Appendix to:
A Plucking Model of Business Cycles

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A. Defining Expansions and Contractions

A.1. Defining Expansions and Contractions

Since our empirical analysis is based on the amplitude and speed of cyclical movements in unemployment, we define business cycle peaks in troughs in such a way that they line up exactly with peaks and troughs of the unemployment rate. This yields business cycle dates that are very similar to but not identical to those identified by the NBER Business Cycle Dating Committee (because the NBER Business Cycle Dating Committee uses a wide variety of cyclical indicators beyond unemployment to date turning points).

We develop a simple algorithm that defines business cycle peak and trough dates for the unemployment rate. The basic idea is to find local minima and maxima of the unemployment rate. However, we ignore small "blips" or "wiggles" in the unemployment rate and focus instead on delineating substantial swings in the unemployment rate in a similar manner as the peaks and troughs identified by the NBER Business Cycle Dating Committee.

Table A.1 presents the peak and trough dates we identify and compares them with the peak and trough dates identified by the NBER.

A.2. An Algorithm for Defining Expansions and Contractions

Let u_t denote the unemployment rate at time t. The algorithm begins by taking the first month of our sample as a candidate for a business cycle peak, cp. If, in all the following months until unemployment becomes X percentage points higher than u_{cp}, unemployment is higher than u_{cp}, we confirm that cp is a business cycle peak. If, instead, the unemployment rate falls below u_{cp} before it is confirmed as a peak, the month in which this happens becomes the new candidate peak. Once we have identified a peak, we switch to looking for a trough (in the analogous manner) and so on until we reach the end of the sample. Formally, starting with $t = 1$ the algorithm is:

1. Set $cp = t$ and set $t = t + 1$ (i.e., move to the next time period).
2. If $u_t < u_{cp}$ go back to step 1
3. If $u_{cp} \leqslant u_t < u_{cp} + X$ set $t = t + 1$ and go back to step 2
4. If $u_t \geqslant u_{cp} + X$ add cp to the set of peaks
5. Set $ct = t$ and set $t = t + 1$
6. If $u_t > u_{ct}$ go back to step 5
7. If $u_{ct} \geqslant u_t > u_{ct} - X$ set $t = t + 1$ and go back to step 6
8. If $u_t \leqslant u_{ct} - X$ add ct to the set of troughs, and go back to step 1

We set $X = 1.5$ percentage points. With this value, our algorithm generates the same set of expansions and contractions as the NBER Business Cycle Dating Committee with one exception: Our algorithm considers the 1979-1982 double-dip recession as a single contraction as opposed to two contractions interrupted by a brief and small expansion (unemployment decreased by 0.6 percentage points in 1980-1981). Values for X between 0.8 and 1.5 percentage points identify exactly the same cycles. Values of X larger than 1.5 drop the 1970-1973 expansion.

An advantage of our algorithm is that it does not impose a duration upon expansions and contractions but only a size X, in contrast to other algorithms based on turning points like the Bry and Boschan (1971) routine. Our algorithm can therefore also be used to define expansions and contractions in our model simulations, even for models that do not match the duration of expansions and contractions in the real-world data.

A.3. Peak and Trough Dates from 1948 to 2020

Table A.1 presents the peak and trough dates we identify. For comparison purposes, we also present the peak and trough dates identified by the NBER. We identify the same set of expansions and contractions as the NBER Business Cycle Dating Committee with one exception: we consider the 1979-1982 double-dip recession as a single contraction as opposed to two contractions interrupted by a brief and small expansion (unemployment decreased by 0.6 percentage points in 1980-1981). The exact timing of the NBER peaks and troughs do not line up exactly with ours for the reasons discussed above. However, in most cases, our dates are quite similar to theirs. The NBER peaks tend to lag our peaks

