The Design of Corporate Debt Structure and Bankruptcy

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Abstract

This paper integrates the problem of designing corporate bankruptcy rules into a theory of optimal debt structure. We show that, in an optimal contracting framework with imperfect renegotiation, having multiple creditors increases a firm's debt capacity while increasing its incentives to default strategically. The optimal debt contract gives creditors claims that are jointly inconsistent in case of default. Bankruptcy rules, therefore, are a necessary part of the overall financing contract, to make claims consistent and to prevent a value reducing run for the assets of the firm. We characterize these rules, with predictions about the allocation of security rights, seniority, the right to trigger bankruptcy and the symmetry of treatment of creditors in bankruptcy.

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JEL Classification: G3, K2.

Bankruptcy law regulates the interaction between debtors and creditors when debtors default and the parties cannot work out their differences outside the courts. The law addresses two main types of conflicts: conflicts between a debtor and her creditors, and conflicts among creditors themselves. Empirically, this latter type of conflict is the major source of complexity in modern bankruptcy law, and has therefore given rise to a substantial literature, much of which in law. This literature typically takes an ex-post perspective: how to sort out claims once the firm is bankrupt, given the contractual and legal arrangements in place. But the way ex-post conflicts are resolved also influences the initial financing and valuation of the firm, which suggests that the two problems should be analyzed jointly. In fact, the problem of bankruptcy is most interesting when posed in an ex-ante framework as it raises what seems like a paradox: if bankruptcy with multiple creditors is so complex, why would a firm contract with several creditors in the first place? Put differently: if conflicts of interest must be resolved ex post anyhow and these resolutions are costly, why create them and how structure them ex ante? We attempt to answer these questions in an optimal contracting approach to corporate debt and bankruptcy.

The paper analyzes a firm's choice of debt structure and its effect on incentives for strategic default. We start from the observation that multiple creditors make contract renegotiations more difficult and emphasize a) the ex-post conflicts among multiple creditors, b) the design of individual claims and their impact on these conflicts and c) the role of bankruptcy rules in resolving such conflicts from an ex ante perspective. In our model, having multiple creditors gives rise to potential ex-post inefficiency, which stems from frictions in multilateral conflict resolution. This corresponds to the well-known inefficiencies in the workouts of financial distress, documented, e.g., by Asquith, Gertner, and Scharfstein (1994). However, in the spirit of the literature on strategic capital structure design,[1] we note that this ex-post inefficiency also has a positive incentive effect, as it forces the firm to honor several claims instead of only one if it wants to avoid the ex-post inefficiency.

To provide an intuition for why two investors are better than one in our model, consider a firm seeking outside finance from two investors. Following Hart and Moore (1998), we describe the moral-hazard problem of the firm in repaying its financiers by assuming that the project generates some unverifiable cash flows next to the verifiable assets. In this setup, repayment is

[1] See, in particular, Hart and Moore (1998), Bolton and Scharfstein (1996), Dewatripont and Tirole (1994), Berglöf and von Thadden (1994).

limited by how much asset value the financiers can credibly threaten to liquidate. Under perfect renegotiation (or with one single creditor), the firm's commitment ability is in principle given by the amount of assets available for foreclosure should the firm default. However, this constraint is relaxed if the firm is forced to renegotiate individually with its creditors. If this is the case, the firm can promise ex ante up to the full amount of available assets to each one of the investors. When the firm only defaults on one investor ex post, this investor has the right to foreclose on the firm's assets to collect her debt. As each creditor has this individual right, the firm must pay out twice its asset value if it wants to protect its assets. Hence, a renegotiation with two creditors creates a "Prisoners' Dilemma", which credibly commits the firm to pay out twice if it wants to evade it. These higher repayment obligations, however, also increase the incentives for strategic default, in which case the inefficiency of the creditor interaction ex post destroys value. This creates a counterveiling force to the commitment effect of high individual debt claims and thus leads to a tradeoff in the design of these claims.

If the firm defaults on both creditors, and one creditor calls the sheriff to enforce the payment, the other creditor can file for bankruptcy. In this case, the sum of the two claims are larger than the available amount of verifiable assets and individual claims must be adjusted. This is the essential role of bankruptcy in our model. Bankruptcy provides a means to pare down individual claims if the debtor is unable or unwilling to honor them and the creditors must confiscate what is available. We thus emphasize the view, shared by many scholars of law and economics, but rarely modeled in full, that "bankruptcy is a situation in which existing claims are inconsistent " (Hart, 1995).

Our model is the first to bring out the distinction between debt collection and bankruptcy that is a fundamental feature of debt finance in practice and legal theory. Debt collection law is basically bilateral and defines the rights of a single creditor in a bilateral conflict with the debtor. In our model, each creditor has the right to collect his debt if the debtor defaults against him, and as every creditor wields this threat, the debtor is under strong pressure to pay out. Yet, if the debtor defaults against several creditors, these threads cannot necessarily be executed simultaneously, individual debt collection ceases, and the debtor is in bankruptcy. In this sense, we model bankruptcy as "collective debt collection" (Jackson, 1986).

Next to the distinction between individual and collective debt collection, our model yields the following empirical predictions.

The model predicts that debt claims should be "undersecured", i.e., that nominal debt claims should be larger than what creditors receive in bankruptcy. While this is a standard consequence of single-creditor models, it is not obvious in a multi-creditor model. In fact, a priori it may be reasonable to "oversecure" certain creditors who then would have strong incentives to trigger bankruptcy should the debtor attempt to default. Our prediction is consistent with standard practice and the empirically documented observation in many countries of systematically low retrieval rates of creditors in bankruptcy (see, e.g., Weiss (1990)).

Further, the model predicts that for a broad range of funding requirements, the debtor should retain some of her assets in bankruptcy. As a corollary, absolute priority - the notion that creditors must be satisfied fully in bankruptcy before owners are to retain something - is violated in the present model. Again, this is consistent with the empirical literature (see, e.g., Franks and Torous (1989)).

Next, the model predicts that all creditors should optimally be treated symmetrically ex post. In particular, side payments from the debtor to one creditor are value-reducing and contracts are designed to exclude them. This corresponds to, and gives an efficiency justification for, the widespread use of “equal treatment” rules in bankruptcy legislation, such as the Trust Indenture Act in the U.S.

Furthermore, our model allows to distinguish between liquidity-driven defaults and value-driven defaults. In the equilibrium of the model, firms default either because they lack liquidity but are fundamentally sound or because they have liquidity but little long-term value. Davydenko (2005) provides evidence that empirically both motives are important and have independent explanatory power.

Our simple model of bankruptcy captures some important elements of existing bankruptcy procedures. We show that each creditor’s right to liquidate assets, which protects him against opportunism by the debtor, must be complemented by the right to trigger bankruptcy, which in turn limits the individual liquidation rights. Bankruptcy is triggered when a creditor files to prevent his claims from being eroded through debt collection of other creditors. The procedure demands an “automatic stay” ensuring that liquidation claims are executed simultaneously, and distributes liquidation values according to a pre-specified rule. Interestingly, however, giving the creditors the right to trigger bankruptcy is not always sufficient to rule out runs for the assets. If individual creditors stand to gain relatively little from bankruptcy, but recover their full debt claim if they are the first to foreclose, they may have an incentive not to trigger bankruptcy ex post, but rather run for the assets. In such a constellation it is optimal to give the debtor the right to trigger bankruptcy. In this respect, our model provides a new efficiency argument for debtor-friendly bankruptcy legislation (see Baird, 1991).

The remainder of this paper is organized as follows. In Section 1, we review some of the literature on debt structure and bankruptcy. Section 2 describes the basic structure of the model. Section 3 studies optimal contracting under the assumption that bankruptcy is triggered automatically. Section 4 extends the model and provides a more detailed institutional analysis of the bankruptcy process, in particular the issues of triggering rights and seniority. Section 5 concludes with a discussion of the role of contracts versus the law and further research avenues. Appendix A contains some technical arguments, and Appendix B extends the analysis to the technically more involved case of efficient ex-post liquidation.

1 Related Literature

Our paper touches on two large strands of the literature that up to now have rarely been brought together: the literature on corporate bankruptcy and on capital structure. Here we review some contributions in both strands that are relevant to our analysis.

An important part of the large literature on bankruptcy law focuses on optimal procedural and substantial rules, taking as given pre-existing debt contracts and the decision to enter bankruptcy (next to the large legal literature, see, e.g., Bebchuk (1988), Aghion, Hart and Moore (1992) or Baird and Bernstein (2005)). This work rightly points out that the choice of capital structure influences what happens and what should happen in bankruptcy. Yet, it is silent on the determinants of debt structure, which is problematic as the bankruptcy procedure has an impact on the firm's capital structure decision. An important exception is the work by Harris and Raviv (1995) who study the impact of different games played ex post on the ex-ante efficiency of the contract. Different from our work, however, Harris and Raviv (1995) are only concerned with games between the debtor and one single investor.

Another interesting avenue of research has looked at the bankruptcy problem from an ex-ante perspective. Building on the early work of Bulow and

Shoven (1978), contributions such as those by Bebchuk (2002), Berkovitch and Israel (1999), Cornelli and Felli (1997), Schwartz (1998) or Acharya, John, and Sundaram (2005) analyse the impact of bankruptcy on debtors' investments, debt levels, and incentives prior to bankruptcy. These ex-ante analyses are not concerned with the key question of our paper, which is the role of multiple creditors in bankruptcy. In fact, all of these articles consider the conflict between a debtor and a single creditor.

Conceptually, our work is closest to the legal literature on bankruptcy, which emphasizes that the main role of bankruptcy law "is that of bankruptcy as a collective debt-collection device, and it deals with the rights of creditors ... inter se" (Jackson, 1986). For example, Kordana and Posner (1999) study the complex voting features associated with Chapter 11 in the U.S. bankruptcy code. Similar to our paper, they discuss the tradeoff between reducing the cost of liquidation by lowering individual pre-bankruptcy entitlements and discouraging strategic default. However, like most of the legal literature, they do not formally model the full ex-ante contracting problem, which makes it difficult to evaluate the effects they are discussing.

There are only very few papers that have modelled multiple creditor problems from an ex-ante perspective. Of interest in our context are, in particular, the contributions by Winton (1995), Bolton and Scharfstein (1996), Bris and Welch (2005), Hege and Mella-Barral (2005), Bisin and Rampini (2006), and Hackbarth, Hennessy, and Leland (2007).

