Appendix to:

The Dynamic Behavior of the Real Exchange Rate in Sticky Price Models

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1 Household Behavior and Market Structure

The world consists of two countries. In each country there is a continuum of household types indexed by x. The home country households have indexes on the interval $N_H = [0,1]$. The foreign country households have indexes on the interval $N_F = (1,2]$. Home households of type x seek to maximize a discounted sum of utilities represented by

$$E_0 \left\{ \sum_{t=0}^{\infty} \beta^t \left[u(C_t) - v(L_t(x), \xi_t) \right] \right\}, \tag{1}$$

where β is a discount factor, ξ_t is a country specific vector of shocks to the household's preferences, C_t denotes household consumption of a composite consumption good, $L_t(x)$ denotes the households' supply of differentiated labor input x. The function $u(C_t)$ is increasing and concave while $v(L_t(x), \xi_t)$ is increasing and convex in $L_t(x)$. There are an equal (large) number of households of each type x.

The consumption index in equation (1) is

$$C_t = \left[\phi_{H,t}^{\frac{1}{\eta}} C_{H,t}^{\frac{\eta-1}{\eta}} + \phi_{F,t}^{\frac{1}{\eta}} C_{F,t}^{\frac{\eta-1}{\eta}} \right]^{\frac{\eta}{\eta-1}}, \tag{2}$$

where $\eta > 0$ denotes the elasticity of substitution between home and foreign goods and the $\phi_{j,t}$'s are preference parameters that determines households' relative preference for home versus foreign goods. If $\phi_{H,t} > \phi_{F,t}$, households preferences are biased toward home produced goods. It is analytically convenient to normalize $\phi_{H,t} + \phi_{F,t} = 1$. I allow the home bias in preferences to vary exogenously over time and refer to such variation as shocks to the world demand for home goods. I assume for simplicity that households in both countries have the same degree of steady state home bias, i.e., $\phi_H^* = \phi_F$.

The subindices, $C_{j,t}$, are in turn CES indices of the differentiated goods produced in the two countries. These indices are given by

$$C_{H,t} = \left[\int_{N_H} c_t(z)^{\frac{\theta_t - 1}{\theta_t}} dz \right]^{\frac{\theta_t}{\theta_t - 1}}, \quad \text{and} \quad C_{F,t} = \left[\int_{N_F} c_t(z)^{\frac{\theta_t^* - 1}{\theta_t^*}} dz \right]^{\frac{\theta_t^*}{\theta_t^* - 1}}. \tag{3}$$

Here the differentiated goods are indexed by z. The consumption by the representative household in the home country of good z in period t is denoted by $c_t(z)$ and $\theta_t > 1$ and $\theta_t^* > 1$ denote the elasticity of substitution at time t between the differentiated goods produced in the home country and foreign country, respectively. I assume that θ_t and θ_t^* vary exogenously. These variations may be interpreted as variation in the monopoly power of firms in the two countries. In the recent literature on monetary policy, these shocks have been referred to as "cost-push" shocks.

All goods produced in the economy are non-durable consumption goods purchased and consumed immediately by households. Investment and capital accumulation play no role in the model. To the extent that capital is used in production, each firm in the economy is endowed with a fixed amount of non-depreciating capital. Labor is immobile and there are a fixed number of firms operating in each country.

Each country has a government. These governments operate fiat currency systems denominated in "home currency" and "foreign currency", respectively. There are independent central banks that conduct monetary policy in each country by controlling the short term nominal interest rate in the domestic currency. The governments finance spending by lump sum taxes.

Households face a decision in each period about how much to consume of each of the differentiated goods produced in the world. The representative household seeks to maximize the value of the composite consumption good, C_t, that it can purchase given its income and given the prices it faces. Prices in the home country are denominated in home currency and are denoted by $p_t(z)$. Prices in the foreign country are denominated in foreign currency and are denoted by $p_t^*(z)$. The demand for home produced good z that results from this optimization by the home and foreign households is

$$c_t(z) = C_{H,t}\left(\frac{p_t(z)}{P_{H,t}}\right)^{-\theta_t} \quad \text{and} \quad c_t^*(z) = C_{H,t}^*\left(\frac{p_t^*(z)}{P_{H,t}^*}\right)^{-\theta_t}, \tag{4}$$

where

$$C_{H,t} = \phi_{H,t}C_t\left(\frac{P_{H,t}}{P_t}\right)^{-\eta} \quad \text{and} \quad C_{H,t}^* = \phi_{H,t}^*C_t^*\left(\frac{P_{H,t}^*}{P_t^*}\right)^{-\eta}. \tag{5}$$

