Approximate Expected Utility Rationalization

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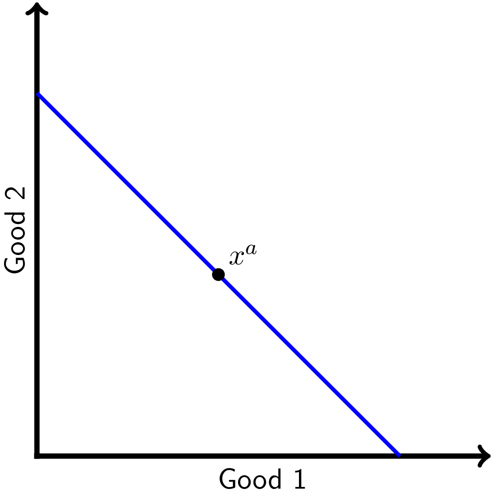
{D,I}ECON, 6 Ago 2019

Revealed Preference Theory

When are agents' choices consistent with utility maximization?

Revealed preference



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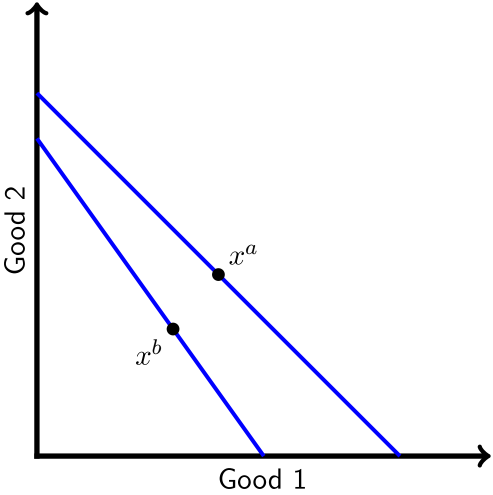

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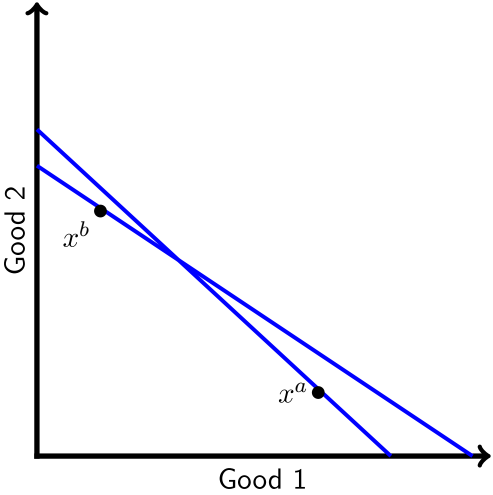

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When are agents' choices consistent with utility maximization?

- Old theory is about *general* utility maximization. Meaning: no structure on the utility function.
- Recent theory is about specific functional forms.

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- Recent theory is about specific functional forms.
- Today: expected utility.
- Our paper: develop practical tools for experiments on choice under risk and uncertainty.

Experiments w/choice from budgets

Andreoni and Sprenger (2012), Andreoni, Kuhn and Sprenger (2015), Ashton (2014), Augenblick, Niederle and Sprenger (2015), Balakrishnan, Haushofer and Jakiela(2015), Barcellos and Carvalho (2014), Brocas, Carrillo and Tarrasó (2016) Carvalho, Prina and Sydnor (2016), Gine, Goldberg, Silverman, and Yang (2017) Janssens Kramer, and Swart (2013,2017) Kuhn Kuhn, and M. C. Villeval (2017) Liu, Meng, and Wang (2014) Lürmann, Serra-Garcia, Winter (2014) Sawada and Kuroishi (2015) Grijalva, Lusk, Rong, and Shaw (2017) Sun and Potters (2016) Andreoni and Harbaugh (2009), Carbone, Dong and Hey (2017), Carvalho, Meier and Wang (2016), Choi, Fisman, Raymond, Gale, and Kariv (2017), Ahn, Choi, Gale, and Kariv (2014), Choi, Kariv, Müller, and Silverman (2014), Carvalho and Silverman (2017), Hey and Pace (2014), Friedman, James, Habib and Crocket (2018), Echenique, Imai and Saito (2019).

Funny

- Many new experiments are on risk and uncertainty.
- But the analysis focuses on general utility max.
- Using Afriat's measure of severity: CCEI
- Given the experiments, it seems natural to ask about exp. utility

Revealed preference

New theory:

- Empirical content of specific utility functions.
- Intertemporal choice (discounted; quasi-hyperbolic utility: see Echenique, Imai and Saito (2017)).
- Risk and uncertainty (EU, max-min, CEU; see Echenique and Saito (2015) and Chambers, Echenique and Saito (2016)).

Theory comes with machinery for measuring and detecting deviations from utility max. (Echenique, Imai and Saito (2017,2018)).

Motivation

- Expected utility

$$U(x) = \sum_{s \in S} \mu_s u(x_s)$$

  - State space S
  - Utility for money u (we assume concave; i.e. risk averse/neutral)
  - Probability μ on S

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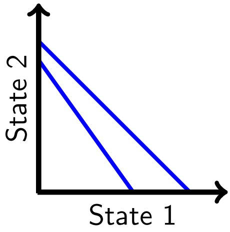
Observable: Market Behavior

$$\max_{x} \sum_{s\in S} \mu_s u(x_s)$$

$$\text{s.t.} \sum_{s\in S} p_s x_s \leq I$$



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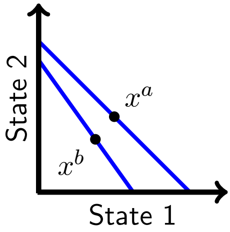

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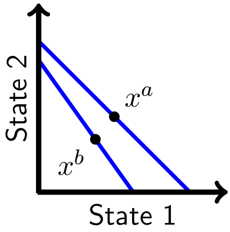
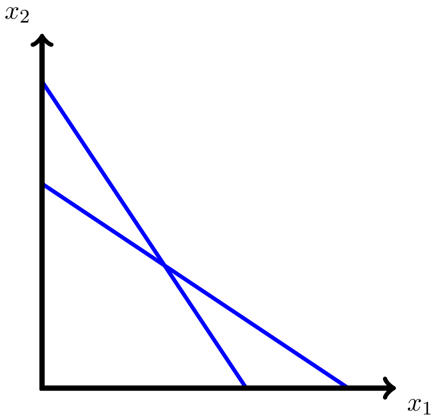


