if } t = \tau + d \\ (\bar{p}, 0) & \text{if } \textit{otherwise} \end{cases}$$

where $\bar{p}$ is high enough that consumption is 0

Back