Demand for foreign produced goods is given by analogous expressions. In these equations $P_{H,t}$, $P_{H,t}^*$, P_t and P_t^* are price indexes given by

$$P_{H,t} = \left[\int_{N_H} p_t(z)^{1-\theta_t}dz\right]^{\frac{1}{1-\theta_t}} \quad , \quad P_{H,t}^* = \left[\int_{N_H} p_t^*(z)^{1-\theta_t}dz\right]^{\frac{1}{1-\theta_t}}, \tag{6}$$

$$P_t = \left[\phi_{H,t}P_{H,t}^{1-\eta} + \phi_{F,t}P_{F,t}^{1-\eta}\right]^{\frac{1}{1-\eta}} \quad \text{and} \quad P_t^* = \left[\phi_{H,t}^*P_{H,t}^{*1-\eta} + \phi_{F,t}^*P_{F,t}^{*1-\eta}\right]^{\frac{1}{1-\eta}}. \tag{7}$$

P_t and P_t^* will be referred to as the home and foreign country price levels, respectively. For simplicity, I assume that the demand of the home and foreign governments—denoted by $g_t(z)$, $g_t^*(z)$, $G_{j,t}$, $G_{j,t}^*$, G_t and G_t^*—is given by analogous equations to equations (4) and (5).

Agents in both countries have access to complete financial markets. There are no impediments to international trade in financial securities. Home households of type x face a flow budget constraint given by

$$P_tC_t + E_t[M_{t,t+1}B_{t+1}(x)] \leq B_t(x) + W_t(x)L_t(x) + \int_{N_H}\Phi_t(z)dz - T_t, \tag{8}$$

where $B_{t+1}(x)$ is a random variable that denotes the state contingent payoff of the portfolio of financial securities held by households of type x at the beginning of period $t+1$, $M_{t,t+1}$ is the stochastic discount factor that prices these payoffs in period t, $W_t(x)$ denotes the wage rate received by home households of type x in period t, $\Phi_t(z)$ is the profit of firm z in period t and T_t denotes lump sum taxes.[1]

A necessary condition for equilibrium in this model is that there exist no arbitrage opportunities. It follows from the absence of arbitrage opportunities that all portfolios of financial securities that pay off in period $t+1$ may be priced in period t using a unique stochastic discount factor, $M_{t,t+1}$, as in equation (8). In order to rule out "Ponzi schemes," households' portfolios of financial wealth must always be large enough that future income suffices to avert default.

[1] In equation (8) financial assets are denominated in the home currency and $M_{t,t+1}$ denotes the home currency nominal stochastic discount factor. It is important to note that the financial assets in equation (8) cannot generally be denominated in "goods". If goods are not freely traded internationally and don't have the same exchange rate adjusted price in the two countries, as will be assumed below, the same good in different countries must be viewed as two different goods. Financial assets can in this case be denominated in "goods for delivery in home country" or "goods for delivery in foreign country" but not "goods".

Home households choose C_t, $L_t(x)$ and $B_t(x)$ in order to maximize expression (1) subject to equation (8). An optimal plan must satisfy

$$u_c(C_t) = P_t \Lambda_t, \tag{9}$$

$$M_{t,T} \Lambda_t = \beta^{T-t} \Lambda_T, \tag{10}$$

$$v_l(L_t(x), \xi_t) = W_t(x) \Lambda_t, \tag{11}$$

where Λ_t denotes the marginal utility of nominal income of households at time t, that is, the Lagrange multiplier of the constrained optimization and subscripts on the functions u and v denote partial derivatives. These three equations should hold for all periods t and all subsequent periods T. The optimal plan must also satisfy a standard trasversality condition.

Foreign households solve an analogous problem. Their optimal plan must satisfy

$$u_c(C_t^*) = P_t^* \Lambda_t^*, \tag{12}$$

$$M_{t,T} \frac{\Lambda_t^*}{\mathcal{E}_t} = \beta^{T-t} \frac{\Lambda_T^*}{\mathcal{E}_T}, \tag{13}$$

$$v_l(L_t^*(x), \xi_t^*) = W_t^*(x) \Lambda_t^*, \tag{14}$$

as well as a trasversality condition. Here $\mathcal{E}_t$ denotes the nominal exchange rate, i.e., the home currency price of foreign currency. Notice that the stochastic discount factor in equation (13) is the same stochastic discount factor as in equation (10). This simply reflects the fact that assets are traded on global markets in which all agents face the same prices.