- When are choices from budget sets consistent with EU?
- Can we find μ and u s.t. for all $k \in \{a, b\}$

$$(x_1^k, x_2^k) \in \text{argmax}_{(x_1,x_2)}\, \mu_1 u(x_1) + \mu_2 u(x_2)$$
$$\text{s.t. } p_1^k x_1 + p_2^k x_2 \leq p_1^k x_1^k + p_2^k x_2^k$$

- Answers: Kubler, Selden and Wei (2014), Echenique and Saito (2015)

2 States – 2 Observations



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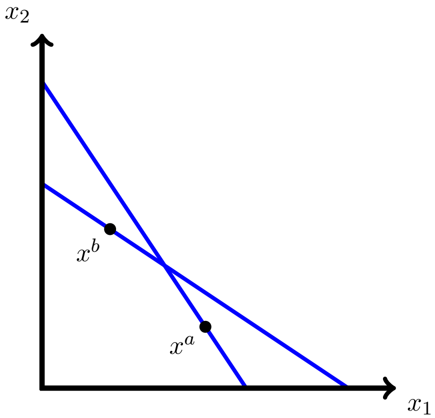

- Violation of WARP

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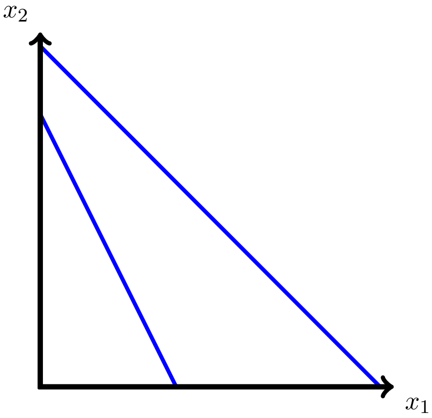

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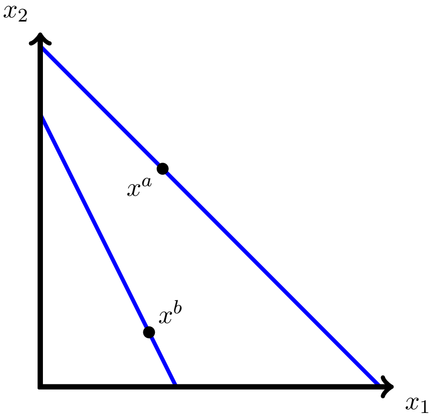

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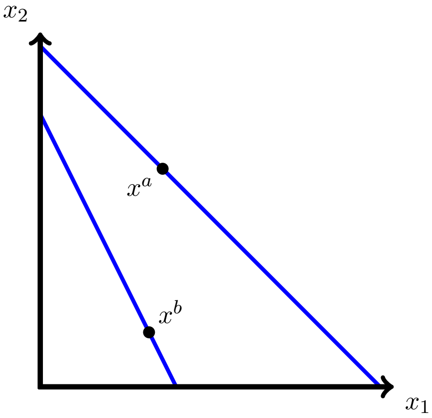

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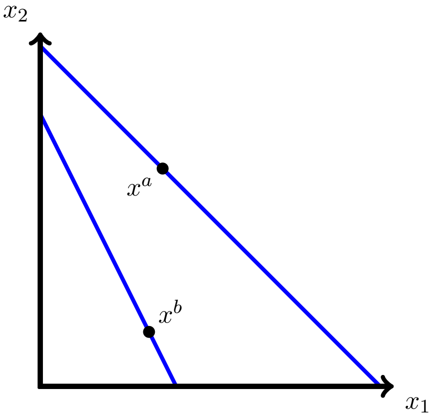

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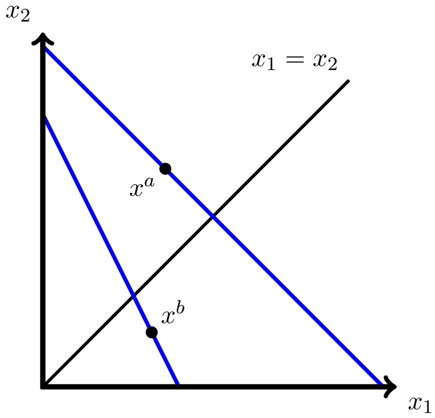

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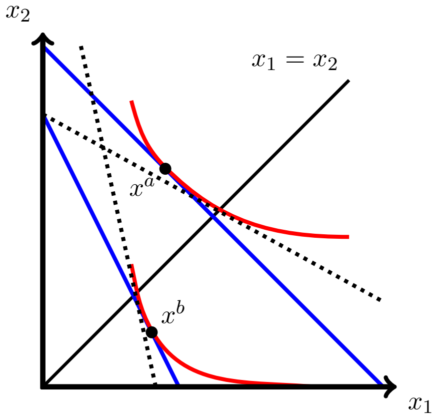

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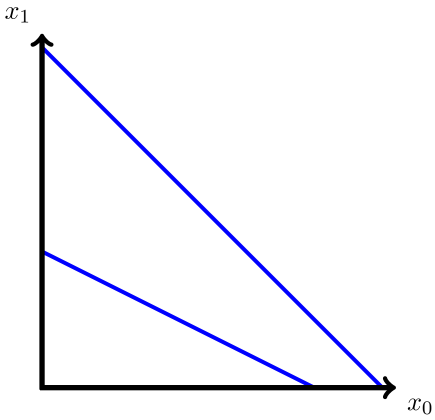
2 Periods – 2 Observations: Case 2



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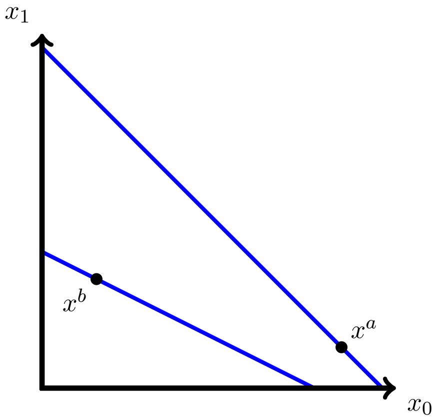

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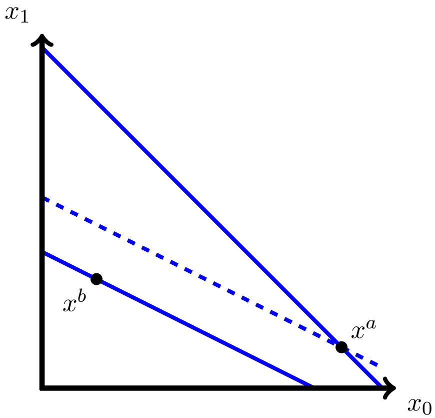

