AGGREGATE RISK

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- Macroeconomists spend a lot of time thinking about policies to dampen business cycles (i.e., stabilization policies)
- But how important in terms of welfare are such policies
- Upper bound: Welfare gains from eliminating all economic fluctuations
- What are the welfare gains from eliminating all economic fluctuations?
Assumes consumer’s consumption stream is trend-stationary:

\[ c_t = Ae^{\mu t} e^{-(1/2)\sigma^2} \epsilon_t \]

with \( \log(\epsilon_t) \sim N(0, \sigma^2) \)

This implies:

\[ E(e^{-(1/2)\sigma^2} \epsilon_t) = 1 \]

\[ E(c_t) = Ae^{\mu t} \]
WELFARE LOSSES FROM ECONOMIC FLUCTUATIONS

- Consumer’s utility function

\[ E \left\{ \sum_{t=0}^{\infty} \beta^t \frac{c_t^{1-\gamma}}{1-\gamma} \right\} \]

- \( \beta \) is subjective discount factor
- \( \gamma \) coefficient of risk aversion
Thought experiment: How much would welfare increase if we could magically eliminate all consumption variation around trend (best case scenario for stabilization policy!)

Represent this as a consumption equivalent gain $\lambda$:

$$E \left\{ \sum_{t=0}^{\infty} \beta^t \frac{((1 + \lambda)c_t)^{1-\gamma}}{1 - \gamma} \right\} = \sum_{t=0}^{\infty} \beta^t \frac{(Ae^{\mu t})^{1-\gamma}}{1 - \gamma}$$

Answer:

$$\lambda \approx \frac{1}{2} \gamma \sigma^2$$
Welfare Losses from Economic Fluctuations

\[ \lambda \approx \frac{1}{2} \gamma \sigma^2 \]

- For 1947-2001, the standard deviation of the log of U.S. real, per capita consumption about a linear trend: 0.032.
- Reasonable values of \( \gamma \) between 1 and 4

\[ \lambda = \frac{1}{2} (0.032)^2 = 0.0005 \]

- Even including the Great Depression and Great Recession (1920-2009) and setting \( \gamma = 4 \):

\[ \lambda = \frac{1}{2} 4 (0.063)^2 = 0.008 \]
Conclusion: Welfare gains from stabilization policy are trivial.

Macroeconomics as originally conceived has succeeded.

Is this convincing?
Conclusion: Welfare gains from stabilization policy are trivial.

Macroeconomics as originally conceived has succeeded.

Is this convincing?

Model used to reach this conclusion may be wrong
- Output/Consumption may not be trend stationary
- Representative consumer view may understate seriousness of recessions

Model Lucas uses does not fit the equity premium!!
Can it be taken seriously for thinking about the costs of risk??
In a simple endowment economy (Mehra-Prescott 85):

\[
\log E_t R_{C,t+1} - \log R_{f,t} = \gamma \text{var}_t(\log \Delta C_{t+1})
\]

Equity Premium Puzzle:

\[
\log E_t R_{e,t+1} - \log R_{f,t} \approx 0.07
\]

\[
\text{var}_t(\log \Delta C_{t+1}) \approx 0.03^2 = 0.0009
\]

(Arguably equity is a leveraged claim to consumption. See, e.g., Barro 06)
Different preferences: Habits (Campbell and Cochrane, 1999)

Incomplete markets / heterogeneous agents  
(Constantinides and Duffie, 1996; Constantinides and Ghosh, 2017)

Different consumption process
- Is trend-stationary consumption process assumed by Lucas or random-walk consumption process assumed in textbook equity premium calculations a good model of consumption growth?  
- Do they accurately capture aggregate risks?  
- What is missing?
How to Model Consumption Dynamics?

Figure: Log Consumption for France
How to Model Consumption Dynamics?

1. Trend Stationary vs. Difference Stationary?

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3. Autocorrelated Growth Rates?

Figure: Log Consumption for France
How to Model Consumption Dynamics?

1. Trend Stationary vs. Difference Stationary?

3. Autocorrelated Growth Rates?

2. Disasters

4. Variation in Uncertainty?

Figure: Log Consumption for France
FIGURE
Growth in U.S. per Capita Consumption

Source: Barro and Ursua (2008)
Is GDP/Consumption a Random Walk?

- Textbook asset pricing model:
  \[ \log C_{t+1} = \mu + \log C_t + \epsilon_{t+1} \]

- What does this imply about \( \frac{\partial \log C_{t+j}}{\partial \epsilon_{t+1}} \) as \( j \to \infty \)?

\[ \frac{\partial \log C_{t+j}}{\partial \epsilon_{t+1}} = 1 \text{ for all } j \]

- What does it imply about \( \text{var} (\log C_{t+j}) \) as \( j \to \infty \)?
  \[ \text{Goes to infinity!} \]
Is GDP/Consumption a Random Walk?

- Textbook asset pricing model:
  \[ \log C_{t+1} = \mu + \log C_t + \epsilon_{t+1} \]

- What does this imply about \( \partial \log C_{t+j} / \partial \epsilon_{t+1} \) as \( j \to \infty \)?
  - \( \partial \log C_{t+j} / \partial \epsilon_{t+1} = 1 \) for all \( j \)?
  - I.e., shocks have permanent effects on GDP
Is GDP/Consumption a Random Walk?

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- What does it imply about \(\text{var}_t(\log C_{t+j})\) as \(j \to \infty\)?

Goes to infinity!!

But does US GDP look like a random walk with drift?

Nakamura-Steinsson
Textbook asset pricing model:

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What does it imply about \( \text{var}_t(\log C_{t+j}) \) as \( j \to \infty \)?

- Goes to infinity!!

But does US GDP look like a random walk with drift?
Fig. 2.—Log real per capita GNP, 1869–1986

Source: Cochrane (1988)
Is GDP/Consumption Trend Stationary?

- Traditional view in macro: GDP is trend stationary
  \[ y_t = bt + \sum_{j=0}^{\infty} a_j \epsilon_{t-j} \]
  where \( a_j \) approaches zero for large \( j \)

- Implies:
  - Long-run forecast invariant to \( \epsilon_t \) (i.e., business cycles are transient)
  - \( \text{var}_t(\log C_{t+j}) \to \sum_{j=0}^{\infty} a_j^2 \sigma < \infty \) as \( j \to \infty \)

- This view was challenged in the 1980s
  (Nelson-Plosser 82; Watson 86; Clark 87; Campbell-Mankiw 87)
**Modeling Time Series**

- Time series are autocorrelated (i.e., correlated with own past)

- Autoregressive (AR) model
  - $Y_t$ a function of $Y_{t-1}$ (and perhaps $Y_{t-2}, \ldots$) plus a shock.
  - AR(1) is a simple case:
    $$Y_t = \alpha + \beta Y_{t-1} + \epsilon_t$$

- Moving Average (MA) model
  - $Y_t$ function of moving average of current and past shocks
    (but not own past). MA(1) is a simple case:
    $$Y_t = \epsilon_t + \gamma \epsilon_{t-1}$$

- ARMA($p,q$) models combine these two features
Estimate an ARMA(p,q) process for GNP growth:

\[ \phi(L) \Delta Y_t = \theta(L) \epsilon_t \]

\( \phi(L) \) and \( \theta(L) \) are polynomials in the lag operator (\( L \Delta Y_t = \Delta Y_{t-1} \))


Estimate by maximum likelihood

Extensive discussion of model selection (i.e., selection of \( p \) and \( q \))

Main result:

\[ \frac{\partial \log C_{t+j}}{\partial \epsilon_{t+1}} \geq 1 \]

for relatively large \( j \)

Relatively robust to \( p \) and \( q \) choice
### TABLE IV
MODEL IMPULSE RESPONSES, In REAL GNP

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Standard errors are in parentheses.

Source: Campbell and Mankiw (1987)
One Shock or Many Shocks?

GDP is driven by many shocks with vastly different dynamics:

- Monetary shocks (transitory?)
- Productivity shocks (permanent?)
- Demographic shocks (build very slowly?)

Makes it very hard to measure “permanent component” of GDP shocks since short-term dynamics not necessarily informative about long-run dynamics (see, e.g., Quah 1992)
Source: FRED. Log GDP per Capita for the U.S.
Cochrane (1988) advocated using variance ratios:

$$VR_{i,k} = \frac{1}{k} \frac{\text{var}(c_{i,t} - c_{i,t-k})}{\text{var}(c_{i,t} - c_{i,t-1})}$$

Non-parametric approach
Cochrane (1988) advocated using variance ratios:

\[ VR_{i,k} = \frac{1}{k} \frac{\text{var}(c_{i,t} - c_{i,t-k})}{\text{var}(c_{i,t} - c_{i,t-1})} \]

Non-parametric approach
- Random walk: \( VR_{i,k} = 1 \) for all \( k \)
- Trend stationary: \( VR_{i,k} \rightarrow 0 \) as \( k \rightarrow \infty \)
- Positively autocorrelated growth: \( VR_{i,k} > 1 \) for large \( k \)
Of the series always returns to the "trend line." Furthermore, that trend line is linear: therefore no "waves" of low-frequency movement. These characteristics drive the finding of a small random walk component. (Note that low-frequency movement generated by a non-linear trend, a shift, etc. would show up as a large random walk component in this and most other estimation techniques based on linear time-series models.)