Table A.1: Business Cycle Peaks and Troughs

	Unemployment		NBER	
	Peak	Trough	Peak	Trough
1	[1/1948]	10/1949	11/1948	10/1949
2	5/1953	9/1954	7/1953	5/1954
3	3/1957	7/1958	8/1957	4/1958
4	2/1960	5/1961	4/1960	2/1961
5	9/1968	12/1970	12/1969	11/1970
6	10/1973	5/1975	11/1973	3/1975
7a	5/1979		1/1980	7/1980
7b		11/1982	7/1981	11/1982
8	3/1989	6/1992	7/1990	3/1991
9	4/2000	6/2003	3/2001	11/2001
10	10/2006	10/2009	12/2007	6/2009
11	9/2019		2/2020	

Note: Business cycle peaks and troughs defined solely based on the unemployment rate and, for comparison, business cycle peaks and troughs as defined by the Business Cycle Dating Committee of the National Bureau of Economic Research.

Table A.2: The Duration of Expansions and Contractions

	Dates		Length in Months	
	Peak	Trough	Expansion	Contraction
1	[1/1948]	10/1949		21
2	5/1953	9/1954	43	16
3	3/1957	7/1958	30	16
4	2/1960	5/1961	19	15
5	9/1968	12/1970	88	27
6	10/1973	5/1975	34	19
7	5/1979	11/1982	48	42
8	3/1989	6/1992	76	39
9	4/2000	6/2003	94	38
10	10/2006	10/2009	40	36
11	9/2019		119	
Mean			59.1	26.9

by a few months and the NBER troughs tend to precede our troughs by a few months. This implies that our estimate of the average duration of contractions is about one year longer than what results from the NBER's dating procedure. We identify September 2019 as a peak as opposed to February 2020 because the unemployment rate first hit 3.5% in September 2019. When several months are tied for the lowest unemployment rate at the end of an expansion, our algorithm picks the first of these months as the peak (and similarly for troughs). Table A.2 lists the duration of all expansions and contractions over our sample period.

B. Solution Method

B.1. Normalization of μ

Recall that the matching function is Cobb-Douglas. The vacancy-filling rate is therefore $q_t = \mu\theta^{-\eta}$. Furthermore, the job finding rate is $f_t = \theta_t q(\theta_t)$. Combining these equations allows us to express the vacancy-filling rate as a function of the job-finding rate:

$$q_t = \mu^{\frac{1}{1-\eta}} (f_t)^{\frac{-\eta}{1-\eta}}. \tag{B.1}$$

We can now see that there is a one-to-one mapping between the cost of hiring a worker $C_t \equiv c/q_t$ and the job-finding rate f_t:

$$C_t \equiv \frac{c}{q_t} = \left(c\mu^{\frac{-1}{1-\eta}}\right) (f_t)^{\frac{\eta}{1-\eta}}. \tag{B.2}$$

This mapping can be used to write the equilibrium conditions of the model in terms of either the cost of hiring a worker or the job-finding rate, without reference to the other (and without reference to labor market tightness). When the model is written in this way (e.g., in terms of the cost of hiring a worker), the parameters c and μ only enter the model though the composite term $c\mu^{\frac{-1}{1-\eta}}$. This implies that we can normalize either c or μ without loss of generality. We choose to normalize $\mu = 1$. Intuitively, only the cost of hiring a worker matters to a firm. It is immaterial to the firm whether this cost consists of posting few vacancies that fill with a high probability but are expensive to post, or of posting many vacancies that fill with a low probability but are inexpensive to post.

B.2. Solving for the Policy Functions

To solve for the policy function under Nash-bargaining, we follow the solution method of Fujita and Ramey (2012) to solve for the functions $J^{Nash}(A, x)$ and $q(A)$. The state-space consists of the two exogenous states A and x. We discretize the AR(1) process for A_t using the Rouwenhorst (1995) method with 11 grid points, and the AR(1) process for x_t using the Tauchen (1986) method with 201 grid points. Combining equations (10) and (11), we can solve for the functions J^{Nash} and $1/q$ by iteration on the policy functions. Specifically, given guesses on the functions J and $1/q$ (and therefore f), we use these guesses to calculate the expected terms on the RHS of equation (11) and update J^{Nash}. We then update $1/q$ using equation (10). We iterate until convergence.