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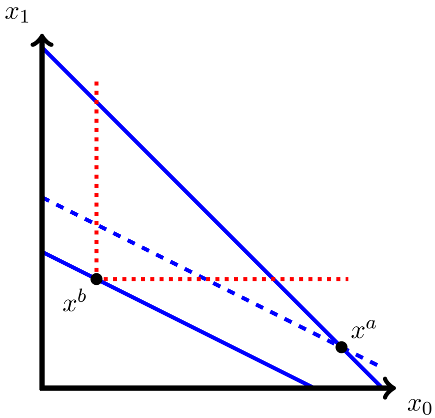

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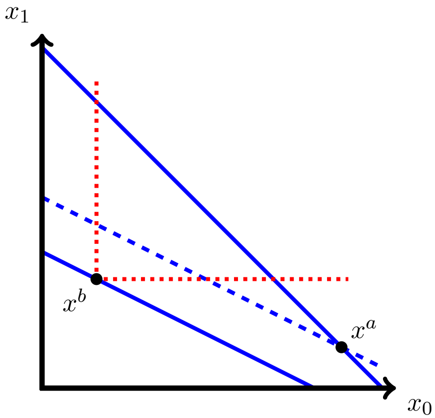
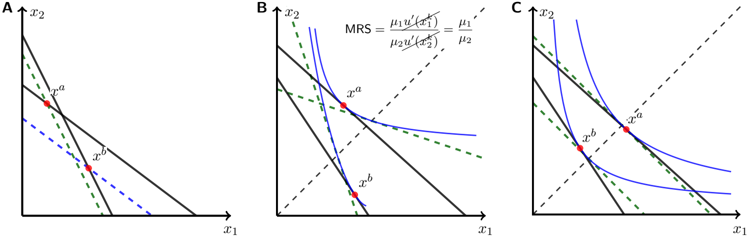


- Violation of normal demand and WARP

EU and CCEI


A
x_2
x^a
x^b
x_1
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This paper

- Propose a measure *“minimal e”* of severity a violation of EU

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- Propose a measure *"minimal e"* of severity a violation of EU
- Applies to objective and subjective EU.
- e-axiom.
- Application to data from three large-scale panel experiments (each over $1,000$ subjects).

This paper: Application

Data from three large-scale panel experiments:

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- Our measure detects these violations of EU, where CCEI does not.

This paper: Application

Data from three large-scale panel experiments:

- confirms that CCEI is not a good indication of compliance with EU.
- Among subjects with high CCEI many subjects make choices that violate monotonicity with respect to FOSD.
- Our measure detects these violations of EU, where CCEI does not.
- Correlation with demographics yields intuitive results.
  - younger subjects, high cognitive abilities, subjects who work, are closer to EU
  - For some of the experiments, we also find that highly educated, high-income, and male subjects, are closer to EU.

This paper

Not rocket science.

$$\frac{\mu_1 u'(x_1)}{\mu_2 u'(x_1)} = \frac{p_1}{p_2}$$

Therefore, **"downward-sloping demand."** Meaning that state-to-state relatively larger prices relate to smaller quantities.

This paper

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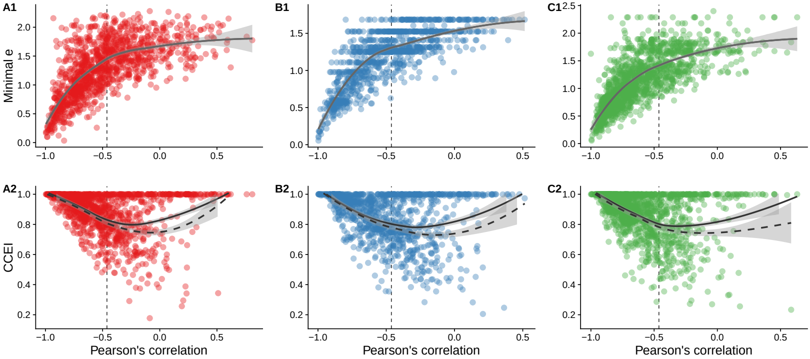

Perturbed OEU

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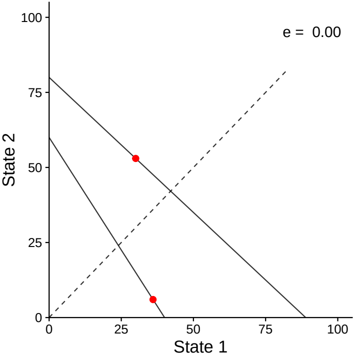

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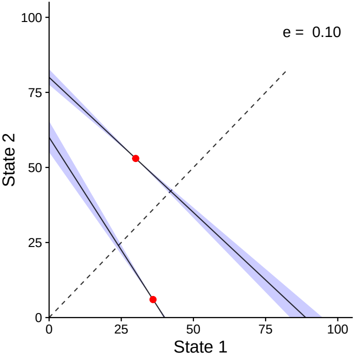

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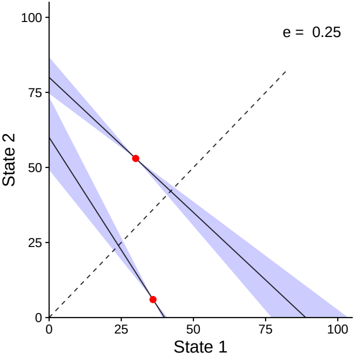

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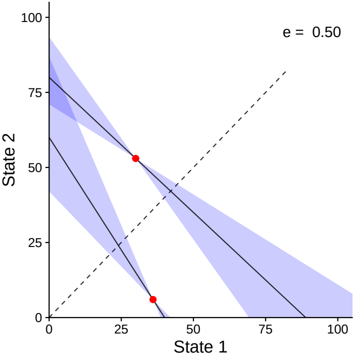

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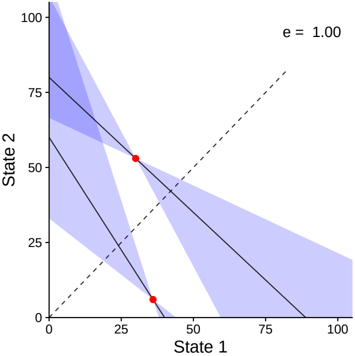

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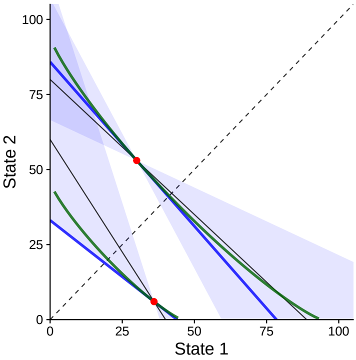

Equivalence

Theorem

Let $e \in \bar{\mathbf{R}}_+$, and D be a dataset. The following statements are equivalent:

- D is e-**belief**-perturbed OEU rational;
- D is e-**utility**-perturbed OEU rational;
- D is e-**price**-perturbed OEU rational.

