Empirical Welfare Economics

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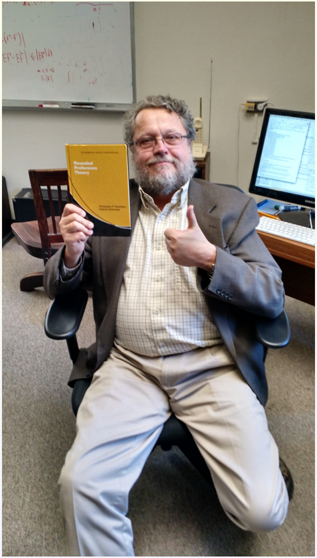
In memory of Kim Border.



Introduction

Our paper:

- Welfare economics
- ...when utilities are unknown
- ...but have data on past choices
- = Empirical Welfare Economics.
- (it is theory, however)

When you face a model, the first thing you should do is understand its Pareto optimal outcomes. Like brushing your teeth: don't think about it; just do it and you'll never regret it.

William Thomson

Overview of results – I

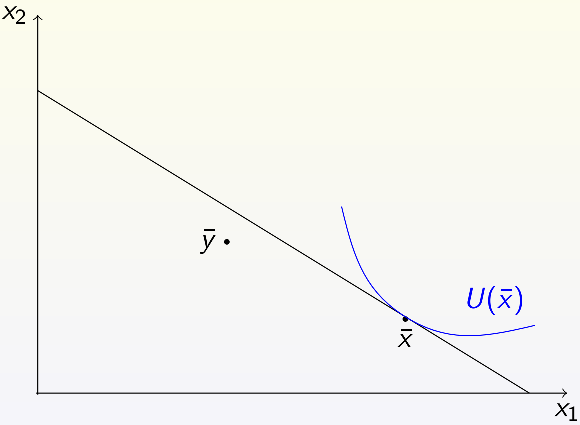
First: individual choice.

Given data on *one* agents' consumption choices.

And given two new, unobserved, bundles, $\bar{x}$ and $\bar{y}$.

Can we infer that the agent would prefer $\bar{x}$ over $\bar{y}$?

When is $\bar{x}$ better than $\bar{y}$?



Overview of results – II

Textbook: Given utility functions, a system of equations characterizes PO allocations, $\mathrm{MRS}_i = \mathrm{MRS}_j$.

But this requires agents' utility functions: We assume utilities are not known.

Instead, have *dataset of choices* made by the agents.

When can we say that an allocation is Pareto optimal for the consumers?

When are there rationalizing utilities for which a proposed allocation is PO?

Reformulate equality of MRS in terms of the revealed preferences defined by consumer data (by way of capturing *existence of common supporting price*).

Overview of results – III

Same methodology yields answers to related questions where we infer preferences, or utility, from data:

Given a proposed aggregate change in the economy, when can winners compensate losers (Kaldor criterion)?

Overview of results – IV

Given data on agents choices, characterize Walrasian equilibrium allocations and prices.
Question related to Brown-Matzkin (1996).

Given an allocation x, are there utilities and prices so that (x, p) constitute a competitive eqm?

Given prices p, are there utilities and allocation so that (x, p) constitutes a competitive eqm?

Definitions

$f : A \subseteq \mathbb{R}^n \to \mathbb{R}$ is *weakly monotone increasing* if $f(x) \leq f(y)$ when $x \leq y$; and *monotone increasing*, if it is weakly monotone increasing and $f(x) < f(y)$ when $x \ll y$.

$u : \mathbb{R}^n_+ \to \mathbb{R}$ is *concave* if, $\forall\ x, y \in \mathbb{R}^n_+$ and $\lambda \in (0, 1)$,

$$u(\lambda x + (1 - \lambda)y) \geq \lambda u(x) + (1 - \lambda)u(y);$$

and *quasiconcave* if, $\forall\ x, y \in \mathbb{R}^n_+$ and $\lambda \in (0, 1)$,

$$u(\lambda x + (1 - \lambda)y) \geq \min\{u(x), u(y)\}.$$

u is *explicitly quasiconcave* if it is quasiconcave and, $\forall\ x, y \in \mathbb{R}^n_+$ and $\lambda \in (0, 1)$, $u(x) \neq u(y)$ implies that

$$u(\lambda x + (1 - \lambda)y) > \min\{u(x), u(y)\}.$$

Model

A pair $(p, x) \in \mathbb{R}^m_{++} \times \mathbb{R}^m_{+}$ is an *observation.*

A finite list of observations $\{(p^k, x^k)\}_{k=1}^K$ is termed an *individual dataset*.