Prewar GNP data are more variable than postwar data, and one might suspect that this characteristic drives the result. However, figure 3 and table 1 present 1/k times the variance of k-differences for postwar GNP, and the same pattern is evident. Both the variance of first differences and the variance of the random walk component are lower, but their proportions do not change much.

The pattern of fig. 2 is sensitive to the precise specification of the variables. First, the variance of quarterly differences of seasonally adjusted GNP is less than one-fourth the variance of yearly differences, so the variance ratio is higher if one uses quarterly rather than annual differences in the denominator. This observation explains most of the difference between fig. 2 and the results reported by Campbell and Mankiw (1988), who use a similar technique on quarterly data. Second, taking the variance of overlapping k-year differences of quarterly data vs. the variance of k-year differences of annual averages, including or excluding population growth, taking logs or not, and even changing the sample by a few years can all change the variance ratio by about one standard error.

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**Fig. 1.**—1/k times the variance of k-differences of log real per capita GNP, 1869–1986, with asymptotic standard errors.

Source: Cochrane (1988)
FIG. 2.-Log real per capita GNP, 1869-1986

FIG. 3.—$1/k$ times the variance of $k$-differences of log real per capita GNP, 1947–86, with asymptotic standard errors.

Source: Cochrane (1988)
Notice that variance ratio initially rises above one

GDP growth positively autocorrelated at short horizons

This is what drives Campbell-Mankiw 87 results

Cochrane’s results reflect slow negative correlation of growth rates at longer horizons which is hard to pick up using low-order ARMA models
If consumption growth is largely trend stationary, then world is even less risky than textbook model assumes

Equity premium puzzle even worse
(and Lucas’ assumptions look good)
Cogley (1990)

- Extends Cochrane’s estimation approach to 9 OECD countries for 1871-1985
- Critiques small sample properties of Cochrane’s asymptotic standard errors
- Presents two estimators for variance ratio:
  - $\hat{V}^f$ based on frequency domain methods
  - $\hat{V}^k$ based on traditional method (i.e., Cochrane’s estimator)
Fig. 1.—Log real per capita GDP, 1871–1985

Source: Cogley (1990)
Table 2

Estimates of the Variance Ratio: Per Capita Output Growth, 1871–1985

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<td></td>
<td>(.27, 1.4)</td>
<td>(.19, 1.2)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>GNP</td>
<td>.49</td>
<td>.41</td>
<td>.60</td>
<td>.53</td>
</tr>
<tr>
<td></td>
<td>(.27, 1.4)</td>
<td>(.22, 1.4)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Note.—Approximate 90 percent confidence intervals are shown in parentheses.

Source: Cogley (1990)
Highly sensitive to the treatment of disasters

Disasters generally involve substantial recoveries

(Nakamura et al., 2010)
### TABLE IV
Variance Ratios in the Data and the Model (k=15)

<table>
<thead>
<tr>
<th>Consumption Growth</th>
<th>Realized Vol. of Cons. Growth</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Data</td>
</tr>
<tr>
<td>France</td>
<td>1.49 3.33 2.56 [1.00, 5.33]</td>
</tr>
<tr>
<td>UK</td>
<td>1.56 2.87 3.84 [1.78, 7.32]</td>
</tr>
<tr>
<td>US</td>
<td>1.08 1.29 1.69 [0.75, 3.65]</td>
</tr>
<tr>
<td>Average</td>
<td>1.11 2.28 2.60 [1.06, 5.29]</td>
</tr>
<tr>
<td>Median</td>
<td>0.87 1.62 2.69 [1.02, 5.47]</td>
</tr>
</tbody>
</table>

Source: Outtakes from Nakamura, Steinsson, and Sergeyev (2017)
How robust is the evidence that macroeconomic time series have a random walk?

Perhaps one or two "structural breaks" account for apparent non-stationarity.

Perron argues that GDP is stationary once one accounts for:

- Great Crash of 1929: Negative level shift
- Oil Price Shock of 1973: Negative trend shift

Data:

- Nelson-Plosser 82 annual data on 14 macro series ending in 1970
- Quarterly real GDP 1947:1-1986:3
Note: The broken straight line is a fitted trend (by OLS) of the form \( \tilde{y}_t = \bar{\mu} + \bar{\gamma} DU_t + \bar{\beta} t \) where \( DU_t = 0 \) if \( t \leq 1929 \) and \( DU_t = 1 \) if \( t > 1929 \).

**FIGURE 1.**—Logarithm of “Nominal Wages.”

Source: Perron (1989)
Note: The broken straight line is a fitted trend (by OLS) of the form: $\hat{y}_t = \hat{\mu} + \hat{\beta}t + \hat{\gamma}DT_t^*$ where $DT_t^* = 0$ if $t \leq 1973:1$ and $DT_t^* = t - T_B$ if $t > 1973:1 = T_B$.

**Figure 2.**—Logarithm of “Postwar Quarterly Real GNP.”

Source: Perron (1989)
Note: The broken straight line is a fitted trend (by OLS) of the form \[ \tilde{y}_t = \tilde{\mu} + \tilde{\gamma}_1 DU_{t} + \tilde{\beta} t + \tilde{\gamma}_2 DT_t \]
where \( DU_t = DT_t = 0 \) if \( t \leq 1929 \) and \( DU_t = 1, DT_t = t \) if \( t > 1929 \).

**FIGURE 3.**—Logarithm of “Common Stock Prices.”

Source: Perron (1989)
### Table I
Regression Analysis for the Wages, Quarterly GNP, and Common Stock Price Series

Regression: $y_t = \mu + \beta t + \alpha y_{t-1} + \sum_{i=1}^{k} \gamma_i \Delta y_{t-i} + \varepsilon_t$

<table>
<thead>
<tr>
<th>Series/Period</th>
<th>$k$</th>
<th>$\mu$</th>
<th>$t_\mu$</th>
<th>$\beta$</th>
<th>$t_\beta$</th>
<th>$\alpha$</th>
<th>$t_\alpha$</th>
<th>$S(\varepsilon)$</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>(a) Wages</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1900–1970$^a$</td>
<td>2</td>
<td>0.566</td>
<td>2.30</td>
<td>0.004</td>
<td>2.30</td>
<td>0.910</td>
<td>-2.09</td>
<td>0.060</td>
</tr>
<tr>
<td>1900–1929</td>
<td>7</td>
<td>4.299</td>
<td>2.84</td>
<td>0.037</td>
<td>2.73</td>
<td>0.304</td>
<td>-2.82</td>
<td>0.0803</td>
</tr>
<tr>
<td>1930–1970</td>
<td>8</td>
<td>1.632</td>
<td>3.60</td>
<td>0.012</td>
<td>2.64</td>
<td>0.735</td>
<td>-3.19</td>
<td>0.0269</td>
</tr>
<tr>
<td><strong>(b) Common stock prices</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1871–1970$^a$</td>
<td>2</td>
<td>0.481</td>
<td>2.02</td>
<td>0.003</td>
<td>2.37</td>
<td>0.913</td>
<td>-2.05</td>
<td>0.158</td>
</tr>
<tr>
<td>1871–1929</td>
<td>3</td>
<td>0.3468</td>
<td>2.13</td>
<td>0.0063</td>
<td>2.70</td>
<td>0.732</td>
<td>-2.29</td>
<td>0.1209</td>
</tr>
<tr>
<td>1930–1970</td>
<td>4</td>
<td>-0.5312</td>
<td>-1.64</td>
<td>0.0166</td>
<td>1.96</td>
<td>0.788</td>
<td>-1.89</td>
<td>0.1376</td>
</tr>
<tr>
<td><strong>(c) Quarterly real GNP</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1947:I–1986:III</td>
<td>2</td>
<td>0.386</td>
<td>2.90</td>
<td>0.0004</td>
<td>2.71</td>
<td>0.946</td>
<td>-2.85</td>
<td>0.010</td>
</tr>
<tr>
<td>1947:I–1973:I</td>
<td>2</td>
<td>0.637</td>
<td>3.04</td>
<td>0.0008</td>
<td>2.99</td>
<td>0.910</td>
<td>-3.02</td>
<td>0.0099</td>
</tr>
<tr>
<td>1973:II–1986:III</td>
<td>1</td>
<td>0.883</td>
<td>2.23</td>
<td>0.0008</td>
<td>2.27</td>
<td>0.878</td>
<td>-2.23</td>
<td>0.0102</td>
</tr>
</tbody>
</table>

$^a$Results taken from Nelson and Plosser (1982, Table 5).