Under DNWR, we first solve for the Nash wage as a function of the state (A, x) by solving the model under Nash-bargaining. This gives the Nash wage under the assumption that wages will be flexible at all future dates, including wages in new matches that are relevant to determine the outside option of workers. Under DNWR, the value function J depends on the two exogenous states x and A, and the new endogenous state of the lagged real wage w_{-1}, $J(A, x, w_{-1})$. Under the assumption on the Nash-bargained wage, J is independent of the state w_{-1}^{new}, so that for numerical considerations, the state-space is only three-dimensional.

The recursion on J is the same as (4), up to the new dependence of the value function on the new state w_{-1}:

$$J(A, x, w_{-1}) = \max\{J^c(A, x, w_{-1}), 0\}, \tag{B.3}$$

where J^c is the value if the match is continued, which solves the recursion

$$J^c(A, x, w_{-1}) = xA - w(A, x, w_{-1}) + \beta(1-\delta)E\left(J(A', x', w)\right), \tag{B.4}$$

where the real wage w is given by equation (13). Equations (B.3)-(B.4) allow to solve for J by iteration. We again use 11 points on the A dimension, 201 on the x dimension, and 401 grid points on the new endogenous dimension w_{-1}. When iterating on equation (B.4), calculating the expected term on the RHS requires to evaluate the value function J at values of the endogenous state w that are not on the grid. We rely on linear interpolation to do so.

Once J is solved for, we can obtain $1/q(A, w_{-1}^{new})$ from J and the free-entry condition (10) which now depends on the new state w_{-1}^{new},

$$J(A, x^{hire}, w_{-1}^{new}) = \frac{c}{q(A, w_{-1}^{new})}. \tag{B.5}$$

B.3. Calculating Separation

Because matches can be endogenously terminated, the destruction rate s_t depends on the cross-sectional distribution of employment across matches' states. Calculating the destruction rate in simulations of the model therefore requires us to keep track of the distribution of employment across matches'states. Under Nash-bargaining, matches' states reduce to match productivity x_t. We follow the method in Fujita and Ramey (2012) to keep track of the distribution of employment across x_t and calculate the destruction rate, only adapting it to any Markovian process for x so that it can accommodate our AR(1) assumption (2) on x_t.

Define $n_t(x)$ the number of employed workers at productivity x, and n_t the vector of $n_t(x)$. (Note that our n_t is the density of the distribution of employment, while Fujita-Ramey's e_t on p.75-77 is the CDF.) We therefore have:

$$u_t = 1 - \sum_x n_t(x). \tag{B.6}$$

Define $n_t^0(x)$ the number of workers employed at productivity x at the beginning of period t, after shocks and exogenous separation have occurred, but before endogenous separation has occurred. Denote n_t^0 the vector of $n_t^0(x)$. We have:

$$n_t^0 = (1-\delta)(T^x)' n_{t-1} \tag{B.7}$$

where T^x is the transition matrix of the Markovian process of x. Define $fired_t$ the number of workers fired at t. It solves:

$$fired_t = \sum_x n_t^0(x) 1_{J_t(x)=0}. \tag{B.8}$$

The job-destruction rate s_t is given by:

$$s_t = \delta + \frac{fired_t}{1-u_{t-1}}. \tag{B.9}$$

The new distribution of employment at t solves the recursion:

$$n_t(x) = n_t^0(x) 1_{J_t(x)>0} + f_t(s_t + (1-s_t)u_{t-1}) 1_{x=x^{hire}}. \tag{B.10}$$

