Online Appendix for:

Monetary Non-Neutrality in a Multi-Sector Menu Cost Model

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A Profit Function

Cost minimization by firm z implies that labor demand and demand for the composite intermediate input be governed by

$$\frac{W_t}{P_t} = (1 - s_m) A_t L_t(z)^{-s_m} M_t(z)^{s_m} \Omega_t(z),$$
$$1 = s_m A_t L_t(z)^{1-s_m} M_t(z)^{s_m - 1} \Omega_t(z),$$

where $\Omega_t(z)$ denotes the marginal costs of firm z at time t. Combining these two equations yields

$$\frac{W_t}{P_t} = \frac{1 - s_m}{s_m} \frac{M_t(z)}{L_t(z)}. \tag{1}$$

The real value of firm z's profits in period t are

$$\Pi_t^R(z) = \left(\frac{p_t(z)}{P_t}\right) y_t(z) - \left(\frac{W_t}{P_t}\right) L_t(z) - M_t(z) - \chi \left(\frac{W_t}{P_t}\right) I_t(z).$$

Using this equation (1) we can rewrite these profits as

$$\Pi_t^R(z) = \left(\frac{p_t(z)}{P_t}\right) y_t(z) - \frac{1}{1 - s_m} \left(\frac{W_t}{P_t}\right) L_t(z) - \chi \left(\frac{W_t}{P_t}\right) I_t(z).$$

Combining the production function—equation (8) in the paper—and equation (1) yields

$$L_t(z) = \left(\frac{y_t(z)}{A_t(z)}\right) \left(\frac{s_m}{1 - s_m}\right)^{-s_m} \left(\frac{W_t}{P_t}\right)^{-s_m}.$$

Using this equation, we can rewrite profits as

$$\Pi_t^R(z) = \left(\frac{p_t(z)}{P_t}\right) y_t(z) - (1 - s_m)^{s_m - 1} s_m^{-s_m} \left(\frac{W_t}{P_t}\right)^{1 - s_m} \left(\frac{y_t(z)}{A_t(z)}\right) - \chi \left(\frac{W_t}{P_t}\right) I_t(z) - U. \tag{2}$$

Using the firm's demand curve—equation (12) in the paper—and the labor supply curve—equation (7) in the paper—we can rewrite profits as

$$\Pi_t^R(z) = Y_t \left(\frac{p_t(z)}{P_t}\right)^{1-\theta} - (1-s_m)^{s_m-1} s_m^{-s_m} \omega^{1-s_m} L_t^{\psi(1-s_m)} C_t^{\gamma(1-s_m)} \left(\frac{1}{A_t(z)}\right) Y_t \left(\frac{p_t(z)}{P_t}\right)^{-\theta} - \chi \omega L_t^{\psi} C_t^{\gamma} I_t(z) - U.$$

Finally, log-linear approximations of $Y_t = C_t + \int_0^1 M_t(z)dz$, the production function and labor supply around the steady state with flexible prices yield $\hat{Y}_t = a_1 \hat{C}_t$ and $\hat{L}_t = a_2 \hat{C}_t$. Here $\hat{Y}_t = \log(Y_t/Y)$ and Y denotes the steady state of Y_t with flexible prices. $\hat{C}_t$ and $\hat{L}_t$ are defined analogously. Using these log-linear approximations and the fact that $C_t = S_t/P_t$, we can rewrite profits as a function of $(A_t(z), p_{t-1}(z)/P_t, S_t/P_t)$ and $p_t(z)$.

B Stationary Distribution

We solve for the stationary distribution over the state space of the firm's problem using the following algorithm:

0. Start with an initial distribution $Q(A(z), p_{-1}(z)/P, S/P)$. We use a uniform distribution as our initial distribution.
1. Map $Q(A(z), p_{-1}(z)/P, S/P)$ into $Q(A(z), p(z)/P, S/P)$ using the policy function F.
2. Map $Q(A(z), p(z)/P, S/P)$ into $Q(A_{+1}(z), p(z)/P, S/P)$ using the transition probability matrix for the technology process.
3. Map $Q(A_{+1}(z), p(z)/P, S/P)$ into $Q(A_{+1}(z), p(z)/P, S_{+1}/P)$ using the probability transition matrix for the nominal aggregate demand process.
4. Map $Q(A_{+1}(z), p(z)/P, S_{+1}/P)$ into $Q(A_{+1}(z), p(z)/P_{+1}, S_{+1}/P_{+1})$ using the function Γ.
5. Check whether $|Q(A_{+1}(z), p(z)/P_{+1}, S_{+1}/P_{+1}) - Q(A(z), p_{-1}(z)/P, S/P)| < \xi$ where $|\cdot|$ denotes a sup-norm. If so, stop. If not, go back to step one.

