Econ 204 – Problem Set 2

Due Tuesday, August 5

1. Prove that a convergent sequence in an arbitrary metric space (X, d) has exactly one cluster point.

2. The decimal expansion of $\frac{1}{7}$ is $0.142857142857142857\ldots$ etc. repeating forever. Suppose we construct the sequence $\{x_n\}$ by, for each n, x_n is the n^{th} decimal place in the infinite expansion of $\frac{1}{7}$. Prove that every sequence made up of the elements from the set $Y = \{1; 4; 2; 8; 5; 7\}$ is a subsequence of $\{x_n\}$.

3. Show whether the following are open, closed, both, or neither:

   (a) The interval $(0, 1)$ as a subset of $\mathbb{R}$.

   (b) The interval $(0, 1)$ embedded in $\mathbb{R}^2$ as the subset $\{(x, 0) : x \in (0, 1)\}$.

   (c) $\mathbb{R}$ as a subset of $\mathbb{R}$.

   (d) $\mathbb{R}$ embedded in $\mathbb{R}^2$ as the subset $\{(x, 0) : x \in \mathbb{R}\}$.

   (e) $\{(x, y, z) : 0 \leq x + y \leq 1, z = 0\}$ as a subset of $\mathbb{R}^3$

   (f) $\{(x, y) : 0 < x + y < 1\}\}$ as a subset of $\mathbb{R}^2$

   (g) $\{\frac{1}{n} : n \in \mathbb{N}\}$ as a subset of $\mathbb{R}$

   (h) $\{\frac{1}{n} : n \in \mathbb{N}\}$ as a subset of the interval $(0, \infty)$

4. Let A be a subset of a metric space. Prove that $\text{int}(\text{int}(A)) = \text{int}(A)$.

5. Let X denote the set of all bounded infinite sequences of real numbers $\{a_n\}_{n=1}^{\infty}$ (hereafter denoted simply as a_n). Define the "distance" between two sequences a_n and b_n to be: $d(a_n, b_n) = \sum_{n=1}^{\infty} 2^{-n}|a_n - b_n|$. Show that (X, d) is a metric space.