Aggregate Risk

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Time Series

- **Stochastic Process**: Sequence of random variables
- **Stationary**: Unconditional distribution not a function of time
- **Trend Stationary**: Stationary after subtracting a trend
- **Difference Stationary**: Stationary after differencing (i.e., $y_t - y_{t-1}$ is stationary)
- **I.i.d sequence**: Sequence of independent and identically distributed random variables

For more detail, see, e.g., Hayashi (2000, ch. 2.2)

TIMES SERIES II

- **Autoregressive model of order 1 (i.e., AR(1))**:

$$y_t = \mu + \rho y_{t-1} + \epsilon_t$$

  where ϵ_t is i.i.d.

- AR(1) is stationary if $|\rho| < 1$
- **Impulse response function**: Response of y_t over time to a shock to ϵ_0
- For AR(1), impulse response at time t is $\rho^t \epsilon_0$

Time Series III

- **Trend Stationary AR(1)**:

$$y_t = \alpha + \mu t + \rho y_{t-1} + \epsilon_t$$

- **Random Walk (with drift)**:

$$y_t = \mu + y_{t-1} + \epsilon_t$$

- A random walk is difference stationary
(but not stationary in levels)

Time Series IV

- **AR(p)**:

$$y_t = \mu + \rho_1 y_{t-1} + ... + \rho_p y_{t-p} + \epsilon_t$$

- **Moving Average of order q (i.e., MA(q))**:

$$y_t = \mu + \epsilon_t + \theta_1 \epsilon_{t-1} + ... + \theta_q \epsilon_{t-q}$$

- Impulse response of MA(q) is:

$$y_0 = \epsilon_0, \quad y_1 = \theta_1 \epsilon_0, \quad ... \quad y_q = \theta_q \epsilon_0, \quad y_{q+1} = 0$$

- **ARMA(p,q)**:

$$y_t = \mu + \rho_1 y_{t-1} + ... + \rho_p y_{t-p} + \epsilon_t + \theta_1 \epsilon_{t-1} + ... + \theta_q \epsilon_{t-q}$$

Are Economic Fluctuations Important?

Lucas (1987, 2003):

- Macroeconomists spend a lot of time thinking about policies to dampen business cycles (i.e., stabilization policies)
- But how important in terms of welfare are such policies
- Upper bound: Welfare gains from eliminating all economic fluctuations
- What are the welfare gains from eliminating all economic fluctuations?

WELFARE LOSSES FROM ECONOMIC FLUCTUATIONS

- Assumes consumer's consumption stream is trend-stationary:

$$c_t = Ae^{\mu t} e^{-(1/2)\sigma^2} \epsilon_t$$

with $\log(\epsilon_t) \sim N(0, \sigma^2)$

- This implies:

$$E(e^{-(1/2)\sigma^2} \epsilon_t) = 1$$

$$E(c_t) = Ae^{\mu t}$$

Welfare Losses from Economic Fluctuations

- Consumer's utility function

$$E\left\{\sum_{t=0}^{\infty}\beta^t \frac{c_t^{1-\gamma}}{1-\gamma}\right\}$$

  - β is subjective discount factor
  - γ coefficient of risk aversion

Welfare Losses from Economic Fluctuations

- Thought experiment: How much would welfare increase if we could magically eliminate all consumption variation around trend (best case scenario for stabilization policy!)
- Represent this as a consumption equivalent gain λ:

$$E\left\{\sum_{t=0}^{\infty}\beta^t\frac{((1+\lambda)c_t)^{1-\gamma}}{1-\gamma}\right\}=\sum_{t=0}^{\infty}\beta^t\frac{(Ae^{\mu t})^{1-\gamma}}{1-\gamma}$$

- Answer:

$$\lambda \simeq \frac{1}{2}\gamma\sigma^2$$

Welfare Losses from Economic Fluctuations

$$\lambda \simeq \frac{1}{2}\gamma\sigma^2$$

- For 1947-2001, the standard deviation of the log of U.S. real, per capita consumption about a linear trend: 0.032.
- Reasonable values of γ between 1 and 4

$$\lambda = \frac{1}{2}(0.032)^2 = 0.0005$$

- Even including the Great Depression and Great Recession (1920-2009) and setting $\gamma = 4$:

$$\lambda = \frac{1}{2}4(0.063)^2 = 0.008$$

Welfare Losses from Economic Fluctuations

- Conclusion: Welfare gains from stabilization policy are trivial.
- Macroeconomics as originally conceived has succeeded.
- Is this convincing?

Welfare Losses from Economic Fluctuations

- Conclusion: Welfare gains from stabilization policy are trivial.
- Macroeconomics as originally conceived has succeeded.
- Is this convincing?
- Model used to reach this conclusion may be wrong
  - Output/Consumption may not be trend stationary
  - Representative consumer view may understate seriousness of recessions
- Model Lucas uses does not fit the equity premium!!
  Can it be taken seriously for thinking about the costs of risk??

Equity Premium Puzzle

- In a simple endowment economy (Mehra-Prescott 85):

$$\log E_t R_{C,t+1} - \log R_{f,t} = \gamma \text{var}_t(\log \Delta C_{t+1})$$

- Equity Premium Puzzle:

$$\log E_t R_{e,t+1} - \log R_{f,t} \approx 0.07$$

$$\text{var}_t(\log \Delta C_{t+1}) \approx 0.03^2 = 0.0009$$

(Arguably equity is a leveraged claim to consumption. See, e.g., Barro 06)

Resolutions of the Equity Premium Puzzle

- Different preferences: Habits (Campbell and Cochrane, 1999)
- Incomplete markets / heterogeneous agents
  (Constantinides and Duffie, 1996; Constantinides and Ghosh, 2017)
- Different consumption process
  - Is trend-stationary consumption process assumed by Lucas or random-walk consumption process assumed in textbook equity premium calculations a good model of consumption growth?
  - Do they accurately capture aggregate risks?
  - What is missing?

How to Model Consumption Dynamics?

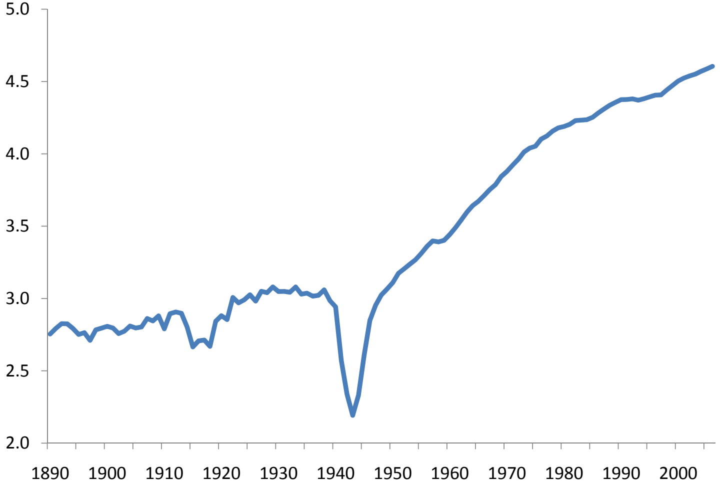
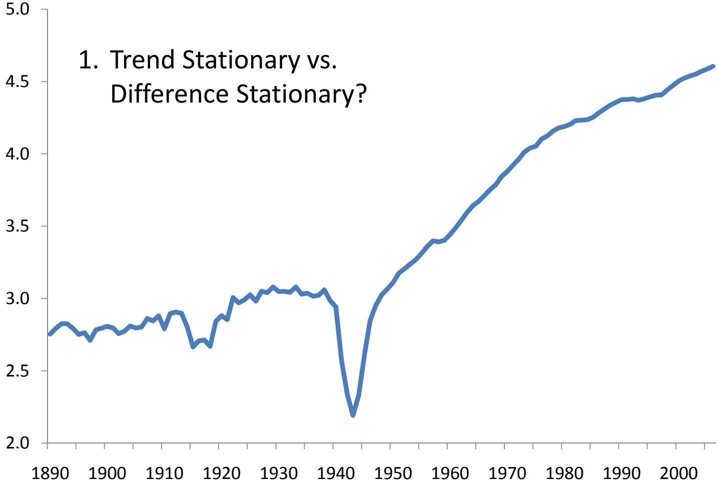


Figure: Log Consumption for France

How to Model Consumption Dynamics?



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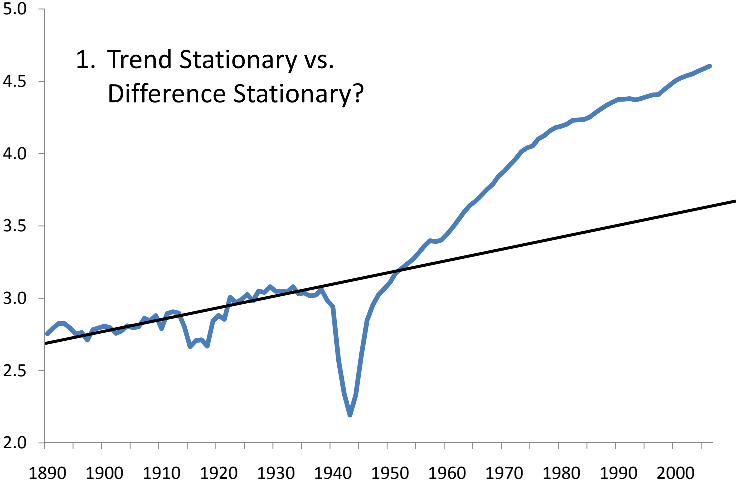

Figure: Log Consumption for France

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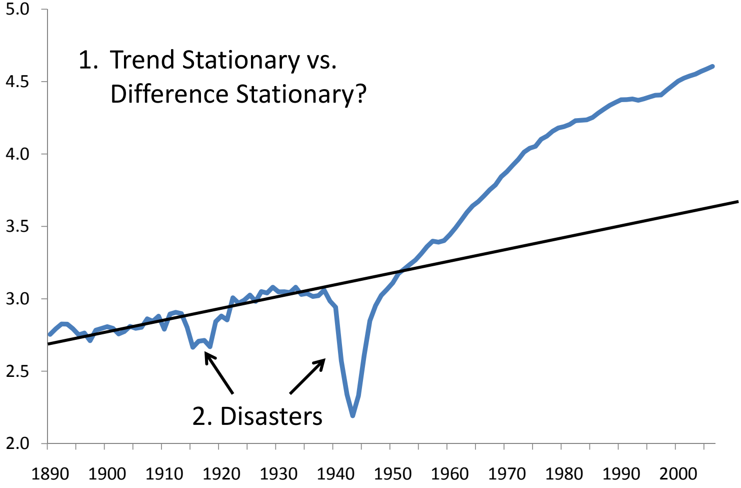