Source: Perron (1989). Dickey-Fuller 2.5% critical value for N = 100, with constant and time trend is -3.7. Corresponding 5% critical value is -3.4.
Perron simulates 10,000 replications of a series $y_t$ of length 100

“Crash” hypothesis:

$$y_t = \mu_1 + (\mu_2 - \mu_1)DU_t + \beta t + e_t$$

where $DU_t = 1$ if $t > 50$, $\mu_1 = 0$, $\beta = 1$, $e_t \sim N(0, 1)$

“Changing Growth” hypothesis:

$$y_t = \mu + \beta_1 t + (\beta_2 - \beta_1)DT_t^* + e_t$$

where $DT_t^* = t - 50$ if $t > 50$, $\mu = 0$, $\beta_1 = 1$, $e_t \sim N(0, 1)$
Estimates misspecified model:

\[ y_t = \tilde{\mu} + \tilde{\beta} t + \tilde{\alpha} y_{t-1} + \tilde{e}_t \]

True \( \alpha = 0 \). But breaks look like a unit root.
TABLE III

MEAN AND VARIANCE OF $\tilde{\alpha}$

(a) Crash Simulations, $\mu_1 = 0$, $\beta = 1$

<table>
<thead>
<tr>
<th>$\mu_2$</th>
<th>$\mu_2 = 0$</th>
<th>$\mu_2 = -2$</th>
<th>$\mu_2 = -5$</th>
<th>$\mu_2 = -10$</th>
<th>$\mu_2 = -25$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
<td>-0.019</td>
<td>0.172</td>
<td>0.558</td>
<td>0.795</td>
<td>0.899</td>
</tr>
<tr>
<td>Variance</td>
<td>0.00986</td>
<td>0.01090</td>
<td>0.00471</td>
<td>0.00089</td>
<td>0.00009</td>
</tr>
</tbody>
</table>

(b) Breaking Trend Simulations, $\beta_1 = 1$, $\mu = 0$

<table>
<thead>
<tr>
<th>$\beta_2$</th>
<th>$\beta_2 = 1.0$</th>
<th>$\beta_2 = 0.9$</th>
<th>$\beta_2 = 0.7$</th>
<th>$\beta_2 = 0.4$</th>
<th>$\beta_2 = 0.0$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
<td>-0.019</td>
<td>0.334</td>
<td>0.825</td>
<td>0.949</td>
<td>0.981</td>
</tr>
<tr>
<td>Variance</td>
<td>0.00986</td>
<td>0.00938</td>
<td>0.00094</td>
<td>0.00009</td>
<td>0.00001</td>
</tr>
</tbody>
</table>

See notes to Figure 4 for case (a) and Figure 5 for case (b).
## Table VII

**Tests for a Unit Root**

(a) Regression (12), Model A; \( y_t = \bar{\mu} + \beta DU_t + \delta T + \Delta y_{t-1} + \Sigma_{t=1}^k \Delta y_{t-1} + \varepsilon_t \)

<table>
<thead>
<tr>
<th>( T_B = 1929 )</th>
<th>( T )</th>
<th>( \lambda )</th>
<th>( k )</th>
<th>( \bar{\mu} )</th>
<th>( t_{\bar{\mu}} )</th>
<th>( \beta )</th>
<th>( t_{\beta} )</th>
<th>( \delta )</th>
<th>( t_{\delta} )</th>
<th>( \bar{\alpha} )</th>
<th>( t_{\bar{\alpha}} )</th>
<th>( S(\bar{\varepsilon}) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Real GNP</td>
<td>62</td>
<td>0.33</td>
<td>8</td>
<td>3.441</td>
<td>5.07</td>
<td>-0.189</td>
<td>-4.28</td>
<td>0.0267</td>
<td>5.05</td>
<td>-0.018</td>
<td>-0.30</td>
<td>0.282</td>
</tr>
<tr>
<td>Nominal GNP</td>
<td>62</td>
<td>0.33</td>
<td>8</td>
<td>5.692</td>
<td>5.44</td>
<td>-0.360</td>
<td>-4.77</td>
<td>0.0359</td>
<td>5.44</td>
<td>0.100</td>
<td>1.09</td>
<td>0.471</td>
</tr>
<tr>
<td>Real per capita GNP</td>
<td>62</td>
<td>0.33</td>
<td>7</td>
<td>3.325</td>
<td>4.11</td>
<td>-0.102</td>
<td>-2.76</td>
<td>0.0111</td>
<td>4.00</td>
<td>-0.070</td>
<td>-1.09</td>
<td>0.531</td>
</tr>
<tr>
<td>Industrial production</td>
<td>111</td>
<td>0.63</td>
<td>8</td>
<td>0.120</td>
<td>4.37</td>
<td>-0.298</td>
<td>-4.58</td>
<td>0.0323</td>
<td>5.42</td>
<td>-0.095</td>
<td>-0.99</td>
<td>0.322</td>
</tr>
<tr>
<td>Employment</td>
<td>81</td>
<td>0.49</td>
<td>7</td>
<td>3.402</td>
<td>4.54</td>
<td>-0.046</td>
<td>-2.65</td>
<td>0.0057</td>
<td>4.26</td>
<td>-0.025</td>
<td>-0.77</td>
<td>0.667</td>
</tr>
<tr>
<td>GNP deflator</td>
<td>82</td>
<td>0.49</td>
<td>5</td>
<td>0.669</td>
<td>4.09</td>
<td>-0.098</td>
<td>-3.16</td>
<td>0.0070</td>
<td>4.01</td>
<td>0.026</td>
<td>0.53</td>
<td>0.776</td>
</tr>
<tr>
<td>Consumer prices</td>
<td>111</td>
<td>0.63</td>
<td>2</td>
<td>0.065</td>
<td>1.12</td>
<td>-0.004</td>
<td>-0.21</td>
<td>0.0005</td>
<td>1.75</td>
<td>-0.036</td>
<td>-0.79</td>
<td>0.978</td>
</tr>
<tr>
<td>Wages</td>
<td>71</td>
<td>0.41</td>
<td>7</td>
<td>2.38</td>
<td>5.45</td>
<td>-0.190</td>
<td>-4.32</td>
<td>0.0197</td>
<td>5.37</td>
<td>0.085</td>
<td>1.36</td>
<td>0.619</td>
</tr>
<tr>
<td>Money stock</td>
<td>82</td>
<td>0.49</td>
<td>6</td>
<td>0.301</td>
<td>4.72</td>
<td>-0.071</td>
<td>-2.59</td>
<td>0.0121</td>
<td>4.18</td>
<td>0.033</td>
<td>0.68</td>
<td>0.812</td>
</tr>
<tr>
<td>Velocity</td>
<td>102</td>
<td>0.59</td>
<td>0</td>
<td>0.050</td>
<td>0.932</td>
<td>-0.005</td>
<td>-0.20</td>
<td>-0.0002</td>
<td>-0.35</td>
<td>-0.136</td>
<td>-2.01</td>
<td>0.941</td>
</tr>
<tr>
<td>Interest rate</td>
<td>71</td>
<td>0.41</td>
<td>2</td>
<td>-0.018</td>
<td>-0.088</td>
<td>-0.343</td>
<td>-2.06</td>
<td>0.0105</td>
<td>2.64</td>
<td>0.197</td>
<td>0.64</td>
<td>0.976</td>
</tr>
</tbody>
</table>

(b) Regression (14), Model C; \( y_t = \bar{\mu} + \beta DU_t + \delta T + \gamma DT_t + \Delta y_{t-1} + \Sigma_{t=1}^k \Delta y_{t-1} + \varepsilon_t \)

<table>
<thead>
<tr>
<th>( T_B = 1929 )</th>
<th>( T )</th>
<th>( \lambda )</th>
<th>( k )</th>
<th>( \bar{\mu} )</th>
<th>( t_{\bar{\mu}} )</th>
<th>( \beta )</th>
<th>( t_{\beta} )</th>
<th>( \gamma )</th>
<th>( t_{\gamma} )</th>
<th>( \delta )</th>
<th>( t_{\delta} )</th>
<th>( \bar{\alpha} )</th>
<th>( t_{\bar{\alpha}} )</th>
<th>( S(\bar{\varepsilon}) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Common stock prices</td>
<td>100</td>
<td>0.59</td>
<td>1</td>
<td>0.353</td>
<td>4.09</td>
<td>-1.051</td>
<td>-4.29</td>
<td>0.0070</td>
<td>4.43</td>
<td>0.0139</td>
<td>3.98</td>
<td>0.128</td>
<td>0.76</td>
<td>0.718</td>
</tr>
<tr>
<td>Real wages</td>
<td>71</td>
<td>0.41</td>
<td>8</td>
<td>2.115</td>
<td>4.33</td>
<td>-0.190</td>
<td>-3.71</td>
<td>0.0107</td>
<td>3.79</td>
<td>0.0066</td>
<td>3.33</td>
<td>0.031</td>
<td>0.78</td>
<td>0.298</td>
</tr>
</tbody>
</table>