Under DNWR, calculating the destruction rate requires to keep track of the distribution of employment along both match productivity x and wages w. We do so in the following way. Let $m_{t-1}(x^*, w^*) = P(x_{t-1} = x^*, \frac{w_{t-1}}{\Pi} \leqslant w^*)$ be the number of matches at $t-1$ with idiosyncratic productivity $x_{t-1} = x^*$ and a real wage less than Πw^*. Considering the number of real wages below Πw^* instead of below w^* is for convenience: This way it gives the number of matches with real wages below w^* at the beginning of period t, after inflation from $t-1$ to t has eroded lagged real wages. Note that $m_{t-1}(x^*, \infty)$ is the number of firms with idiosyncratic productivity $x_{t-1} = x^*$ at $t-1$, and $\sum_{x*} m_{t-1}(x^*, \infty)$ is employment at $t-1$.

Denote $m_t^0(x^*, w^*) = P(x_t = x^*, \frac{w_{t-1}}{\Pi} \leqslant w^*)$ the number of matches with idiosyncratic productivity $x_t = x^*$ and inherited real wage less than w^* at the beginning of period t, after match-specific productivity shocks and exogenous separation shocks have hit but before any wage-adjustment. It is given by

$$m_t^0 = (1-\delta) T_x' m_{t-1} \tag{B.11}$$

where T_x is again the transition matrix of the Markovian process of x.

We now keep track of how wage adjustments change the distribution of wages under DNWR. Denote $m_t^1(x^*, w^*)$ the number of wages with $x_t = x^*$ and inherited real wage less than w^* after wage adjustments. It is the same as m_t^0, except that all wages below $w_t^{Nash}(x^*)$ are reset to $w_t^{Nash}(x^*)$, i.e.

$$m_t^1(x^*, w^*) = 0 \text{ for all } w^* \leqslant w_t^{Nash}(x^*). \tag{B.12}$$

We now calculate the number of endogenously terminated matches, and keep track of how it affects the distribution of wages. Denote $w^{thresh}(x^*)$ the threshold on wages above which matches with productivity x^* are terminated. It is defined as the lowest wage w such that $J(x^*, w) = 0$. The number $fired_t(x^*)$ of matches with productivity x^* that are terminated is $m_t^1(x^*, \infty) - m_t^1(x^*, w^{thresh}(x^*))$. Knowing the number of exogenously and endogeneously separated matches we can calculate the separation rate as:

$$s_t = \frac{\sum_x fired_t(x)}{N_{t-1}} + \delta. \tag{B.13}$$

Denote $m_t^2(x^*, w^*)$ the number of wages with $x_t = x^*$ and inherited real wage less than w^* of wages after endogenous separation. It is the same as m_t^1, except that it no longer includes wages above $w^{thresh}(x^*)$, i.e.

$$m_t^2(x^*, w^*) = m_t^1(x^*, w^{thresh}(x^*)) \text{ for all } w^* \geqslant w^{thresh}(x^*). \tag{B.14}$$

We now keep track of how new hires affect the distribution of wages. Denote $m_t^3(x^*, w^*)$ the number of wages with $x_t = x^*$ and inherited real wage less than w^* after hiring. It is the same as m_t^2, except that it adds the number of new hires at productivity x^{hire} and hiring wage w_t^{hire}, i.e.

$$m_t^3(x^{hire}, w^*) = m_t^2(x^{hire}, w^*) + f_t(1 - (1 - s_t)N_{t-1})\text{for all} w^* \geqslant w_t^{hire}(x^{hire}). \tag{B.15}$$

This is the distribution of effective real wage in period t. Employment at t is therefore given by $\sum_{x^*} m_t^3(x^*, \infty)$.