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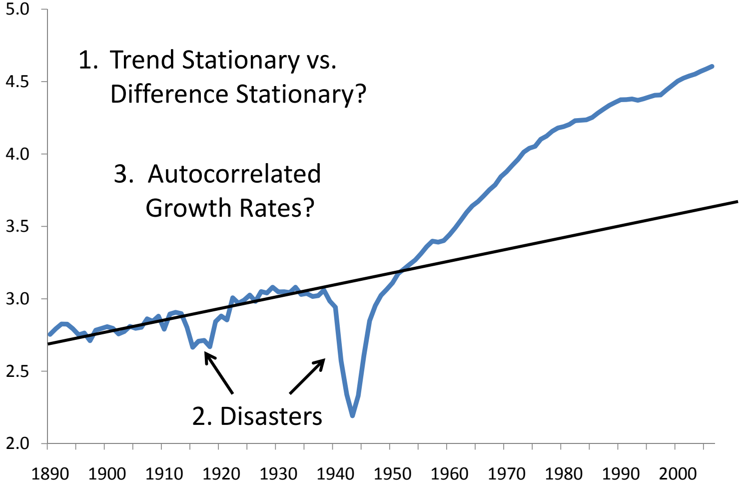

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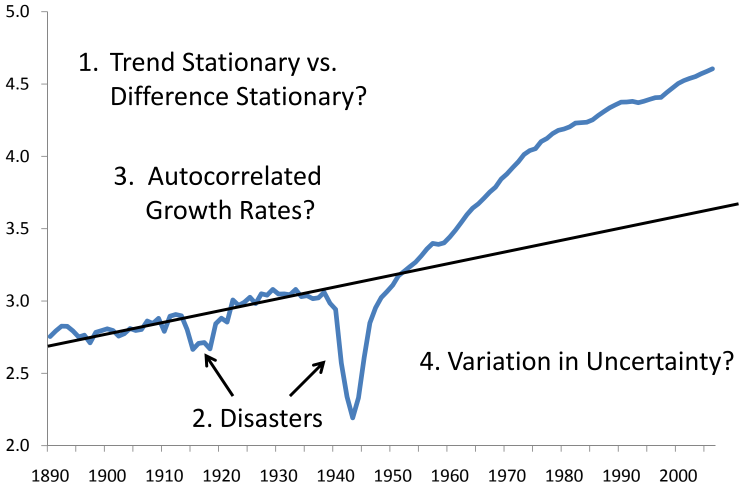

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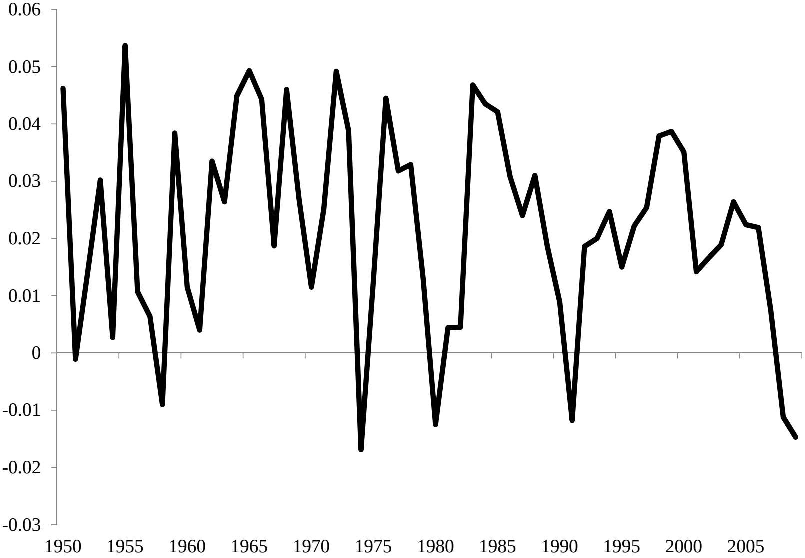

FIGURE
Growth in U.S. per Capita Consumption

Source: Barro and Ursua (2008)

Is GDP/Consumption a Random Walk?

- Textbook asset pricing model:

$$\log C_{t+1} = \mu + \log C_t + \epsilon_{t+1}$$

- What does this imply about $\partial \log C_{t+j} / \partial \epsilon_{t+1}$ as $j \to \infty$?

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  - $\partial \log C_{t+j} / \partial \epsilon_{t+1} = 1$ for all j?
  - I.e., shocks have permanent effects on GDP

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IS GDP/CONSUMPTION A RANDOM WALK?

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- What does this imply about $\partial \log C_{t+j}/\partial \epsilon_{t+1}$ as $j \to \infty$?
  - $\partial \log C_{t+j}/\partial \epsilon_{t+1} = 1$ for all j?
  - I.e., shocks have permanent effects on GDP
- What does it imply about $\text{var}_t(\log C_{t+j})$ as $j \to \infty$?
  - Goes to infinity!!
- But does US GDP look like a random walk with drift?

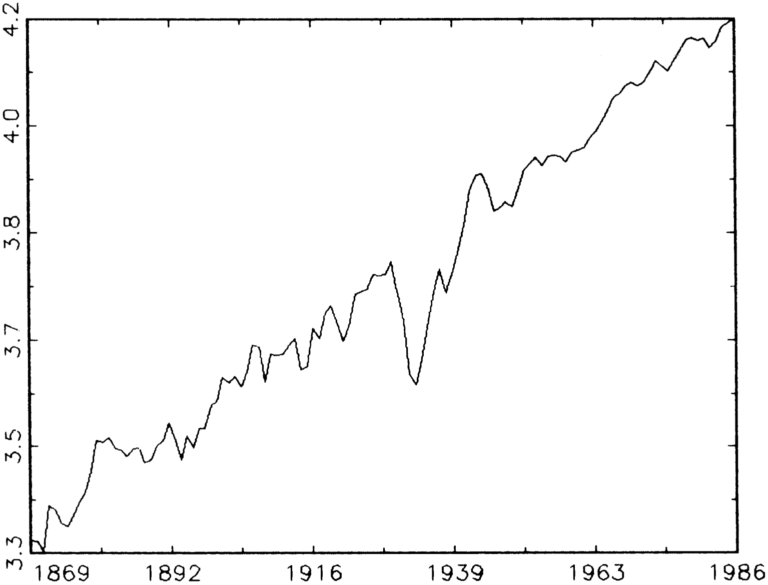


FIG. 2.—Log real per capita GNP, 1869–1986

Source: Cochrane (1988)

Is GDP/Consumption Trend Stationary?

- Traditional view in macro: GDP is trend stationary

$$y_t = bt + \sum_{j=0}^{\infty} a_j \epsilon_{t-j}$$

  where a_j approaches zero for large j

- Implies:
  - Long-run forecast invariant to ϵ_t (i.e., business cycles are transient)
  - $\text{var}_t(\log C_{t+j}) \rightarrow \sum_{j=0}^{\infty} a_j^2 \sigma < \infty$ as $j \rightarrow \infty$

- This view was challenged in the 1980s

  (Nelson-Plosser 82; Watson 86; Clark 87; Campbell-Mankiw 87)

Campbell and Mankiw (1987)

- Estimate an ARMA(p,q) process for GNP growth:

$$\phi(L)\Delta Y_t = \theta(L)\epsilon_t$$

  $\phi(L)$ and $\theta(L)$ are polynomials in the lag operator ($L\Delta Y_t = \Delta Y_{t-1}$)
- Sample period: 1947:1 - 1985:4 (quarterly data)
- Estimate by maximum likelihood
- Extensive discussion of model selection (i.e., selection of p and q)
- Main result:
  - $\partial \log Y_{t+j} / \partial \epsilon_{t+1} \geq 1$ for relatively large j
  - Relatively robust to p and q choice

TABLE IV
MODEL IMPULSE RESPONSES, ln REAL GNP

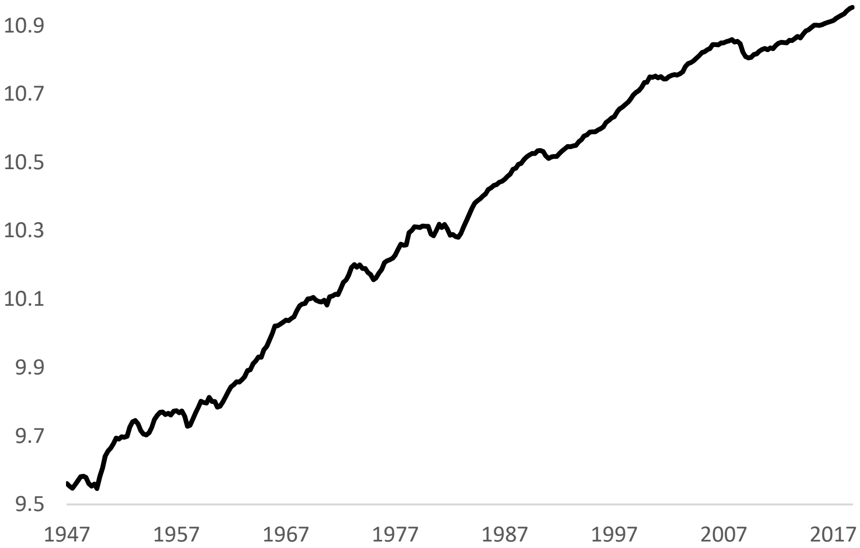
Model p,q	1	2	4	8	16	20	40	80
0,1	1.261	1.261	1.261	1.261	1.261	1.261	1.261	1.261
	(0.072)	(0.072)	(0.072)	(0.072)	(0.072)	(0.072)	(0.072)	(0.072)
0,2	1.305	1.573	1.573	1.573	1.573	1.573	1.573	1.573
	(0.073)	(0.123)	(0.123)	(0.123)	(0.123)	(0.123)	(0.123)	(0.123)
0,3	1.323	1.647	1.754	1.754	1.754	1.754	1.754	1.754
	(0.077)	(0.128)	(0.170)	(0.170)	(0.170)	(0.170)	(0.170)	(0.170
1,0	1.363	1.496	1.561	1.571	1.571	1.571	1.571	1.571
	(0.070)	(0.120)	(0.161)	(0.171)	(0.172)	(0.172)	(0.172)	(0.172)
1,1	1.344	1.523	1.666	1.715	1.719	1.719	1.719	1.719
	(0.077)	(0.119)	(0.202)	(0.268)	(0.278)	(0.279)	(0.279)	(0.279)
1,2	1.322	1.635	1.728	1.734	1.734	1.734	1.734	1.734
	(0.075)	(0.130)	(0.206)	(0.222)	(0.222)	(0.222)	(0.222)	(0.222)
1,3	1.271	1.488	1.341	1.090	0.721	0.586	0.208	0.026
	(0.119)	(0.269)	(0.572)	(1.110)	(1.895)	(2.177)	(2.958)	(3.338)
2,0	1.314	1.547	1.730	1.804	1.812	1.812	1.812	1.812
	(0.073)	(0.116)	(0.201)	(0.264)	(0.276)	(0.276)	(0.276)	(0.276)
2,1	1.321	1.591	1.731	1.770	1.772	1.772	1.772	1.772
	(0.071)	(0.122)	(0.198)	(0.242)	(0.248)	(0.248)	(0.248)	(0.248)
2,2	1.302	1.621	1.572	1.532	1.517	1.517	1.517	1.517
	(0.078)	(0.128)	(0.193)	(0.142)	(0.162)	(0.160)	(0.161)	(0.161)
2,3	1.289	1.561	1.502	1.115	0.592	0.431	0.088	0.004
	(0.119)	(0.268)	(0.596)	(1.178)	(1.921)	(2.140)	(2.599)	(2.720)
3,0	1.336	1.632	1.641	1.568	1.571	1.571	1.571	1.571
	(0.076)	(0.132)	(0.207)	(0.230)	(0.223)	(0.222)	(0.222)	(0.222)
3,1	1.320	1.614	1.604	1.334	1.364	1.360	1.360	1.360
	(0.077)	(0.131)	(0.206)	(0.327)	(0.288)	(0.297)	(0.297)	(0.297)
3,2	1.318	1.624	1.630	1.626	1.595	1.596	1.597	1.597
	(0.078)	(0.127)	(0.210)	(0.196)	(0.206)	(0.203)	(0.203)	(0.203)
3,3	1.279	1.563	1.416	1.095	0.720	0.584	0.207	0.026
	(0.122)	(0.267)	(0.602)	(1.141)	(1.929)	(2.213)	(3.001)	(3.389)