In Bolton and Scharfstein's (1996) seminal theory of debt structure, multiple lending relationships can be optimal, but need not. In their model, multiple (two) creditors increase firm value on the one hand because of increased bargaining pressure in strategic default, but decrease firm value on the other hand because of less efficient continuation choices in liquidity default. The optimal number of creditors emerges as a trade-off between these two tendencies. Their analysis does not consider the key problem of our paper, the ex-post tension between individual and collective liquidation rights of creditors. As a consequence, they are not concerned with the optimal allocation of individual collection and security rights and their impact on default and bankruptcy.

Bris and Welch (2005) take a different approach to the coordination problem between multiple creditors in ex-post bargaining. They note that such bargaining is wasteful because of lobbying and other dead-weight losses, but that free-riding reduces the incentives for such wasteful activities when the number of creditors increases. Hence, although payments to creditors in financial distress decrease with the number of creditors, this is priced in the debt claims ex ante and only the beneficial effect of reduced influence costs ex post matters, which creates a tendency to contract with multiple creditors. Differently from our theory, Bris and Welch (2005) ignore the problem of strategic default, by simply assuming that the debtor pays out in good states of nature. Our theory is consistent with theirs in that we emphasize the advantage of contracting with multiple creditors, but we differ from their theory by pointing out the downside of multiple creditors when it comes to strategic default. Our theory of the design of individual debt claims and the role of bankruptcy is driven by the tension between collective and individual creditor rights, which is absent in their model.

Winton (1995) approaches the problem of multiple creditors from the perspective of costly state verification, generalizing the work of Townsend (1979) and Gale and Hellwig (1985). His results provide a theoretical rationale for seniority and absolute priority, and predict an ordering of monitoring activities among investors. These monitoring activities are reactions to financial distress and can therefore be interpreted as gradual bankruptcy provisions. Different from our work, in Winton (1995) the debtor borrows from several creditors by assumption.[2]

Bisin and Rampini (2006) are interested in the ex-ante incentive effects of bankruptcy in an environment in which a debtor can borrow from several lenders to smooth consumption. They show that a bankruptcy-like contract allows the main lender to relax the debtor's incentive compatibility constraint, because it is a means for the main lender to commit to confiscate returns in low-return states (which is not optimal for consumption smoothing reasons, but increases the borrower's effort incentives). Different from our model, in their model lending is for consumption smoothing and not production (so the focus is rather on consumer bankruptcy), and exclusive lending contracts are superior to contracts with multiple creditors, but cannot be enforced by assumption.

Hege and Mella-Barral (2005) and Hackbarth, Hennessy and Leland (2007) consider continuous-time models of debt renegotiation with multipe creditors. Like in our model, equity can make opportunistic debt exchange offers to force concessions on coupon payments. Different from our paper, however,

[2] It is actually interesting to note that Winton (1995) himself plays down the link of his theory to bankruptcy. He rather stresses examples such as asset securitization or reinsurance, where the individual verification activities do not necessarily imply that a joint "bankruptcy" decision is negotiated and implemented.

Hege and Mella-Barral (2005) assume that all debt claims are ex-ante identical and do not study the design of individual claims and its relationship with bankruptcy. Hackbarth, Hennessy, and Leland (2007) extend the work by Hege and Mella-Barral (2005) by assuming that there are two types of debt, market debt and bank debt. Bank debt can be costlessly and efficiently renegotiated, while market debt cannot be renegotiated at all. Ideally, firms would only contract bank debt, but that claim is limited by the firm's collateral value. In order to increase debt-capacity, firms must take out market debt. The paper shows in particular, that optimally both types of debt coexist. Thus, Hackbarth, Hennessy, and Leland (2007) study how exogenously different types of debt coexist, while our paper makes no a priori assumptions about debt characteristics and derives the differentiation of individual claims and their treatment in bankruptcy endogenously.

2 The Model

A firm can invest the fixed amount I at date 0 and lives for two periods after that date. As in the "incomplete contracts" literature on debt contracts following Hart and Moore (1998), at date 1, the firm has assets in place worth A that generate a cash flow Y. Asset value A at date 1 is verifiable and deterministic, known to everybody in advance. Cash flow, Y, is observable, but not verifiable, and accrues to the firm's management. The difference between A and Y is that foreclosure on the firm's property by the sheriff can only reach A, not Y. Hence, while contractual transfers of assets in place can be enforced by courts, transfers of cash must be incentive-compatible for the firm.

If the firm is not liquidated at date 1, final firm value V is realized at date 2, where V is a continuous random variable with cumulative distribution function $F(V)$ and support $[\underline{V}, \overline{V}]$. We assume that F is differentiable on $(\underline{V}, \overline{V})$ with density f, and will extend the definition of F and f to all of $[0, \overline{V}]$ in the obvious way.[3] In this paper we shall assume for simplicity that all of V is non-verifiable, i.e. that management cannot credibly promise to transfer value to creditors at date 2. Hence, we focus on short-term debt and ignore issues such as debt maturity structure and debt rescheduling.[4]

At date 1, there is a public signal v about V. To simplify the presentation

[3] With this extension management can always claim at date 2 that it has nothing to pay out (although this is a zero-probability event, it can occur).

[4] See Gertner and Scharfstein (1991) and Berglöf and von Thadden (1994) for models that address some of these issues.

(no loss of generality here), we assume that the signal is perfect, i.e. that V is known already at date 1. At date 1, assets in place can be liquidated. If $L \leq A$ is the total amount liquidated, the firm continues on the scale $(1 - L/A)$. This means that the firm and its owners obtain $(1 - L/A)V$ at date 2. This assumption amounts to assuming that long-term firm value is produced with constant returns to scale.[5]

Interest rates across periods are normalized to 0. In order to simplify the presentation, we assume that it is always inefficient to liquidate assets in place:

$$\underline{V} \geq A. \tag{1}$$

This assumption describes the more interesting case of the bankruptcy problem: although it is inefficient ex post, the parties may be forced to liquidate assets to motivate the debtor to repay. All our results hold in the more general case $\underline{V} \geq 0$.

Short term cash flow Y, realized at date 1, is given by

$$Y = \begin{cases} 0 & \text{with proba } 1 - q \\ Y_H & \text{with proba } q. \end{cases}$$

Here $Y = 0$ describes the situation in which the firm is liquidity-constrained,

[5] As for example in Hart and Moore (1998) and Harris and Raviv (1995).

and $Y = Y_H$ the case of no liquidity constraints. To simplify, we assume that cash flows in the the good state are sufficiently high so as to avoid any liquidity constraints in that state; specifically, we impose

$$Y_H \geq 2A \tag{2}$$

Without this assumption, liquidity constraints would play a role even in the good cash flow state, which would complicate the analysis without any gain in insights.

At date 0, the firm is run by a risk-neutral owner/manager who has no funds and raises them from external investors. Investors are risk-neutral and competitive, they each put up $I_i > 0, \sum I_i = I$, $i = 1, ..., n$. We shall focus here on the case of two creditors, the case of more than two creditors being a simple extension.

According to standard incentive-contracting theory, a financial contract is a state-contingent repayment/liquidation decision at date 1, $(R_i(Y), B_i(Y))$, $i = 1, 2$, where $R_i(Y)$ is the payment made by the firm out of cash (Y), and $B_i(Y)$ the liquidation of asset value (A) if the cash-flow state is Y. In this framework, limited liability implies $R_i(0) = 0$ for all i, and (second-best) optimality $B_i(Y_H) = 0$ for all i (because liquidation is inefficient and the firm is not cash-constraint in the good state by (2)). By a slight abuse of notation, let $R_i = R_i(Y_H)$ denote the repayments in the good state and $B_i = B_i(0)$ the collective liquidations in the bad state. An incentive-efficient contract then is a vector (R_1, R_2, B_1, B_2) that satisfies the usual incentive and limited-liability constraints.

In order to capture more of the richness of multilateral debt contracts in practice, we go beyond the simple incentive-efficient contracting framework and allow for renegotation. We model renegotiation as a simple bilateral take-it-or-leave-it offer game between the firm and each creditor, a set up that corresponds to a simple model of exchange offers.[6] The fact that bargaining is bilateral represents the frictions that are inherent in typical out-of-court bargaining and means that the creditors as a group cannot get together with the firm to negotiate their way efficiently around bankruptcy. There is substantial empirical evidence on the difficulty of out-of-court agreements. Among others, Gilson, John, and Lang (1990), and Asquith, Gertner, and Scharfstein (1994) find that out-of-court restructurings have a substantial risk of failure, and this risk is the higher, the larger the number of creditors, in particular public debtholders. Gilson (1997) argues that the relatively

[6]See Detriagache and Garella (1996) for an ex-post analysis of exchange offers (that does not address the ex-ante problems we discuss here).

high transactions costs of private restructurings that he finds (compared to Chapter 11 cases) are the result of unanimity requirements and coordination problems in out-of-court settlements.

Formally, the extensive form game between the firm and its creditors at date 1 is the following:

1. Nature determines Y and V.

2. The firm pays out $p_i \leq R_i$, $i = 1, 2$.

3. If $p_i < R_i$ for one or both creditors, these creditors simultaneously choose to accept the payment (strategy a) or to foreclose on the firm's assets (strategy f).

Different from the case of standard incentive-compatible contracting, this renegotiation game requires that one specifies individual liquidation rights, which determine payoffs in the case in which one creditor accepts the firm's cash offer in the renegotiation and the other does not. Denote by $D_i \leq A$ these individual collection rights. As before, denote by B_i the individual claims under collective debt collection. Clearly, $B_1 + B_2 \leq A$.[7]

[7] In full generality, the liquidation righs depend on the proposed repayment, $D_i(p_i)$ and $B_i(p_i)$. However, it is easy to show that the optimal contract will always feature maximum punishment, $B_i(p_i) = B_i(0) \equiv B_i$, $D_i(p_i) = D_i(0) \equiv D_i$.