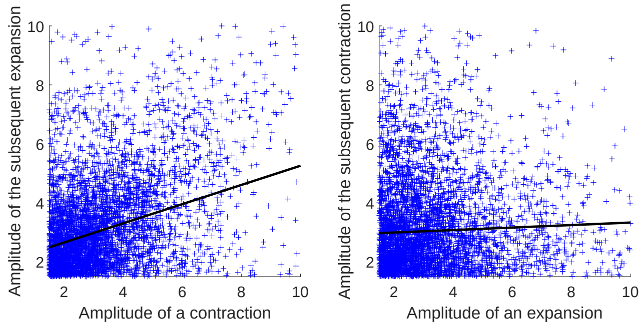
Finally, we keep track of the eroding effect of inflation and growth from t to $t+1$ to get $m_t(x^*, w^*)$ and be able to start the whole process in period $t+1$. We have that

$$m_t(x^*, w^*) = m_t^3(x^*, \Pi \times w^*). \tag{B.16}$$

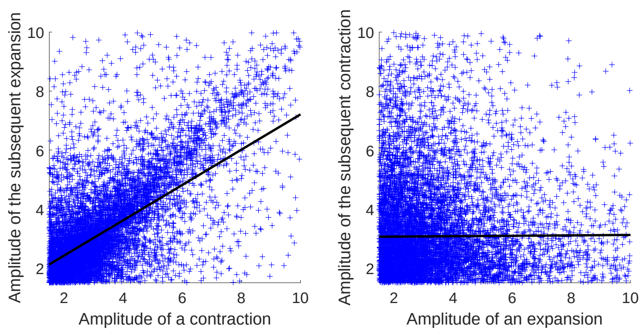
We calculate it by linear interpolation.

C. Plucking Scatter Plots

Figure C.1 below illustrates our plucking regression results graphically. It presents scatter plots for the regressions discussed in Table 3.



(a) Fujita-Ramey Model under Flexible Wages (Nash Bargaining)



(b) Fujita-Ramey Model under DNWR

Figure C.1: Plucking Scatter Plots

Note: The figure displays the scatter plots associated with the plucking regressions for the Fujita-Ramey model under flexible wages (panel a) and the Fujita-Ramey model under DNWR (panel b). The plots feature all the expansion/contraction pairs obtained by pooling together 500 samples of 866 months. OLS regression lines are plotted in each panel.

D. The Volatility of Aggregate Shocks and the Average Level of Unemployment

Figure D.2 plots the average level of the unemployment rate in our plucking model as a function of the volatility of aggregate shocks. The model has the property that average unemployment increases with the volatility of the aggregate shocks, from a steady-state level of 3.1%. Average unemployment increases steeply with the volatility of shocks. A previous version of the paper showed than with decreasing returns to labor instead of constant returns to labor, the increase was not as strong because decreasing returns to labor make firms able to withstand larger shocks under DNWR without being willing to lay off all their workers, as explained in Appendix H.