From equation (9)-(10) and (12)-(13) it follows that

$$\frac{u_c(C_T)}{u_c(C_t)} = \frac{M_{t,T}}{\beta^{T-t}} \frac{P_T}{P_t} \quad \text{and} \quad \frac{u_c(C_T^*)}{u_c(C_t^*)} = \frac{M_{t,T}}{\beta^{T-t}} \frac{\mathcal{E}_T P_T^*}{\mathcal{E}_t P_t^*}. \tag{15}$$

Combining these equations yields

$$Q_t = \frac{u_c(C_t^*)}{u_c(C_t)} \tag{16}$$

where $Q_t = \mathcal{E}_t P_t^* / P_t$ is the real exchange rate at time t and for simplicity $Q_0 = 1$.

2 Firm Behavior

In each country there is a continuum of firm types indexed by z. The home country firms have indexes on the interval $N_H = [0, 1]$. The foreign country firms have indexes on the interval $N_F =$

(1, 2]. Firms of type z specializes in the production of a differentiated good, $y_t(z)$. There are an equal (large) number of firms of each type.

In the following two subsections, I will describe two environments and the resulting firm behavior in each environment. I will refer to these two environments as the heterogeneous factor markets model and the homogeneous factor markets model. In both the heterogeneous factor markets model and the homogeneous factor markets model, I assume that firms are able to price discriminate between consumers in the two countries. In other words, they price-to-market (see, e.g., Krugman, 1987). Furthermore, firms denominate the price of their good in the home and foreign country in the local currency of each country. In other words, they practice local-currency pricing (see, e.g., Devereux, 1997). Prices are sticky in both countries. Price setting is assumed to be synchronized within each firm type but staggered between firm types.[2] In each period firms of type z can change their prices with probability $1 - \alpha$. With probability α they must keep their prices unchanged. This model of price stickiness was first proposed in Calvo (1983). The fact that a firm's ability to change its prices is independent of the state of the economy makes this model simple and tractable.

2.1 The Heterogeneous Factor Market Model

All inputs to production except labor are fixed for each firm. Firms of type z must hire labor of type $x = z$. Other types of labor are not useful in the production of goods of type z. In other words, the labor market is highly segmented. This may be due to the fact that specific skills are required to produce each type of good. In this case, x denotes the skills each type of household is endowed with or has invested in. The production function of firms of type z is

$$y_t(z) = A_t f(L_t(z)) \tag{17}$$

where A_t denotes an exogenous technology factor and $L_t(z)$ denotes the amount of labor input used by firms of type z in period t. The function f is increasing and concave. It is concave because there are diminishing marginal returns to labor given the fixed amount of other inputs employed at the firm. Firms act to maximize their value in domestic currency.

In order to maximize profits a home country firm of type z that is able to change its prices at

[2]See Woodford (2003, section 3.1.) for an argument for why this assumption is reasonable.

time t chooses $p_t(z)$, $p_t^*(z)$ and $L_T(z)$ to maximize

$$E_t \sum_{T=t}^{\infty} \alpha^{T-t} M_{t,T} \Phi_T(z), \tag{18}$$

where

$$\Phi_T(z) = p_t(z)(C_{H,T}+G_{H,T}) \left(\frac{p_t(z)}{P_{H,T}}\right)^{-\theta_T} + \mathcal{E}_T p_t^*(z)(C_{H,T}^*+G_{H,T}^*) \left(\frac{p_t^*(z)}{P_{H,T}^*}\right)^{-\theta_T} - W_T(z)L_T(z) \tag{19}$$

subject to the constraint that it produces at least as much as it sells,

$$(C_{H,T}+G_{H,T}) \left(\frac{p_t(z)}{P_{H,T}}\right)^{-\theta_T} + (C_{H,T}^*+G_{H,T}^*) \left(\frac{p_t^*(z)}{P_{H,T}^*}\right)^{-\theta_T} \leq A_T f(L_T(z)). \tag{20}$$

Necessary conditions for an optimal plan are

$$E_t \sum_{T=t}^{\infty} \alpha^{T-t} M_{t,T}(C_{H,T}+G_{H,T}) P_{H,T}^{\theta_T}(1-\theta_T)[p_t(z) - \frac{\theta_T}{\theta_T - 1} S_T(z)] = 0, \tag{21}$$

$$E_t \sum_{T=t}^{\infty} \alpha^{T-t} M_{t,T}(C_{H,T}^*+G_{H,T}^*) P_{H,T}^{*\theta_T}(1-\theta_T)[\mathcal{E}_T p_t^*(z) - \frac{\theta_T}{\theta_T - 1} S_T(z)] = 0, \tag{22}$$

for each period t at which firms of type z are able to change their prices,

$$W_t(z) = A_t f_l(L_t(z)) S_t(z) \tag{23}$$

for all t and equation (20) with equality for all t. Here $S_t(z)$ is the marginal cost of production, i.e. the Lagrange multiplier of the firm's constrained optimization problem. Foreign firms solve an analogous optimization problem.