(c) Regression (10), Model B; \( y_t = \bar{\mu} + \beta T + \gamma T + \bar{\gamma} \bar{y}_{t-1} + \Sigma_{t=1}^k \bar{\gamma} \bar{y}_{t-1} + \varepsilon_t \)

| \( T_B = 1973:1 \) | \( T \) | \( \lambda \) | \( k \) | \( \bar{\mu} \) | \( t_{\bar{\mu}} \) | \( \beta \) | \( t_{\beta} \) | \( \gamma \) | \( t_{\gamma} \) | \( \bar{\alpha} \) | \( t_{\bar{\alpha}} \) | \( S(\bar{\varepsilon}) \) |
|-----------------|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Quarterly real GNP | 159 | 0.66 | 10 | 6.977 | 1160.51 | 0.0087 | 97.73 | -0.0031 | -12.06 | 0.86 | -3.98^c | 0.0097 |

**Note:** a, b, and c denote statistical significance at the 1%, 2.5%, and 5% level respectively.
Perron argues that after allowing for Great Crash of 1929 and 1973 Growth Slowdown, many macro series are stationary (i.e., he rejects the null of a unit root).
Perron argues that after allowing for Great Crash of 1929 and 1973 Growth Slowdown, many macro series are stationary (i.e., he rejects the null of a unit root)

But he chooses the break dates ex post

Perhaps it is normal for a unit root of that length to look like it has a break and is otherwise stationary

Main lesson: Hard to distinguish trends from unit roots in the presence of breaks.

What is a break? Infrequent unit root shock.
Recent literature has moved beyond trend vs. difference stationary debate

Three types of risks have been emphasized:
- Rare disasters (Ritz, 1988; Barro, 2006)
- Growth rate shocks (Bansal and Yaron, 2004)
- Stochastic volatility (Bansal and Yaron, 2004)
Same setup as Mehra-Prescott, except

\[ \log C_{t+1} = \mu + \log C_t + u_{t+1} + v_{t+1} \]

- \( u_{t+1} \sim N(0, \sigma^2) \)
- \( v_{t+1} \) reflects disasters:
  - Probability \( e^{-p} \): \( v_{t+1} = 0 \)
  - Probability \( 1 - e^{-p} \): \( v_{t+1} = \log(1 - b) \)
Key parameters: $p$ and $b$

- Measure declines in per capita GDP (Data: Maddison, 2003)
- Disaster: Cumulative drop of 15% or greater
- $p$ frequency of such drops: 1.7%
- $b$ peak-to-trough decline (e.g. WWII 1939-1945)
  - $E(b) = 0.29$ (mean size of disasters)
  - Huge amount of heterogeneity in disaster size
Panel A: Contractions in Table I

Panel B: Contractions in Table I adjusted for trend growth

FIGURE I
Frequency Distribution of Economic Disasters

The histograms apply to the 35 countries covered over the twentieth century in Table I. The horizontal axis has intervals for declines in real per capita GDP. The vertical axis shows the number of economic contractions in each interval. The five war aftermaths shown in Table I are excluded; therefore, 60 events are used. The bottom panel adjusts for trend growth at 0.0252 per year.
Barro (2006)

- What is the impact of heterogeneity in disaster size?
- Why focus on disasters and ignore bonanzas?
Barro (2006): Asset Pricing

- Representative consumer
- Power utility
- Assets to price:
  - Unlevered consumption claim
  - One period, bond (occasional default during disasters)
- Empirical moments:
  - Equity Premium: Stocks: 7.1%, Bills: -0.1%
  - Leverage ratio for equity of 1.5
  - Target for unlevered equity: 7.2%/1.5 = 4.8%
### TABLE V
**Calibrated Model for Rates of Return**

<table>
<thead>
<tr>
<th>Parameters</th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
<th>(5)</th>
<th>(6)</th>
<th>(7)</th>
</tr>
</thead>
<tbody>
<tr>
<td>No disasters baseline</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>θ</strong> (coeff. of relative risk aversion)</td>
<td>4</td>
<td>4</td>
<td>3</td>
<td>4</td>
<td>4</td>
<td>4</td>
<td>4</td>
</tr>
<tr>
<td><strong>σ</strong> (s.d. of growth rate, no disasters)</td>
<td>0.02</td>
<td><strong>0.02</strong></td>
<td>0.02</td>
<td>0.02</td>
<td>0.02</td>
<td>0.02</td>
<td>0.02</td>
</tr>
<tr>
<td><strong>ρ</strong> (rate of time preference)</td>
<td>0.03</td>
<td><strong>0.03</strong></td>
<td>0.03</td>
<td>0.03</td>
<td>0.03</td>
<td>0.03</td>
<td><strong>0.02</strong></td>
</tr>
<tr>
<td><strong>γ</strong> (growth rate, deterministic part)</td>
<td>0.025</td>
<td><strong>0.025</strong></td>
<td>0.025</td>
<td>0.025</td>
<td>0.025</td>
<td><strong>0.020</strong></td>
<td>0.025</td>
</tr>
<tr>
<td><strong>p</strong> (disaster probability)</td>
<td>0</td>
<td><strong>0.017</strong></td>
<td>0.017</td>
<td><strong>0.025</strong></td>
<td>0.017</td>
<td>0.017</td>
<td>0.017</td>
</tr>
<tr>
<td><strong>q</strong> (bill default probability in disaster)</td>
<td>0</td>
<td><strong>0.4</strong></td>
<td>0.4</td>
<td>0.4</td>
<td><strong>0.3</strong></td>
<td>0.4</td>
<td>0.4</td>
</tr>
</tbody>
</table>

**Variables**

- Expected equity rate: 0.128, 0.071, 0.076, 0.044, 0.071, 0.051, 0.061
- Expected bill rate: 0.127, 0.035, 0.061–0.007, 0.029, 0.015, 0.025
- Equity premium: 0.0016, 0.036, 0.016, 0.052, 0.042, 0.036, 0.036

Source: Barro (2006)
Barro (2006): Simple disaster model can match
- A high equity premium
- A low risk-free rate

Barro (2009): What does this same model imply about:
- Welfare costs of business cycles?
- Welfare costs of disasters?
and earns the common real wage rate, \( w_t \). Since the labor market is competitive, \( w_t \) equals the marginal product of labor, determined from equation (29).

Each person is endowed with one unit of time, which can be allocated between leisure and market work. Utility now depends on each period's consumption, \( C_t \), and leisure, \( L_t \). One straightforward way to model preferences is to use the Epstein-Zin-Weil formulation of utility from equation (9), but replace \( C_t \) by \( \lambda C_t + \rho L_t \).16 The new parameter \( \lambda \) is the constant elasticity of substitution between consumption and leisure at a point in time. This form is consistent with the prescription of Robert G. King, Charles I. Plosser, and Sergio Rebelo (1988) that preferences accord with the property that work effort, \( L_t \), be constant in the long run, that is, when \( w_t \) and \( C_t \) advance at the same rate due to steady productivity growth. In the present setting, which lacks capital accumulation, this property also holds in the short run, so that \( L_t \) ends up constant in equilibrium.

