Econ 204 – Problem Set 4

Due Tuesday, August 12

1. Similarly as it's defined in class, let $C([0,1])$ be the set of all continuous functions whose domain is the unit interval $[0,1]$ and range is $\mathbb{R}$. Let Φ be the subset consisting of all real polynomials (whose domain is restricted to the unit interval) of degree at most two:

$$\Phi \equiv \{\, a + bx + cx^2 \,|\, a, b, c \in \mathbb{R} \}$$

   Note that the set $C([0,1])$ is a vector space over the field of real numbers and the subset Φ is a proper subspace.

   (a) Are the vectors $\{\, x,\ (x^2 - 1),\ (x^2 + 2x + 1) \,\}$ linearly independent over $\mathbb{R}$?

   (b) Find a Hamel basis for the subspace Φ.

   (c) What is the dimension of Φ ? Show that $C([0,1])$ is not finite dimensional!

2. Let be λ a given eigenvalue of A. Let be the eigenspace corresponding to λ the set of the eigenvectors corresponding to λ. Prove that the eigenspace of A for a given eigenvalue is a vectorspace.

3. Let T be an invertible linear transformation. Prove that its inverse is a linear transformation.

4. Let V have finite dimension greater than 1. Prove whether or not the set of non-invertible operators is a subspace of $L(V, V)$.

5. Let A be an nxn matrix with n equal eigenvalues. Show that A is diagonalizable iff A is already diagonal.

6. Suppose that V is finite dimensional and $T, S \in L(V, V)$. Prove that TS is invertible if and only if both T and S are invertible.

7. Prove that λ is an eigenvalue of a matrix A iff it is an eigenvalue of the transpose of A.