Standard errors are in parentheses.

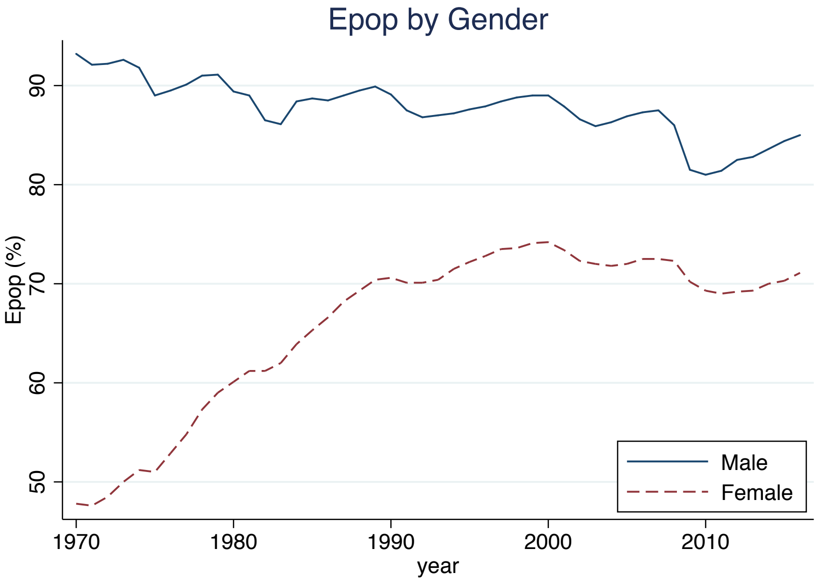
Source: Campbell and Mankiw (1987)

One Shock or Many Shocks?

- GDP is driven by many shocks with vastly different dynamics:
  - Monetary shocks (transitory?)
  - Productivity shocks (permanent?)
  - Demographic shocks (build very slowly?)
- Makes it very hard to measure “permanent component” of GDP shocks since short-term dynamics not necessarily informative about long-run dynamics (see, e.g., Quah 1992)



Source: FRED. Log GDP per Capita for the U.S.



Source: Fukui, Nakamura, and Steinsson (2019)

Variance Ratios for Consumption Growth

- Cochrane (1988) advocated using variance ratios:

$$\mathrm{VR}_{i,k} = \frac{1}{k} \frac{\mathrm{var}(c_{i,t} - c_{i,t-k})}{\mathrm{var}(c_{i,t} - c_{i,t-1})}$$

  Non-parametric approach

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  Non-parametric approach

- Random walk: $VR_{i,k} = 1$ for all k
- Trend stationary: $VR_{i,k} \to 0$ as $k \to \infty$
- Positively autocorrelated growth: $VR_{i,k} > 1$ for large k

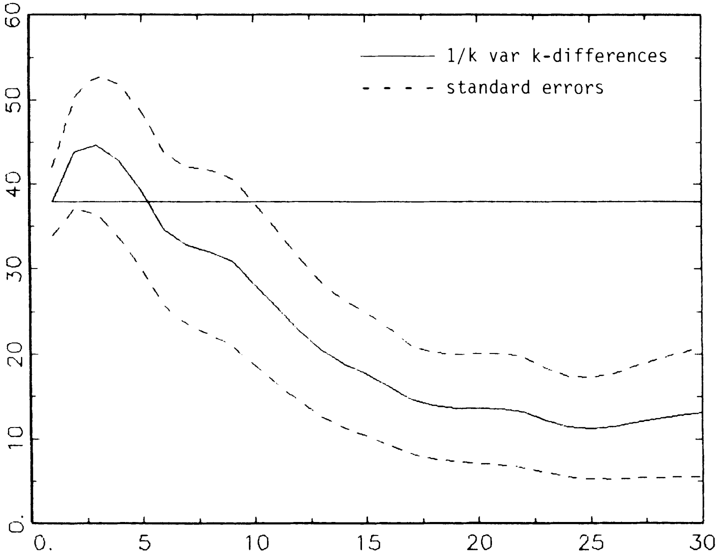


FIG. 1.—$1/k$ times the variance of k-differences of log real per capita GNP, 1869–1986, with asymptotic standard errors.

Source: Cochrane (1988)

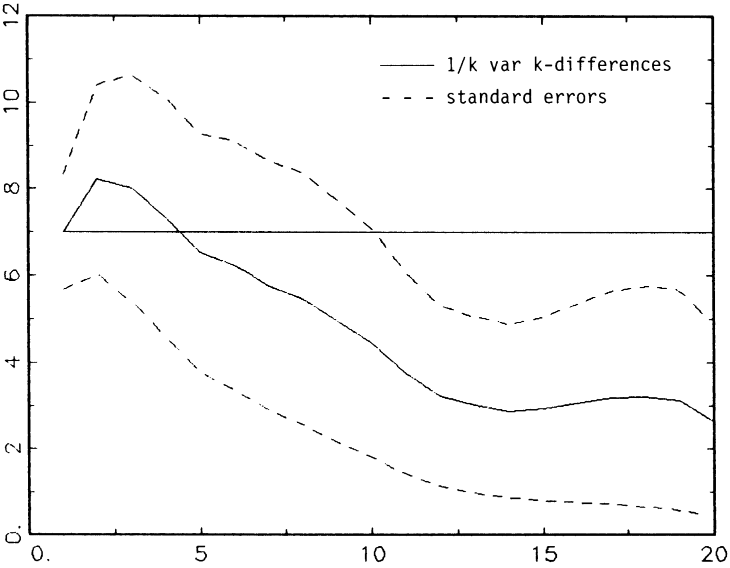


FIG. 3.—$1/k$ times the variance of k-differences of log real per capita GNP, 1947–86, with asymptotic standard errors.

Source: Cochrane (1988)

Campbell-Mankiw vs. Cochrane

- Notice that variance ratio initially rises above one
- GDP growth positively autocorrelated at short horizons
- This is what drives Campbell-Mankiw 87 results
- Cochrane's results reflect slow negative correlation of growth rates at longer horizons which is hard to pick up using low-order ARMA models

Equity Premium Puzzle Worse!

- If consumption growth is largely trend stationary, then world is even less risky than textbook model assumes
- Equity premium puzzle even worse
  (and Lucas' assumptions look good)

Cogley (1990)

- Extends Cochrane's estimation approach to 9 OECD countries for 1871-1985
- Critiques small sample properties of Cochrane's asymptotic standard errors
- Presents two estimators for variance ratio:
  - $\hat{V}^f$ based on frequency domain methods
  - $\hat{V}^k$ based on traditional method (i.e., Cochrane's estimator)

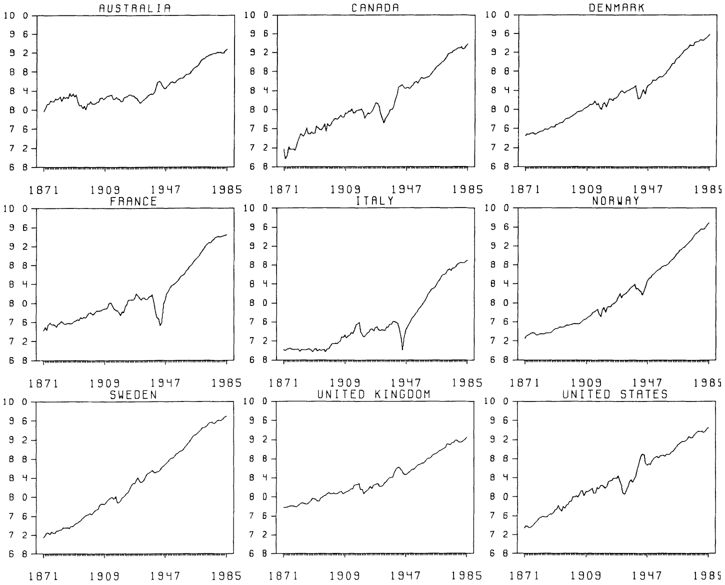


FIG. 1.—Log real per capita GDP, 1871–1985

Source: Cogley (1990)

TABLE 2

ESTIMATES OF THE VARIANCE RATIO: PER CAPITA OUTPUT GROWTH, 1871–1985

	$\hat{V}^f$		$\hat{V}^k$	
	$k = 15$	$k = 20$	$k = 15$	$k = 20$
Australia	1.15	1.21	1.25	1.40
	(.63, 3.2)	(.64, 4.1)		
Canada	.64	.64	.72	.77
	(.35, 1.8)	(.34, 2.2)		
Denmark	.92	.97	1.00	1.09
	(.51, 2.6)	(.51, 3.3)		
France	1.57	1.55	1.78	1.84
	(.86, 4.4)	(.82, 4.9)		
Italy	1.60	1.80	1.75	2.02
	(.88, 4.5)	(.96, 6.1)		
Norway	1.21	1.39	1.24	1.39
	(.67, 3.4)	(.74, 4.7)		
Sweden	.90	.89	.99	.97
	(.50, 2.5)	(.47, 3.0)		
United Kingdom	.77	.85	.94	1.03
	(.43, 2.2)	(.45, 2.9)		
United States:				
GDP	.48	.36	.62	.51
	(.27, 1.4)	(.19, 1.2)		
GNP	.49	.41	.60	.53
	(.27, 1.4)	(.22, 1.4)		