The new set of first-order conditions involves substitution between leisure and consumption at each point in time:

\[
0 u/0 (1/2) L_t = 0 w_t.
\]

Table 3—Effects of Preference Parameters on Rates of Return and Welfare Costs

<table>
<thead>
<tr>
<th>( \gamma )</th>
<th>( \theta )</th>
<th>( \rho )</th>
<th>( \rho^* )</th>
<th>( r^e )</th>
<th>( r^f )</th>
<th>( V )</th>
<th>Welfare effects (percent)</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>0.25</td>
<td>0.054</td>
<td>0.027</td>
<td>0.069</td>
<td>0.010</td>
<td>20.7</td>
<td>1.65</td>
</tr>
<tr>
<td>4</td>
<td><strong>0.50</strong></td>
<td><strong>0.052</strong></td>
<td><strong>0.027</strong></td>
<td><strong>0.069</strong></td>
<td><strong>0.010</strong></td>
<td><strong>20.7</strong></td>
<td><strong>1.65</strong></td>
</tr>
<tr>
<td>4</td>
<td>1</td>
<td>0.048</td>
<td>0.027</td>
<td>0.069</td>
<td>0.010</td>
<td>20.7</td>
<td>1.64</td>
</tr>
<tr>
<td>4</td>
<td>4</td>
<td>0.027</td>
<td>0.027</td>
<td>0.069</td>
<td>0.010</td>
<td>20.7</td>
<td>1.60</td>
</tr>
<tr>
<td>3.5</td>
<td>0.25</td>
<td>0.062</td>
<td>0.027</td>
<td>0.074</td>
<td>0.035</td>
<td>18.7</td>
<td>1.31</td>
</tr>
<tr>
<td>3.5</td>
<td>0.50</td>
<td>0.059</td>
<td>0.027</td>
<td>0.074</td>
<td>0.035</td>
<td>18.7</td>
<td>1.30</td>
</tr>
<tr>
<td>3.5</td>
<td>1</td>
<td>0.054</td>
<td>0.027</td>
<td>0.074</td>
<td>0.035</td>
<td>18.7</td>
<td>1.30</td>
</tr>
<tr>
<td>3.5</td>
<td>4</td>
<td>0.022</td>
<td>0.027</td>
<td>0.074</td>
<td>0.035</td>
<td>18.7</td>
<td>1.27</td>
</tr>
<tr>
<td>3</td>
<td>0.25</td>
<td>0.063</td>
<td>0.027</td>
<td>0.074</td>
<td>0.048</td>
<td>18.7</td>
<td>1.12</td>
</tr>
<tr>
<td>3</td>
<td>0.50</td>
<td>0.060</td>
<td>0.027</td>
<td>0.074</td>
<td>0.048</td>
<td>18.7</td>
<td>1.12</td>
</tr>
<tr>
<td>3</td>
<td>1</td>
<td>0.053</td>
<td>0.027</td>
<td>0.074</td>
<td>0.048</td>
<td>18.7</td>
<td>1.12</td>
</tr>
<tr>
<td>3</td>
<td>4</td>
<td>0.014</td>
<td>0.027</td>
<td>0.074</td>
<td>0.048</td>
<td>18.7</td>
<td>1.10</td>
</tr>
<tr>
<td>1</td>
<td>0.25</td>
<td>0.041</td>
<td>0.027</td>
<td>0.047</td>
<td>0.044</td>
<td>37.1</td>
<td>0.74</td>
</tr>
<tr>
<td>1</td>
<td>0.50</td>
<td>0.036</td>
<td>0.027</td>
<td>0.047</td>
<td>0.044</td>
<td>37.1</td>
<td>0.74</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>0.027</td>
<td>0.027</td>
<td>0.047</td>
<td>0.044</td>
<td>37.1</td>
<td>0.74</td>
</tr>
<tr>
<td>1</td>
<td>4</td>
<td>−0.030</td>
<td>0.027</td>
<td>0.047</td>
<td>0.044</td>
<td>37.1</td>
<td>0.73</td>
</tr>
</tbody>
</table>

Notes: The baseline results are in bold, \( \gamma \) is the coefficient of relative risk aversion, \( \theta \) is the reciprocal of the IES in the formula for utility in equation (9), \( \rho \) is the rate of time preference, and \( \rho^* \) is the effective rate of time preference, given in equation (12); \( \rho = \rho^* \) holds when \( \gamma = \theta \). The formulas for the expected rate of return on equity, \( r^e \), the risk-free rate, \( r^f \), and the price-dividend ratio, \( V \), are given in equations (6), (7), and (5), respectively, after replacing \( \rho \) by \( \rho^* \). The value of \( \rho^* \) is set at 0.027 to generate \( r^f = 0.010 \) with the baseline parameters. The value for \( \rho \) (0.052 in the baseline specification) is then varied in each case to maintain \( \rho^* = 0.027 \) (in equation (12)). Since \( \rho^* \) is held constant, the values for \( r^e \), \( r^f \), and \( V \) depend on \( \gamma \) but not on \( \theta \). Each welfare effect gives the percentage reduction in initial output, \( 1 - (Y_t)^*/Y_t \), that maintains attained utility while setting to zero either the standard deviation, \( \sigma \), of normal economic fluctuations or the disaster probability, \( p \). The effects are for a given expected growth rate, \( g^* \), given in equation (2). The values for \( 1 - (Y_t)^*/Y_t \) come from equation (23).

Source: Barro (2009)
Barro’s model:

$$\log C_{t+1} = \mu + \log C_t + u_{t+1} + v_{t+1}$$

- $u_{t+1} \sim N(0, \sigma^2)$
- $v_{t+1}$:
  - Probability $e^{-p}$: $v_{t+1} = 0$
  - Probability $1 - e^{-p}$: $v_{t+1} = \log(1 - b)$

Is this a realistic model of disasters?
Barro (2006): Stylized Disaster Model

- All disasters are completely permanent
- Disasters occur instantaneously
- Timing of disasters uncorrelated across countries
- Informal estimation procedure
Consumption:
\[ c_{i,t} = x_{i,t} + z_{i,t} + \epsilon_{i,t} \]

Potential Consumption:
\[ \Delta x_{i,t} = \mu_{i,t} + \eta_{i,t} + I_{i,t}\theta_{i,t} \]

The Disaster Gap
\[ z_{i,t} = \rho_z z_{i,t-1} - I_{i,t}\theta_{i,t} + I_{i,t}\phi_{i,t} + \nu_{i,t} \]

\[ \epsilon_{i,t} \sim N(0, \sigma^2_{\epsilon_i}) \quad \eta_{i,t} \sim N(0, \sigma^2_{\eta_i}) \quad \nu_{i,t} \sim N(0, \sigma^2_{\nu_i}) \]
\[ \theta_{i,t} \sim N(\theta, \sigma^2_{\theta}) \quad \phi_{i,t} \sim \text{truncN}(\phi, \sigma^2_{\phi}, [-\infty, 0]) \]
Two disaster shocks:

1. $\phi_{i,t}$: Short run effect but no long run effect
2. $\theta_{i,t}$: Long run effect but no short run effect

Examples:

- Transitory effects ($\phi_{i,t}$):
  - Destruction of capital, military spending crowds out consumption, financial stress

- Permanent effects ($\theta_{i,t}$):
  - Loss of time spent on R&D, change in institutions
$\theta = 0, \phi = -0.1$
\[ \theta = -0.1, \ \phi = -0.1 \]
$\theta = -0.1, \phi = -0.2$
Our model is difficult to estimate by ML
  - Many unobserved state variables
Relatively simple to estimate by Bayesian MCMC estimation
Allow for breaks in:
  - $\sigma_{\eta,i}$, $\sigma_{\epsilon,i}$ in 1946. (change in data quality)
  - $\mu_i$ in 1946 and 1973. (captures high post-WWII growth)
Source: Nakamura, Steinsson, Barro, and Ursua (2013)
Source: Nakamura, Steinsson, Barro, and Ursua (2013)
Source: Nakamura, Steinsson, Barro, and Ursua (2013)
Source: Nakamura, Steinsson, Barro, and Ursua (2013)
Source: Nakamura, Steinsson, Barro, and Ursua (2013)
<table>
<thead>
<tr>
<th>CRRA</th>
<th>4.5</th>
<th>6.5</th>
<th>8.5</th>
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<tbody>
<tr>
<td>IES</td>
<td>2.0</td>
<td>2.0</td>
<td>2.0</td>
</tr>
</tbody>
</table>

Log Expected Return

<table>
<thead>
<tr>
<th></th>
<th>Equity</th>
<th>Bond</th>
<th>Equity Premium</th>
</tr>
</thead>
<tbody>
<tr>
<td>CRRA 4.5</td>
<td>0.050</td>
<td>0.032</td>
<td>0.018</td>
</tr>
<tr>
<td>CRRA 6.5</td>
<td>0.058</td>
<td>0.009</td>
<td>0.048</td>
</tr>
<tr>
<td>CRRA 8.5</td>
<td>0.066</td>
<td>-0.023</td>
<td>0.088</td>
</tr>
</tbody>
</table>

Log Expected Return (Cond. on No Disasters)

<table>
<thead>
<tr>
<th></th>
<th>Equity</th>
<th>Bond</th>
<th>Equity Premium</th>
</tr>
</thead>
<tbody>
<tr>
<td>CRRA 4.5</td>
<td>0.051</td>
<td>0.034</td>
<td>0.017</td>
</tr>
<tr>
<td>CRRA 6.5</td>
<td>0.058</td>
<td>0.010</td>
<td>0.048</td>
</tr>
<tr>
<td>CRRA 8.5</td>
<td>0.066</td>
<td>-0.025</td>
<td>0.091</td>
</tr>
</tbody>
</table>

Source: Nakamura, Steinsson, Barro, and Ursua (2013). Equity is unleveraged.
Source: Nakamura, Steinsson, Barro, and Ursua (2013)
TABLE
Asset Prices with CRRA=4 and IES=1/4

<table>
<thead>
<tr>
<th>asset type</th>
<th>Baseline</th>
<th>Barro (2006)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Equity</td>
<td>0.112</td>
<td>0.071</td>
</tr>
<tr>
<td>Bond</td>
<td>0.103</td>
<td>0.035</td>
</tr>
<tr>
<td>Equity Premium</td>
<td>0.009</td>
<td>0.036</td>
</tr>
</tbody>
</table>

Log Expected Return (Cond. on No Disasters)

<table>
<thead>
<tr>
<th>asset type</th>
<th>Baseline</th>
<th>Barro (2006)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Equity</td>
<td>0.097</td>
<td>0.076</td>
</tr>
<tr>
<td>Bond</td>
<td>0.106</td>
<td>0.037</td>
</tr>
<tr>
<td>Equity Premium</td>
<td>-0.009</td>
<td>0.039</td>
</tr>
</tbody>
</table>
Source: Nakamura, Steinsson, Barro, and Ursua (2013)
EZW utility: Stock market crash at onset of disaster
  - Assuming IES>1

Power utility: Stock market boom!