Combining equations (9), (11) and (23) in order to eliminate $\Lambda_t(z)$ and $W_t(z)$ gives

$$\frac{S_t(z)}{P_t} = \frac{v_l(L_t(z), \xi_t)}{A_t f_l(L_t(z)) u_c(C_t)}. \tag{24}$$

Notice that $L_t(z) = f^{-1}(y_t(z)/A_t)$. Using this relation, $S_t(z)/P_t$ can be written without reference to $L_t(z)$ as

$$\frac{S_t(z)}{P_t} = \frac{v_l(f^{-1}(y_t(z)/A_t), \xi_t)}{A_t f_l(f^{-1}(y_t(z)/A_t)) u_c(C_t)}. \tag{25}$$

Here the marginal costs of firms of type z have been written in terms of their level of output and the level of domestic consumption. This is useful since it simplifies the model by eliminating both $W_t(z)$ and $L_t(z)$.

2.2 The Homogeneous Factor Markets Model

There exists a fixed amount of non-depretiating capital in the economy that is owned by the firms. For simplicity, I assume that firms can rent their capital stock to other firms but not sell it. All workers are identical from each firm's perspective. Firms are therefore indifferent regarding which workers they hire and all workers receive the same wage W_t in equilibrium. The production function of firms of type z is

$$y_t(z) = A_t f(L_t(z), K_t(z)) \tag{26}$$

where A_t denotes an exogenous technology factor and $L_t(z)$ denotes the amount of labor input used by firms of type z in period t and $K_t(z)$ denotes the amount of capital used by firms of type z in period t. The function f is increasing in both its arguments and homogeneous of degree one. Firms act to maximize their value in domestic currency.

In order to maximize profits a home country firms of type z that are able to change its prices at time t chooses $p_t(z)$, $p_t^*(z)$, $L_T(z)$ and $K_t(z)$ to maximize (18) where

$$\begin{aligned}\Phi_T(z) = p_t(z)(C_{H,T} + G_{H,T})\left(\frac{p_t(z)}{P_{H,T}}\right)^{-\theta_T} &+ \mathcal{E}_T p_t^*(z)(C_{H,T}^* + G_{H,T}^*)\left(\frac{p_t^*(z)}{P_{H,T}^*}\right)^{-\theta_T} \\ &- W_T L_T(z) - R_T(K_T(z) - K(z))\end{aligned} \tag{27}$$

subject to the constraint that it produces at least as much as it sells,

$$(C_{H,T} + G_{H,T})\left(\frac{p_t(z)}{P_{H,T}}\right)^{-\theta_T} + (C_{H,T}^* + G_{H,T}^*)\left(\frac{p_t^*(z)}{P_{H,T}^*}\right)^{-\theta_T} \leq A_T f(L_T(z), K_T(z)), \tag{28}$$

where R_T denotes the rental rate on capital in period T and $K(z)$ denotes the capital endowment of firms of type z.

Necessary conditions for an optimal plan are equations (21)-(22) for each period t at which firms of type z are able to change their prices,

$$W_t = A_t f_l(L_t(z), K_t(z)) S_t(z) \tag{29}$$

$$R_t = A_t f_k(L_t(z), K_t(z)) S_t(z) \tag{30}$$

for all t and equation (28) with equality for all t. Notice that equations (29)-(30) imply that

$$\frac{W_t}{R_t} = \frac{f_l(L_t(z), K_t(z))}{f_k(L_t(z), K_t(z))}.$$

Since f is homogeneous of degree one, this implies that all firms choose the same labor-capital ratio in period t even though they produce different amounts. This, in turn, implies that equation (29) can be rewritten as

$$S_t = \frac{W_t}{A_t f_l(h_t, 1)},$$

where h_t denotes the common labor-capital ratio of all firms. Notice that this equation implies that the marginal cost of all firms is equal. I have denoted this common marginal cost as S_t.