NOTE.—Approximate 90 percent confidence intervals are shown in parentheses

Source: Cogley (1990)

Variance Ratios and Disasters

- Highly sensitive to the treatment of disasters
- Disasters generally involve substantial recoveries (Nakamura et al., 2010)

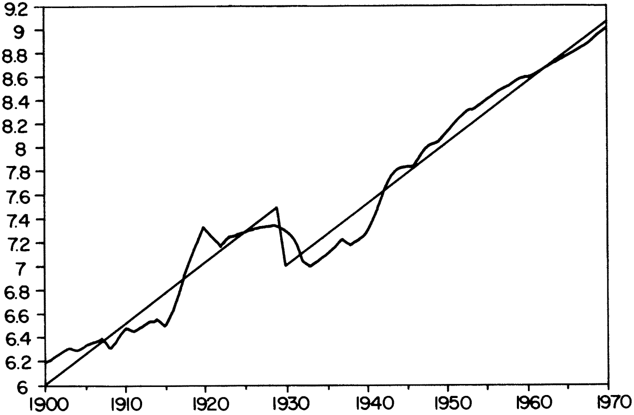
TABLE IV
Variance Ratios in the Data and the Model (k=15)

	Consumption Growth				Realized Vol. of Cons. Growth			
	Data		Full Model		Data		Full Model	
	Incl.Dis.	Excl.Dis.	Med.	[5%, 95%]	Incl.Dis.	Excl.Dis.	Med.	[5%, 95%]
France	1.49	3.33	2.56	[1.00, 5.33]	4.60	2.26	2.40	[1.04, 4.39]
UK	1.56	2.87	3.84	[1.78, 7.32]	1.60	1.26	1.22	[0.55, 2.57]
US	1.08	1.29	1.69	[0.75, 3.65]	4.70	1.80	1.87	[0.76, 3.96]
Average	1.11	2.28	2.60	[1.06, 5.29]	3.48	2.17	1.82	[0.79, 3.56]
Median	0.87	1.62	2.69	[1.02, 5.47]	3.16	2.14	1.72	[0.66, 3.62]

Source: Outtakes from Nakamura, Steinsson, and Sergeyev (2017)

PERRON (1989)

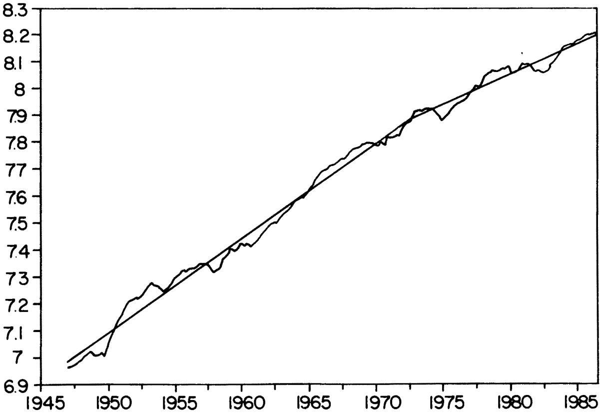
- How robust is the evidence that macroeconomic time series have a random walk?
- Perhaps one or two "structural breaks" account for apparent non-stationarity
- Perron argues that GDP is stationary once one accounts for:
  - Great Crash of 1929: Negative level shift
  - Oil Price Shock of 1973: Negative trend shift
- Data:
  - Nelson-Plosser 82 annual data on 14 macro series ending in 1970
  - Quarterly real GDP 1947:1-1986:3



Note: The broken straight line is a fitted trend (by OLS) of the form $\tilde{y}_t = \tilde{\mu} + \tilde{\gamma} DU_t + \tilde{\beta} t$ where $DU_t = 0$ if $t \leqslant 1929$ and $DU_t = 1$ if $t > 1929$.

FIGURE 1.—Logarithm of "Nominal Wages."

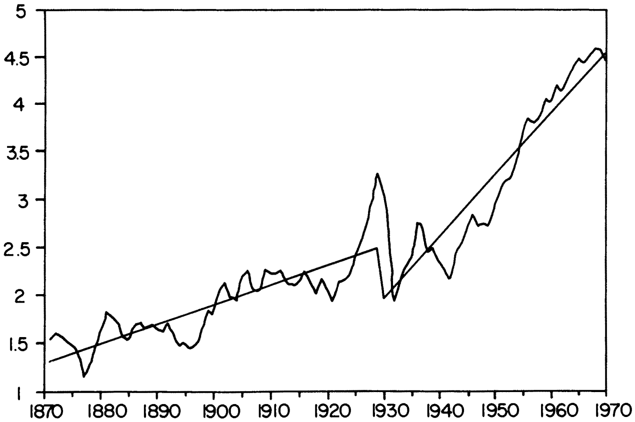
Source: Perron (1989)



Note: The broken straight line is a fitted trend (by OLS) of the form: $\tilde{y}_t = \tilde{\mu} + \tilde{\beta} t + \tilde{\gamma} DT_t^*$ where $DT_t^* = 0$ if $t \leqslant 1973{:}\mathrm{I}$ and $DT_t^* = t - T_B$ if $t > 1973{:}\mathrm{I} = T_B$.

FIGURE 2.—Logarithm of "Postwar Quarterly Real GNP."

Source: Perron (1989)



Note: The broken straight line is a fitted trend (by OLS) of the form $\tilde{y}_t = \tilde{\mu} + \tilde{\gamma}_1 DU_t + \tilde{\beta} t + \tilde{\gamma}_2 DT_t$ where $DU_t = DT_t = 0$ if $t \leqslant 1929$ and $DU_t = 1$, $DT_t = t$ if $t > 1929$.

FIGURE 3.—Logarithm of "Common Stock Prices."

Source: Perron (1989)

TABLE I

Regression Analysis for the Wages, Quarterly GNP, and Common Stock Price Series

Regression: $y_t = \tilde{\mu} + \tilde{\beta} t + \tilde{\alpha} y_{t-1} + \Sigma_{i=1}^{k} \tilde{c}_i \Delta y_{t-i} + \tilde{e}_t$

Series/Period	k	$\tilde{\mu}$	$t_{\tilde{\mu}}$	$\tilde{\beta}$	$t_{\tilde{\beta}}$	$\tilde{\alpha}$	$t_{\tilde{\alpha}}$	$S(\tilde{e})$
(a) Wages								
1900–1970[a]	2	0.566	2.30	0.004	2.30	0.910	−2.09	0.060
1900–1929	7	4.299	2.84	0.037	2.73	0.304	−2.82	0.0803
1930–1970	8	1.632	3.60	0.012	2.64	0.735	−3.19	0.0269
(b) Common stock prices								
1871–1970[a]	2	0.481	2.02	0.003	2.37	0.913	−2.05	0.158
1871–1929	3	0.3468	2.13	0.0063	2.70	0.732	−2.29	0.1209
1930–1970	4	−0.5312	−1.64	0.0166	1.96	0.788	−1.89	0.1376
(c) Quarterly real GNP								
1947:I–1986:III	2	0.386	2.90	0.0004	2.71	0.946	−2.85	0.010
1947:I–1973:I	2	0.637	3.04	0.0008	2.99	0.910	−3.02	0.0099
1973:II–1986:III	1	0.883	2.23	0.0008	2.27	0.878	−2.23	0.0102

[a] Results taken from Nelson and Plosser (1982, Table 5).

Source: Perron (1989). Dickey-Fuller 2.5% critical value for N = 100, with constant and time trend is -3.7. Corresponding 5% critical value is -3.4.

Confusing Breaks for Unit Roots

- Perron simulates 10,000 replications of a series y_t of length 100
- "Crash" hypothesis:

$$y_t = \mu_1 + (\mu_2 - \mu_1)DU_t + \beta t + e_t$$

- where $DU_t = 1$ if $t > 50$, $\mu_1 = 0$, $\beta = 1$, $e_t \sim \mathrm{N}(0, 1)$

- "Changing Growth" hypothesis:

$$y_t = \mu + \beta_1 t + (\beta_2 - \beta_1)DT_t^* + e_t$$

- where $DT_t^* = t - 50$ if $t > 50$, $\mu = 0$, $\beta_1 = 1$, $e_t \sim \mathrm{N}(0, 1)$

CONFUSING BREAKS FOR UNIT ROOTS

- Estimates misspecified model:

$$y_t = \tilde{\mu} + \tilde{\beta} t + \tilde{\alpha} y_{t-1} + \tilde{e}_t$$

- True $\alpha = 0$. But breaks look like a unit root.

TABLE III

MEAN AND VARIANCE OF $\tilde{\alpha}$

(a) Crash Simulations, $\mu_1 = 0$, $\beta = 1$					
	$\mu_2 = 0$	$\mu_2 = -2$	$\mu_2 = -5$	$\mu_2 = -10$	$\mu_2 = -25$
Mean	−0.019	0.172	0.558	0.795	0.899
Variance	0.00986	0.01090	0.00471	0.00089	0.00009
(b) Breaking Trend Simulations, $\beta_1 = 1$, $\mu = 0$					
	$\beta_2 = 1.0$	$\beta_2 = 0.9$	$\beta_2 = 0.7$	$\beta_2 = 0.4$	$\beta_2 = 0.0$
Mean	−0.019	0.334	0.825	0.949	0.981
Variance	0.00986	0.00938	0.00094	0.00009	0.00001

See notes to Figure 4 for case (a) and Figure 5 for case (b).