Intuition: FOC

$$\mu_s^* u'(x_s^k) = \lambda^k p_s^k$$

Risk-neutral prices

Definition

Given data $(p^k, x^k)_{k=1}^K$, the *risk neutral price* $\rho_s^k \in \mathbf{R}_{++}^S$ in choice problem k at state s is

$$\rho_s^k = \frac{p_s^k}{\mu_s^*}.$$

Test sequences

Definition

A sequence of pairs $(x^{k_i}_{s_i}, x^{k'_i}_{s'_i})^n_{i=1}$ is a *test sequence* if

1. $x^{k_i}_{s_i} > x^{k'_i}_{s'_i}$;
2. each k appears as k_i (on the left of the pair) the same number of times it appears as k'_i (on the right).

SAROEU

Axiom (Strong Axiom for Revealed Objective Expected Utility (SAROEU))

For any test sequence $(x_{s_i}^{k_i}, x_{s_i'}^{k_i'})_{i=1}^{n}$,

$$\prod_{i=1}^{n} \frac{\rho_{s_i}^{k_i}}{\rho_{s_i'}^{k_i'}} \leq 1. \quad (1)$$

Characterization

OEU rational is $e = 0$-belief perturbed OEU rational.

Theorem

A dataset is OEU rational iff it satisfies SAROEU

Kubler-Selden-Wei (2014) and Echenique-Saito (2015).

Notation

Consider any sequence $(x^{k_i}_{s_i}, x^{k'_i}_{s'_i})_{i=1}^n$ of pairs. Let $(x^{k_i}_{s_i}, x^{k'_i}_{s'_i})_{i=1}^n \equiv \sigma$.
For any $k \in K$ and$s \in S$,

$$d(\sigma, k, s) = \#\{i \mid x^k_s = x^{k_i}_{s_i}\} - \#\{i \mid x^k_s = x^{k'_i}_{s'_i}\}.$$

and

$$m(\sigma) = \sum_{s \in S} \sum_{k \in K : d(\sigma,k,s) > 0} d(\sigma, k, s).$$

e-PSAROEU

Axiom (e-Perturbed Strong Axiom for Revealed Objective Expected Utility (e-PSAROEU))

For any test sequence of pairs $(x^{k_i}_{s_i}, x^{k'_i}_{s'_i})^n_{i=1} \equiv \sigma$, *we have*

$$\prod_{i=1}^{n} \frac{\rho^{k_i}_{s_i}}{\rho^{k'_i}_{s'_i}} \leq (1+e)^{m(\sigma)}.$$

Characterization

Theorem

Given $e \in \mathbf{R}_+$, and let D be a dataset. The following are equivalent:

- *D is e-belief-perturbed OEU rational.*
- *D satisfies e-PSAROEU.*

How severe is a EU violation?

Our measure: *minimal* e.

How severe is a EU violation?

- Bounds on belief distortions in e-belief-perturbed OEU

$$\frac{1}{1+e} \le \frac{\mu_s^k/\mu_t^k}{\mu_s^*/\mu_t^*} \le 1+e$$

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$e = 0$ $\mu_s^k = \mu_s^*$ for all $s \in S$, $k \in K$

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- What is the **smallest** e that makes non-OEU dataset e-perturbed OEU rational?

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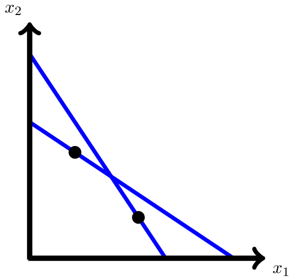
- What is the smallest e that makes non-OEU dataset e-perturbed OEU rational?

Definition

Minimal e, denoted e_*, is the smallest $e \geq 0$ for which the data is e-perturbed OEU rational.

Related measure of rationality

- Critical Cost Efficiency Index (CCEI)
  - how far budget sets would need to be shifted to remove any violations of GARP



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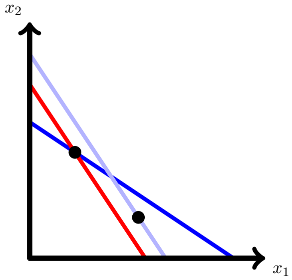
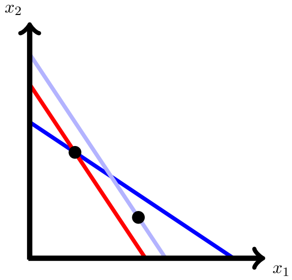


Related measure of rationality

- Critical Cost Efficiency Index (CCEI)
  - how far budget sets would need to be shifted to remove any violations of GARP
- Note:

e_* rotating the budget lines to remove EU-violating obs

CCEI shifting the budget lines to remove GARP-violating obs



Application

Three large-scale panel experiments

Application: Data

- Datasets from three studies:

  CKMS Choi, Kariv, Müller, and Silverman (2014) *AER*

  CMW Carvalho, Meier, and Wang (2016) *AER*

  CS Carvalho and Silverman (2017) *mimeo*

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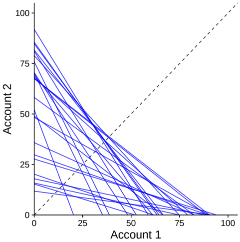

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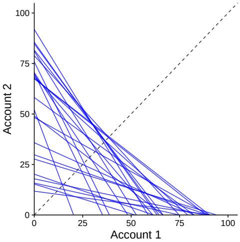
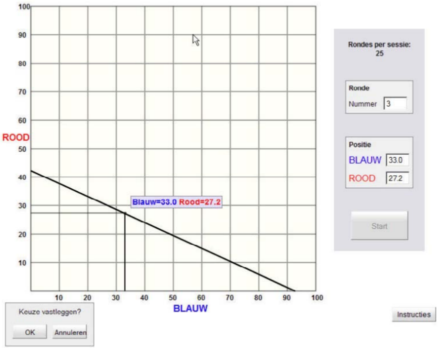