Combining this last equations with equations (9), (11) and (29) in order to eliminate $\Lambda_t(z)$ and W_t yields

$$\frac{S_t}{P_t} = \frac{v_l(L_t, \xi_t)}{A_t f_l(h_t, 1) u_c(C_t)}, \tag{31}$$

where L_t is the amount of labor supplied by the representative household. Unlike in the heterogeneous markets case, all households supply the same amount of labor when the labor market is homogeneous.

3 Log-Linearization of Heterogeneous Factor Markets Model

In this section, I work out a log-linear approximation of the heterogeneous factor markets model. A log-linear approximation of the homogeneous factor markets model may be derived in an analogous fashion.

First, consider the left equation in (15). The expectation of the $T = t + 1$ version of this equation may be written

$$I_t = E_t \left[\frac{1}{\beta} \frac{u_c(C_t)}{u_c(C_{t+1})} \frac{P_{t+1}}{P_t} \right],$$

since the gross short term nominal interest rate is given by $I_t = 1/E_t M_{t,t+1}$. A log-linear approximation of this equations is

$$c_t = E_t c_{t+1} - \sigma(i_t - E_t \pi_{t+1}), \tag{32}$$

where $\sigma = -u_c/u_{cc}C$, lower case letters denote percentage deviations from steady state of the same upper case letters unless otherwise noted, uppercase letters without a time subscript denote steade state values and $\pi_t = \log(P_t/P_{t-1})$. The foreign consumption Euler equation yields an analogous log-linear approximation.

A log-linear approximation of equation (16) is

$$c_t - c_t^* = \sigma q_t. \tag{33}$$

Log-linear approximations of the equations in (7) are

$$\phi_H p_{H,t} + \phi_F p_{F,t} = 0, \tag{34}$$

$$\phi_F p^*_{H,t} + \phi_H p^*_{F,t} = 0, \tag{35}$$

where $p_{j,t} = \log(P_{j,t}/P_t)$ and I have made use of the fact that the normalization $\phi_{H,t} + \phi_{F,t} = 1$ implies that all relative prices are 1 in steady state. Notice that these last two equations imply that

$$\pi_t = \phi_H \pi_{H,t} + \phi_F \pi_{F,t} \tag{36}$$

$$\pi^*_t = \phi_F \pi^*_{H,t} + \phi_H \pi^*_{F,t} \tag{37}$$

A log-linear approximation of equation (6) is

$$\pi_{H,t} = \frac{1-\alpha}{\alpha}(p_{h,t} - p_{H,t}). \tag{38}$$

$$\pi_{F,t} = \frac{1-\alpha}{\alpha}(p_{f,t} - p_{F,t}). \tag{39}$$

where $\pi_{j,t} = \log(P_{j,t}/P_{j,t-1})$.

Define c^M_t and c^{M*}_t as $c^M_t = \phi_H c_t + \phi_F c^*_t$ and $c^{M*}_t = \phi_F c_t + \phi_H c^*_t$, respectively and M and $M*$ superscripts on other variables denote the analogous weighted averages. Given this notation, a log-linear approximation of (20), (25) and their foreign counterparts are

$$y_{t,T} = c^M_T + g^M_T + (\theta - \eta)p^M_{H,T} - \theta p^M_{h,t} + \theta \sum_{\tau=t+1}^{T} \pi^M_\tau + \phi^M_{H,T},$$

$$y^*_{t,T} = c^{M*}_T + g^{M*}_T + (\theta - \eta)p^{M*}_{F,T} - \theta p^{M*}_{f,t} + \theta \sum_{\tau=t+1}^{T} \pi^{M*}_\tau + \phi^{M*}_{F,T},$$

$$s_{t,T} = \left(\frac{v_{ll}Y}{v_l f_l A} + \frac{\Psi_y Y}{\Psi A}\right) y_{t,T} - \frac{u_{cc}C}{u_c} c_T + \frac{v_{l\xi}}{v_l}\xi_T - \left(\frac{v_{ll}Y}{v_l f_l A} + \frac{\Psi_y Y}{\Psi A} + 1\right) a_T,$$

$$s^*_{t,T} = \left(\frac{v_{ll}Y}{v_l f_l A} + \frac{\Psi_y Y}{\Psi A}\right) y^*_{t,T} - \frac{u_{cc}C}{u_c} c^*_T + \frac{v_{l\xi}}{v_l}\xi^*_T - \left(\frac{v_{ll}Y}{v_l f_l A} + \frac{\Psi_y Y}{\Psi A} + 1\right) a^*_T.$$

where $s_{t,T}$ denotes the percent deviation from steady state of the real marginal cost in period T of the firms that set their prices in period t, $y_{t,T}$ denotes the percent deviation from steady state in period T of the level of output of firms that set their prices in period t and $\Psi = 1/f_l(f^{-1}(y/A))$. Also, I assume that $C = C^* = Y$.