TABLE VII

Tests for a Unit Root

(a) Regression (12), Model A; $y_t = \hat{\mu} + \hat{\theta} DU_t + \hat{\beta} t + \hat{d} D(TB)_t + \hat{\alpha} y_{t-1} + \sum_{i=1}^{k} \hat{c}_i \Delta y_{t-i} + \hat{e}_t$

$T_B = 1929$	T	λ	k	$\hat{\mu}$	$t_{\hat{\mu}}$	$\hat{\theta}$	$t_{\hat{\theta}}$	$\hat{\beta}$	$t_{\hat{\beta}}$	$\hat{d}$	$t_{\hat{d}}$	$\hat{\alpha}$	$t_{\hat{\alpha}}$	$S(\hat{e})$
Real GNP	62	0.33	8	3.441	5.07	−0.189	−4.28	0.0267	5.05	−0.018	−0.30	0.282	−5.03[a]	0.0509
Nominal GNP	62	0.33	8	5.692	5.44	−0.360	−4.77	0.0359	5.44	0.100	1.09	0.471	−5.42[a]	0.0694
Real per capita GNP	62	0.33	7	3.325	4.11	−0.102	−2.76	0.0111	4.00	−0.070	−1.09	0.531	−4.09[b]	0.0555
Industrial production	111	0.63	8	0.120	4.37	−0.298	−4.58	0.0323	5.42	−0.095	−0.99	0.322	−5.47[a]	0.0875
Employment	81	0.49	7	3.402	4.54	−0.046	−2.65	0.0057	4.26	−0.025	−0.77	0.667	−4.51[a]	0.0295
GNP deflator	82	0.49	5	0.669	4.09	−0.098	−3.16	0.0070	4.01	0.026	0.53	0.776	−4.04[b]	0.0438
Consumer prices	111	0.63	2	0.065	1.12	−0.004	−0.21	0.0005	1.75	−0.036	−0.79	0.978	−1.28	0.0445
Wages	71	0.41	7	2.38	5.45	−0.190	−4.32	0.0197	5.37	0.085	1.36	0.619	−5.41[a]	0.0532
Money stock	82	0.49	6	0.301	4.72	−0.071	−2.59	0.0121	4.18	0.033	0.68	0.812	−4.29[b]	0.0440
Velocity	102	0.59	0	0.050	0.932	−0.005	−0.20	−0.0002	−0.35	−0.136	−2.01	0.941	−1.66	0.0663
Interest rate	71	0.41	2	−0.018	−0.088	−0.343	−2.06	0.0105	2.64	0.197	0.64	0.976	−0.45	0.2787

(b) Regression (14), Model C; $y_t = \hat{\mu} + \hat{\theta} DU_t + \hat{\beta} t + \hat{\gamma} DT_t + \hat{d} D(TB)_t + \hat{\alpha} y_{t-1} + \sum_{i=1}^{k} \hat{c}_i \Delta y_{t-i} + \hat{e}_t$

$T_B = 1929$	T	λ	k	$\hat{\mu}$	$t_{\hat{\mu}}$	$\hat{\theta}$	$t_{\hat{\theta}}$	$\hat{\beta}$	$t_{\hat{\beta}}$	$\hat{\gamma}$	$t_{\hat{\gamma}}$	$\hat{d}$	$t_{\hat{d}}$	$\hat{\alpha}$	$t_{\hat{\alpha}}$	$S(\hat{e})$
Common stock prices	100	0.59	1	0.353	4.09	−1.051	−4.29	0.0070	4.43	0.0139	3.98	0.128	0.76	0.718	−4.87[b]	0.1402
Real wages	71	0.41	8	2.115	4.33	−0.190	−3.71	0.0107	3.79	0.0066	3.33	0.031	0.78	0.298	−4.28[c]	0.0330

(c) Regression (10), Model B; $y_t = \tilde{\mu} + \tilde{\beta} t + \tilde{\gamma} DT_t^* + \tilde{y}_t$; $\tilde{y}_t = \tilde{\alpha} \tilde{y}_{t-1} + \sum_{i=1}^{k} \tilde{c}_i \Delta \tilde{y}_{t-i} + \tilde{e}_t$

$T_B = 1973{:}\mathrm{I}$	T	λ	k	$\tilde{\mu}$	$t_{\tilde{\mu}}$	$\tilde{\beta}$	$t_{\tilde{\beta}}$	$\tilde{\gamma}$	$t_{\tilde{\gamma}}$	$\tilde{\alpha}$	$t_{\tilde{\alpha}}$	$S(\tilde{e})$
Quarterly real GNP	159	0.66	10	6.977	1160.51	0.0087	97.73	−0.0031	−12.06	0.86	−3.98[c]	0.0097

NOTE: a, b, and c denote statistical significance at the 1%, 2.5%, and 5% level respectively.

Perron (1989)

- Perron argues that after allowing for Great Crash of 1929 and 1973 Growth Slowdown, many macro series are stationary (i.e., he rejects the null of a unit root)

Perron (1989)

- Perron argues that after allowing for Great Crash of 1929 and 1973 Growth Slowdown, many macro series are stationary (i.e., he rejects the null of a unit root)
- But he chooses the break dates ex post
- Perhaps it is normal for a unit root of that length to look like it has a break and is otherwise stationary
- Main lesson: Hard to distinguish trends from unit roots in the presence of breaks.
- What is a break? Infrequent unit root shock.

AGGREGATE RISKS

- Recent literature has moved beyond trend vs. difference stationary debate
- Three types of risks have been emphasized:
  - Rare disasters (Ritz, 1988; Barro, 2006)
  - Growth rate shocks (Bansal and Yaron, 2004)
  - Stochastic volatility (Bansal and Yaron, 2004)

Barro (2006)

- Same setup as Mehra-Prescott, except

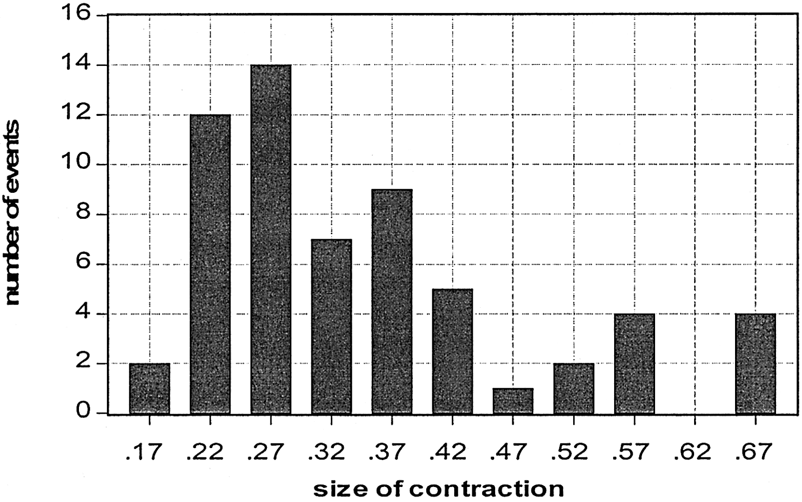
$$\log C_{t+1} = \mu + \log C_t + u_{t+1} + v_{t+1}$$

- $u_{t+1} \sim \mathrm{N}(0, \sigma^2)$
- v_{t+1} reflects disasters:
  - Probability e^{-p}: v_{t+1} =0
  - Probability $1 - e^{-p}$: $v_{t+1} = \log(1 - b)$

Barro (2006): Calibration of Disasters

- Key parameters: p and b
- Measure declines in per capita GDP (Data: Maddison, 2003)
- Disaster: Cumulative drop of 15% or greater
- p frequency of such drops: 1.7%
- b peak-to-trough decline (e.g. WWII 1939-1945)
  - E (b) = 0.29 (mean size of disasters)
  - Huge amount of heterogeneity in disaster size

Panel A: Contractions in Table I



Panel B: Contractions in Table I adjusted for trend growth

FIGURE I

Frequency Distribution of Economic Disasters

BARRO (2006)

- What is the impact of heterogeneity in disaster size?
- Why focus on disasters and ignore bonanzas?

Barro (2006): Asset Pricing

- Representative consumer
- Power utility
- Assets to price:
  - Unlevered consumption claim
  - One period, bond (occasional default during disasters)
- Empirical moments:
  - Equity Premium: Stocks: 7.1%, Bills: -0.1%
  - Leverage ratio for equity of 1.5
  - Target for unlevered equity: 7.2%/1.5 = 4.8%

TABLE V
Calibrated Model for Rates of Return

	(1)	(2)	(3)	(4)	(5)	(6)	(7)
			Parameters				
	No disasters	Baseline	Low θ	High p	Low q	Low γ	Low ρ
θ (coeff. of relative risk aversion)	4	**4**	**3**	4	4	4	4
σ (s.d. of growth rate, no disasters)	0.02	**0.02**	0.02	0.02	0.02	0.02	0.02
ρ (rate of time preference)	0.03	**0.03**	0.03	0.03	0.03	0.03	**0.02**
γ (growth rate, deterministic part)	0.025	**0.025**	0.025	0.025	0.025	**0.020**	0.025
p (disaster probability)	**0**	**0.017**	0.017	**0.025**	0.017	0.017	0.017
q (bill default probability in disaster)	**0**	**0.4**	0.4	0.4	**0.3**	0.4	0.4
			Variables				
Expected equity rate	0.128	0.071	0.076	0.044	0.071	0.051	0.061
Expected bill rate	0.127	0.035	0.061	−0.007	0.029	0.015	0.025
Equity premium	0.0016	0.036	0.016	0.052	0.042	0.036	0.036

Source: Barro (2006)

Barro (2009): Welfare Costs of Disasters

- Barro (2006): Simple disaster model can match
  - A high equity premium
  - A low risk-free rate
- Barro (2009): What does this same model imply about:
  - Welfare costs of business cycles?
  - Welfare costs of disasters?

TABLE 3—EFFECTS OF PREFERENCE PARAMETERS ON RATES OF RETURN AND WELFARE COSTS

γ	θ	ρ	ρ^*	r^e	r^f	V	Welfare effects (percent) $\sigma = 0$	$p = 0$
4	0.25	0.054	0.027	0.069	0.010	20.7	1.65	24.7
4	**0.50**	**0.052**	**0.027**	**0.069**	**0.010**	**20.7**	**1.65**	**24.0**
4	1	0.048	0.027	0.069	0.010	20.7	1.64	22.6
4	4	0.027	0.027	0.069	0.010	20.7	1.60	17.3
3.5	0.25	0.062	0.027	0.074	0.035	18.7	1.31	16.5
3.5	0.50	0.059	0.027	0.074	0.035	18.7	1.30	16.1
3.5	1	0.054	0.027	0.074	0.035	18.7	1.30	15.5
3.5	4	0.022	0.027	0.074	0.035	18.7	1.27	12.7
3	0.25	0.063	0.027	0.074	0.048	18.7	1.12	12.0
3	0.50	0.060	0.027	0.074	0.048	18.7	1.12	11.8
3	1	0.053	0.027	0.074	0.048	18.7	1.12	11.5
3	4	0.014	0.027	0.074	0.048	18.7	1.10	9.9
1	0.25	0.041	0.027	0.047	0.044	37.1	0.74	4.7
1	0.50	0.036	0.027	0.047	0.044	37.1	0.74	4.6
1	1	0.027	0.027	0.047	0.044	37.1	0.74	4.6
1	4	−0.030	0.027	0.047	0.044	37.1	0.73	4.3

Notes: The baseline results are in bold, γ is the coefficient of relative risk aversion, θ is the reciprocal of the IES in the formula for utility in equation (9), ρ is the rate of time preference, and ρ^* is the effective rate of time preference, given in equation (12); ($\rho = \rho^*$ holds when $\gamma = \theta$). The formulas for the expected rate of return on equity, r^e, the risk-free rate, r^f, and the price-dividend ratio, V, are given in equations (6), (7), and (5), respectively, after replacing ρ by ρ^*. The value of ρ^* is set at 0.027 to generate $r^f = 0.010$ with the baseline parameters. The value for ρ (0.052 in the baseline specification) is then varied in each case to maintain $\rho^* = 0.027$ (in equation (12)). Since ρ^* is held constant, the values for r^e, r^f, and V depend on γ but not on θ. Each welfare effect gives the percentage reduction in initial output, $1 - (Y_t)^*/Y_t$, that maintains attained utility while setting to zero either the standard deviation, σ, of normal economic fluctuations or the disaster probability, p. The effects are for a given expected growth rate, g^*, given in equation (2). The values for $1 - (Y_t)^*/Y_t$ come from equation (23).