N a finite set of agents.

A *group dataset* is a collection of individual datasets, one for each $i \in N$.

So $\{(p_i^k, x_i^k)\}_{k=1}^{K_i}$ denotes the individual dataset for individual $i \in N$.

Rational data

An individual dataset is *rationalizable* if there is an increasing utility function $u_i : \mathbf{R}^m_+ \to \mathbf{R}$ s.t

$$u_i(x) > u_i(x_i^k) \Longrightarrow p_i^k \cdot x > p_i^k \cdot x_i^k$$

We say that u_i *rationalizes* the individual dataset.

A group dataset is rationalizable if each individual dataset is rationalizable.

Revealed preferences

$x \succeq_i^R y$ if either

- $x \geq x_i^k$ for some k and $p_i^k \cdot x_i^k \geq p_i^k \cdot y$
- $x = y$

$x \succ_i^R y$ if $x = x_i^k$ and $p_i^k \cdot x_i^k > p_i^k \cdot y$, or $x > x_i^k \succeq_i^R y$, for some k.

indirect revealed preference $\succeq_i^I$ is the transitive closure of $\succeq_i^R$.

indirect revealed strict preference $x \succ_i^I y$ when there's a chain $x = z_1 \succeq_i^R \ldots \succeq_i^R z_L = y$, with at least one instance of $\succeq_i^R$ is $\succ_i^R$.

Dataset satisfies *GARP* when there's no $x \succeq_i^I y$ and $y \succ_i^I x$.

Afriat's theorem

Given an individual dataset $\{(x^k, p^k) : 1 \leq k \leq K\}$ the following statements are equivalent:

1. The dataset is rationalizable.
2. The dataset satisfies GARP
3. There are numbers $\lambda^k > 0$ and U^k that solve the linear inqualities

$$U^\ell \leq U^k + \lambda^k p^k \cdot (x^\ell - x^k)$$

4. There is a strictly monotone increasing and concave rationalization.

Note: there's always a concave rationalization.

One consumer

First, welfare comparisons of a single consumer: Alice.

We have observed her past choices: $\{(x^k, p^k) : 1 \leq k \leq K\}$.

Now we have to choose for her between $\bar{x}$ and $\bar{y}$.

Alice

Given a dataset $\{(x^k, p^k) : 1 \leq k \leq K\}$ and two bundles $\bar{x}$ and $\bar{y}$.

When can we say that $u(\bar{x}) > u(\bar{y})$ for all monotone and concave u that rationalize the data?

Varian '82 gave an answer in the form of an LP. We'll get a condition in terms of the revealed preference relation (actually using his LP to get it).

Alice

First, revealed preference. Maybe $\bar{x}$ was chosen at some observation, $\bar{x} = x^k$, when $\bar{y}$ was affordable.

Second, transitivity: $\bar{x}$ revealed prefererred to x^k, which is revealed preferred to $\bar{y}$...
Third, monotonicity. For example $\bar{x} > x^k$, which is revealed preferred to $\bar{y}$...

Fourth, we can combine monotonicity and transitivity...

Fifth, use convexity: Suppose $\bar{x} = \sum_j \lambda_j z_j$, a convex combination, and, by the previous criteria, each z_j is better than $\bar{y}$.

Sixth, combine convexity, monotonicity, and transitivity...

Alice

Given a dataset $\{(x^k, p^k) : 1 \leq k \leq K\}$ and two unobserved bundles $\bar{x}$ and $\bar{y}$, say that $\bar{x}$ *bests* $\bar{y}$ if $\bar{x}$ can be written as a convex combination of bundles z^ℓ, where for each ℓ

$$z^\ell \succeq^I \bar{x} \text{ or } z^\ell \succeq^I \bar{y},$$

and at least one occurrence of the latter.

Say that $\bar{x}$ *strictly bests* $\bar{y}$ if it weakly bests it, and one of the inequalities is strict ($\succ^I$ for $\succeq^I$).

Alice

If $\bar{x} = \sum_\ell \lambda_\ell z^\ell$ strictly bests $\bar{y}$, for any concave increasing rationalizing utility:

$$\begin{aligned} u(\bar{x}) &\geq \sum_\ell \lambda_\ell u(z^\ell) \\ &\geq \sum_\ell \lambda_\ell u(x^{k_\ell}) \\ &\geq \alpha u(\bar{x}) + (1-\alpha) u(\bar{y}) \end{aligned}$$

with $\alpha < 1$ and some inequality strict.