C A Model with Capital

Consider an extension of the model presented in section 2 of the paper in which firms use capital as well as labor and intermediate inputs to produce goods. The presence of capital affects the equilibrium behavior of this type of model primarily by affecting the cyclicality of marginal costs. If the marginal product of capital is highly variable over the cycle, this will raise the cyclicality of firms' marginal costs and thereby reduce the amount of monetary non-neutrality generated by the model. In the language of section 5 of the paper, capital may generate Ω-type strategic substitutability.

Capital adjustment costs make the capital stock adjust sluggishly to variations in the marginal product of capital. Such adjustment costs thus increase the variability of the marginal product of capital and the variability of firms' marginal costs (Christiano et al. 2005). The capital stock being fixed is a limiting case as capital adjustment costs become large. Other things equal, the effect of capital in reducing monetary non-neutrality in our model is thus maximized if the aggregate capital stock in the economy is fixed. To simplify our analysis, we assume that the aggregate capital stock is fixed and analyze the effect that introducing capital has on the cyclicality of marginal costs. We interpret our results as an upper bound on the effect that capital would have on the cyclicality of marginal costs. A model with smaller adjustment costs would imply a smaller response of marginal cost to output and thus greater monetary non-neutrality.

C.1 Household Behavior

Households own the capital stock and rent it to firms each period in a competitive capital market. Since capital is fixed, households make no choices regarding capital. The household budget constraint becomes

$$P_tC_t + E_t[D_{t,t+1}B_{t+1}] \leq B_t + W_tL_t + P_tR_tK + \int_0^1 \Pi_t(z)dz, \tag{3}$$

where R_t denotes the real rental rate on capital and K denotes the fixed amount of capital owned by the households. Other assumptions regarding household behavior are identical to our baseline model.

C.2 Firm Behavior

The production function of firm z is given by

$$y_t(z) = A_t(z)(L_t(z)^\alpha K_t(z)^{1-\alpha})^{1-s_m} M_t(z)^{s_m}. \tag{4}$$

Cost minimization by firms implies

$$\frac{W_t}{P_t} = (1-s_m)\alpha L_t(z)^{\alpha(1-s_m)-1} K_t(z)^{(1-\alpha)(1-s_m)} M_t(z)^{s_m} \Omega_t(z),$$

$$1 = s_m A_t (L_t(z)^{\alpha} K_t(z)^{1-\alpha})^{1-s_m} M_t(z)^{s_m - 1} \Omega_t(z),$$

$$R_t = (1-s_m)(1-\alpha) L_t(z)^{\alpha(1-s_m)} K_t(z)^{(1-\alpha)(1-s_m)-1} M_t(z)^{s_m} \Omega_t(z),$$

where $\Omega_t(z)$ denotes the marginal costs of firm z at time t. Eliminating $\Omega(z)$ from these three equations yields

$$\frac{W_t/P_t}{R_t} = \frac{\alpha}{1-\alpha}\frac{K_t(z)}{L_t(z)}, \tag{5}$$

$$\frac{W_t}{P_t} = \alpha\frac{1-s_m}{s_m}\frac{M_t(z)}{L_t(z)}. \tag{6}$$

These two equations imply that all firms have the same capital-labor ratio and the same materials-labor ratio.