Study	# subjects
CKMS	1,182
CMW	1,119
CS	1,423

Screenshot


100
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ROOD
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90
100
BLAUW
Blauw=33.0 Rood=27.2
Keuze vastleggen?
OK
Annuleren
Rondes per sessie:
25
Ronde
Nummer
3
Positie
BLAUW
33.0
ROOD
27.2
Start
Instructies

Application: Implementation

- Use price-perturbed OEU
  - condition: for all $k \in K$ and $s, t \in S$

$$\frac{1}{1+e} \leq \frac{\varepsilon_s^k}{\varepsilon_t^k} \leq 1 + e$$

- Problem:

$$\min_{(\varepsilon_s^k, v_s^k, \lambda^k)_{s,k}} \max_{k,s,t} \ \varepsilon_s^k / \varepsilon_t^k$$

$$\text{s.t. } \mu_s^* v_s^k = \lambda^k \varepsilon_s^k p_s^k$$

$$x_s^k > x_t^{k'} \ \Rightarrow \ \log v_s^k \leq \log v_t^{k'}$$

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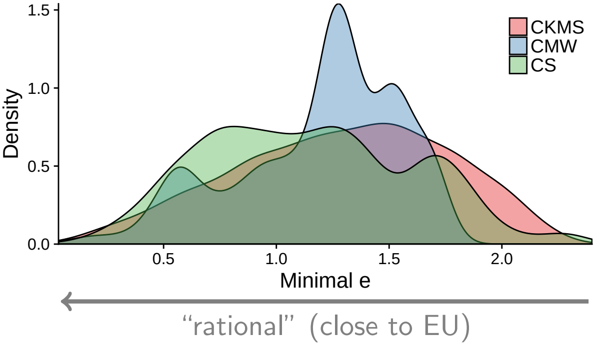
$$\frac{1}{1+e} \leq \frac{\varepsilon_s^k}{\varepsilon_t^k} \leq 1+e$$

- Problem:

$$\begin{aligned} \min_{(v_s^k)_{s,k}} \quad \max_{k,s,t} \; & (\log \mu_s^* + \log v_s^k - \log p_s^k) \\ & - (\log \mu_t^* + \log v_t^k - \log p_t^k) \\ \text{s.t. } & x_s^k > x_t^{k'} \;\Rightarrow\; \log v_s^k \leq \log v_t^{k'} \end{aligned}$$

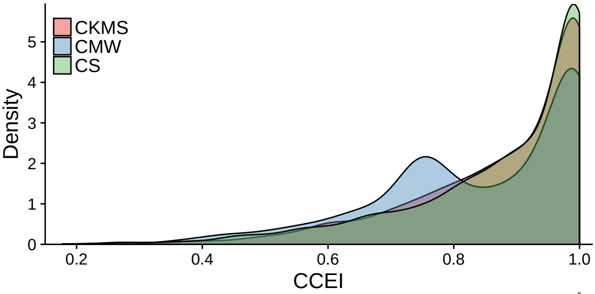
Result: Distribution

- Distribution of e_*



Result: Distribution

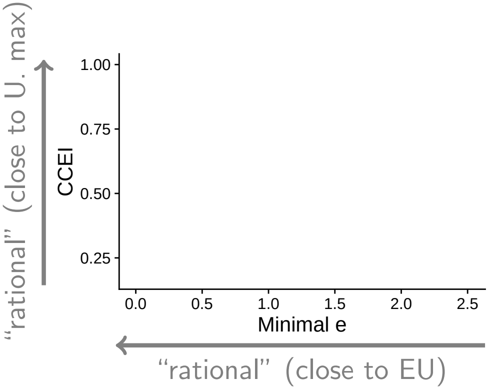
- Distribution of CCEI



"rational" (close to U. max)

Result: Correlation between two measures

- Correlation between e_* and CCEI
  - CKMS dataset



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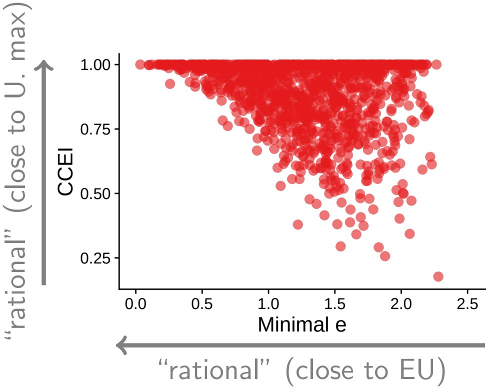

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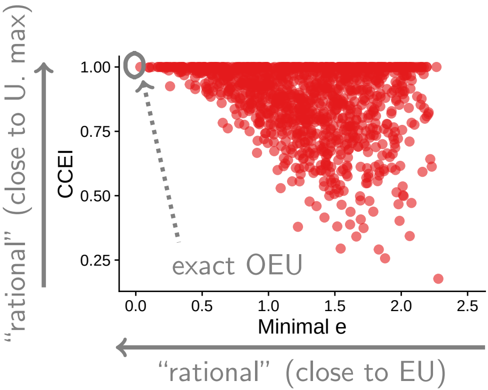

Result: Correlation between two measures

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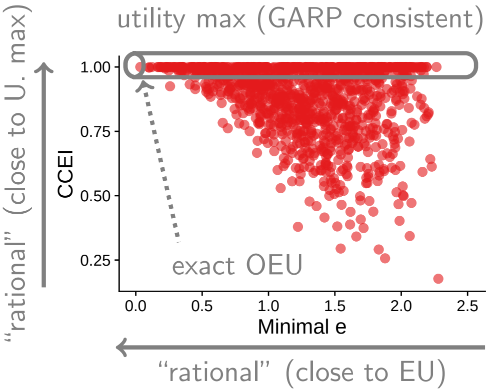
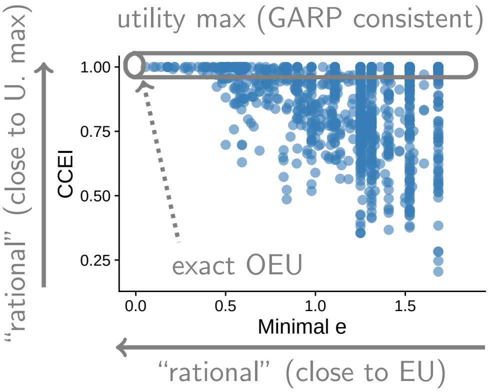