So

$$u(\bar{x}) > u(\bar{y}).$$

Alice

Punchline: this is necessary and sufficient.

Theorem

Let $\{(x^k, p^k) : 1 \leq k \leq K\}$ be a a dataset and $\bar{x}, \bar{y} \in \mathrm{R}^m_+$ be two unobserved bundles. Then $u(\bar{x}) > u(\bar{y})$ for all concave and monotone rationalizing u iff $\bar{x}$ strictly bests $\bar{y}$.

So "besting" is the right empirical counterpart to this theoretical welfare comparison.

Introducing domination

One consumer again, Bob.
Similar exercise:

Given individual data $\{(x_i^k, p_i^k)\}$ and an unobserved bundle $\bar{x}$, when can we say that $u_i(\bar{x}) \geq u_i(x_i^k)$ for all k, for *some* rationalizing u_i.

So we want to know if Bob could rank $\bar{x}$ above any of his past choices.

Domination

A bundle y *weakly dominates* $\bar{x}$ if it is a convex combination of a collection z^ℓ of bundles, $1 \leq \ell \leq L$, s.t. $\forall \ell \; z^\ell \succeq^I \bar{x}$.

A bundle y *strictly dominates* $\bar{x}$ for agent i if it weakly dominates it and, moreover, if in the defining convex combination there is ℓ with $z^\ell \succ^I \bar{x}$.

Domination

Let $\{(x^k, p^k) : 1 \leq k \leq K\}$ be a dataset and $\bar{x} \in \mathrm{R}^m_+$ an arbitrary bundle.

Theorem

There exists a rationalizing utility for which $u(\bar{x}) \geq \max\{u(x^k) : 1 \leq k \leq K\}$ iff once we add $\bar{x} \succeq^R x^k$ for all k to the revealed preference relation, as well as any revealed preference comparisons required by the sign of $p^k \cdot (x^k - \bar{x})$, we have

- GARP is satisfied.
- There is no bundle $\bar{y} \leq \bar{x}$ that strictly dominates $\bar{x}$.

Multiple agents

Now we go beyond the single-agent exercise.

Alice, Bob, Carol, David, . . .

An *allocation* is a vector $x \in \mathbf{R}_+^{N \times m}$.

Pareto optimality

Let $x = (x_i)_{i \in N}$ and $y = (y_i)_{i \in N}$ be two allocations.

Given $(u_i)_{i \in N}$ mon. inc.
y *Pareto dominates* x
if

- $\sum_i y_i \leq \sum_i x_i$,
- $u_i(x_i) \leq u_i(y_i)$ for all i,
- and $u_i(x_i) < u_i(y_i)$ for (at least) one i

Given a group dataset.
y *empirically dominates* x
if

- $\sum_i y_i \leq \sum_i x_i$,
- y_i weakly dominates x_i for all i,
- and st. dom. it for (at least) one i.

Pareto optimality

Let $x = (x_i)_{i \in N}$ be an allocation.

Given $(u_i)_{i \in N}$ mon. inc.

x is *Pareto optimal* if there is no y that Pareto dominates x.

Given a group dataset.

x is *empirically undominated* if there is no y that empirically dominates x.

Efficiency

Let $\{(p_i^k, x_i^k)\}_{k=1}^{K_i}$ be a rationalizable dataset.

Theorem

Let x be an allocation. The following statements are equivalent:

1. x is not empirically dominated.
2. There are rationalizing, increasing, and explicitly quasi-concave utilities s.t x is Pareto optimal.
3. There are rationalizing, increasing, and concave utilities s.t x is Pareto optimal.

Caveats

Empirical efficiency asks whether a specified allocation could be Pareto efficient for some preferences consistent with the observed data.

So preferences consistent with:
observed choices + transitivity + monotoncity + convexity.

Note two restrictions:

- May not be PO for *arbitrary* selection of rationalizing utilities. Confident when rejected.
- Exercise is restricted to convex preferences.

Kaldor criterion

Given utility functions u_i for agents $i \in N$,

Given an allocation x and a proposed project that would change the aggregate consumption from $X = \sum x_i$ to $Z \in \mathbf{R}^m$.

When can we say if the change is desirable?

