Economics 204
Fall 2013
Problem Set 2
Due Tuesday, August 6 in Lecture

1. Show that the sequence $\{x_n\}$ in a metric space X converges to x if and only if every subsequence has x as a cluster point.

2. Let $A \subset \mathbf{R}^n$ be uncountable. Prove that there is a sequence of distinct points in A converging to a point of A.

3. Some practice with "relative" openness.

   (a) Given a metric space X, let Y be a metric subspace of X, and take any $A \subset Y$. Show that A is open in Y if and only if $A = O \cap Y$ for some open subset O of X, and is closed if $A = C \cap Y$ for some closed subset C of Y.

   (b) Let Y be open in X, prove that
   $$A \text{ is open in } X \text{ iff } A \text{ is open in } Y.$$

   (c) Can you given a example of either side of the implication in (b) not holding when Y is not necessarily open in X?

4. Prove the following
$$\overline{\operatorname{Int}\overline{\operatorname{Int} A}} = \overline{\operatorname{Int} A}$$

5. How many pairwise disjoint sets can one obtain using operators of closure and interior?

6. Some practice with continuity

   (a) Use the "pre-image of a closed set is closed" definition of continuity to show that $\{(x, y) |\ x^2 + y^2 \leq 1\} \subset \mathbf{R}^2$ is closed.

   (b) Suppose that $f : X \to Y$ is continuous. If x is a limit point of $A \subset X$, is it necessarily true that $f(x)$ is a limit point of $f(A)$? (Recall that a limit point of a set $A \subset X$ is defined as a point $x \in X$ such that $B_\varepsilon(x)$ contains some element of $A \setminus \{x\}$ for any $\epsilon > 0$.)

7. Let $f : \mathbf{R} \to \mathbf{R}$ be continuous function such that $|f(x) - f(y)| \geq |x - y|$ for all x and y. Prove that the range of f is all of $\mathbf{R}$.

8. Let (X, ρ) and (Y, σ) be metric spaces. Let $\{ f_n\}$ be a sequence of bijective functions from X to Y and $\{ g_n\}$ be the sequence of their uniformly continuous inverses. Prove that uniform convergence of $f_n \to f$ implies uniform convergence of $g_n \to g$, where g is a uniformly continuous inverse of f.

9. Show that if x_n and y_n are Cauchy sequences from a metric space X, then $d(x_n, y_n)$ converges.