Source: Barro (2009)

Barro (2006): Realistic Model of Disasters?

- Barro's model:

$$\log C_{t+1} = \mu + \log C_t + u_{t+1} + v_{t+1}$$

  - $u_{t+1} \sim \mathrm{N}(0, \sigma^2)$
  - v_{t+1}:
    - Probability e^{-p}: v_{t+1} =0
    - Probability $1 - e^{-p}$: $v_{t+1} = \log(1-b)$

- Is this a realistic model of disasters?

Barro (2006): Stylized Disaster Model

- All disasters are completely permanent
- Disasters occur instantaneously
- Timing of disasters uncorrelated across countries
- Informal estimation procedure

Nakamura, Steinsson, Barro, Ursua (2013)

- Consumption:

$$c_{i,t} = x_{i,t} + z_{i,t} + \epsilon_{i,t}$$

- Potential Consumption:

$$\Delta x_{i,t} = \mu_{i,t} + \eta_{i,t} + I_{i,t}\theta_{i,t}$$

- The Disaster Gap

$$z_{i,t} = \rho_z z_{i,t-1} - I_{i,t}\theta_{i,t} + I_{i,t}\phi_{i,t} + \nu_{i,t}$$

$$\epsilon_{i,t} \sim \mathrm{N}(0, \sigma^2_{\epsilon,i}) \quad \eta_{i,t} \sim \mathrm{N}(0, \sigma^2_{\eta,i}) \quad \nu_{i,t} \sim \mathrm{N}(0, \sigma^2_{\nu,i})$$

$$\theta_{i,t} \sim \mathrm{N}(\theta, \sigma^2_{\theta}) \quad \phi_{i,t} \sim \mathrm{truncN}(\phi, \sigma^2_{\phi}, [-\infty, 0])$$

What Happens in a Disaster?

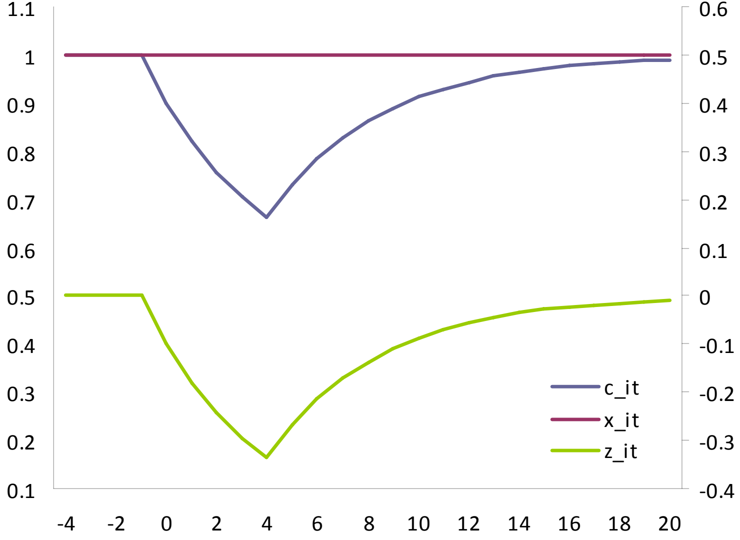
Two disaster shocks:

1. $\phi_{i,t}$: Short run effect but no long run effect
2. $\theta_{i,t}$: Long run effect but no short run effect

Examples:

- Transitory effects ($\phi_{i,t}$):
  - Destruction of capital, military spending crowds out consumption, financial stress
- Permanent effects ($\theta_{i,t}$):
  - Loss of time spent on R&D, change in institutions

θ = 0, ϕ = -0.1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0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
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0.5
0.4
0.3
0.2
0.1
0
-0.1
-0.2
-0.3
-0.4
-4
-2
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c_it
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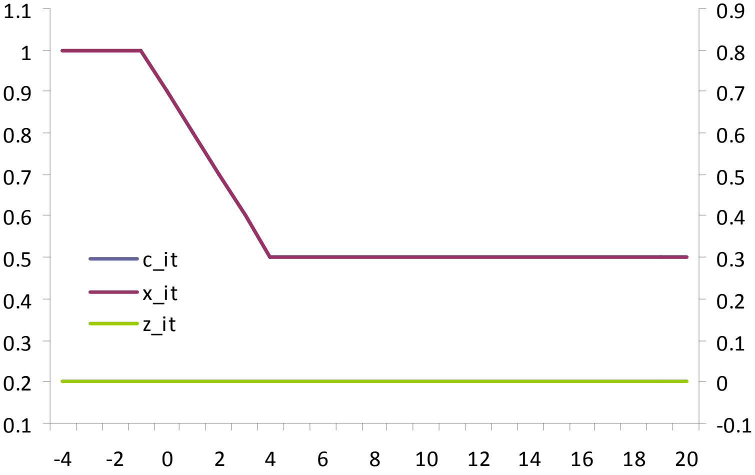
z_it

θ = -0.1, ϕ = -0.1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-0.1
-4
-2
0
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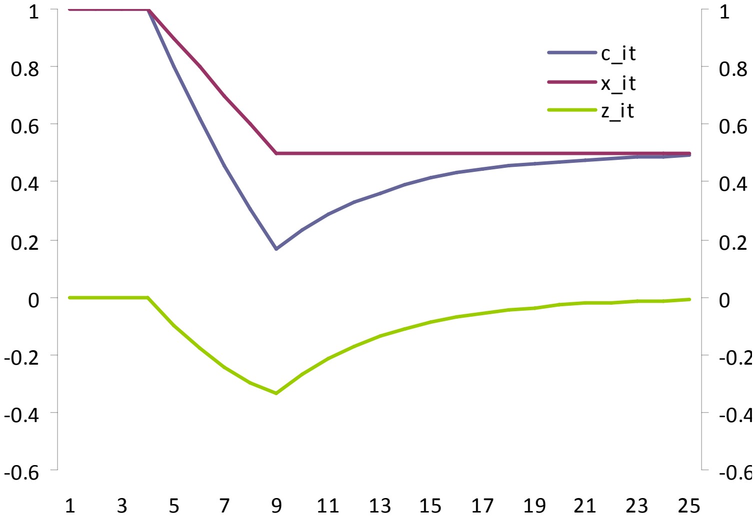
c_it
x_it
z_it

θ = -0.1, ϕ = -0.2



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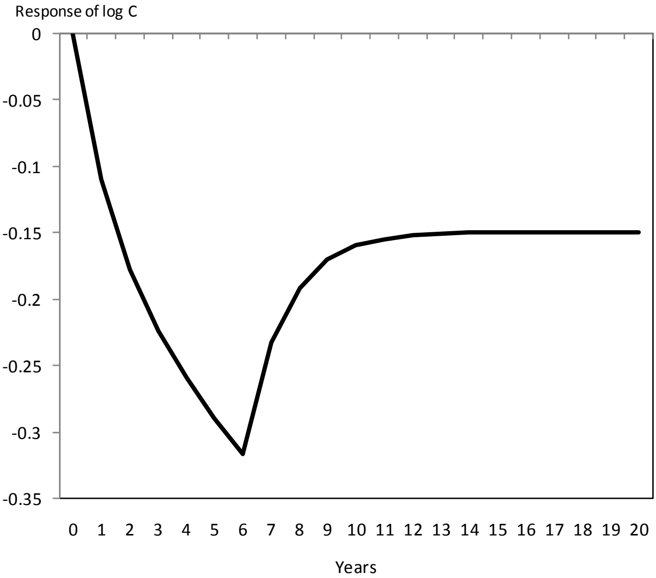
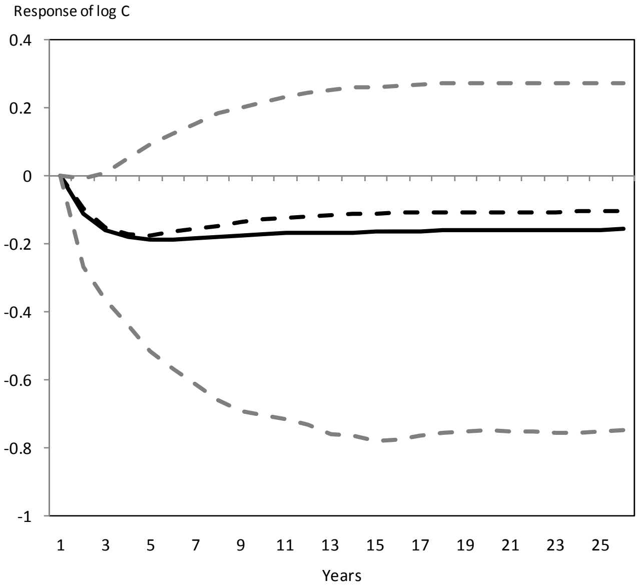
17
19
21
23
25
c_it
x_it
z_it

Empirical Methods

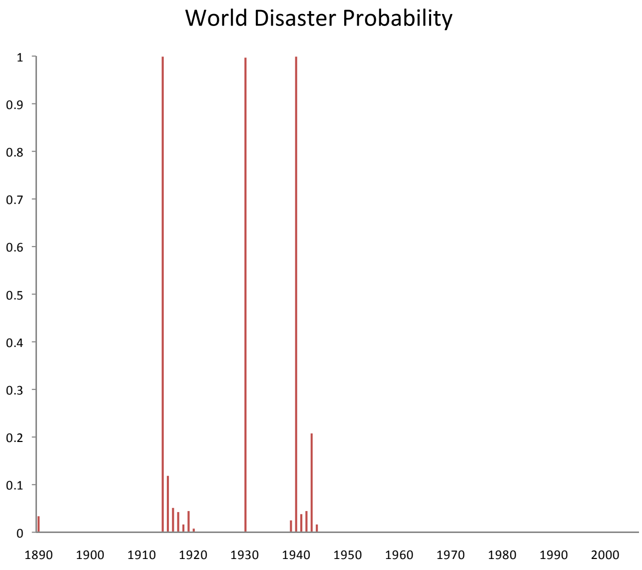
- Our model is difficult to estimate by ML
  - Many unobserved state variables
- Relatively simple to estimate by Bayesian MCMC estimation
- Allow for breaks in:
  - $\sigma_{\eta,i}$, $\sigma_{\epsilon,i}$ in 1946. (change in data quality)
  - μ_i in 1946 and 1973. (captures high post-WWII growth)