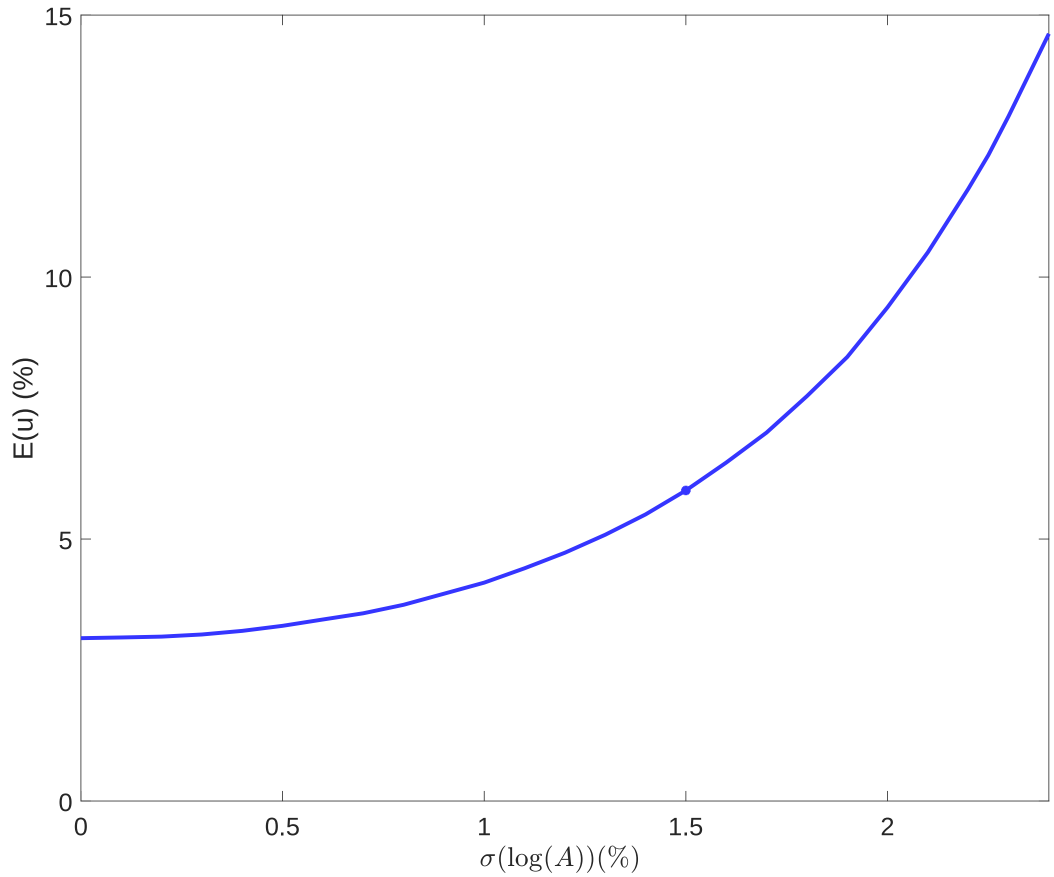


Figure D.2: Average Unemployment and the Volatility of Aggregate Shocks

Note: The figure gives the average rate of unemployment as a function on the standard deviation of aggregate shocks in the DNWR model of section 3.

E. Results for Untruncated Cycles

Table E.3 reports the same simulation results as Table 3 when using all expansions and contractions, including those of more than 6.5 percentage points.

F. Robustness to a Lower Value of δ

Table F.4 gives the same statistics as in Table 3 for the model under AR(1) with the alternative calibration of $\delta = 1\%$. In this alternative calibration, all other parameters are the same, except that we re-calibrate γ, σ_a and σ_x in order to still match an average unemployment rate of 5.7%, a standard deviation of unemployment of 1.6pp, and a rate of exit from employment of 2.0%. This gives $\gamma = 0.62$, $\sigma_a = 2.0\%$ and $\sigma_x = 3.9\%$ with

Table E.3: Simulation Results with Untruncated Cycles: Plucking Property, Speed, and Duration

	Data	AR(1)		AR(2)		AR(2) + Job Ladder	
		Nash	DNWR	Nash	DNWR	Nash	DNWR
Subsequent expansion	1.12	0.51	0.80	0.74	0.80	0.68	0.76
on contraction, β		(0.41)	(0.25)	(0.27)	(0.21)	(0.26)	(0.21)
Subsequent contraction	-0.38	-0.08	-0.07	-0.04	-0.04	-0.02	-0.03
on expansion, β		(0.39)	(0.24)	(0.20)	(0.10)	(0.22)	(0.15)
Subsequent expansion	0.59	0.28	0.69	0.55	0.62	0.47	0.64
on contraction, R^2		(0.29)	(0.27)	(0.28)	(0.27)	(0.26)	(0.26)
Subsequent contraction	0.22	0.05	0.03	0.02	0.00	0.02	0.01
on expansion, R^2		(0.13)	(0.07)	(0.06)	(0.02)	(0.07)	(0.03)
Speed of expansions	0.87	1.61	2.77	0.83	2.50	0.65	1.33
(pp / year)		(0.71)	(0.82)	(0.27)	(0.42)	(0.13)	(0.33)
Speed of contractions	1.89	1.61	5.10	0.80	4.20	0.64	1.86
(pp / year)		(0.87)	(1.72)	(0.29)	(0.88)	(0.13)	(0.39)
Duration of expansions	59.1	37.5	28.4	90.4	39.6	99.6	63.5
(months)		(11.7)	(6.3)	(15.6)	(3.3)	(15.8)	(6.8)
Duration of contractions	26.9	38.0	18.7	91.6	24.7	99.1	42.4
(months)		(11.9)	(5.2)	(16.0)	(2.5)	(15.9)	(5.9)