This model of individual versus collective liquidation rights adds a new, more realistic dimension to the contracting problem. In practice, debt contracts typically specify individual, non-interactive collection rights, which are governed by debt collection law, and are less specific concerning collective debt collection. This problem is addressed by covenants, priority rules or collateral assignments, but many interactive collection rights of creditors are left to bankruptcy law and judges, as a way to implement multilateral debt collection. Our approach to modelling these decisions in this and the next section is to ask what a contract would optimally stipulate if it included complete provisions for multilateral debt collection and if these provisions were executed automatically as soon as both creditors decide to foreclose on the firm's assets. Such multilateral debt collection therefore is akin to bankruptcy. In Section 4, we will investigate the bankruptcy problem more deeply by asking how such multilateral debt collection can be implemented if it does not happen automatically.

A contract in this renegotiation framework thus is given by six non-negative numbers $(R_1, D_1, B_1, R_2, D_2, B_2)$ with $D_i \leq A$, $B_1 + B_2 \leq A$, $R_1 + R_2 \leq Y_H$. Let $D = D_1 + D_2$ and $B = B_1 + B_2$ denote aggregate claims. Furthermore, we normalize notation by assuming that creditor 1 is the one with a (weakly) higher individual foreclosure right:

$$D_1 \geq D/2 \geq D_2. \tag{3}$$

Note that, once (3) is fixed, a further such normalization is not possible with respect to the collective liquidation claims B_i.

Date-1 outcomes are payments by the firm of $(0, 0)$, resp. (R_1, R_2) in the two cash-flow states if there is no renegotiation, and are given by the following payment matrix if there is renegotiation:

	a	f
a	p_1, p_2	p_1, D_2
f	D_1, p_2	B_1, B_2

(4)

3 Optimal Contracts

We first analyze the interaction at date 1 for any given contract and then study the choice of contract at date 0, assuming that the contracting parties anticipate how the contract influences their behavior later on. We maintain the assumption that the liquidation outcome (B_1, B_2) obtains automatically if both creditors want to foreclose simultaneously in game (4)). In Section 4, we investigate what game structure can achieve this ex-post outcome.

3.1 Date 1 interaction:

Consider first the low cash-flow state, $Y = 0$. The firm has nothing to pay out, and whether or not the firm attempts to renegotiate, the payments to the creditors will be $(0, 0)$ and liquidation (B_1, B_2)

Now consider the case $Y = Y_H$. If the firm renegotiates with both creditors, the outcome of the foreclosure game between the two creditors depends on the offered payments. Clearly, the best way for the firm to induce the outcome (f, f) is to offer $p_1 = p_2 = 0$. A necessary condition for (a, a) to be an equilibrium (i.e. for no liquidation to take place), is that $p_1 \geq D_1$ and $p_2 \geq D_2$. If there is equality in one of these inequalities, this equilibrium may not be unique for reasons of indifference. We rule out this possibility by the standard assumption that indifferences are resolved in such a way that the ex-ante optimization problem has a solution. In the present context this means that creditors accept the firm's payment whenever it is weakly greater than their liquidation return. Under this assumption, (a, a) is the unique equilibrium of (4) if and only if

$$p_i \geq D_i, i = 1, 2, \tag{5}$$

and

$$p_1 > B_1 \text{ or } p_2 > B_2. \tag{6}$$

We shall see that at the optimal contract the latter condition will be slack and can therefore ignore it now. Thus (5) implies that the best way for the firm to induce outcome (a, a) is to set $p_i = D_i$, $i = 1, 2$.

By a similar reasoning, the firm can induce the asymmetric outcome (a, f) by offering $p_1 = B_1$ and $p_2 = 0$, and analogously (f, a). Here, the firm defaults and treats its creditors asymmetrically: it does not pay creditor 2 and has him collect his debt, and it pays creditor 1 just enough to prevent him from sending the firm into bankruptcy. Note that if $B_2 < D_2$ creditor 1 exerts a positive externality on creditor 2, because if creditor 1 refused the firm's reduced payout, the firm would go bankrupt and creditor 2 would obtain less.[8]

Going back one stage in the bargaining game, which of the four bargaining outcomes in matrix (4) does the debtor want to induce if she chooses to

[8]Such asymmetry of treatment between the two creditors is typically illegal in most jurisdictions. As we construct the interaction from first principles, we cannot rule out such behaviour. Rather, true to our approach, we shall show that optimality considerations will rule out such asymmetries.

renegotiate at date 1?[9] Clearly, this depends on the parameters of the original contract and on the realization of V. The following lemma describes this decision by parametrizing the original contract by B, D, B_1, D_1.

Lemma 1: *The renegotiation game at date 1 has the following outcomes:*

1. *If*

$$B_1 > \frac{D}{B}D_1 - D + B \tag{7}$$

   the firm optimally induces

   - *outcome* (a, a) *if and only if* $\frac{V}{A} \geq \frac{D-B+B_1}{D_1}$
   - *outcome* (f, a) *if and only if* $\frac{B-B_1}{B-D_1} \leq \frac{V}{A} < \frac{D-B+B_1}{D_1}$
   - *outcome* (f, f) *if and only if* $\frac{V}{A} < \frac{B-B_1}{B-D_1}$

2. *If*

$$B_1 < \frac{D}{B}D_1 + D - \frac{D^2}{B} \tag{8}$$

   the firm optimally induces

   - *outcome* (a, a) *if and only if* $\frac{V}{A} \geq \frac{D-B_1}{D-D_1}$

[9] Of course, at date 0 management prefers the (a, a) outcome because liquidation is inefficient. But at date 1, its preferences are guided by the D_i, B_i, and no longer by overall efficiency considerations.

- *outcome* (a, f) *if and only if* $\frac{B_1}{B-D+D_1} \leq \frac{V}{A} < \frac{D-B_1}{D-D_1}$
- *outcome* (f, f) *if and only if* $\frac{V}{A} < \frac{B_1}{B-D+D_1}$

3. *If*

$$\frac{D}{B}D_1 + D - \frac{D^2}{B} \leq B_1 \leq \frac{D}{B}D_1 - D + B \tag{9}$$

the firm optimally induces

- *outcome* (a, a) *if and only if* $\frac{V}{A} \geq \frac{D}{B}$
- *outcome* (f, f) *if and only if* $\frac{V}{A} < \frac{D}{B}$.

The proof of the lemma follows by a straightforward comparison of alternatives. The lemma states that the firm will default against both creditors if future firm value is low, it will repay both creditors (D_1, D_2) in cash if future firm value is high, and it will default partially in an intermediate region. This intermediate region is empty in the third case, given by (9). What is remarkable is that the firm's choice of renegotiation outcomes is so well-structured. There are three possible regimes defined by the relative weight of individual claims. Note that these three regimes are exhaustive and exclusive.[10]

If we fix the values of aggregate claims, B and D, the three regimes identified in Lemma 1 can be graphically described in D_1-B_1-space. The conditions

[10] This is because

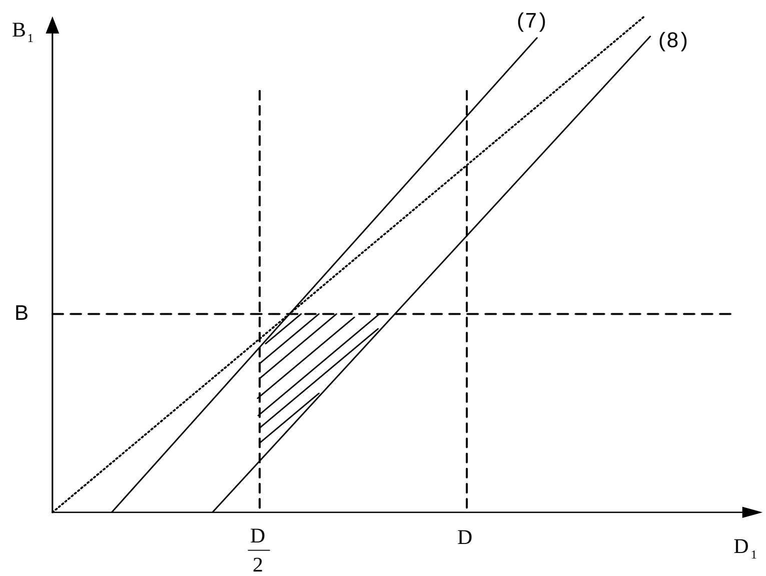

Figure 1: The three regimes if $B < D < 2B$

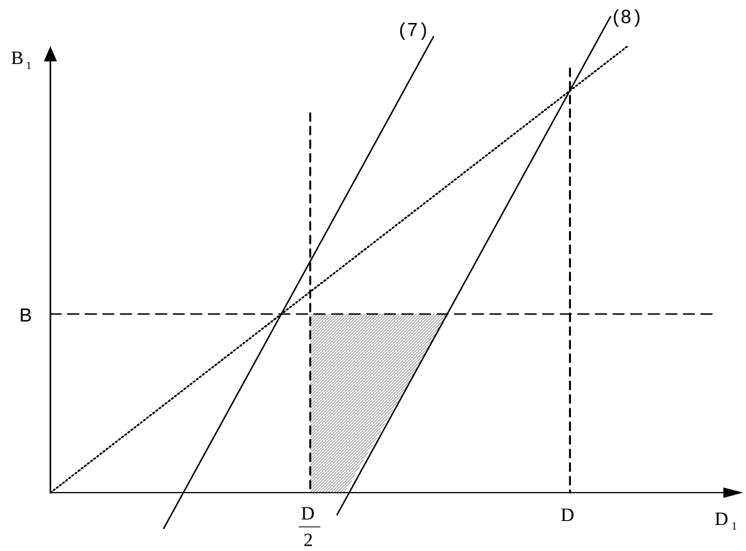

Figure 2: The three regimes if $D > 2B$

(7) and (8) define two parallel straight lines in B_1-D_1 space. Depending on the relative sizes of B and D the two straight lines partition this space in different ways. Figures 1 and 2 show the two cases ($B < D < 2B$ and $D > 2B$) that will later turn out to be the relevant ones and indicate regime 3 as hatched surfaces.

In regime 1 (points above line (7), below the line $B_1 = B$ and between the lines $D_1 = D/2$ and $D_1 = D$), creditor 1's bankruptcy claim B_1 is high compared to his individual claim D_1 (and to that of creditor 2). This regime is empty if $D > 2B$. In this regime it is never optimal to induce partial liquidation by creditor 2, because that would require paying off creditor 1's bankruptcy claim in cash and still have a sizeable foreclosure by creditor 2. Yet, all the other three outcomes are possible, depending on the firm's long-term value V.