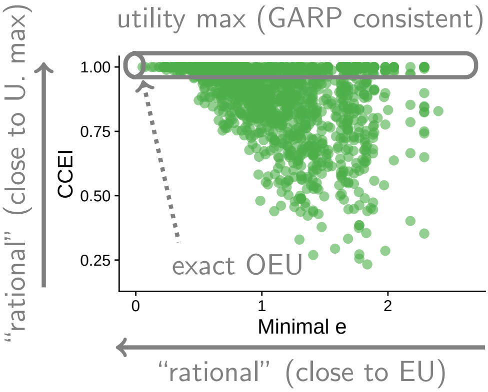
Result: Correlation between two measures

- Correlation between e_* and CCEI
  - CMW dataset



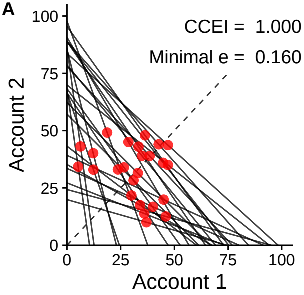
Result: Correlation between two measures

- Correlation between e_* and CCEI
  - CS dataset



Result: Sample choice pattern

- Three examples with CCEI $= 1$



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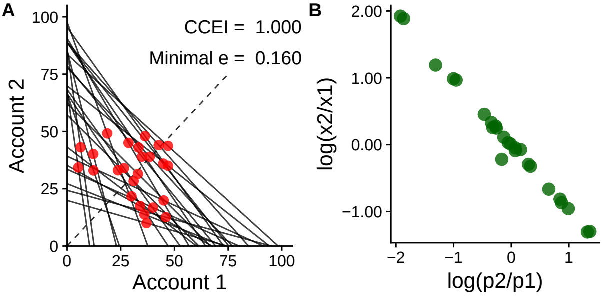

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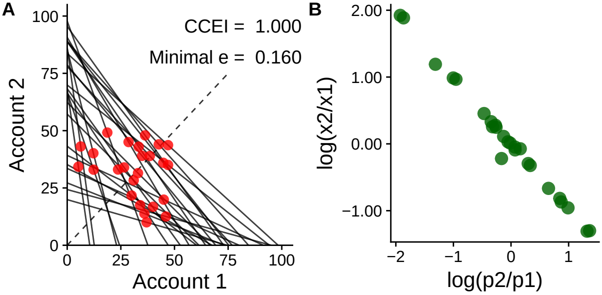
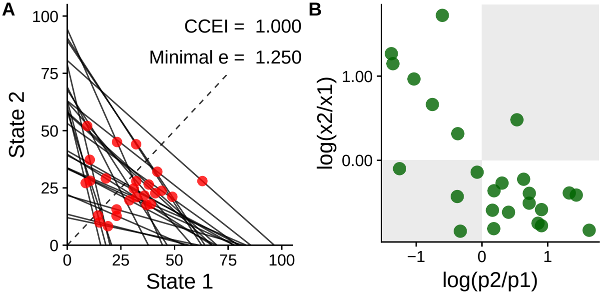


- e_* captures deviation from **downward sloping demand**

Result: Sample choice pattern

- Three examples with CCEI $= 1$



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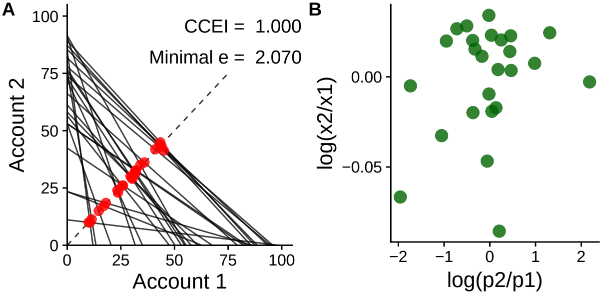

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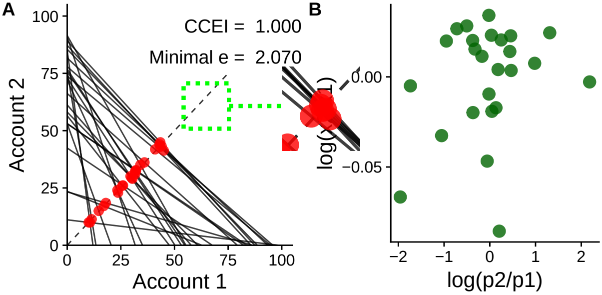

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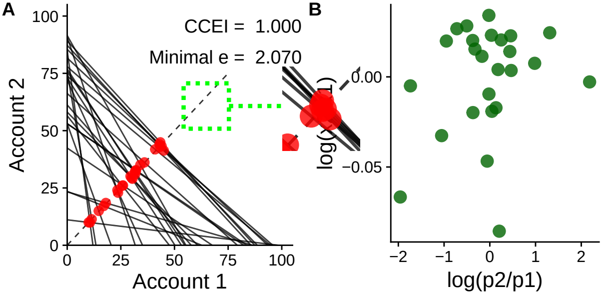
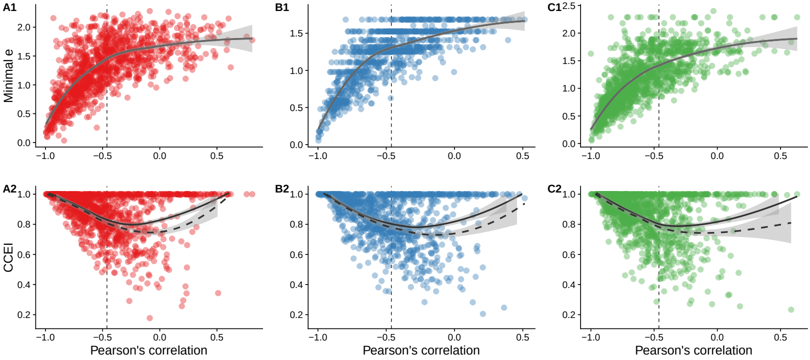


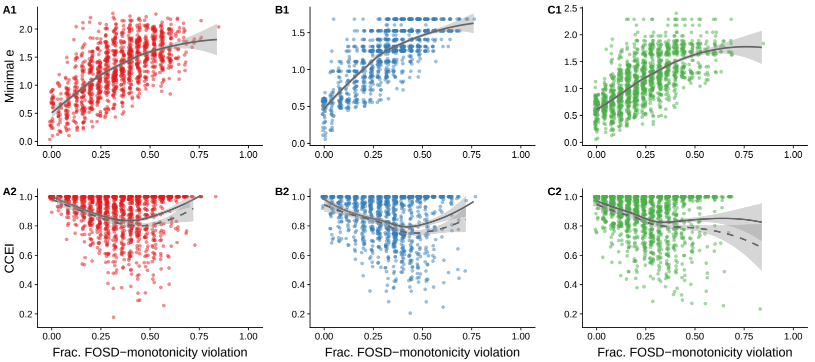
- e_* captures deviation from downward sloping demand
- Look at correlation between $\log(p_2/p_1)$ and $\log(x_2/x_1)$