Note: The table compares data from the model under AR(1) shocks, from the model under AR(2) shocks, and from the model under AR(2) shocks and a job ladder, in each case both with Nash bargaining and with downward nominal wage rigidity (DNWR). The first (third) row reports the coefficient (R^2) in an OLS regression of the size of an expansion (percentage point fall in unemployment rate) on the size of the previous contraction (percentage point increase in unemployment rate). The second (fourth) row report the coefficient (R^2) in an analogous regression of the size of a contraction on the size of the previous expansion. The next two rows report the average speed of expansions and contractions, measured in percentage points of unemployment per year. The final two rows report the average duration of expansions and contractions, measured in months. For the version of the model without a job ladder, the reported point estimate is the median value of the statistic over 5000 samples of 866 periods each (the length of our sample of real-world data). For the model under AR(2) shocks and a job ladder, results are shown for 1000 samples of 5*866 periods each, to avoid samples with no or few cycles of less than 6.5 percentage points. The standard error reported in parentheses is the standard deviation of the estimates across the 5000 (or 1000) samples.

Nash bargaining, and $\gamma = 0.41$, $\sigma_a = 1.6\%$ and $\sigma_x = 2.7\%$ with DNWR. The results with $\delta = 1.9\%$ are added to the table for comparison. The calibration with $\delta = 1\%$ generates if anything more plucking than with $\delta = 1.9\%$ under DNWR, and less plucking under Nash bargaining.

G. Job Ladder

Recall T_x is the $n_x \times n_x$ transition matrix for the discretized process for idiosyncratic productivity (2) in the baseline version of the model. We add $M + 1$ new states to capture the trial period of new hires. New hires start at the middle productivity, but for M

Table F.4: Simulation Results: Robustness to $\delta = 1\%$

	Data	Fujita-Ramey Model, AR(1)			
		$\delta = 1\%$		$\delta = 1.9\%$	
		DNWR	Nash	DNWR	Nash
Subsequent expansion	1.12	0.69	0.30	0.64	0.35
on contraction, β		(0.23)	(0.44)	(0.27)	(0.43)
Subsequent contraction	-0.38	-0.04	-0.03	-0.05	-0.04
on expansion, β		(0.25)	(0.48)	(0.31)	(0.44)
Subsequent expansion	0.59	0.49	0.14	0.42	0.16
on contraction, R^2		(0.25)	(0.24)	(0.26)	(0.24)
Subsequent contraction	0.22	0.03	0.08	0.04	0.07
on expansion, R^2		(0.08)	(0.18)	(0.11)	(0.17)
Speed of expansions	0.87	3.07	1.46	2.53	1.56
(pp / year)		(0.76)	(0.69)	(0.73)	(0.64)
Speed of contractions	1.89	5.39	1.45	4.56	1.56
(pp / year)		(1.28)	(0.82)	(1.51)	(0.78)
Duration of expansions	59.1	22.4	37.6	27.4	35.8
(months)		(4.4)	(12.5)	(6.7)	(12.1)
Duration of contractions	26.9	13.4	38.5	18.3	36.1
(months)		(3.4)	(12.7)	(5.5)	(12.1)