In regime 2, things are similar, with the roles of creditor 1 and 2 reversed. In regime 3 (the hatched surfaces in Figures 1 and 2), the individual claims of

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$$\frac{D}{B}D_1 - D + B \geq \frac{D}{B}D_1 + D - \frac{D^2}{B}$$
$$\Leftrightarrow \quad (B - D)^2 \geq 0.$$

the two creditors are not too different from each other, and, if $B < D < 2B$, not too different from their bankruptcy claims. Therefore, it is never optimal to induce partial liquidation: the creditor experiences a sizeable liquidation by whoever he would choose to default upon and still needs to pay some cash to the other creditor to keep her from sending the firm into bankruptcy.

In regime 3, the choice among the firm's two alternatives can easily be understood by comparing the respective payoffs. With (a, a) the debtor's payoff is

$$Y_H - D + V, \tag{10}$$

while the payoff under (f, f) is

$$Y_H + (1 - \frac{B}{A})V. \tag{11}$$

Comparing (10) and (11) shows that the debtor prefers (a, a) over (f, f) iff

$$\frac{D}{B} \leq \frac{V}{A}. \tag{12}$$

In other words, in regime 3 the firm prefers to pay out if the continuation value V is higher than the threshold DA/B and prefers complete strategic default otherwise. In the other two regimes of Lemma 1 there is an interval around DA/B such that there is partial default for values of V in this interval.

The above discussion shows that if the firm pays out to both creditors then it does not have to pay more than (D_1, D_2). Hence, without loss of generality we can assume that the contract is renegotiation-proof and that $(R_1, R_2) = (D_1, D_2)$. This is consistent with our earlier definition of D_1 and D_2 as individual foreclosure rights, because in all jurisdictions we know of, an individual creditor has the right to sue the debtor for the full amount of her due debt, unless the firm seeks bankruptcy protection.

3.2 Date 0 contracting:

We now turn to the contract design problem at date 0, which consists of choosing the optimal individual claims D_i (the face values) and bankruptcy claims B_i. A first important observation is that it is never optimal to induce default and partial liquidation.

Proposition 1 (Individual Debt Design): *Any optimal contract satisfies (9) and therefore does not induce asymmetric default.*

The proof of Proposition 1 is given in the appendix. To get an intuition for what condition (9) implies for the distribution of individual debt and bankruptcy claims, consider first the case $B < D < 2B$ (Figure 1), when aggregate bankruptcy claims are relatively high (more than 50 percent of total face value). In this case, the larger creditor must get some bankruptcy liquidation rights (B_1 is bounded away from 0). This prevents the debtor from strategically defaulting against the smaller creditor, because this would cost $D_2 = D - D_1$ in asset value and B_1 in cash to prevent creditor 1 from triggering bankruptcy. If both creditors are of similar size (D_1 close to $D/2$), then creditor 2 also must hold some bankruptcy liquidation claims; if creditor 2 is much smaller than creditor 1 (D_1 sufficiently large) then creditor 2 does not need to receive anything in bankruptcy. The reason is that in this case asymmetric default against creditor 1 is not attractive because of his high individual liquidation rights (remember that in an asymmetric default creditor 2 gets his bankruptcy payment B_2 in cash). On the other hand, if $D > 2B$ (Figure 2), bankruptcy liquidation rights are relatively less important, so that it is even possible for creditor 1 (but not both creditors) to receive nothing in bankruptcy ($B_1 = 0$). Again, what prevents partial default and liquidation in this case are the high individual foreclosure rights.

Proposition 1 is of interest for several reasons. First, it shows that optimal contracts rule out a too unequal division of nominal claims among the creditors. In particular, holding total nominal claims, D, fixed, creditor 1's claim, D_1, is bounded away from D. Creditor 1 can only have a significantly higher nominal claim than creditor 2 if total debt is high, overall bankruptcy claims low ($D > 2B$), and if this claim is held up well in bankruptcy (B_1 relatively high, to make partial default against creditor 2 costly).

Second, optimal contracts exclude default with partial liquidation. In this sense, optimal contracts are symmetric: creditors are treated equally ex post. This feature is in line with the many provisions in different bankruptcy codes that prohibit the unequal treatment of creditors.[11] What is interesting is that the traditional justifications for such equal treatment rules are ethical: it is deemed unjust to treat similar creditors differently. Our model justifies such behavior on efficiency grounds: unequal treatment ex-post provides a way to pay off one creditor cheaply, which relaxes the ex-ante discipline of the liquidation threat.[12]

The third point of interest of Proposition 1 is that it shows that optimal

[11] The *par conditio omnium creditorum* (equal treatment of all creditors) in Roman law is the mother of these clauses.

[12] The cash transfer of B_i made to prevent creditor i from triggering bankruptcy, can be seen as a fraudulent conveyance, ruled out in most bankruptcy codes. A payment to one creditor is a fraudulent conveyance if other creditors of equal or higher seniority are being paid less without being compensated in bankruptcy.

contracts only depend on the aggregate claims B and D. This is because an optimal contract uses the prisonner's-dilemma game between the creditors to maximize the debtor's repayment incentives: either the debtor repays in full or she goes bankrupt; intermediate deals, which would involve the comparison of individual debt terms, are ruled out by contract design. Of course, the individual debt contracts (D_1, B_1) and (D_2, B_2) can be tailored freely and will be priced accordingly (as long as the limits of (9) for the individual claims are respected). But the firm's repayment incentives and debt capacity depend only on aggregate debt and collateral.

For rest of this section we therefore consider only the aggregate values D and B. The firm gets $(1 - B/A)V$ in the bad cash flow state and either $Y_H - D + V$ (if $V \geq DA/B$) or $Y_H + (1 - \frac{B}{A})V$ (if $V < DA/B$) in the good cash flow state. Letting

$$\theta = \text{Prob}\ (V \geq \frac{D}{B}A), \tag{13}$$

the firm's expected profit at date 0 is, therefore,

$$\begin{aligned} S_0 &= (1-q)(1-\frac{B}{A})EV + q\theta(Y_H - D + E[V \Big| V \geq \frac{D}{B}A]) \quad (14) \\ &\quad + q(1-\theta)(Y_H + (1-\frac{B}{A})E[V \Big| V < \frac{D}{B}A]) \\ &= qY_H + EV - q\theta D - (1-q)\frac{EV}{A}B - q(1-\theta)\frac{B}{A}E[V \Big| V < \frac{D}{B}A] \quad (15) \end{aligned}$$

The last two terms of (15) show the two sources of inefficiency in the contracting problem. First, there is liquidation in the bad cash flow state (occuring with probability $1-q$), and second there is liquidation in the good cash flow state following strategic default (occuring with probability $q(1-\theta)$). The first inefficiency is minimized by making bankruptcy liquidation B as small as possible. Yet, this tends to aggravate the second inefficiency, because it increases the threshold for strategic default in the last term of (15), thus making (ex ante) wasteful strategic default more likely. The optimal contract must strike a balance between these two payouts and the average cash payout $q\theta D$, which a priori has no negative efficiency effects but must be supported by liquidation threats to be effective.

The investors' participation constraints are

$$(1-q)B_i + q\theta D_i + q(1-\theta)B_i = I_i, i = 1, 2, \tag{16}$$

where the I_i are, in fact, defined by (16), and $I_1 + I_2 \geq I$. Furthermore, the feasibility constraints

$$0 \leq D_1, D_2, B_1, B_2, B \leq A \tag{17}$$

and the constraints (9) and $D_1 \geq D/2$ must hold.

Summing (16), the investors' aggregate participation constraint is

$$(1 - q\theta)B + q\theta D \geq I. \tag{18}$$

Clearly, the participation constraint binds. Hence, the contract optimization problem at date 0 is

$$\max_{D,B} \quad S_0 \tag{19}$$

$$\text{s.t.} \quad (1 - q\theta)B + q\theta D = I \tag{20}$$

$$0 \leq B \leq A \tag{21}$$

$$0 \leq D \leq 2A. \tag{22}$$

Here, (21) is the constraint on bankruptcy liquidation and (22) the constraint on cash payout. Because equilibrium cash payout is supported by a double out-of-equilibrium liquidation threat, the latter constraint is weaker than the former.

The left-hand side of (20) is the equilibrium repayment to investors. Its maximum is the firm's debt capacity. Clearly, the firm can raise any amount $I \leq A$ by setting $B = D = I$. In this case the firm has no incentive for strategic default ($\theta = 1$) and its repayment is I for sure: with probability q in cash, with probability $1 - q$ through bankruptcy liquidation. The interesting question is whether the firm's debt capacity can be strictly greater than its asset base.

The following estimate first provides an upper bound for the debt capacity:

$$\max_{B,D}(1-q\theta)B+q\theta D \leq (1-q\theta)A+2q\theta A \tag{23}$$

$$\leq (1+q)A \tag{24}$$

This is intuitive because $(1+q)A = (1-q)A+q2A$: the creditors can never get more than all assets in the bad state and cash of double that value in the good state. But (13) shows that this reasoning actually gives the exact debt capacity if $\underline{V} \geq 2A$, i.e. if long-term firm value is always very high. In this case, increasing B and D all the way up to their maximum values maximises the payments to investors without creating incentives for strategic default for the debtor. On the other hand, if $\underline{V} < 2A$, the debt capacity is lower than $(1+q)A$ because the incentive for strategic default at low values of V provides a countervailing effect.

In order to analyze the contracting problem more fully, we first introduce the variable $t = D/B$, the inverse of the bankruptcy recovery ratio. By substituting $D = tB$, dropping additive terms, and multiplying by $-A$, the contracting problem can be equivalently written as the following payout minimization problem:

$$\min_{t,B} \quad [(1-q)EV + q\theta tA + q(1-\theta)E[V\,|V < tA]]\,B \tag{25}$$

$$\text{s.t.} \quad (1 + q\theta(t-1))B = I \tag{26}$$

$$0 \leq B \leq A \tag{27}$$

$$0 \leq tB \leq 2A \tag{28}$$

We can use the participation constraint (26) to express B as a function of t (remembering that $\theta = 1 - F(At)$):

$$B = \varphi(t) \equiv \frac{I}{1 + q(1 - F(At))(t-1)} \tag{29}$$

φ is defined for $t > 0$, continuous, and is differentiable for $t > 0$ except at $t = \underline{V}/A$ where it has a kink. Denoting the objective function (25) by

$$H(B,t) = \left[(1-q)EV + q(1-F(At))At + q\int_0^{At} V dF(V)\right] B \tag{30}$$

the problem can finally be written as the following constrained minimization problem in one real variable:

$$\min_{t} \quad H(\varphi(t), t) \tag{31}$$

$$\text{s.t.} \quad t \geq 0 \tag{32}$$

$$\varphi(t) \leq A \tag{33}$$

$$t\varphi(t) \leq 2A \tag{34}$$

Here, (33) is the upper constraint on B (the right hand side of (21)) and (34) the upper constraint on D (the right hand side of (22)).