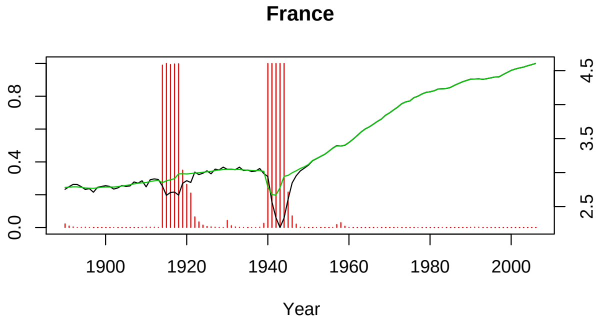
Source: Nakamura, Steinsson, Barro, and Ursua (2013)

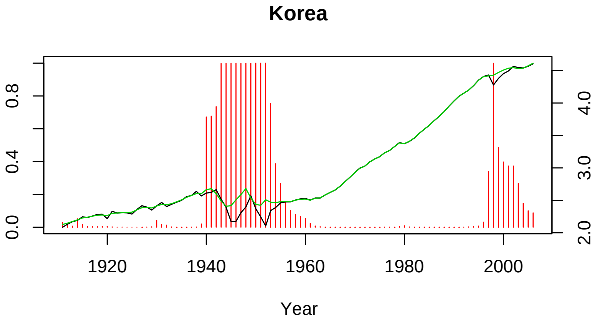


Source: Nakamura, Steinsson, Barro, and Ursua (2013)

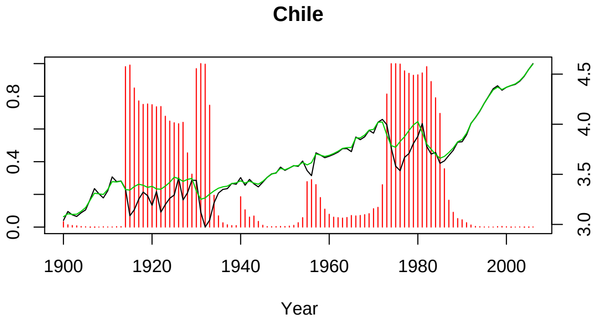


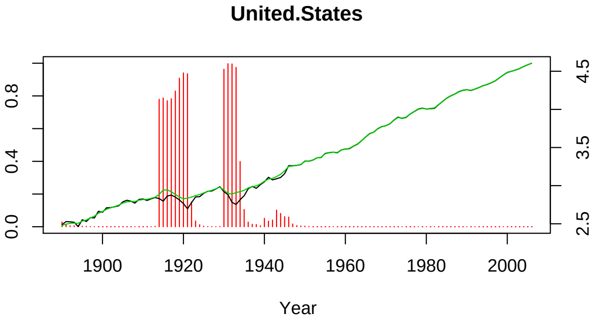
Source: Nakamura, Steinsson, Barro, and Ursua (2013)





Source: Nakamura, Steinsson, Barro, and Ursua (2013)



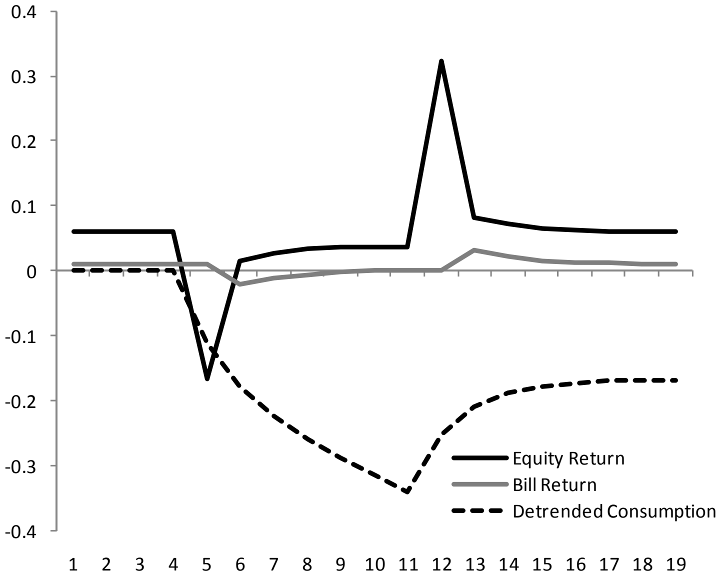


Source: Nakamura, Steinsson, Barro, and Ursua (2013)

TABLE
Asset Prices in Baseline Model with EZW Preferences

CRRA	4.5	6.5	8.5
IES	2.0	2.0	2.0
Log Expected Return			
Equity	0.050	0.058	0.066
Bond	0.032	0.009	-0.023
Equity Premium	0.018	0.048	0.088
Log Expected Return (Cond. on No Disasters)			
Equity	0.051	0.058	0.066
Bond	0.034	0.010	-0.025
Equity Premium	0.017	0.048	0.091

Source: Nakamura, Steinsson, Barro, and Ursua (2013). Equity is unleveraged.

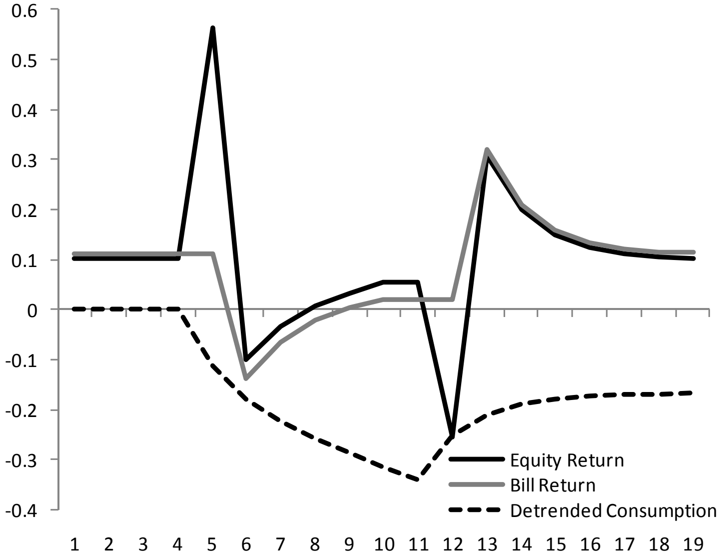


Source: Nakamura, Steinsson, Barro, and Ursua (2013)

TABLE

Asset Prices with CRRA=4 and IES=1/4

	Baseline	Barro (2006)
Log Expected Return		
Equity	0.112	0.071
Bond	0.103	0.035
Equity Premium	0.009	0.036
Log Expected Return (Cond. on No Disasters)		
Equity	0.097	0.076
Bond	0.106	0.037
Equity Premium	-0.009	0.039



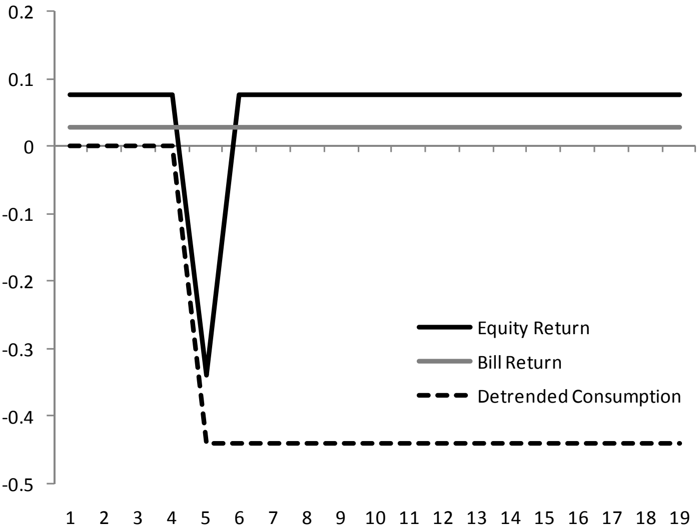
Source: Nakamura, Steinsson, Barro, and Ursua (2013)

The Role of EZW Preferences

- EZW utility: Stock market crash at onset of disaster
  - Assuming IES>1
- Power utility: Stock market boom!
- Why?

The Role of EZW Preferences

- EZW utility: Stock market crash at onset of disaster
  - Assuming IES>1
- Power utility: Stock market boom!
- Why?
  - At onset of disaster, expected growth is negative, uncertainty increases
  - Leads to high savings in a model with low IES (Power Utility)
- Contrast vs. Barro (2006) with permanent shocks



Source: Nakamura, Steinsson, Barro, and Ursua (2013)

How to Model Consumption Dynamics?

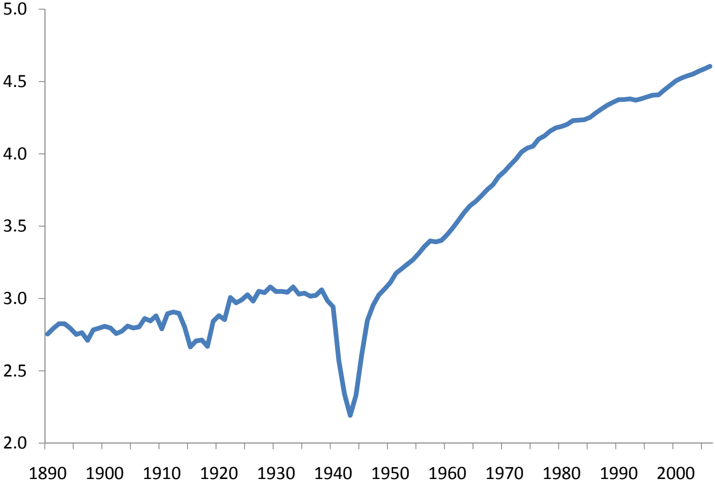


Figure: Log Consumption for France

Bansal and Yaron (2004): Long-run Risks

$$\begin{aligned}
\Delta c_{t+1} &= \mu + x_t + \chi \sigma_t \eta_{t+1}, \\
x_{t+1} &= \rho x_t + \sigma_t \epsilon_{t+1}, \\
\sigma^2_{t+1} &= \sigma^2 + \gamma(\sigma^2_t - \sigma^2) + \sigma_\omega \omega_{t+1},
\end{aligned}$$

Idea:

- x_t and σ^2_t small but persistent
- Small enough that they are hard to observe (can't be rejected)

Main Result:

- Even small "long run risks" makes a big difference for asset pricing

Are Long Run Risks Priced?

- Seems intuitive that long-run risks to growth and uncertainty would raise equity premium
- But does this work in benchmark model?
- I.e.: Are long run risks priced?