Why?
EZW utility: Stock market crash at onset of disaster
  - Assuming IES>1
Power utility: Stock market boom!
Why?
  - At onset of disaster, expected growth is negative, uncertainty increases
  - Leads to high savings in a model with low IES (Power Utility)
Contrast vs. Barro (2006) with permanent shocks
Source: Nakamura, Steinsson, Barro, and Ursua (2013)
How to Model Consumption Dynamics?

Figure: Log Consumption for France
\[ \Delta c_{t+1} = \mu + x_t + \chi \sigma_t \eta_{t+1}, \]
\[ x_{t+1} = \rho x_t + \sigma \epsilon_{t+1}, \]
\[ \sigma_{t+1}^2 = \sigma^2 + \gamma (\sigma_t^2 - \sigma^2) + \sigma \omega \omega_{t+1}, \]

Idea:
- \( x_t \) and \( \sigma_t^2 \) small but persistent
- Small enough that they are hard to observe (can’t be rejected)

Main Result:
- Even small “long run risks” makes a big difference for asset pricing
Seems intuitive that long-run risks to growth and uncertainty would raise equity premium

But does this work in benchmark model?

I.e.: Are long run risks priced?
IN POWER UTILITY MODEL: LRR NOT PRICED

\[ \Delta c_{t+1} = \mu + x_t + \chi \sigma_t \eta_{t+1}, \]
\[ x_{t+1} = \rho x_t + \sigma_t \epsilon_{t+1}, \]
\[ \sigma^2_{t+1} = \sigma^2 + \gamma (\sigma^2_t - \sigma^2) + \sigma \omega \omega_{t+1}, \]

- Notice that \( \epsilon_{t+1} \) and \( \omega_{t+1} \) affect:
  - \( R_{e,t+1} \)
  - \( \Delta c_{t+j} \) for \( j > 1 \)
  - But not \( \Delta c_{t+1} \)

- With power utility, long run risks:
  - Don’t create correlation between returns and stochastic discount factor
  - Have no effect on asset prices

- Timing issue implies that EZW preferences are crucial in LRR model
EZW preferences with:
- CRRA: $\gamma = 10$
- IES: $\psi = 1.5$

Two assets:
- One period, risk-free bond
- “Equity” with dividend growth rate:

$$\Delta d_{t+1} = \mu + \phi x_t + \varphi_d \sigma_t u_t$$

- Leverage: $\phi = 3$
- Dividend volatility: $\varphi_d = 4.5$
BANSAL AND YARON (2004): CALIBRATION

\[
\begin{align*}
\Delta c_{t+1} &= \mu + x_t + \chi \sigma t \eta_{t+1}, \\
x_{t+1} &= \rho x_t + \sigma_t \epsilon_{t+1}, \\
\sigma^2_{t+1} &= \sigma^2 + \gamma (\sigma^2_t - \sigma^2) + \sigma \omega \omega_{t+1},
\end{align*}
\]

- Calibrate long-run risks parameters:

  \[\mu = 0.0015, \quad \rho = 0.979, \quad \sigma = 0.078, \quad \varphi_e = 0.044\]

- No formal macro calibration targets
- Parameters largely viewed as free parameters
- Chosen largely to fit asset prices
BANSAL AND YARON (2004): CALIBRATION

\[
\begin{align*}
\Delta c_{t+1} &= \mu + x_t + \chi \sigma t \eta_{t+1}, \\
x_{t+1} &= \rho x_t + \sigma t \epsilon_{t+1}, \\
\sigma_{t+1}^2 &= \sigma^2 + \gamma (\sigma_t^2 - \sigma^2) + \sigma \omega \omega_{t+1},
\end{align*}
\]

- Calibrate long-run risks parameters:

  \[
  \mu = 0.0015, \quad \rho = 0.979, \quad \sigma = 0.078, \quad \sigma_e = 0.044
  \]

- No formal macro calibration targets
- Parameters largely viewed as free parameters
- Chosen largely to fit asset prices
- Why is this viable?
  - Long-run risks small enough they don’t seriously affect model’s fit to data on macro aggregates
In order to isolate the economic effects of persistent expected growth rates from those of fluctuating economic uncertainty, we report our results first for Case I, where fluctuating economic uncertainty has been shut off (,w is set to zero), and then consider the model specification where both channels are operational.

### A. Persistent Expected Growth

In Table I we display the time-series properties of the model given in (4). The specific parameters are given below the table. In spite of a persistent growth component, the model's implied time-series properties are largely consistent with the data.

Barsky and DeLong (1993) rely on a persistence parameter p equal to 1. We calibrate p at 0.979; this ensures that expected consumption growth rates are stationary and permits the possibility of large dividend elasticity of equity prices and equity risk premia. Our choice of Ce and a is motivated to ensure that we match the unconditional variance and the autocorrelation function of annual consumption growth. The standard deviation of the one-step ahead innovation in consumption, that is a, equals 0.0078. This parameter configuration implies that the predictable variation in monthly consumption growth, that is, the R2, is

<table>
<thead>
<tr>
<th>Variable</th>
<th>Estimate</th>
<th>SE</th>
<th>Mean</th>
<th>95%</th>
<th>5%</th>
<th>p-Val</th>
<th>Pop</th>
</tr>
</thead>
<tbody>
<tr>
<td>σ(g)</td>
<td>2.93</td>
<td>(0.69)</td>
<td>2.72</td>
<td>3.80</td>
<td>2.01</td>
<td>0.37</td>
<td>2.88</td>
</tr>
<tr>
<td>AC(1)</td>
<td>0.49</td>
<td>(0.14)</td>
<td>0.48</td>
<td>0.65</td>
<td>0.21</td>
<td>0.53</td>
<td>0.53</td>
</tr>
<tr>
<td>AC(2)</td>
<td>0.15</td>
<td>(0.22)</td>
<td>0.23</td>
<td>0.50</td>
<td>-0.17</td>
<td>0.70</td>
<td>0.27</td>
</tr>
<tr>
<td>AC(5)</td>
<td>-0.08</td>
<td>(0.10)</td>
<td>0.13</td>
<td>0.46</td>
<td>-0.13</td>
<td>0.93</td>
<td>0.09</td>
</tr>
<tr>
<td>AC(10)</td>
<td>0.05</td>
<td>(0.09)</td>
<td>0.01</td>
<td>0.32</td>
<td>-0.24</td>
<td>0.80</td>
<td>0.01</td>
</tr>
<tr>
<td>VR(2)</td>
<td>1.61</td>
<td>(0.34)</td>
<td>1.47</td>
<td>1.69</td>
<td>1.22</td>
<td>0.17</td>
<td>1.53</td>
</tr>
<tr>
<td>VR(5)</td>
<td>2.01</td>
<td>(1.23)</td>
<td>2.26</td>
<td>3.78</td>
<td>0.79</td>
<td>0.63</td>
<td>2.36</td>
</tr>
<tr>
<td>VR(10)</td>
<td>1.57</td>
<td>(2.07)</td>
<td>3.00</td>
<td>6.51</td>
<td>0.76</td>
<td>0.77</td>
<td>2.96</td>
</tr>
<tr>
<td>σ(g_δ)</td>
<td>11.49</td>
<td>(1.98)</td>
<td>10.96</td>
<td>15.47</td>
<td>7.79</td>
<td>0.43</td>
<td>11.27</td>
</tr>
<tr>
<td>AC(1)</td>
<td>0.21</td>
<td>(0.13)</td>
<td>0.33</td>
<td>0.57</td>
<td>0.09</td>
<td>0.53</td>
<td>0.39</td>
</tr>
<tr>
<td>corr(g, g_δ)</td>
<td>0.55</td>
<td>(0.34)</td>
<td>0.31</td>
<td>0.60</td>
<td>-0.03</td>
<td>0.07</td>
<td>0.35</td>
</tr>
</tbody>
</table>

Source: Bansal and Yaron (2004)
The entries are model population values of asset prices. The model incorporates fluctuating economic uncertainty (i.e., Case II) using the process in equation (8). In addition to the parameter values given in Panel A of Table II ($\theta = 0.998$, $\mu = -fd = 0.0015$, $p = 0.979$, $a = 0.0078$, $\alpha = 3$, $\epsilon = 0.044$, and $pd = 4.5$), the parameters of the stochastic volatility process are $v_1 = 0.987$ and $a_v = 0.23 \times 10^{-5}$.