Result: Downward sloping demand

- Response to prices: **corr**$(\log(p_2/p_1), \log(x_2/x_1))$

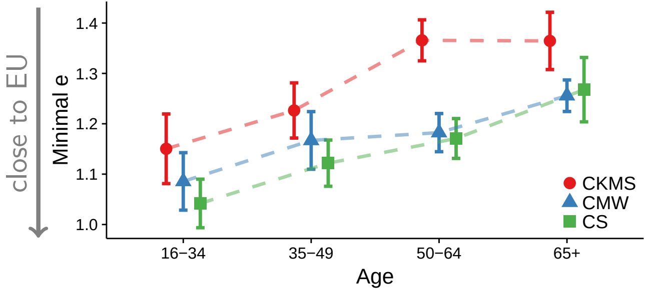


Monotonicity wrt FOSD



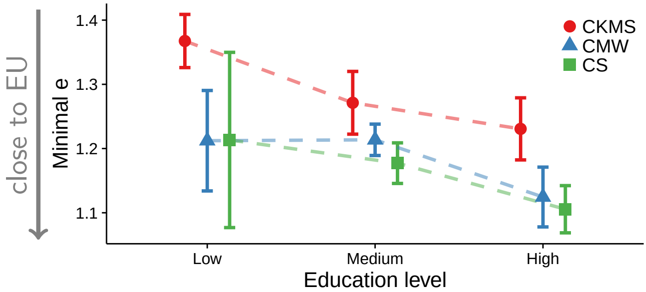
Result: Demographic characteristics

- Age (younger subjects ⇝ closer to EU)



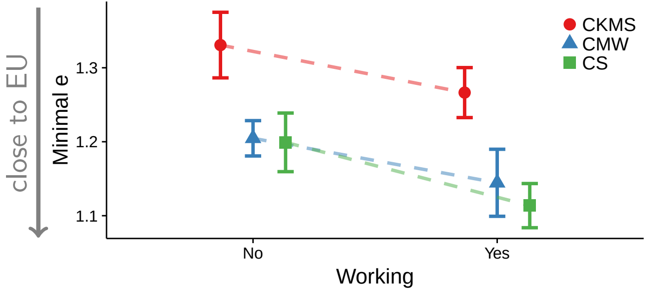
Result: Demographic characteristics

- Education level (higher education $\rightsquigarrow$ closer to EU)



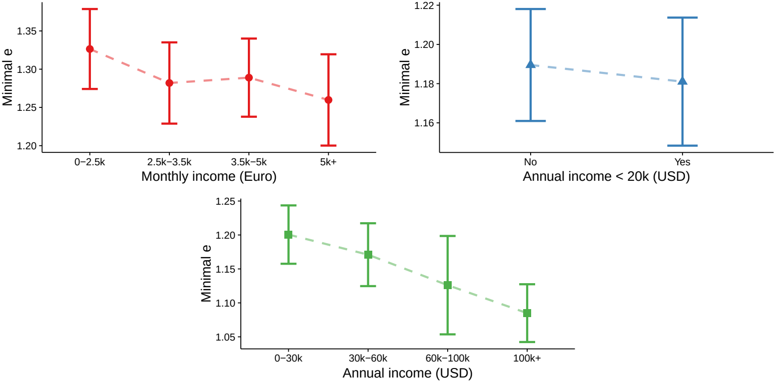
Result: Demographic characteristics

- Occupation (currently working ⇝ closer to EU)



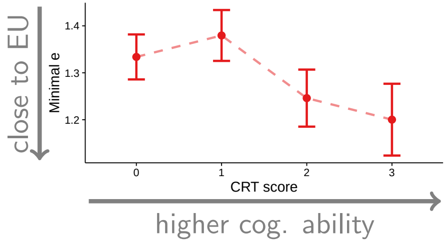
Result: Demographic characteristics

- Income



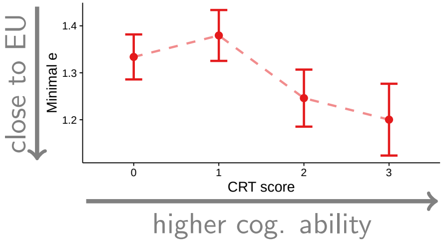
Result: Demographic characteristics

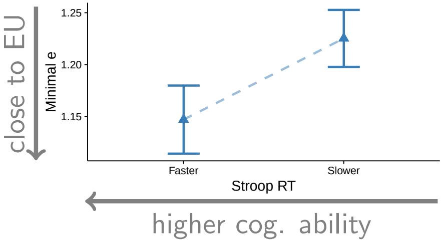
- Cognitive ability (higher c.a. ⇝ closer to EU)
  - Cognitive Reflection Test score



Result: Demographic characteristics

- Cognitive ability (higher c.a. $\rightsquigarrow$ closer to EU)
  - Cognitive Reflection Test score
  - response time in Stroop task





Additional results

- ▶ dropping 1 or 2 “fatal” mistakes does not change results dramatically

Additional results

-
  - dropping 1 or 2 “fatal” mistakes does not change results dramatically
- Understanding the size of e_*
  - comparison with random choice benchmark
  - hypothesis testing

Minimum Perturbation Test

What is the meaning of e_*?

- We provide a statistical test.
- Reject H_0 of EU behavior at given significance.
- Main drawback: one degree of freedom.
- A possible solution based on measurement error.

Minimum Perturbation Test

- $D_{\text{true}} = (p^k, x^k)_{k=1}^K$ a dataset;
- $D_{\text{pert}} = (\tilde{p}^k, x^k)_{k=1}^K$ a "perturbed" dataset.
- Prices $\tilde{p}^k$ are prices p^k measured with error, or misperceived:

$$\tilde{p}_s^k = p_s^k \varepsilon_s^k \text{ for all } s \in S \text{ and } k \in K$$

- $\varepsilon_s^k > 0$ is a random variable.

Minimum Perturbation Test

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- $\varepsilon_s^k > 0$ is a random variable.