Note: The table gives the same results as Table 3 in the paper for the Fujita-Ramey model, both under the calibration $\delta = 1.9\%$ used in the paper, and for a lower calibration of $\delta = 1\%$. The first (third) row reports the coefficient (R^2) in an OLS regression of the size of an expansion (percentage point fall in unemployment rate) on the size of the previous contraction (percentage point increase in unemployment rate). The second (fourth) row report the coefficient (R^2) in an analogous regression of the size of a contraction on the size of the previous expansion. The next two rows report the average speed of expansion and contractions, measured in percentage points of unemployment per year. The final two rows report the average duration of expansions and contractions, measured in months. For the models, the reported point estimate is the median value of the statistic over 5000 samples of 866 periods each (the length of our sample of real-world data). Expansions and contractions of more than 6.5 percentage points are excluded from the samples. The standard error reported in parentheses is the standard deviation of the estimates across the 5000 samples.

periods face a probability d of seeing their productivity shrink to zero (the firm will then endogenously choose to lay them off). During their trial periods, their productivity remains constant at the middle productivity level at which they started when hired—this allows to keep the number of states manageable. But once they have spent M periods at the firm, their trial period is over and their productivity evolves according to the transition matrix T_x.

The corresponding $(n_x + S + 1) \times (n_x + S + 1)$ transition matrix $\tilde{T}_x$ is $\tilde{T}_x =$

$[A_{(M+1)*(M+1)}, B_{(M+1)*n_x}; 0_{n_x*(M+1)}, T_x]$, where the matrix A is

$$A = \begin{bmatrix} 1 & 0 & ... & ... & ... & ... & 0 \\ d & 0 & (1-d) & 0 & ... & ... & 0 \\ d & 0 & 0 & (1-d) & 0 & ... & 0 \\ ... & ... & ... & ... & ... & ... & ... \\ d & 0 & ... & ... & 0 & (1-d) & 0 \\ d & 0 & ... & ... & ... & 0 & 0 \end{bmatrix}, \quad \text{(G.1)}$$

and the matrix B is the matrix with zeros everywhere except in the middle column of the last row where it is $(1-d)$. State 1 has productivity 0, states 2 to $(M+1)$ have average productivity (since we assume newly hired workers start at average productivity), and new hires start in state 2.

H. Sensitivity to Large Shocks under Constant Returns to Labor

Under DNWR and constant returns to labor, large negative shocks lead firms to lay off a large number of their workers instead of maintaining them at a frozen wage. Indeed, under constant returns to labor, the firm's flow value of labor is $A - w$, which is independent of the number of employees working for the firm. If the firm faces a large negative shock to A that is quite persistent (as shocks are when shocks follow an AR(2) in particular) the firm will want to lay off not just a few workers but all its workers. The feature is specific to constant returns to labor and disappears under decreasing returns to labor. With decreasing returns, the marginal product of workers rises as the firm lays off workers. As a result, the firm only lays off a portion of its workforce and keeps the other at a frozen wage.

Decreasing returns to labor also allow to move away from the Hagedorn and Manovskii (2008) calibration while keeping hiring pro-cyclical. Recall that in the standard DMP model low values of the flow value of unemployment z imply that the firms' value function increases little with productivity, and hiring is therefore close to acyclical—the Shimer (2005) unemployment volatility puzzle. With DNWR this problem can become even worse, making hiring *counter-cyclical*. In this case higher productivity has two effects: it increases the current flow value to firms $A - w$, but it also increases wages, increasing the probability that the DNWR constraint will bind in the future. For a low value of z, the first effect is small—the root of the Shimer puzzle—and the second effect can dominate.

Decreasing returns bring a third effect into play. While higher productivity today still increases wages and makes it more likely that the DNWR constraint will bind in the future, the flow value of firms is now $AF'(N) - w$. If the DNWR constraint will be binding in the future, employment N will also be lower at that point, raising the marginal productivity $F'(N)$ of workers. The job-finding rate is therefore easily procyclical under decreasing returns to labor, even away from the Hagedorn Manovskii calibration.