Combining these last four equations to eliminate $y_{t,T}$ and $y^*_{t,T}$ yields

$$s_{t,T} = \omega(c^M_T + g^M_T) + \omega(\theta-\eta)p^M_{H,T} - \omega\theta p^M_{h,t} + \omega\theta \sum_{\tau=t+1}^{T} \pi^M_\tau + \phi^M_{H,T} + \sigma^{-1}c_T - \tilde{a}_T, \tag{40}$$

$$s^*_{t,T} = \omega(c^{M*}_T + g^{M*}_T) + \omega(\theta-\eta)p^{M*}_{F,T} - \omega\theta p^{M*}_{f,t} + \omega\theta \sum_{\tau=t+1}^{T} \pi^{M*}_\tau - \phi^M_{H,T} + \sigma^{-1}c^*_T - \tilde{a}^*_T, \tag{41}$$

where

$$\omega = \left(\tfrac{v_{ll}Y}{v_l f_l A} + \tfrac{\Psi_y Y}{\Psi A}\right) \quad \text{and} \quad \tilde{a}_t = (\omega+1)a_t - \tfrac{v_{l\xi}}{v_l}\xi_t$$

and where we use the fact that $\phi^M_{H,t} = -\phi^{M*}_{F,t}$.

Log-linear approximations of equations (21) and (22) and their foreign counterparts are given by

$$p_{ht} = (1-\alpha\beta)\sum_{j=0}^{\infty}(\alpha\beta)^j E_t(s_{t,t+j} - \hat{\theta}_{t+j}) + \sum_{j=1}^{\infty}(\alpha\beta)^j E_t\pi_{t+j}, \tag{42}$$

$$p^*_{ht} = (1-\alpha\beta)\sum_{j=0}^{\infty}(\alpha\beta)^j E_t(s_{t,t+j} - q_{t+j} - \hat{\theta}_{t+j}) + \sum_{j=1}^{\infty}(\alpha\beta)^j E_t\pi^*_{t+j}, \tag{43}$$

$$p^*_{ft} = (1-\alpha\beta)\sum_{j=0}^{\infty}(\alpha\beta)^j E_t(s^*_{t,t+j} - \hat{\theta}^*_{t+j}) + \sum_{j=1}^{\infty}(\alpha\beta)^j E_t\pi^*_{t+j}, \tag{44}$$

$$p_{ft} = (1-\alpha\beta)\sum_{j=0}^{\infty}(\alpha\beta)^j E_t(s^*_{t,t+j} + q_{t+j} - \hat{\theta}^*_{t+j}) + \sum_{j=1}^{\infty}(\alpha\beta)^j E_t\pi_{t+j}, \tag{45}$$

where $\hat{\theta}_t = (\theta/(\theta-1)^2)\theta_t$.

Combining equations (38), (40) and (42) yields

$$\pi_{H,t} + \frac{1-\alpha}{\alpha}p_{H,t} = \kappa\sum_{j=0}^{\infty}(\alpha\beta)^j E_t\Big(\omega(c^M_{t+j} + g^M_{t+j}) + \omega(\theta-\eta)p^M_{H,t+j} - \omega\theta p^M_{h,t} + \omega\theta\sum_{\tau=t+1}^{t+j}\pi^M_\tau$$

$$+\sigma^{-1}c^M_{t+j} + \phi_F\sigma^{-1}c^R_{t+j} + \phi^M_{H,t+j} - \tilde{a}_{t+j} - \hat{\theta}_{t+j}\Big) + \frac{1-\alpha}{\alpha}\sum_{j=1}^{\infty}(\alpha\beta)^j E_t\pi_{t+j} \tag{46}$$

Notice that

$$\sum_{j=0}^{\infty}(\alpha\beta)^j \sum_{\tau=t+1}^{t+j}\pi^M_\tau = \frac{1}{1-\alpha\beta}\sum_{j=1}^{\infty}(\alpha\beta)^j\pi^M_{t+j}.$$