Kaldor criterion

Given utility functions u_i for agents $i \in N$,

and an allocation $x = (x_i)$, the *Scitovsky contour* at x is

$$S(x) = \{\sum_i z_i : u_i(z_i) \geq u_i(x_i) \text{ for all } i \in N\}$$

Idea: the economy should move from x to the aggregate bundle Z if $Z \in S(x)$. Winners could compensate losers.

If a price q supports all individual upper contour sets at (x_i) and $q \cdot Z < q \cdot \sum_i x_i$, then $Z \notin S(x)$.

Kaldor criterion

Given utility functions u_i for agents $i \in N$,

An allocation x *weakly Kaldor dominates* an allocation y if $\sum_i y_i \notin S(x)$.

Kaldor criterion

Given is a pair of allocations $\overline{x}$ and $\overline{y}$. Think of $\bar{x}$ as the status quo, and $\bar{y}$ as an alternative.

Corollary

Suppose that for every allocation $\overline{z}$ and every $\theta \geq 0$, if $\sum_i \overline{z}_i \leq \sum_i \overline{x}_i + \theta \left(\sum_i \overline{y}_i - \sum_i \overline{x}_i \right)$, if $\overline{z}_i$ weakly dominates $\overline{x}_i$ for each $i \in N$, then the following two statements are false:

1. $\sum_i \overline{z}_i \ll \sum_i \overline{x}_i + \theta \left(\sum_i \overline{y}_i - \sum_i \overline{x}_i \right)$
2. There is $i \in N$ for which $\overline{z}_i$ strictly dominates $\overline{x}_i$.

Then $\overline{x}$ is possibly efficient and possibly Kaldor dominates $\overline{y}$.

Walrasian eqm

We've looked at when an allocation can be PO.

Similar ideas can answer when an allocation can be a Walrasian eqm.

Suppose that we have access to individual endowments (ω_i), for which $\sum_i \omega_i = \sum_i \bar{x}_i$.

Want to know if there are prices q for which $(\bar{x}, q)$ constitutes a Walrasian equilibrium of the exchange economy defined by the endowments and some rationalizing utilities.

Walrasian eqm

Say that $\bar{y}_i$ *ω_i-dominates* $\bar{x}_i$ if $\bar{y}_i$ is the convex combination of bundles z_i^ℓ where, for each ℓ, either $z_i^\ell = \omega_i$ or $z_i^\ell \succeq_i^I \bar{x}_i$.

$\bar{y}_i$ *strictly ω_i-dominates* $\bar{x}_i$ if $\bar{y}_i$ ω_i-dominates $\bar{x}_i$ and one of the inequalities in the convex combination is strict: so there is ℓ with

$$z_i^\ell \succ_i^I \bar{x}_i.$$

Walrasian eqm

Let $\{(x_i^k, p_i^k) : 1 \leq k \leq K_i\}$, for $i \in N$, be a rationalizable group dataset.

Suppose given endowmens $(\omega_i)_{i \in N}$ and an allocation $(\bar{x}_i)_{i \in N}$ of $(\omega_i)_{i \in N}$.

Theorem

There exists a price vector q, and inc. and concave rationalizing utilities $(u_i)_{i \in N}$ so that $(q, (\bar{x}_i))$ is a Walrasian equilibrium of $(u_i, \omega_i)_{i \in N}$ iff there's no allocation $(\bar{y}_i)_{i \in N}$ of the endowments s.t

1. $\bar{y}_i$ ω_i-dominates $\bar{x}_i$ for all i, and
2. strictly ω_i-dominates it for some i.

Ideas in the proofs of these results.

CALIFORNIA INSTITUTE OF TECHNOLOGY

Division of the Humanities and Social Sciences

Alternative Linear Inequalities

KC Border

October 2007

v. 9–26–08 11:08

Alice again

Theorem

Let $\{(x^k, p^k) : 1 \leq k \leq K\}$ be a a dataset and $\bar{x}, \bar{y} \in \mathrm{R}^m_+$ be two unobserved bundles. Then $u(\bar{x}) > u(\bar{y})$ for all concave and monotone rationalizing u iff $\bar{x}$ strictly bests $\bar{y}$.