The predictable variation of realized volatility is 5.5%. The expressions $E(R_m - R_f)$ and $E(R_f)$ are, respectively, the annualized equity premium and mean risk-free rate. The expressions $\sigma(R_m)$, $\sigma(R_f)$, and $\sigma(p - d)$ are the annualized volatilities of the market return, risk-free rate, and the log price-dividend, respectively. The expressions AC1 and AC2 denote, respectively, the first and second autocorrelation. Standard errors are Newey and West (1987) corrected using 10 lags.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Data Estimate</th>
<th>Data SE</th>
<th>Model $\gamma = 7.5$</th>
<th>Model $\gamma = 10$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
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<td></td>
<td></td>
</tr>
<tr>
<td>Returns</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$E(r_m - r_f)$</td>
<td>6.33</td>
<td>(2.15)</td>
<td>4.01</td>
<td>6.84</td>
</tr>
<tr>
<td>$E(r_f)$</td>
<td>0.86</td>
<td>(0.42)</td>
<td>1.44</td>
<td>0.93</td>
</tr>
<tr>
<td>$\sigma(r_m)$</td>
<td>19.42</td>
<td>(3.07)</td>
<td>17.81</td>
<td>18.65</td>
</tr>
<tr>
<td>$\sigma(r_f)$</td>
<td>0.97</td>
<td>(0.28)</td>
<td>0.44</td>
<td>0.57</td>
</tr>
<tr>
<td>Price Dividend</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$E(\exp(p - d))$</td>
<td>26.56</td>
<td>(2.53)</td>
<td>25.02</td>
<td>19.98</td>
</tr>
<tr>
<td>$\sigma(p - d)$</td>
<td>0.29</td>
<td>(0.04)</td>
<td>0.18</td>
<td>0.21</td>
</tr>
<tr>
<td>AC1$(p - d)$</td>
<td>0.81</td>
<td>(0.09)</td>
<td>0.80</td>
<td>0.82</td>
</tr>
<tr>
<td>AC2$(p - d)$</td>
<td>0.64</td>
<td>(0.15)</td>
<td>0.65</td>
<td>0.67</td>
</tr>
</tbody>
</table>

Source: Bansal and Yaron (2004)
Role of Epstein-Zin Preferences

- Stochastic discount factor with Epstein-Zin-Weil preferences:
  \[
  \log M_{t+1} = \theta \log \beta - \frac{\theta}{\psi} \Delta c_{t+1} + (\theta - 1) R_{c,t+1}
  \]

- Current marginal utility depends on news about future consumption growth (through \( R_{c,t+1} \))
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  - Increase in future uncertainty raises current marginal utility
    (If CRRA > 1 and IES > 1)

- IES > 1 crucial for LRRs to increase equity premium
Large literature argues stock returns are predictable
(Campbell-Shiller, 1988; Fama-French, 1988, Cochrane, 2008,
van Binsbergen-Koijen, 2010)

Idea: High P/D ratio predicts low returns
Table VI
Predictability of Returns, Growth Rates, and Price-Dividend Ratios

This table provides evidence on predictability of future excess returns and growth rates by price-dividend ratios, and the predictability of price-dividend ratios by consumption volatility. The entries in Panel A correspond to regressing $r_{t+1} + r_{t+2} + \ldots + r_{t+j} = a(j) + B(j) \log(P_t/D_t) + v_{t+j}$, where $r_{t+1}$ is the excess return, and $j$ denotes the forecast horizon in years. The entries in Panel B correspond to regressing $g_{t+1} + g_{t+2} + \ldots + g_{t+j} = a(j) + B(j) \log(P_t/D_t) + v_{t+j}$, and $g_{t}^{a}$ is annualized consumption growth. The entries in Panel C correspond to $\log(P_{t+j}/D_{t+j}) = a(j) + B(j) l_{E_{g,t}} + v_{t+j}$, where $l_{E_{g,t}}$ is the volatility of consumption defined as the absolute value of the residual from regressing $g_t = 1/5 A_j g_{j-1} + e_{g,t}$.

The model is based on the process in equation (8), with parameter configuration given in Table IV and $y = 10$. The entries for the model are based on 1,000 simulations each with 840 monthly observations that are time-aggregated to an annual frequency. Standard errors are Newey and West (1987) corrected using 10 lags.

### Panel A: Excess Returns

<table>
<thead>
<tr>
<th>Variable</th>
<th>Data</th>
<th>SE</th>
<th>Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>$B(1)$</td>
<td>-0.08</td>
<td>(0.07)</td>
<td>-0.18</td>
</tr>
<tr>
<td>$B(3)$</td>
<td>-0.37</td>
<td>(0.16)</td>
<td>-0.47</td>
</tr>
<tr>
<td>$B(5)$</td>
<td>-0.66</td>
<td>(0.21)</td>
<td>-0.66</td>
</tr>
<tr>
<td>$R^2(1)$</td>
<td>0.02</td>
<td>(0.04)</td>
<td>0.05</td>
</tr>
<tr>
<td>$R^2(3)$</td>
<td>0.19</td>
<td>(0.13)</td>
<td>0.10</td>
</tr>
<tr>
<td>$R^2(5)$</td>
<td>0.37</td>
<td>(0.15)</td>
<td>0.16</td>
</tr>
</tbody>
</table>

In Panel B of Table VI, we provide regression results where the dependent variable is the sum of annual consumption growth rates. In the data it seems that price-dividend ratios have little predictive power, particularly at longer horizons. The slope coefficients and $R^2$s of these regressions are quite low both in the data and the model. The $R^2$s are relatively small in the model for two reasons. First, price-dividend ratios are determined by expected growth rates, and the variation in expected growth rates is quite small. Recall that the monthly $R^2$ for consumption dynamics is less than 5%. Second, price-dividend ratios are also affected by independent movements in economic uncertainty, which lowers their ability to predict future growth rates. Overall, the model, like the data, suggests that growth rates at long horizons are not predicted by price-dividend ratios in any economically sizeable manner.

14 Consistent with Lettau and Ludvigson (2001), predictability coefficients and $R^2$s based on the wealth-consumption ratio follow the same pattern and are slightly larger than those based on price-dividend ratios.

15 Our model can be easily modified to further lower the predictability of growth rates. Consider an augmented model (as in Cecchetti et al. (1993)) that allows for additional predictable movements in dividend growth rates that are unrelated to consumption. This will not affect the risk-free rate and the risk premia in the model, but will additionally lower the ability of price-dividend ratios to predict future consumption growth rates.

Source: Bansal and Yaron (2004)
P/D ratio is stationary

A decrease in P/D therefore implies:
- High returns going forward, or ...
- Low Dividend growth going forward, or ...
- Both

Uncertainty shock in LRR model implies:
- Stock prices fall (if CRRA > 1 and IES > 1)
- No effect on expected dividends
- So, expected returns must rise
What about growth rate shocks?

In LLR model, high growth rate shocks raise P/D and predict future consumption growth.