Using this and equations (33), (38) and (46) may be written

$$(1+\omega\theta)\left(\pi_{H,t} + \frac{1-\alpha}{\alpha}p_{H,t}\right) - \phi_F\omega\theta\left(\pi^R_{H,t} + \frac{1-\alpha}{\alpha}p^R_{H,t}\right) = \kappa\sum_{j=0}^{\infty}(\alpha\beta)^j(\omega+\sigma^{-1})E_t c^M_{t+j}$$

$$+\kappa\sum_{j=0}^{\infty}(\alpha\beta)^j E_t\left(\omega(\theta-\eta)p^M_{H,t+j} + \phi_F(q_{t+j} + \epsilon^R_{t+j}) + \phi^M_{H,t+j} - \tilde{a}_{t+j} + \omega g^M_{t+j} - \hat{\theta}_{t+j}\right)$$

$$+(1+\omega\theta)\frac{1-\alpha}{\alpha}\sum_{j=1}^{\infty}(\alpha\beta)^j E_t\pi_{t+j} - \phi_F\omega\theta\frac{1-\alpha}{\alpha}\sum_{j=1}^{\infty}(\alpha\beta)^j E_t\pi^R_{t+j}$$

Now, using the fact that $p_{H,t} - p_{H,t-1} = \pi_{H,t} - \pi_t$ and defining

$$\kappa = \tfrac{(1-\alpha)(1-\alpha\beta)}{\alpha} \quad \text{and} \quad \zeta = \tfrac{\omega+\sigma^{-1}}{1+\omega\theta},$$

this equation can be rewritten as

$$\pi_{H,t} - \beta E_t \pi_{H,t+1} + \kappa p_{H,t} - \phi_F \frac{\omega\theta}{1+\omega\theta}\left(\pi^R_{H,t} - \beta E_t \pi^R_{H,t+1} + \kappa p^R_{H,t}\right)$$
$$= \kappa\zeta c^M_t + \kappa\frac{\omega(\theta-\eta)}{1+\omega\theta}p^M_{H,t} + \kappa\frac{\phi_F}{1+\omega\theta}(q_t + \epsilon^R_t) - \frac{\kappa}{1+\omega\theta}(\tilde{a}_t - \omega g^M_t - \phi^M_{H,t+j} + \hat{\theta}_t).$$

A similar set of manipulations involving $\pi^*_{H,t}$ yields

$$\pi^*_{H,t} - \beta E_t \pi^*_{H,t+1} + \kappa p^*_{H,t} + \phi_H \frac{\omega\theta}{1+\omega\theta}\left(\pi^R_{H,t} - \beta E_t \pi^R_{H,t+1} + \kappa p^R_{H,t}\right)$$
$$= \kappa\zeta c^M_t + \kappa\frac{\omega(\theta-\eta)}{1+\omega\theta}p^M_{H,t} - \kappa\frac{1-\phi_F}{1+\omega\theta}(q_t + \epsilon^R_t) - \frac{\kappa}{1+\omega\theta}(\tilde{a}_t - \omega g^M_t - \phi^M_{H,t+j} + \hat{\theta}_t).$$

Combining the last two equations yields

$$\pi^R_{H,t} = \beta E_t \pi^R_{H,t+1} + \kappa q_t - \kappa p^R_{H,t},$$

$$\pi^M_{H,t} = \beta E_t \pi^M_{H,t+1} + \kappa\zeta c^M_t - \kappa\frac{1+\omega\eta}{1+\omega\theta}p^M_{H,t} + \kappa\frac{2\phi_H\phi_F}{1+\omega\theta}q_t - \frac{\kappa}{1+\omega\theta}(\tilde{a}_t - \omega g^M_t - \phi^M_{H,t+j} + \hat{\theta}_t),$$

$$\pi_{H,t} = \beta E_t \pi_{H,t+1} + \kappa\zeta c^M_t - \kappa\frac{1+\omega\eta}{1+\omega\theta}p^M_{H,t} - \kappa\phi_F p^R_{H,t} + \kappa\phi_F q_t - \frac{\kappa}{1+\omega\theta}(\tilde{a}_t - \omega g^M_t - \phi^M_{H,t+j} + \hat{\theta}_t),$$

$$\pi^*_{H,t} = \beta E_t \pi^*_{H,t+1} + \kappa\zeta c^M_t - \kappa\frac{1+\omega\eta}{1+\omega\theta}p^M_{H,t} + \kappa\phi_H p^R_{H,t} - \kappa\phi_H q_t - \frac{\kappa}{1+\omega\theta}(\tilde{a}_t - \omega g^M_t - \phi^M_{H,t+j} + \hat{\theta}_t).$$