An LP

Varian '82:

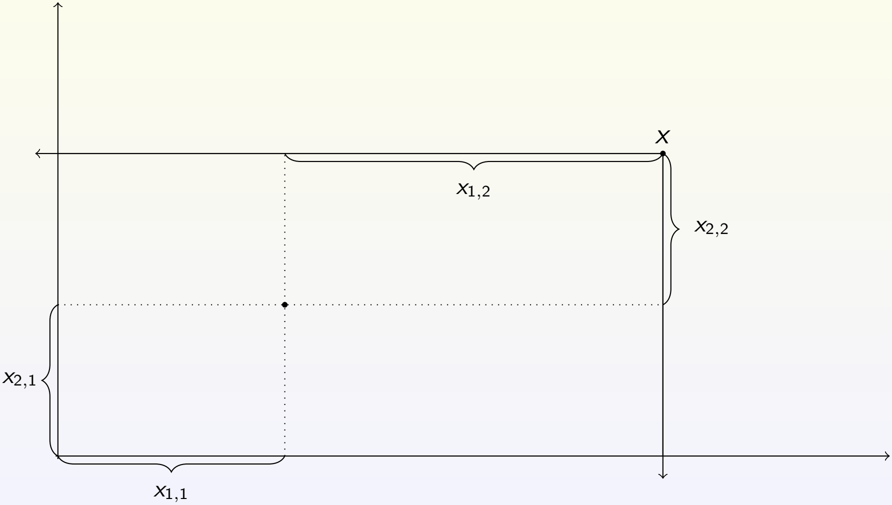
$\bar{y}$ is revealed worse than $\bar{x}$ if and only if there is no solution $q > 0$ to the system of linear inequalities comprised by:

1. $q \cdot \bar{x} \leq q \cdot x^k$ for all k with $x^k \succeq^I \bar{x}$
2. $q \cdot \bar{x} \leq q \cdot x^k$ for all k with $x^k \succeq^I \bar{y}$
3. $q \cdot \bar{x} < q \cdot x^k$ for all k with $x^k \succ^I \bar{x}$
4. $q \cdot \bar{x} < q \cdot x^k$ for all k with $x^k \succ^I \bar{y}$

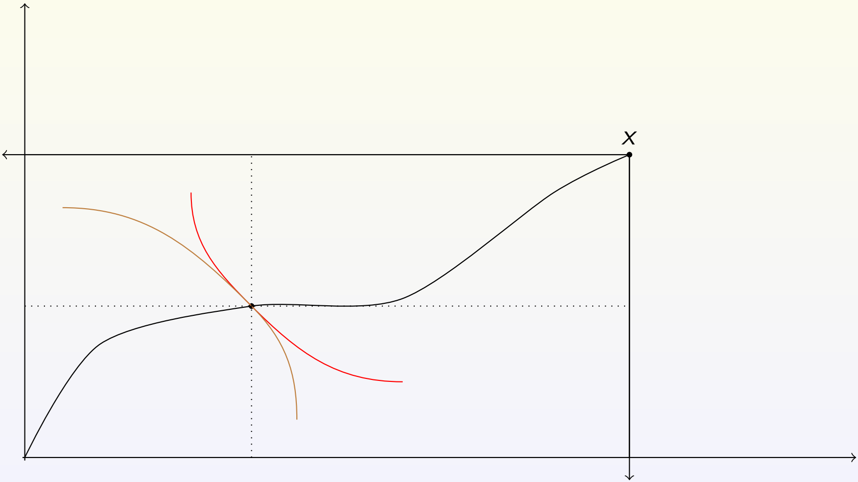
Obs: there's always q solving 1 and 3.

Digression

Efficiency and Scitovsky contours



Efficiency and Scitovsky contours



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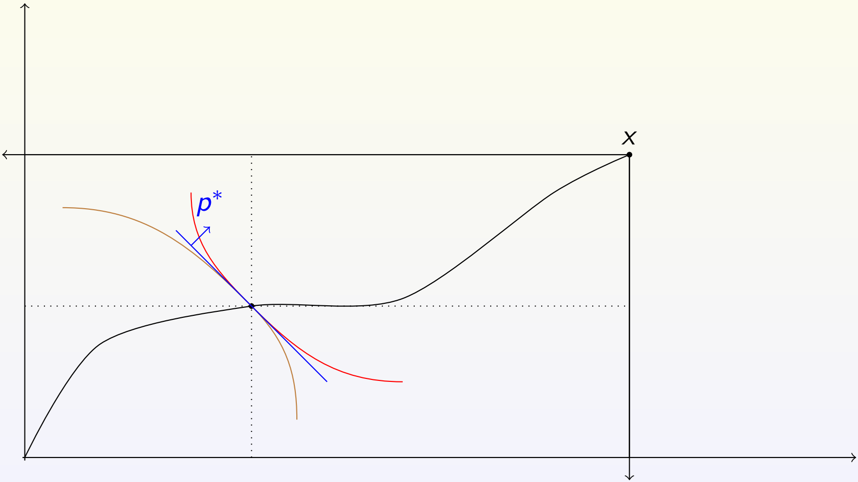