Not in the data.
The Long-Run Risks Model and Aggregate Asset Prices: An Empirical Assessment

\[ \sum_{j=1}^{J} (r_{m,t+j} - r_{f,t+j}) = \alpha + \beta (p_t - d_t) + \epsilon_{t+j} \]

<table>
<thead>
<tr>
<th>( \beta )</th>
<th>( t )</th>
<th>( \hat{R}^2 )</th>
<th>( R^2(50%) )</th>
<th>( %(\hat{R}^2) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>data</td>
<td>data</td>
<td>data</td>
<td>BY</td>
<td>BKY</td>
</tr>
<tr>
<td>1 Y</td>
<td>-0.093</td>
<td>-1.803</td>
<td>0.044</td>
<td>0.007</td>
</tr>
<tr>
<td>3 Y</td>
<td>-0.264</td>
<td>-3.231</td>
<td>0.170</td>
<td>0.017</td>
</tr>
<tr>
<td>5 Y</td>
<td>-0.413</td>
<td>-3.781</td>
<td>0.269</td>
<td>0.025</td>
</tr>
<tr>
<td>4 Q</td>
<td>-0.119</td>
<td>-2.625</td>
<td>0.090</td>
<td>0.008</td>
</tr>
<tr>
<td>12 Q</td>
<td>-0.274</td>
<td>-3.191</td>
<td>0.187</td>
<td>0.022</td>
</tr>
<tr>
<td>20 Q</td>
<td>-0.424</td>
<td>-3.365</td>
<td>0.257</td>
<td>0.033</td>
</tr>
</tbody>
</table>

\[ \sum_{j=1}^{J} (\Delta c_{t+j}) = \alpha + \beta (p_t - d_t) + \epsilon_{t+j} \]

<table>
<thead>
<tr>
<th>( \alpha )</th>
<th>( \beta )</th>
<th>( \gamma )</th>
<th>( %(\hat{R}^2) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 Y</td>
<td>0.011</td>
<td>1.586</td>
<td>0.060</td>
</tr>
<tr>
<td>3 Y</td>
<td>0.010</td>
<td>0.588</td>
<td>0.013</td>
</tr>
<tr>
<td>5 Y</td>
<td>-0.001</td>
<td>-0.060</td>
<td>0.000</td>
</tr>
<tr>
<td>4 Q</td>
<td>0.000</td>
<td>0.140</td>
<td>0.000</td>
</tr>
<tr>
<td>12 Q</td>
<td>-0.002</td>
<td>-0.296</td>
<td>0.001</td>
</tr>
<tr>
<td>20 Q</td>
<td>-0.003</td>
<td>-0.296</td>
<td>0.002</td>
</tr>
</tbody>
</table>

Source: Beeler and Campbell (2012)
Key LRR parameters are macro parameters

- How important are changes in trend growth rates (e.g., productivity slowdown)
- How important are fluctuations in macro volatility? (e.g. Great Moderation)

However, in LRR literature, key parameters are calibrated or estimated to fit asset pricing data

Since model has no other way to fit asset pricing data, it concludes that LRR are there

But are these features really “there” in macro data?
Estimate long-run risks model using only macro data

Use data on aggregate consumption from 16 countries over 120 years

Pool data across countries to better estimate key parameters

Advantage of using macroeconomic data alone:
- Results not driven by need to explain asset prices
- Results provide direct evidence for the mechanism
\[ c_{i,t+1} = \tilde{c}_{i,t+1} + \sigma_{i,\nu} \nu_{i,t+1} + I_{i,t+1}^d \sigma_{i,\psi} \psi_{i,t+1} \]
\[ \Delta \tilde{c}_{i,t+1} = \mu_i + x_{i,t} + \xi_i x_{W,t} + \chi_i \eta_{i,t+1}, \]
\[ x_{i,t+1} = \rho x_{i,t} + \epsilon_{i,t+1}, \]
\[ \sigma_{i,t+1}^2 = \sigma_i^2 + \gamma (\sigma_{i,t}^2 - \sigma_i^2) + \omega_{i,t+1}, \]
\[ x_{W,t+1} = \rho_W x_{W,t} + \epsilon_{W,t+1}, \]
\[ \sigma_{W,t+1}^2 = \sigma_W^2 + \gamma (\sigma_{W,t}^2 - \sigma_W^2) + \omega_{W,t+1}, \]

- Volatility of \( \epsilon_{W,t+1} \) is \( \sigma_{W,t}^2 \)
- Volatility of \( \epsilon_{i,t+1} \) and \( \eta_{i,t+1} \) is \( \sigma_{i,t}^2 + \sigma_{W,t}^2 \)
- Corr\( (\epsilon_{W,t+1}, \omega_{W,t+1}) = \lambda_W \), Corr\( (\epsilon_{i,t+1}, \omega_{i,t+1}) = \lambda \)
- Pooled parameters: \( \rho_W, \rho, \gamma, \sigma_{W}^2, \sigma_{\omega}^2, \lambda_W, \lambda \)
- Country-specific parameters: \( \mu_i, \xi_i, \chi_i, \sigma_i^2 \)
Consumer expenditure data from Barro and Ursua (2008)

Focus on 16 developed countries:
- Australia, Belgium, Canada, Denmark, Finland, France, Germany, Italy, Netherlands, Norway, Portugal, Spain, Sweden, Switzerland, United Kingdom, United States

Sample period: 1890 - 2009
- Unbalanced panel
- All countries start before 1914

Asset prices: Global Financial Data
- Total returns on equity and government bills
- Price-dividend ratios on equity
results

- Large and persistent world growth-rate process,
- Less persistent country-specific growth-rate process
- High volatility correlated with low growth

- Match equity premium with CRRA = 6.5
- Also consistent with high volatility of stock returns, low and stable risk-free rate, predictability of stock returns based on P/D, volatility of P/D
The figure plots the posterior mean value of $x_{w,t}$ for each year in our sample.

Source: Nakamura, Sergeyev, Steinsson (2017)
FIGURE III
World Stochastic Volatility
The figure plots the posterior mean value of $\sigma_{w,t}$ for each year in our sample.

Source: Nakamura, Sergeyev, Steinsson (2017)
FIGURE IV
Stochastic Volatility for the United States, the United Kingdom and Canada

Source: Nakamura, Sergeyev, Steinsson (2017)
## Correlations between Growth-Rate and Uncertainty Shocks

<table>
<thead>
<tr>
<th>Country-Specific ($\lambda$)</th>
<th>Baseline</th>
</tr>
</thead>
<tbody>
<tr>
<td>Country-Specific ($\lambda$)</td>
<td>-0.47</td>
</tr>
<tr>
<td>World ($\lambda_w$)</td>
<td>-0.42</td>
</tr>
</tbody>
</table>

Source: Nakamura, Sergeyev, Steinsson (2017)
## Properties of Consumption Growth

<table>
<thead>
<tr>
<th></th>
<th>Data Median</th>
<th>[2.5%, 97.5%]</th>
<th>Model Median</th>
<th>[2.5%, 97.5%]</th>
</tr>
</thead>
<tbody>
<tr>
<td>AC(1)</td>
<td>0.13</td>
<td>-0.01</td>
<td>[0.17, 0.17]</td>
<td></td>
</tr>
<tr>
<td>AC(2)</td>
<td>0.14</td>
<td>0.13</td>
<td>[0.03, 0.27]</td>
<td></td>
</tr>
<tr>
<td>AC(3)</td>
<td>0.04</td>
<td>0.10</td>
<td>[0.01, 0.25]</td>
<td></td>
</tr>
<tr>
<td>AC(4)</td>
<td>0.07</td>
<td>0.07</td>
<td>[-0.01, 0.22]</td>
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</tr>
<tr>
<td>AC(5)</td>
<td>0.00</td>
<td>0.06</td>
<td>[-0.02, 0.20]</td>
<td></td>
</tr>
<tr>
<td>AC(10)</td>
<td>0.12</td>
<td>0.02</td>
<td>[-0.05, 0.13]</td>
<td></td>
</tr>
</tbody>
</table>

Source: Nakamura, Sergeyev, Steinsson (2017)
<table>
<thead>
<tr>
<th></th>
<th>Data</th>
<th></th>
<th>Model</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Median</td>
<td>U.S.</td>
<td>Median</td>
<td>U.S.</td>
</tr>
<tr>
<td>$E(R_m-R_f)$</td>
<td>6.87</td>
<td>7.10</td>
<td>6.60</td>
<td>6.90</td>
</tr>
<tr>
<td>$\sigma(R_m-R_f)$</td>
<td>21.82</td>
<td>17.37</td>
<td>13.85</td>
<td>13.91</td>
</tr>
<tr>
<td>$E(R_m-R_f)/\sigma(R_m-R_f)$</td>
<td>0.32</td>
<td>0.41</td>
<td>0.48</td>
<td>0.50</td>
</tr>
<tr>
<td>$E(R_m)$</td>
<td>9.10</td>
<td>8.23</td>
<td>7.74</td>
<td>8.03</td>
</tr>
<tr>
<td>$\sigma(R_m)$</td>
<td>21.99</td>
<td>17.89</td>
<td>13.84</td>
<td>13.88</td>
</tr>
<tr>
<td>$E(R_f)$</td>
<td>1.43</td>
<td>1.13</td>
<td>0.92</td>
<td>1.13</td>
</tr>
<tr>
<td>$\sigma(R_f)$</td>
<td>4.57</td>
<td>3.33</td>
<td>1.55</td>
<td>1.55</td>
</tr>
<tr>
<td>$E(p-d)$</td>
<td>3.30</td>
<td>3.30</td>
<td>2.94</td>
<td>2.92</td>
</tr>
<tr>
<td>$\sigma(p-d)$</td>
<td>0.41</td>
<td>0.40</td>
<td>0.27</td>
<td>0.27</td>
</tr>
<tr>
<td>$AC1(p-d)$</td>
<td>0.85</td>
<td>0.90</td>
<td>0.90</td>
<td>0.90</td>
</tr>
</tbody>
</table>

Source: Nakamura, Sergeyev, Steinsson (2017)