To simplify the presentation we assume from now on that the distribution of V satisfies the monotone likelihood ratio property:

Assumption (MLRP): The likelihood ratio of V, $\frac{f(V)}{1-F(V)}$, is non-decreasing.

(MLRP) is a frequent assumption in contracting problems that is met by most standard distributions. It simplifies the analysis of the present problem because it implies that the auxiliary function φ has a unique minimum.

Lemma 2: *Under Assumption (MLRP),*

(a) *φ has exactly one minimum t_0.*

(b) *If $f(\underline{V})(\underline{V} - A) \geq 1$, then $t_0 = \underline{V}/A$. Otherwise $t_0 > \underline{V}/A$ and $\varphi'(t_0) = 0$.*

(c) *$t_0 > 1$.*

(d) *The function $t\varphi(t)$ is strictly increasing for all $t > 0$.*

Proof: For $t < \underline{V}/A$, φ is strictly decreasing. For $t > \underline{V}/A$, the derivative of φ is

$$\varphi'(t) = -\frac{1 - F(At) - Af(At)(t-1)}{(1 + q(1 - F(At))(t-1))^2} qI$$

This derivative is negative iff

$$\frac{f(At)}{1 - F(At)} < \frac{1}{At - A}$$

Hence, (MLRP) implies statements (a) and (b).

If $f(\underline{V})(\underline{V} - A) < 1$, (b) implies that $t_0 > \underline{V}/A \geq 1$. If $f(\underline{V})(\underline{V} - A) \geq 1$, then $\underline{V} > A$, hence $t_0 = \underline{V}/A > 1$, which proves (c).

Statement (d) follows by differentiation for $t \neq \underline{V}/A$.¥

Comparing the two constraints of the optimization problem (31) - (34), we have $t\varphi(t)/2 < \varphi(t) \Leftrightarrow t < 2$. Hence, constraint (34) on D is redundant if $t < 2$, and constraint (33) on B is redundant if $t > 2$. Constraint (32) is always redundant. Since the constraint set defined by (32) - (34) is compact and the objective function is continuous, this characterizes the existence of a solution.

Proposition 2 (Debt Capacity): *The contracting problem has a solution if and only if*

$$\varphi(\min(2, t_0)) \leq A \tag{35}$$

where t_0 is the unique minimum of φ.

Proof: The problem has a solution if and only if constraints (33) and (34) are compatible. If $t_0 < 2$ the two constraints are compatible iff $\varphi(t_0) \leq A$. If $t_0 > 2$ they are compatible iff $\varphi(2) \leq A$. ¥

If $t_0 > 2$ condition (35) is equivalent to $[1 + q(1-(F(2A))]A \geq I$. If $t_0 < 2$ (35) is equivalent to $[1 + q(1-(F(t_0A))(t_0-1)]A \geq I$. Hence, because $t_0 > 1$, the firm's debt capacity is strictly greater than A iff $\min(2, t_0)A < \overline{V}$. This reasoning also shows that the debt capacity can be as small as A, despite the double liquidation threat with two creditors. This is the case if the firm's long-term returns are concentrated at values close to A. In this case, any attempt to provide high-powered repayment incentives lead to strategic default.

Even without knowing the optimal t in the one-dimensional problem (31) - (34), this reformulation of the original problem yields an interesting qualitative insight concerning optimal leverage:

Corollary 1 (Linear Pricing): Optimal leverage D^* is linear investment I.

Proof: An inspection of (30) shows that the objective function of the optimization problem is linear in I. Therefore its solution t^* is independent of I , and $D^* = t^*\varphi(t^*)$ is linear in I.

Hence, debt pricing is linear in the sense that the interest rate $D^*/I - 1$ is independent of the borrowing level I.

We now turn to the determination of the optimal contract, i.e. the determination of the value t^* that solves problem (31) - (34). It is easy to check that both partial derivatives of the objective function H are positive:

$$\begin{aligned} H_B(B,t) &= (1-q)EV + q(1-F(At))At + q\int_0^{At} VdF(V) > 0 \\ H_t(B,t) &= qAB(1-F(At)) \geq 0 \end{aligned}$$

This is intuitive: increasing collective liquidation (B) and total payout obligations ($D = t\varphi(t)$, which is increasing in t by Lemma 2-(d), increases the firm's loss. Given that t_0 is the unique minimum of φ, this implies that $\frac{d}{dt}H(\varphi(t),t) > 0$ if $t > t_0$. Hence, the solution to the minimization problem (31) - (34) satisfies $t^* \leq t_0$. Furthermore, by explicit calculation one finds that for $t < \underline{V}/A$ (which implies $F(At) = f(At) = 0$),

$$\frac{d}{dt}H(\varphi(t),t) = -\frac{q(1-q)I}{(1+q(t-1))^2}[EV - A] < 0 \tag{36}$$

This implies that the solution satisfies

$$\underline{V}/A \leq t^* \leq t_0 \tag{37}$$

Proposition 3 (Bankruptcy recovery): *The solution to the minimization* *problem (31) - (34) satisfies* $t^* > 1$. *Hence, under the optimal contract, creditors are not fully repaid in bankruptcy:* $B^* < D^*$.

Proof: If $\underline{V} > A$, (37) implies the desired result. If $\underline{V} = A$ we have $t^* > 1$ iff the right-hand derivative of $H(\varphi(t), t)$ at $\underline{V}/A$ is strictly negative. Direct calculation shows that this derivative is

$$\frac{qA^2 I}{(A + q(\underline{V} - A))^2}[A + q(\underline{V} - A) - ((1 - q)EV + q\underline{V})(1 - f(\underline{V})(\underline{V} - A))] \tag{38}$$

For $\underline{V} = A$, this derivative is $-q(1 - q)I[EV - A] < 0$, which proves the proposition.¥

The proposition states that creditors should get less than the face value of their claims in bankruptcy. This result is not trivial because in the model it is quite possible to have full or even excessive recovery of claims in bankruptcy if $D < A$. Yet, our model highlights that bankruptcy liquidation has the cost of destroying value ex post, which is an effect the parties want to minimize ex ante by relying more strongly on.individual liquidation threats than on aggregate ones. Proposition 3 only concerns aggregate values of debt and repayment. Individual claims will be analyzed in the next section.

Proposition 3 has an immediate consequence for the optimal number of creditors in our model. If the firm contracted with only one creditor, this creditor would hold a liquidation claim of D regardless of the type of default. This implies that the creditor would be repaid D in both states: in the good state in cash (because the firm prefers paying out to liquidating the same amount), and in the bad state through liquidation (because the firm is cash constrained). Such a contract can be replicated in the two creditor case by setting $D = B$ (Lemma 1 implies that in this case the firm never defaults strategically, just as with one creditor). Proposition 3 implies that such a contract is not optimal.

Proposition 4 (Optimal Number of Creditors): The firm strictly prefers to have two creditors rather than one.

Hence, the ability of the debtor to pledge his assets individually to more than one creditor strictly improves the terms of contracting. Because the debtor can credibly promise a higher cash payout than his bankruptcy liquidation value, the creditors can lower the amount of liquidation in bankruptcy. Although this strengthens the debtor's incentives for strategic default, these incentives are outweighed by the improved repayment incentives. The next proposition provides a sufficient condition for strategic default to occur in equilibrium.

Proposition 5 (Strategic Default): *Strategic default occurs with positive probability if*

$$f(\underline{V})(\underline{V} - A)((1 - q)EV + q\underline{V}) < (1 - q)(EV - A) \qquad (39)$$

Proof: Condition (39) implies that the right-hand derivative of $H(\varphi(t), t)$ at $t = \underline{V}/A$, (38), is strictly negative. Hence, $t^* > \underline{V}/A$. This implies the claim, because by (12), the firm defaults strategically iff $V < At$.¥

Proposition 5 illustrates the basic tradeoff faced by the debtor. In the original two-dimensional contracting problem (25)-(28), on the one hand, minimising bankruptcy liquidation B is desirable. However, the participation constraint (26) shows that this comes at the cost of increasing t, which means that the face value of debt ($D = tB = t\varphi(t)$) rises, which in turn favors strategic default. Yet, as long as it is possible to decrease B such that t does not exceed $\underline{V}/A$, strategic default is no issue and decreasing B is unambiguously advantageous. Otherwise, we have $t^* > \underline{V}/A$, and the contract induces some strategic default. This will happen, as condition (39) shows, if $\underline{V}$ is sufficiently small. In this case, there are some realizations of long-term firm value such that the firm prefers to keep the current cash rather than the right to future returns.

Interestingly, whether or not strategic default occurs in equilibrium does not depend on the investment level I. There is strategic default iff $t^* > \underline{V}/A$, and t^* does not depend on I. Hence, strategic default is not a problem of too much debt. It arises if long-term firm values are low, which make the firm unwilling rather than unable to repay.

We finally investigate under what conditions the firm is not fully liquidated in bankruptcy.

Proposition 6 (Deviation from Absolute Priority): *If $I < q\underline{V} + (1 - q)A$ the bankruptcy liquidation constraint (27) is slack: $B^* < A$.*

Proof: If $\underline{V}/A > 2$ we have $t^* \geq \underline{V}/A > 2$. Therefore, $\varphi(t^*) < \frac{t^*}{2}\varphi(t^*) \leq A$ by (34).