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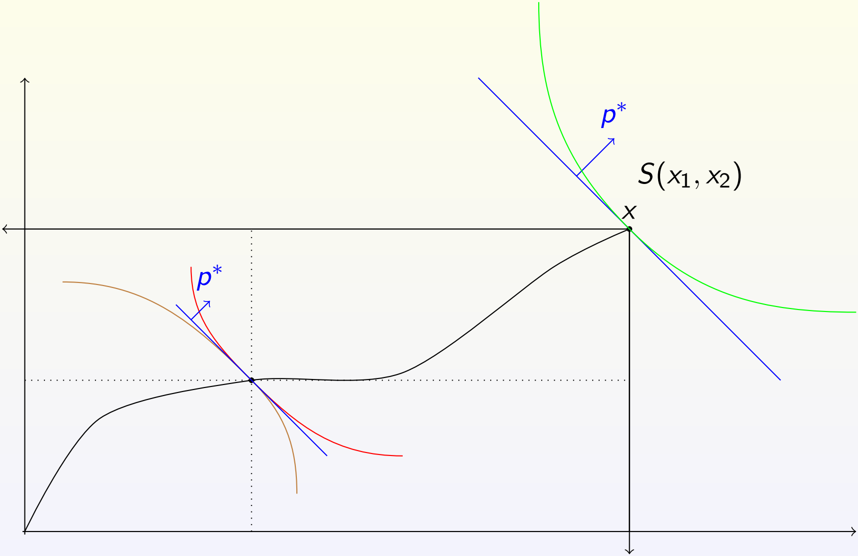

Afriat's theorem

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4. There is a strictly monotone increasing and concave rationalization.

Afriat inequalities

There are numbers $\lambda^k > 0$ and U^k that solve the linear inqualities

$$U^\ell \leq U^k + \lambda^k p^k \cdot (x^\ell - x^k)$$

Can choose one inequality and set $\lambda = 1$.

This allows us to have prices be an unknown.

Then we reason as in the second welfare theorem.

Idea: Afriat inequalities

$$U_i^\ell \leq U_i^k + \lambda_i^k p_i^k \cdot (x_i^\ell - x_i^k)$$

- Unknowns: U_i^k and $\lambda_i^k > 0$
- $1 \leq \ell, k \leq K$

Equivalently:

$$U_i^\ell \leq U_i^k + \lambda_i^k p_i^k \cdot (x_i^\ell - x_i^k)$$

For $1 \leq \ell \leq K$ and $1 \leq k \leq K-1$ and

$$U_i^\ell \leq U_i^K + p_i^K \cdot (x_i^\ell - x_i^K)$$

for $1 \leq \ell \leq K$.

Suggesting

$$U_i^\ell \le U_i^k + \lambda_i^k p_i^k \cdot (x_i^\ell - x_i^k)$$

For $1 \le \ell \le K$ and $1 \le k \le K-1$ and

$$U_i^\ell \le U_i^K + q \cdot (x_i^\ell - x_i^K)$$

for $1 \le \ell \le K$.
Unknowns: U_i^k and $\lambda_i^k > 0$ and $q \in \mathrm{R}_+^m$

Related Literature

- Afriat (1967)
- Counterfactual demand: Varian (1982), Blundell, Browning, and Crawford (2007), Blundell, Browning, and Crawford (2008),Blundell, Browning, Crawford, De Rock, Vermeulen, and Cherchye (2015)
- Equilibrium: Brown and Matzkin (1996),Carvajal, Ray, and Snyder (2004),Bachmann (2004),Bachmann (2006),Bossert and Sprumont (2002),Cherchye et al. (2011b)
- Aggregation: Cherchye, Crawford, De Rock, and Vermeulen (2011a),Cherchye, Crawford, De Rock, and Vermeulen (2016)
- Welfare under incomplete preferences: Carroll (2010)
- Welfare and RP: Brown and Calsamiglia (2007),Cherchye, Demuynck, and De Rock (2011b)

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