And a similar set of manipulations involving $\pi_{F,t}$ and $\pi^*_{F,t}$ yields

$$\pi^R_{F,t} = \beta E_t \pi^R_{F,t+1} + \kappa q_t - \kappa p^R_{F,t} - \kappa\hat{\theta}^R_t,$$

$$\pi^{M*}_{F,t} = \beta E_t \pi^{M*}_{F,t+1} + \kappa\zeta c^{M*}_t - \kappa\frac{1+\omega\eta}{1+\omega\theta}p^{M*}_{F,t} - \kappa\frac{2\phi_F\phi_H}{1+\omega\theta}q_t - \frac{\kappa}{1+\omega\theta}(\tilde{a}^*_t - \omega g^{M*}_t + \phi^M_{H,t+j} + \hat{\theta}^*_t),$$

$$\pi_{F,t} = \beta E_t \pi_{F,t+1} + \kappa\zeta c^{M*}_t - \kappa\frac{1+\omega\eta}{1+\omega\theta}p^{M*}_{F,t} - \kappa\phi_H p^R_{F,t} + \kappa\phi_H q_t - \frac{\kappa}{1+\omega\theta}(\tilde{a}^*_t - \omega g^{M*}_t + \phi^M_{H,t+j} + \hat{\theta}^*_t),$$

$$\pi^*_{F,t} = \beta E_t \pi^*_{F,t+1} + \kappa\zeta c^{M*}_t - \kappa\frac{1+\omega\eta}{1+\omega\theta}p^{M*}_{F,t} + \kappa\phi_F p^R_{F,t} - \kappa\phi_F q_t - \frac{\kappa}{1+\omega\theta}(\tilde{a}^*_t - \omega g^{M*}_t + \phi^M_{H,t+j} + \hat{\theta}^*_t).$$

These equations along with equations (36) and (37) imply that

$$\pi_t = \beta E_t \pi_{t+1} + \kappa\zeta(\phi_H c^M_t + \phi_F c^{M*}_t) - \kappa\frac{1+\omega\eta}{1+\omega\theta}(\phi_H p^M_{H,t} + \phi_F p^{M*}_{F,t}) - \kappa\phi_H\phi_F(p^R_{H,t} + p^R_{F,t})$$
$$+\kappa 2\phi_H\phi_F q_t - \frac{\kappa}{1+\omega\theta}(\tilde{a}^M_t - \omega(\phi_H g^M_t + \phi_F g^{M*}_t) - (\phi_H - \phi_F)\phi^M_{H,t} + \theta^M_t),$$

$$\pi_t^* = \beta E_t \pi_{t+1}^* + \kappa\zeta(\phi_F c_t^M + \phi_H c_t^{M*}) - \kappa\frac{1+\omega\eta}{1+\omega\theta}(\phi_F p_{H,t}^M + \phi_H p_{F,t}^{M*}) + \kappa\phi_H\phi_F(p_{H,t}^R + p_{F,t}^R)$$
$$-\kappa 2\phi_H\phi_F q_t - \frac{\kappa}{1+\omega\theta}(\tilde{a}_t^{M*} - \omega(\phi_F g_t^M + \phi_H g_t^{M*}) - (\phi_F - \phi_H)\phi_{H,t}^M + \theta_t^{M*}).$$

Using equations (34) and (35), these equations may be simplified:

$$\pi_t = \beta E_t \pi_{t+1} + \kappa\zeta(\phi_H c_t^M + \phi_F c_t^{M*}) - \kappa\frac{(\phi_H - \phi_F)\omega(\theta - \eta)}{1+\omega\theta} p_{F,t}^{M*} + \kappa 2\phi_H\phi_F q_t$$
$$-\frac{\kappa}{1+\omega\theta}(\tilde{a}_t^M - \omega(\phi_H g_t^M + \phi_F g_t^{M*}) - (\phi_H - \phi_F)\phi_{H,t}^M + \theta_t^M),$$

$$\pi_t^* = \beta E_t \pi_{t+1}^* + \kappa\zeta(\phi_F c_t^M + \phi_H c_t^{M*}) + \kappa\frac{(\phi_H - \phi_F)\omega(\theta - \eta)}{1+\omega\theta} p_{F,t}^{M*} - \kappa 2\phi_H\phi_F q_t$$
$$-\frac{\kappa}{1+\omega\theta}(\tilde{a}_t^{M*} - \omega(\phi_F g_t^M + \phi_H g_t^{M*}) - (\phi_F - \phi_H)\phi_{H,t}^M + \theta_t^{M*}).$$

Notice, furthermore, that if $\theta = \eta$ the $p_{F,t}^{M*}$ terms drop out of these equations.

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