If $\underline{V}/A \leq 2$ (33) is slack if $\varphi(\underline{V}/A) < A$, because φ is decreasing on $[\underline{V}/A, t_0]$ and because of (37). This last condition is equivalent to $I < q\underline{V} + (1 - q)A$.¥

Proposition 6 only provides a sufficient condition, but this suffices to make the point. The point is that the constraint on B, (21) or (33), does not need to bind at the optimum. If this is the case, the debtor retains some of the assets after bankruptcy. This is due to the fact that continuation in the present model is always efficient. Therefore, the parties have a strong ex ante incentive not to punish the debtor too hard in case of bankruptcy. The point continues to be true if continuing the firm ex post is efficient only with a sufficiently high probabability ($\underline{V} < A$), because bankruptcy liquidation must be independent of V. On the other hand, if the initial investment I, and therefore total leverage, is sufficiently high, it is optimal to make maximum use of the liquidation threat and set $B = A$.

The basic message of Proposition 6 is fairly general: if the debtor has a comparative advantage using the assets, it is ex ante costly to separate her from them ex post, and, therefore, an optimal contract will aim at reducing this incidence as much as possible. This insight, simple as it is, is in sharp contrast with traditional legal reasoning that demands to satisfy creditors first in case of bankruptcy (absolute priority).

4 Bankruptcy versus Debt Collection

In the base model of the last section, we have assumed that a simultaneous attempt by creditors to collect their debt automatically triggers bankruptcy. In reality, of course, bankruptcy must be triggered by someone, and the base model is silent on this issue. In this section, we generalize the base model to a model in which bankruptcy is not an automatic consequence of simultaneous debt collection, but the result of deliberate individual decisions. For these decisions the individual payoffs are important, which were left largely unspecified in the optimal payment/liquidation schedule of the last section.

Lemma 1 and Proposition 1 already imply some interesting structure for individual claims.

Proposition 7 (No Over-Collateralization): The creditors' individual claims satisfy $B_i^* \leq D_i^*$ for $i = 1, 2$.

Proof: By Proposition 1, B_1^*, D_1^*, B^*, D^* must satisfy constraint (9). Suppose first that $B_1^* > D_1^*$. Then (9) implies $D_1^* < \frac{D^*}{B^*} D_1^* - D^* + B^*$ which is equivalent to $D_1^* > B^*$. Hence, $D_1^* > B_1^*$, a contradiction. Now suppose that $B_2^* > D_2^*$. This is equivalent to $B_1^* < D_1^* - D^* + B^*$. Now (9) implies

$D_1^* < D^* - B^*$, which implies $B_1^* < 0$, a contradiction.¥

Proposition 7 makes a familiar but not obvious statement. From Proposition 3 we know that total bankruptcy liquidation must be strictly smaller than the total face value of debt. Proposition 7 states that this must even hold at the individual level: no single creditor must have the right to liquidate more in bankruptcy than his nominal debt claim. This is what is true in practice, but upon reflection not a priori clear. In fact, our model provides one possible reason why it might be optimal to have one creditor who is over-collateralized, i.e. who holds a bankruptcy claim $B_i > D_i$: such a creditor would always trigger bankruptcy when the firm defaults strategically. Since bankruptcy by Proposition 3 protects the firm, this is ex ante desirable and would also have a dissuasive effect on strategic default. However, Proposition 7 also shows why such an arrangement is impossible: Giving one creditor a high bankruptcy claim would imply that the other creditor gets such a small bankruptcy claim that there are ex-post firm values V for which the firm will default partially, which is not optimal ex ante.

We now generalize the contracting model of the last two sections by asking what type of interaction at date 1 can bring about the (second-best) optimal aggregate repayment/liquidation schedule (B^*, D^*) if bankruptcy does not obtain automatically. For this, we must reconsider the assumption about simultaneous foreclosure in the debt collection game (4). While bankruptcy is a coordinated attempt to recover funds from the debtor in default, simultaneous foreclosure is a priori uncoordinated. Hence, instead of the collective debt collection procedure assumed in game (4) in Section 2, we will now assume that simultaneous foreclosure results in an uncoordinated run for the assets, in which the first to collect his debt liquidates D_i, and the second receives $\min(D_j, A - D_i)$ where $j \neq i$ and i is the creditor who collects first. Assuming that each creditor has the same probability of being first, the payoff matrix becomes

	a	f
a	p_1, p_2	p_1, D_2
f	D_1, p_2	$\frac{1}{2}D_1 + \frac{1}{2}\min(D_1, A - D_2), \frac{1}{2}D_2 + \frac{1}{2}\min(D_2, A - D_1)$

(40)

where $p_1 = p_2 = 0$ in the case of $Y = 0$.

In this framework, which represents a "pre-bankruptcy", primitive state, ex-post interactions and ex-ante contracting will be as in Section 3, with the exception that B is fixed exogenously at $\overline{B} = \min(A, D)$. This simplifies the problem of Section 3 considerably, but, as Propositions 3 and 6 have shown, the resulting contract will typically not be optimal. The reason is that the deadweight loss from high liquidation in the bad state more than outweighs the improved incentives for payout in the good state. Ex ante it is therefore optimal to reduce the threat of liquidation from $\min(A, D)$ to B^*.

This can be done by introducing bankruptcy into the model; more precisely, by giving each creditor or the firm the right to trigger bankruptcy when the firm defaults on (some of) its payments. Bankruptcy then means that individual debt collection is suspended, and creditors receive B_i instead of D_i.

The possibility of triggering bankruptcy changes nothing in the analysis of Section 3 if the firm has defaulted partially. In fact, the creditor who is defaulted upon recuperates his full claim through individual debt collection (which is individually optimal by Proposition 7), whereas the other creditor weakly prefers to accept the renegotiated payment from the firm. The firm prefers partial default to bankruptcy by revealed preferences.

If the firm has defaulted on both creditors, be it for liquidity or strategic reasons, the analysis changes. In this case, creditor i when observing the attempt to foreclose by creditor j, will call bankruptcy if this makes him better off than waiting, i.e. if

$$B_i > \min(D_i, A - D_j). \quad (41)$$

Proposition 7 implies that for optimal values of B_i, D_i condition (41) is equivalent to

$$B_i^* > A - D_j^*. \quad (42)$$

Since either of the two creditors may be last in line for debt collection, condition (42) must hold for $i = 1, 2$, if an (inefficient) run for the assets shall be avoided for sure. Adding up (42) for $i = 1, 2$ yields the joint condition

$$B^* + D^* > 2A. \quad (43)$$

Whether or not the optimal (B^*, D^*) satisfy this condition depends on the parameters of the model. Using the definition of φ and t of the previous section, (43) is equivalent to

$$(1 + t^*)\varphi(t^*) > 2A \quad (44)$$

Because the left-hand side of (44) is linear in investment I (remember that t^* is independent of I), it is easy to show that (44) holds if I is large and does not hold if I is small.

If the optimal aggregate repayment/liquidation schedule (D^*, B^*) satisfies the joint condition, the next question is whether one can find values $B_1 \leq D_1$ such that $D_1 \geq D^*/2$ and (9) hold. If this is the case, the solution (D^*, B^*) can be implemented by giving each creditor the right to trigger bankruptcy. If this is not the case, or if $B^* + D^* < 2A$, this will not be possible, and at least one creditor will have an incentive to run for the assets even if he has the right to trigger bankruptcy.

In this case, however, another simple contractual remedy is available: allowing the firm to trigger bankruptcy. In fact, if $B^* < A$, the firm strictly prefers bankruptcy to uncoordinated liquidation. If on the other hand $B^* = A$, the firm is indifferent, but then there is no efficiency case for bankruptcy in the first place.

The following proposition shows that the condition $B^* + D^* > 2A$ is indeed sufficient for creditor-initiated bankruptcy to implement the optimal repayment/liquidation schedule.

Proposition 8 (Implementation): *If* $B^* + D^* > 2A$, *optimal repayment/ liquidation scheme* (B^*, D^*) *can be implemented by giving each creditor the individual right to trigger bankruptcy following default. If* $B^* + D^* < 2A$, *the firm must have the right to trigger bankruptcy.*

The proof in the appendix provides the restrictions on B_1 and D_1 that must hold in order to implement (B^*, D^*) in the case $B^* + D^* > 2A$. Hence, by the discussion preceding the proposition, creditor-initiated bankruptcy is sufficient to get (second-best) optimal bankruptcy decisions if the leverage ratio is high (i.e. if I is sufficiently large compared to A), and it is not sufficient otherwise.

Proposition 8 is interesting because it provides a new efficiency justification for debtor-friendly bankruptcy rules. There is a broad consensus in academia and practice that creditors should have the right to trigger bankruptcy. The consensus is less developed with respect to debtor rights. While most of the criticism of debtor-friendly rules concerns debtor-in-possession rules such as those of Chapter 11 in the U.S., it is also being argued that the individual right of a debtor to evade the discipline of debt collection is harmful. Proposition 8 shows that there is an efficiency reason for such a right: if the creditors do not have the ex-post incentives to select the efficient continuation decision, the debtor may have these incentives. Hence, any reform of possibly excessive managerial discretion rights under Chapter 11 should be careful not to throw out the child with the bathwater and scrap the debtor's right to trigger bankruptcy.[13]

A final question that is interesting to address in the framework of the present model concerns seniority. Usually a claim is said to be senior if it is satisfied before other claims. In our model, this means that the claim of creditor i is senior if $B_i = D_i$ or $B_j = 0$: either creditor i receives his face value in bankruptcy or he gets all liquidation proceeds.

Using the argument of the proof of Proposition 8 one can show that it is not optimal to set $B_i = D_i$ for any i. The reason is that this would induce the other creditor (j) not to trigger bankruptcy if the firm defaults, because he would get more from an uncoordinated liquidation than from bankruptcy. Hence, it is never optimal for one creditor to hold a safe claim. However, it can be possible to allocate all the bankruptcy liquidation rights to the larger creditor if this creditor is sufficiently large. A re-examination of the proof of Proposition 8 shows that conditions (9) of Proposition 1 and (42) of Proposition 8 can jointly be satisfied by setting $B_1 = B^*$ and D_1 sufficiently large, iff $(D^* - B^*)(D^* - A) \geq B^*(A - B^*)$. In terms of the solution of the optimal contracting problem of Section 3, this latter condition can be shown

[13] On a related note, see Berkovitch, Israel and Zender (1998) and Povel (1999). An excellent discussion is given by Baird (1991).

to be equivalent to

$$((t^*)^2 - t + 1)\varphi(t^*) \geq A \tag{45}$$

As discussed above, because t^* is independent of I and φ is linear in I, this shows that creditor 1 can be made senior iff total leverage (which is linearly increasing in I by Corollary 1) is sufficiently high. This is intuitive because when total leverage is high and the larger creditor holds a large share of it, it is very costly to default strategically against this creditor, even if it takes no side payments to the smaller creditor to keep him from triggering bankruptcy. The proof of Proposition 8 shows furthermore that it is impossible to set $B_1 = 0$. Hence, the larger creditor must always receive something in bankruptcy.