If the *variance* of ε is large, it will be easy to accommodate the data.

Minimum Perturbation Test

So one degree of freedom: the variance of ε.

This problem is not new in revealed preference theory (see Varian 1990).

Our solution is to interpret the error as misperception and use the story of misperception to quantify the variance.

Minimum Perturbation Test

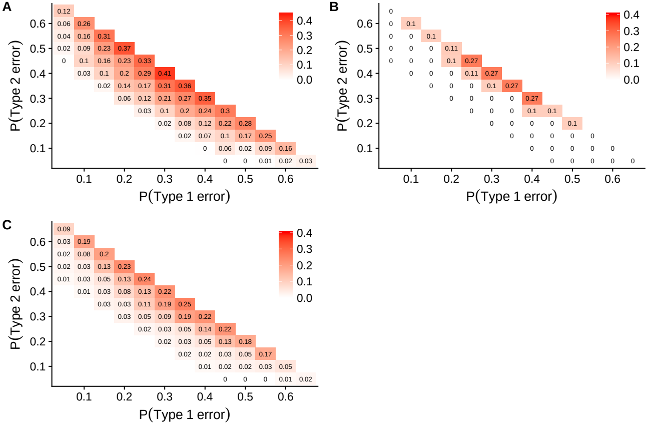
Imagine an agent who conducts a statistical test for the variance of prices.

- True variance of prices is σ_0^2
- Implied variance of $\tilde{p}$ is $\sigma_1^2 > \sigma_0^2$.
- The agent would conduct a test for the null of $\sigma^2 = \sigma_0^2$ against the alternative of $\sigma^2 = \sigma_1^2$.
- We want the variances to be close enough that the agent might reasonably get inconclusive results from such a test.
- *Specifically, we assume the sum of type I and type II errors in this test is relatively large*.

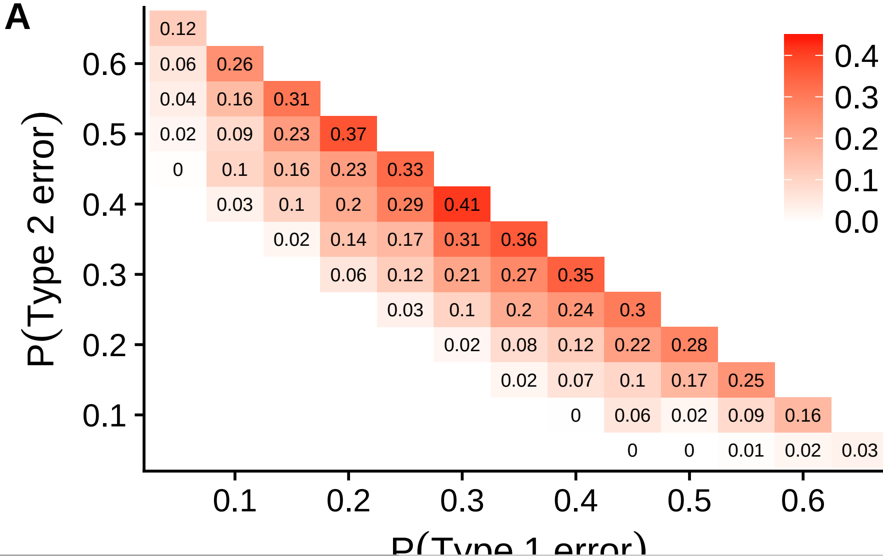
Rejection rates

- For the *largest values* of $\eta^I + \eta^{II}$
- 25% for CS, 27% for CMW, and 41% for CKMS.
- Many subjects in the CS, CMW and CKMS experiments are inconsistent with OEU, but for most subjects the rejections could be attributed to mistakes.

Minimum Perturbation Test



Minimum Perturbation Test



Minimum Perturbation Test

Detailed description of the test.

Minimum Perturbation Test

Given $D_{\text{true}} = (p^k, x^k)_{k=1}^K$ and fixing ε_s^k, let $\mathcal{E}^*$ be the value of:

$$\begin{aligned} \min_{(v_s^k, \lambda^k, \varepsilon_s^k)_{s,k}} \quad & \max_{k \in K, s,t \in S} \frac{\varepsilon_s^k}{\varepsilon_t^k} \\ \text{s.t.} \quad & \log \mu_s^* + \log v_s^k - \log \lambda^k - \log p_s^k - \log \varepsilon_s^k = 0 \\ & x_s^k > x_{s'}^{k'} \Longrightarrow \log v_s^k \leq \log v_{s'}^{k'}. \end{aligned} \tag{2}$$

Minimum Perturbation Test

$$\mathcal{E}^* \left((p^k, x^k)_{k=1}^K \right) \leq \max_{k \in K, s, t \in S} \frac{\varepsilon_s^k}{\varepsilon_s^k} = \widehat{\mathcal{E}}$$

under the null hypothesis.

Minimum Perturbation Test

Construct a test as follows:

$$\begin{cases} \text{reject } H_0 & \text{if } \displaystyle\int_{\mathcal{E}^*\left((p^k,x^k)_{k=1}^K\right)}^{\infty} f_{\widehat{\mathcal{E}}}(z)dz < \alpha \\ \text{accept } H_0 & \text{otherwise} \end{cases},$$

where α is the size of the test and $f_{\widehat{\mathcal{E}}}$ is the density function of the distribution of $\widehat{\mathcal{E}} = \max_{k,s,t} \varepsilon_s^k / \varepsilon_t^k$.

Minimum Perturbation Test

Given nominal size α, find a critical value C_α satisfying

$$\Pr[\widehat{\mathcal{E}} > C_\alpha] = \alpha;$$

Set $C_\alpha = F_{\widehat{\mathcal{E}}}^{-1}(1-\alpha)$, where $F_{\widehat{\mathcal{E}}}$ denotes the cumulative distribution function of $\widehat{\mathcal{E}}$.

Minimum Perturbation Test

However, because $\mathcal{E}^*\left((p^k, x^k)_{k=1}^K\right) \leq \widehat{\mathcal{E}}$, the true size of the test is better than α. Concretely, size $= \Pr[\mathcal{E}^* > C_\alpha] \leq \Pr[\widehat{\mathcal{E}} > C_\alpha] = \alpha$.

What we did

- Introduce and characterize **"perturbed"** EU
- Develop a **measure of consistency** with EU
- Apply the method using data from three large survey experiments

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- Companion paper: Measuring e_* **for SEU** using choice data from the lab and the large-scale panel survey