The real value of firm z's profits in period t are

$$\Pi_t^R(z) = \left(\frac{p_t(z)}{P_t}\right) y_t(z) - \left(\frac{W_t}{P_t}\right) L_t(z) - M_t(z) - R_t K_t(z) - \chi\left(\frac{W_t}{P_t}\right) I_t(z) - U.$$

Using equations (5)-(6) we can rewrite these profits as

$$\Pi_t^R(z) = \left(\frac{p_t(z)}{P_t}\right) y_t(z) - \frac{1}{\alpha}\frac{1}{1-s_m}\left(\frac{W_t}{P_t}\right) L_t(z) - \chi\left(\frac{W_t}{P_t}\right) I_t(z) - U.$$

Combining equations (4)-(6) yields

$$L_t(z) = \left(\frac{y_t(z)}{A_t(z)}\right)\left(\frac{\alpha}{1-\alpha}\right)^{(1-\alpha)(1-s_m)}\left(\alpha\frac{1-s_m}{s_m}\right)^{s_m}\left(\frac{R_t}{W_t/P_t}\right)^{(1-\alpha)(1-s_m)}\left(\frac{W_t}{P_t}\right)^{-s_m}.$$

Combining these last two equations yields

$$\Pi_t^R(z) = \left(\frac{p_t(z)}{P_t}\right) y_t(z) - \Psi\left(\frac{y_t(z)}{A_t(z)}\right)\left(\frac{W_t}{P_t}\right)^{1-s_m}\left(\frac{R_t}{W_t/P_t}\right)^{(1-\alpha)(1-s_m)} - \chi\left(\frac{W_t}{P_t}\right) I_t(z) - U, \tag{7}$$

where

$$\Psi = \left(\frac{1}{\alpha}\frac{1}{1-s_m}\right)\left(\frac{\alpha}{1-\alpha}\right)^{(1-\alpha)(1-s_m)}\left(\alpha\frac{1-s_m}{s_m}\right)^{s_m}.$$

Equation (7) is almost identical to equation (2). There are two differences. First, the constant Ψ is different from the corresponding constant in equation (2). Second, the second term in equation (7) has an additional piece involving the ratio of the rental rate and the real wage. Notice that the average real marginal cost is pinned down by the markup.

The difference in the elasticity of marginal cost between the model with capital and the model without capital stems from the potential cyclicality of

$$\left(\frac{R_t}{W_t/P_t}\right)^{(1-\alpha)(1-s_m)}.$$

If R_t is more cyclical than W_t/P_t, the model with capital will have more cyclical marginal costs than the model without capital.

Combining equations (7) of the paper and (5) and adopting the our calibration of $\gamma = 1$ and $\psi = 0$ yields

$$\omega C_t = \frac{\alpha}{1-\alpha} R_t \frac{K_t(z)}{L_t(z)}.$$

If we log-linearize this equation, aggregate the resulting equation and use the fact that aggregate capital is fixed, we get $\hat{C}_t + \hat{L}_t = \hat{R}_t$.

Log-linearizing equations (7) of the paper and (4)-(6) and solving for the relationship between output and labor supply yields

$$\hat{L}_t = \frac{1-s_m}{(1-s_m)\alpha + s_m/\theta}\hat{C}_t \equiv a_2\hat{C}_t,$$

Combining this equation with $\hat{C}_t + \hat{L}_t = \hat{R}_t$ yields $\hat{R}_t = (1 + a_2)\hat{C}_t$. Since, the real wage in our model has a unit elasticity with respect to output, this shows that the rental rate is more cyclical than the real wage.

The equations above imply that the overall elasticity of marginal cost with respect to output in the model with capital is $(1-s_m)(1+a_2(1-\alpha))$. If we assume that the capital share is 1/3 and the intermediate input share is 0.7, then the elasticity of marginal cost is 0.38. Adding capital to the model thus increases the cyclicality of marginal costs from 0.3 to 0.38. The empirical results of Solon, Barsky, and Parker (1994) on the cyclicality of real wages suggest that for the U.S. economy the elasticity of real wages with respect to output is in fact only about 0.6. Our calibration without capital thus somewhat overstates the elasticity of real wages with respect to output. If we redo the elasticity calculation for the model with capital using the real wage elasticity from Solon, Barsky, and Parker (1994), we get an elasticity of marginal cost of 0.28. This is almost exactly equal to the elasticity of 0.3 that we assume in our baseline model. In other words, our baseline specification (without capital) implies an elasticity of marginal costs similar to what is implied by a model with capital and calibrated to match the empirical evidence presented in Solon, Barsky, and Parker

(1994). The addition of these two features—capital and a realistic value for the elasticity of real wages with respect to output—thus roughly cancel each other out and yield a model with the same amount our real rigidities as our baseline model.

References

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