Proposition 9 (Seniority): *Under the optimal contract, it is impossible to collateralize one claim fully, i.e. to make the claim riskless. If total leverage is sufficiently large (such that the optimal contract satisfies condition (45)), the larger creditor (creditor 1) can be made senior in the sense that he receives all the bankruptcy proceeds. The smaller creditor cannot be made senior.*

5 Conclusion

We have analyzed the design of bankruptcy rules and debt structure in an optimal-contracting perspective. If cash flow is not verifiable and only the asset value of the firm is verifiable, then when a firm borrows from a single creditor and has all bargaining power, its debt capacity is limited to the value of its asset base. The reason is that the creditor can never expect to receive more than the asset value in liquidation and in renegotiation. However, when a firm borrows from more than one creditor, it can increase its debt capacity by pledging its asset base to more than one creditor by giving each the right to foreclose individually. If the debt structure of the firm is designed appropriately, this creates a commitment for the firm to pay out more in good states to prevent the exercise of individual foreclosure rights and thus raises the firm's debt capacity. Having multiple creditors thus helps to reduce the negative effects of the lack of commitment in contracting by distinguishing between individual foreclosure rights and joint liquidation rights achieved under bankruptcy.

Our theory provides a bridge between corporate finance and the legal theory of debtor-creditor law. The key distinction in debtor-creditor law in most jurisdictions is that between debt collection law and bankruptcy law.

The former governs the interaction between the debtor and a single creditor, the latter the interaction between the debtor and several creditors.[14] Our analysis shows how this same distinction can be made in a contract-theoretic approach to debt. Individual foreclosure rights (corresponding to debt-collection law) are crucial to generate repayment incentives, but need to be complemented by collective liquidiation rights (corresponding to bankruptcy law) in order to maximize ex-post efficiency.

Our results on debt structure and overleverage under multiple creditors depend on the fact that creditors have unilateral foreclosure rights that they can exercise in case of default, independently of what other creditors decide. These rights should be seen as an important element of investor protection. The renegotiation procedure modeled in this paper emphasises the effect of these rights since renegotiation is assumed to happen on an individual basis. The ensuing prisoner's dilemma situation forces the debtor to respect contractual claims as given by individual foreclosure rights whenever he wants to avoid default. The key assumption in this approach is that it is difficult and costly to bring the creditors together to renegotiate the debt contract

[14] The U.S. is a particularly telling example of this distinction. Here, the two bodies of law are even governed by different authorities: debt-collection law is state law, bankruptcy law is federal law.

collectively. Only bankruptcy brings all the contracting parties together at one table, but in bankruptcy the debtor has given up his residual ownership rights, and the procedure is mostly concerned with the reconciliation of individual liquidation claims. This is the classical "vis attractiva" of bankruptcy.[15]

Yet, it is theoretically conceivable that the debtor can unite the group of creditors and extract from them joint concessions under the threat of bankruptcy. If such workouts are frictionless, our theory becomes trivial. As documented, e.g., by Asquith, Gertner, and Scharfstein (1994) and Gilson (1997), however, frictions in such negotiations are usually substantial and increase with the number of creditors. One important reason for these frictions is the hold-up problem of individual creditors, which is precisely the reason for institutionalised bankruptcy rules as discussed in Section 4. Further reasons (which we ignore in this paper) include the aggregation of asymmetric information among multiple creditors and the legal uncertainty accompanying

[15] The notion of "vis attractiva" ("attracting force") is one of the classical principles of bankruptcy theory going back to Roman times. It formulates the historical experience that creditors typically fail to reach agreement when left to their own devices, and that only bankruptcy - when the debtor gives up the right to his estate - has the force to bring them together.

out-of-court debt renegotiations, if individual creditors have the possibility of contesting the new arrangement in court.

Our model has been kept deliberately simple and parsimonious, but can easily be generalized to make it more realistic. The assumption that long-term asset value V is not verifiable has allowed us to focus solely on short-term debt contracts. In a more general model, V would have a verifiable and an unverifiable component. The verifiable component would give rise to long-term debt and to the possibility to reschedule short-term debt in debt renegotiation. The assumption that $\underline{V} \geq A$ has focused the analysis on the case of efficient ex-post continuation. Allowing more generally that $\underline{V} < A$ would allow us to include situations in which ex post there is no conflict of interest and the only question is how to divide the liquidation value of the firm. The fact that asset value A has been assumed to be deterministic has provided a simple reference point for the ex-post analysis. More generally, A should be stochastic, and all ex-post variables should be indexed on A.

In our model, all results are derived as parts of an optimal ex-ante contract between debtor and creditors. Strictly speaking this suggests that there is no need for a law. In practice, however, there may well be, if individuals are unable to join the initial grand contract and write contracts specifying procedures of collective behavior. In fact, this is the classical Rawlsian justification of legislation as a substitute for contracting in the "original position" (Rawls, 1971), an approach to law, and bankruptcy law in particular, that is wide-spread in legal thinking.[16] The classic text of Jackson (1986), for example, when exploring the foundations of bankruptcy law, only argues that a "collective system of debt collection law" is needed, relegating the issue of private contracting to a footnote.[17] Conceptually, our approach to the foundations of bankruptcy law does not go beyond this, our contribution is to make the hypothetical private contract explicit.

There still remains the question what bankruptcy courts actually do in contractual models like ours. One obvious such role of bankruptcy courts with great practical importance is that of a certification agency that verifies the firm's asset value A and the outstanding claims against the firm. We have gone beyond that and shown that courts are important in imposing an automatic stay to prevent a run for the assets and supervise the contractually agreed collective liquidation procedure. In fact, our model shows that there is an important conceptual distinction between out-of-bankruptcy negotia-

[16] See, e.g., Schwartz (1998) and more generally Schwartz and Scott (2003).

[17] "As such, it reflects the kind of contract that creditors would agree to if they were able to negotiate with each other before extending credit" (Jackson, 1986, p. 17).

tions and bankruptcy. These two events are both inefficient ex-post, but for different reasons: negotiations out of bankruptcy are inefficient because the parties have difficulties to coordinate and avoid free-riding. Bankruptcy has no coordination costs because the law forces the parties around one table, but since the court enforces the liquidation promise of the original contract, asset value is destroyed inefficiently. Going one step further, one could ask whether courts can also have a role in determining the distribution of the asset value to the creditors. If, for example, it is difficult ex ante to fix bankruptcy payouts B_i contractually because not everybody has joined the initial grand contract, the model might provide guidelines of the type $B_i = g_i(D_1, D_2)$ that determine bankruptcy payments as a function of nominal claims. Note that aggregate bankruptcy payout in our model is linked to total debt by $B = \varphi(\psi^{-1}(D))$ where $\psi(t) = t\varphi(t)$ (which is strictly monotone by Lemma 2). Bankruptcy courts might be used to implement such rules for individual payments.

Another line of research to develop our model further is to analyze the effect of different renegotiation procedures. We have considered a fixed bargaining game between debtor and creditors, and analyzed its implications for debt structure and bankruptcy. This bargaining game, bilateral exchange offers, captures some important features and frictions of debt renegotiations, but, in the spirit of Harris and Raviv (1995), it would be useful to compare this structure to others. For these and other extensions, our model may be a useful building block that allows to develop a more comprehensive theory of bankruptcy and ultimately contribute to a broader comparative analysis of the workings and effects of different bankruptcy laws.

6 Appendix

6.1 Proofs:

Proof of Proposition 1: We will first show that it is not optimal to set $B_1 < \frac{D}{B}(D_1 + B - D)$ (which is condition (8) in Lemma 1). Define the bounds derived in Lemma 1 as

$$V_L = \frac{B_1}{B - D + D_1} A$$
$$V_H = \frac{D - B_1}{D - D_1} A$$

and recall that if the contract satisfies (8), ex post the firm optimally induces

- outcome (a, a) if and only if $V \geq V_H$
- outcome (a, f) if and only if $V_L \leq V < V_H$
- outcome (f, f) if and only if $V < V_L$.

We need to show that setting (D_1, B_1) in the region defined by (8) makes the parties ex ante worse off than setting (D_1, B_1) in the area defined by (9). To do this, we vary B_1, D_1 for $0 < B_1 < B$ and $\frac{B}{D}B_1 + D - B < D_1 < D$ such that the investors stay on their participation constraint.

The investors' (aggregate) participation constraint is

$$(1 - q + qF(V_L))B + q(1 - F(V_H))D + q(F(V_H) - F(V_L))(B_1 + D - D_1) = I. \tag{46}$$

Hence, small changes dB_1 and dD_1 keep the investors indifferent iff

$$(F(V_H)-F(V_L))(dB_1-dD_1) = (D_1-B_1)f(V_H)dV_H+(D-B-D_1+B_1)f(V_L)dV_L \tag{47}$$

where $dV_i = \partial_{B_1}V_i dB_1 + \partial_{D_1}V_i dD_1$.

The total ex-ante deadweight loss under any choice (D_1, B_1) is

$$\begin{aligned} L &= q\left[\int_{\underline{V}}^{V_L}\left(\frac{B}{A}V - B\right)dF(V) + \int_{V_L}^{V_H}\left(\frac{D-D_1}{A}V - (D-D_1)\right)dF(V)\right] \\ &\quad +(1-q)(\frac{B}{A}EV - B) \end{aligned}$$

(remember that cash transfers constitute no loss). Hence, under the small change (dD_1, dB_1) we have, after some manipulations,

$$\begin{aligned} \frac{1}{q}dL &= (D-D_1)(\frac{V_H}{A} - 1)f(V_H)dV_H + (B + D_1 - D)(\frac{V_L}{A} - 1)f(V_L)dV_L \\ &\quad + \left(F(V_H) - F(V_L) - \int_{V_L}^{V_H}\frac{V}{A}dF(V)\right)dD_1 \\ &= (D-D_1)(\frac{V_H}{A} - 1)f(V_H)dV_H + (B + D_1 - D)(\frac{V_L}{A} - 1)f(V_L)dV_L \\ &\quad - \left((\frac{V_H}{A} - 1)F(V_H) - (\frac{V_L}{A} - 1)F(V_L) - \frac{1}{A}\int_{V_L}^{V_H}F(V)dV\right)dD_1 \end{aligned}$$

where we have used partial integration for the last equality. Combining the last formula with (47) shows, after some straightforward manipulations, that increasing D_1 increases L if (8) holds. Hence, at the optimum (D_1, B_1) is either on or to the left of the line defined by (8) or on the line segment $B_1 = 0$, $D_1 > D - B$. The latter possibility can easily be discarded. Hence, a contract satisfying (8) is not optimal.

A similar argument shows that (7) is not possible under an optimal contract.¥

Proof of Proposition 8: Assume that

$$B^* + D^* > 2A. \tag{48}$$

Because $B^* \leq A$ (feasibility), this condition immediately implies $D^* > A$. As argued in the main text, if none of the two creditors shall prefer a run to bankruptcy, we must have

$$B_1 > A - D^* + D_1 \tag{49}$$

$$B_1 < B^* - A + D_1. \tag{50}$$

Note that (49) and (50) imply $B_i \leq D_i$, $i = 1, 2$. The question is: Are conditions (49) and (50), plus the normalization $D_1 \geq D^*/2$, consistent with condition (9),

$$\frac{D^*}{B^*}D_1 + D^* - \frac{D^{*2}}{B^*} \leq B_1 \leq \frac{D^*}{B^*}D_1 - D^* + B^*,$$

which we know from Proposition 1 an optimal contract must satisfy?

Consider first the case $D^* > 2B^*$, for which Figure 4, which uses Figure 2, depicts condition (9) graphically. Condition (50) is compatible with (9) iff the straight line defined by (50) in $D_1 - B_1$ - space intersects the line $B_1 = \frac{D^*}{B^*}D_1 + \frac{D^*}{B^*}(B^* - D^*)$ in the positive orthant (see Figure 4). The intersection is given by

$$\begin{aligned} B^* - A + D_1 &= \frac{D^*}{B^*}D_1 + D^* - \frac{D^{*2}}{B^*} \\ \Leftrightarrow \quad D_1 &= D^* - B^* \frac{A - B^*}{D - B^*} \\ &> D^* - B^* \end{aligned}$$

Hence, the intersection is indeed in the positive orthant.

Condition (49) is compatibel with (9) iff the intersection of the straight line defined by (49) with the line $D_1 = D/2$ lies below the line $B_1 = B^*$ (see Figure 4). The intersection is given by

$$B_1 = A - D^* + \frac{D^*}{2} = A - \frac{D^*}{2}$$

and we have $A - D^*/2 < B^*$ by (48).

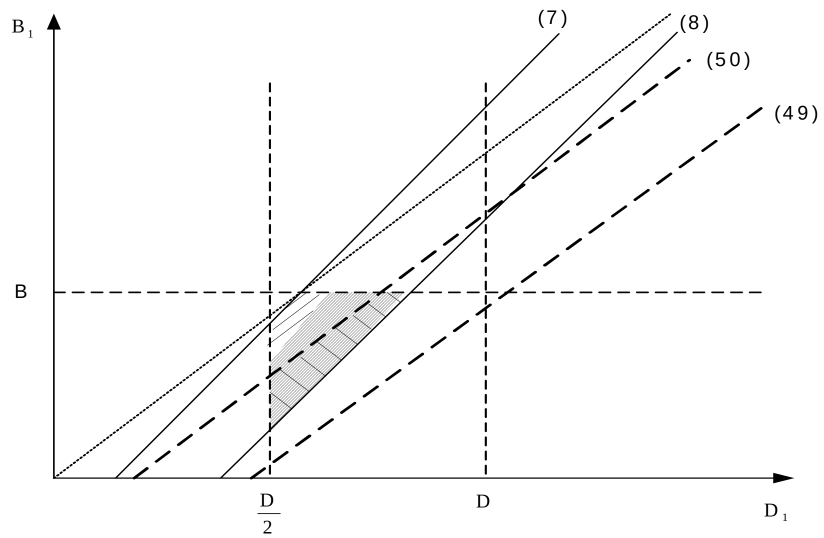

Figure 3: Compatibility of (49) and (50) with (9): $B < D < 2B$

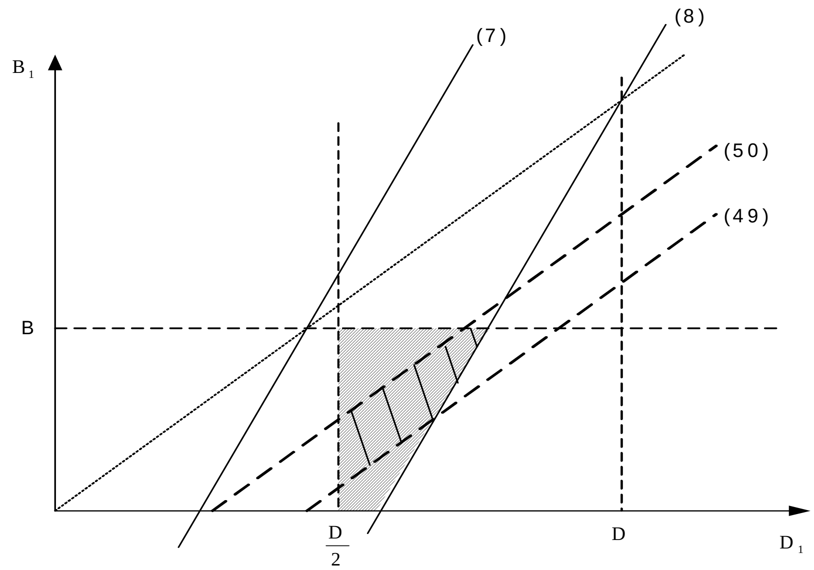

Figure 4: Compatibility of (49) and (50) with (9): $D > 2B$

The case $B^* < D^* \leq 2C^*$ is similar and omitted (see Figure 3 for an illustration).¥

6.2 Inefficient continuation: the case $\underline{V} < A$

We now briefly sketch how the analysis changes if the continuation of the firm can be inefficient ex post. Hence, we replace assumption (1) by the assumption $\underline{V} < A < \overline{V}$.

The main results do not change in this setting. However, some of the analysis becomes more complicated. Already Lemma 1 changes significantly, which reflects the different continuation incentives when $V < A$. In fact, the main insight for the case $V < A$ is that the debtor will always liquidate all his assets voluntarily ex post. This means that the debtor values an asset loss exactly like a cash loss. In particular, if $V < A$ the debtor prefers repayment (the cell (a, a)) over strategic bankruptcy (the cell (f, f)) iff $D < B$. This condition, and the analogous ones for asymmtric default, are different from the conditions in Lemma 1 and must therefore be added to the (already complicated) set of conditions in Lemma 1. However, a closer inspection shows that for the case of regime 3 (no partial liquidation), defined by (9), the additional restrictions mean very little. More precisely, if $D > B$, the default condition in regime 3 remains unchanged (i.e., the new conditions are slack); and for $D < B$, it becomes even simpler, becaus there never is strategic default.

Proposition 1 does not change at all when $\underline{V} < A$. The reason for why regime 3 is optimal ex ante is essentially the same as before: As seen in Proposition 1, it is strictly suboptimal to induce partial liquidation when $V > A$ for sure, because it induces too much liquidation compared to what it achieves on the incentives front. However, if $V < A$ ex post, liquidation is efficient and the parties are therefore indifferent ex ante whether to induce partial liquidation or not. Hence, as long as there is some probability that $V > A$, the parties will strictly prefer to induce regime 3.

For the analysis of Propositions 2 - 6, one additional argument is necessary when $\underline{V} < A$. As seen above, if $D < B$ the debtor will always repay D when solvent (regardless of V). Hence, there is no concern with strategic bankruptcy ex ante. This implies that it is optimal to increase D at the margin (which is costless in terms of incentives and liquidation loss) and lower B in exchange (which brings an efficiency gain if $V > A$ ex post). It follows that the optimal contract has $D \geq B$.

This now implies that the analysis remains essentially unchanged in the more general model. In fact, the debtor will default strategically if $V < A$ (because this costs him cash of B instead of cash of D - remember that he transforms his assets into cash, anyhow), and will default strategically if $A \leq V < tA$ (for the reasons discussed in Section 3.1). Taken together this means that the debtor will default strategically iff $V < tA$, exactly as before. The debtor's ex ante objective therefore is to maximize

$$(1-q)F(A)(A-B) + (1-q)\int_A^{\overline{V}}(1-\frac{B}{A})VdF(V) + qF(A)(Y + A - B)$$
$$+q\int_A^{At} Y + (1-\frac{B}{A})VdF(V) + q\int_{At}^{\overline{V}} Y - D + VdF(V)$$

which is a direct generalization of S_0 in (14). Hence, the deadweight loss to be minimized is

$$H(B,t) = \left[K + (1 - F(At))At + \int_A^{At} VdF(V)\right] B \tag{51}$$

where

$$K = \frac{F(A)}{q}A + \frac{1-q}{q}\int_A^{\overline{V}} VdF(V)$$

The function H as defined in (51) is exactly the same as in (30), taking into account that $F(A) = 0$ in the case of Section 3.2 and that liquidation for $V < A$ is no deadweight loss. Similarly, the participation constraint does not change at all, and therefore, the φ - function linking B and t remains unchanged in the more general framework. Hence, the analysis can be conducted as before. Intuitively, what happens is that all the realizations $V < A$ lead to voluntary liquidation by the debtor, which gives him verifiable funds of A. This is the same as assuming that the distribution of V has $\underline{V} = A$ with a mass point at A.

Put differently, liquidation is efficient for $V < A$ and inefficient for $V > A$. In the former case, liquidation by the creditors simply is a transfer, with no ex-post efficiency consequences. Qualitatively, in the design of the initial contract, the efficiency consideration of the case $\underline{V} \geq A$ is therefore the only one that matters. Quantitatively, shifting weight in the distribution of V to the left of A will, of course, change things. For example, an inspection of the φ - function in (29) shows that this shift will shift φ uniformly upward. But this is only to be expected: if the firm's prospects become worse, its debt capacity goes down, and its bankruptcy loss will increase.

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