In Power Utility Model: LRR Not Priced

$$
\begin{aligned}
\Delta c_{t+1} &= \mu + x_t + \chi \sigma_t \eta_{t+1}, \\
x_{t+1} &= \rho x_t + \sigma_t \epsilon_{t+1}, \\
\sigma_{t+1}^2 &= \sigma^2 + \gamma(\sigma_t^2 - \sigma^2) + \sigma_\omega \omega_{t+1},
\end{aligned}
$$

- Notice that ϵ_{t+1} and ω_{t+1} affect:
  - $R_{e,t+1}$
  - Δc_{t+j} for $j > 1$
  - But not Δc_{t+1}
- With power utility, long run risks:
  - Don't create correlation between returns and stochastic discount factor
  - Have no effect on asset prices
- Timing issue implies that EZW preferences are crucial in LRR model

BANSAL AND YARON (2004): ASSET PRICING

- EZW preferences with:
  - CRRA: $\gamma = 10$
  - IES: $\psi = 1.5$
- Two assets:
  - One period, risk-free bond
  - "Equity" with dividend growth rate:

$$\Delta d_{t+1} = \mu + \phi x_t + \varphi_d \sigma_t u_t$$

- Leverage: $\phi = 3$
- Dividend volatility: $\varphi_d = 4.5$

Bansal and Yaron (2004): Calibration

$$
\begin{aligned}
\Delta c_{t+1} &= \mu + x_t + \chi \sigma_t \eta_{t+1}, \\
x_{t+1} &= \rho x_t + \sigma_t \epsilon_{t+1}, \\
\sigma_{t+1}^2 &= \sigma^2 + \gamma(\sigma_t^2 - \sigma^2) + \sigma_\omega \omega_{t+1},
\end{aligned}
$$

- Calibrate long-run risks parameters:

$$\mu = 0.0015, \quad \rho = 0.979, \quad \sigma = 0.078, \quad \varphi_e = 0.044$$

- No formal macro calibration targets
- Parameters largely viewed a free parameters
- Chosen largely to fit asset prices

Bansal and Yaron (2004): Calibration

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- No formal macro calibration targets
- Parameters largely viewed a free parameters
- Chosen largely to fit asset prices
- Why is this viable?
    - Long-run risks small enough they don't seriously affect model's fit to data on macro aggregates

	Data		Model				
Variable	Estimate	SE	Mean	95%	5%	p-Val	*Pop*
$\sigma(g)$	2.93	(0.69)	2.72	3.80	2.01	0.37	2.88
$AC(1)$	0.49	(0.14)	0.48	0.65	0.21	0.53	0.53
$AC(2)$	0.15	(0.22)	0.23	0.50	−0.17	0.70	0.27
$AC(5)$	−0.08	(0.10)	0.13	0.46	−0.13	0.93	0.09
$AC(10)$	0.05	(0.09)	0.01	0.32	−0.24	0.80	0.01
$VR(2)$	1.61	(0.34)	1.47	1.69	1.22	0.17	1.53
$VR(5)$	2.01	(1.23)	2.26	3.78	0.79	0.63	2.36
$VR(10)$	1.57	(2.07)	3.00	6.51	0.76	0.77	2.96
$\sigma(g_d)$	11.49	(1.98)	10.96	15.47	7.79	0.43	11.27
$AC(1)$	0.21	(0.13)	0.33	0.57	0.09	0.53	0.39
$corr(g, g_d)$	0.55	(0.34)	0.31	0.60	−0.03	0.07	0.35

Source: Bansal and Yaron (2004)

	Data		Model	
Variable	Estimate	SE	$\gamma = 7.5$	$\gamma = 10$
	Returns			
$E(r_m - r_f)$	6.33	(2.15)	4.01	6.84
$E(r_f)$	0.86	(0.42)	1.44	0.93
$\sigma(r_m)$	19.42	(3.07)	17.81	18.65
$\sigma(r_f)$	0.97	(0.28)	0.44	0.57
	Price Dividend			
$E(\exp(p - d))$	26.56	(2.53)	25.02	19.98
$\sigma(p - d)$	0.29	(0.04)	0.18	0.21
$AC1(p - d)$	0.81	(0.09)	0.80	0.82
$AC2(p - d)$	0.64	(0.15)	0.65	0.67

Source: Bansal and Yaron (2004)

Role of Epstein-Zin Preferences

- Stochastic discount factor with Epstein-Zin-Weil preferences:

$$\log M_{t+1} = \theta \log \beta - \frac{\theta}{\psi} \Delta c_{t+1} + (\theta - 1) R_{c,t+1}$$

- Current marginal utility depends on news about future consumption growth (through $R_{c,t+1}$)

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  - Decrease in future expected growth raise current marginal utility (If IES > 1 and CRRA > 1/IES)
  - Increase in future uncertainty raises current marginal utility (If CRRA > 1 and IES > 1)
- IES > 1 crucial for LRRs to increase equity premium

Predictability of Stock Returns

- Large literature argues stock returns are predictable (Campbell-Shiller, 1988; Fama-French, 1988, Cochrane, 2008, van Binsbergen-Koijen, 2010)
- Idea: High P/D ratio predicts low returns

	Panel A: Excess Returns		
Variable	Data	SE	Model
$B(1)$	-0.08	(0.07)	-0.18
$B(3)$	-0.37	(0.16)	-0.47
$B(5)$	-0.66	(0.21)	-0.66
$R^2(1)$	0.02	(0.04)	0.05
$R^2(3)$	0.19	(0.13)	0.10
$R^2(5)$	0.37	(0.15)	0.16

Source: Bansal and Yaron (2004)

Intuition for Predictability in LRR Model

- P/D ratio is stationary
- A decrease in P/D therefore implies:
  - High returns going forward, or ...
  - Low Dividend growth going forward, or ...
  - Both
- Uncertainty shock in LRR model implies:
  - Stock prices fall (if CRRA > 1 and IES > 1)
  - No effect on expected dividends
  - So, expected returns must rise

- What about growth rate shocks?
- In LLR model, high growth rate shocks raise P/D and predict future consumption growth
- Not in the data

	$\widehat{\beta}$	t	$\widehat{R}^2$	$R^2(50\%)$		$\%(\widehat{R}^2)$	
	data	data	data	BY	BKY	BY	BKY
			$\sum_{j=1}^{J}(r_{m,t+j} - r_{f,t+j}) = \alpha + \beta(p_t - d_t) + \varepsilon_{t+j}$				
1 Y	−0.093	−1.803	0.044	0.007	0.011	0.918	0.841
3 Y	−0.264	−3.231	0.170	0.017	0.028	**0.980**	0.940
5 Y	−0.413	−3.781	0.269	0.025	0.043	**0.990**	**0.956**
4 Q	−0.119	−2.625	0.090	0.008	0.012	**0.980**	**0.952**
12 Q	−0.274	−3.191	0.187	0.022	0.033	**0.970**	0.933
20 Q	−0.424	−3.365	0.257	0.033	0.050	**0.969**	0.926
			$\sum_{j=1}^{J}(\Delta c_{t+j}) = \alpha + \beta(p_t - d_t) + \varepsilon_{t+j}$				
1 Y	0.011	1.586	0.060	0.324	0.145	**0.006**	0.202
3 Y	0.010	0.588	0.013	0.350	0.109	**0.002**	0.132
5 Y	−0.001	−0.060	0.000	0.285	0.085	**0.001**	**0.015**
4 Q	0.000	0.140	0.000	0.237	0.063	**0.000**	**0.023**
12 Q	−0.002	−0.296	0.001	0.269	0.068	**0.003**	0.069
20 Q	−0.003	−0.296	0.002	0.213	0.060	**0.014**	0.089

Source: Beeler and Campbell (2012)

Dark Matter

- Key LRR parameters are macro parameters
  - How important are changes in trend growth rates (e.g., productivity slowdown)
  - How important are fluctuations in macro volatility? (e.g. Great Moderation)
- However, in LRR literature, key parameters are calibrated or estimated to fit asset pricing data
- Since model has no other way to fit asset pricing data, it concludes that LRR are there
- But are these features really "there" in macro data?

Nakamura, Sergeyev, and Steinsson (2017)

- Estimate long-run risks model using **only** macro data

- Use data on aggregate consumption from 16 countries over 120 years
- Pool data across countries to better estimate key parameters
- Advantage of using macroeconomic data alone:
  - Results not driven by need to explain asset prices
  - Results provide direct evidence for the mechanism

$$
\begin{aligned}
c_{i,t+1} &= \tilde{c}_{i,t+1} + \sigma_{i,\nu}\nu_{i,t+1} + I^d_{i,t+1}\sigma_{i,\psi}\psi^d_{i,t+1} \\
\Delta\tilde{c}_{i,t+1} &= \mu_i + x_{i,t} + \xi_i x_{W,t} + \chi_i \eta_{i,t+1}, \\
x_{i,t+1} &= \rho x_{i,t} + \epsilon_{i,t+1}, \\
\sigma^2_{i,t+1} &= \sigma^2_i + \gamma(\sigma^2_{i,t} - \sigma^2_i) + \omega_{i,t+1}, \\
x_{W,t+1} &= \rho_W x_{W,t} + \epsilon_{W,t+1}, \\
\sigma^2_{W,t+1} &= \sigma^2_W + \gamma(\sigma^2_{W,t} - \sigma^2_W) + \omega_{W,t+1},
\end{aligned}
$$

- Volatility of $\epsilon_{W,t+1}$ is $\sigma^2_{W,t}$
- Volatility of $\epsilon_{i,t+1}$ and $\eta_{i,t+1}$ is $\sigma^2_{i,t} + \sigma^2_{W,t}$
- Corr$(\epsilon_{W,t+1}, \omega_{W,t+1}) = \lambda_W$, Corr$(\epsilon_{i,t+1}, \omega_{i,t+1}) = \lambda$
- Pooled parameters: ρ_W, ρ, γ, σ^2_W, $\sigma^2_{\omega,W}$, σ^2_ω, λ_W, λ
- Country-specific parameters: μ_i, ξ_i, χ_i, σ^2_i

DATA

- Consumer expenditure data from Barro and Ursua (2008)
- Focus on 16 developed countries:
  - Australia, Belgium, Canada, Denmark, Finland, France, Germany, Italy, Netherlands, Norway, Portugal, Spain, Sweden, Switzerland, United Kingdom, United States
- Sample period: 1890 - 2009
  - Unbalanced panel
  - All countries start before 1914
- Asset prices: Global Financial Data
  - Total returns on equity and government bills
  - Price-dividend ratios on equity

Results

- Large and persistent world growth-rate process,
- Less persistent country-specific growth-rate process
- High volatility correlated with low growth

- Match equity premium with CRRA = 6.5
- Also consistent with high volatility of stock returns, low and stable risk free rate, predictability of stock returns based on P/D, volatility of P/D

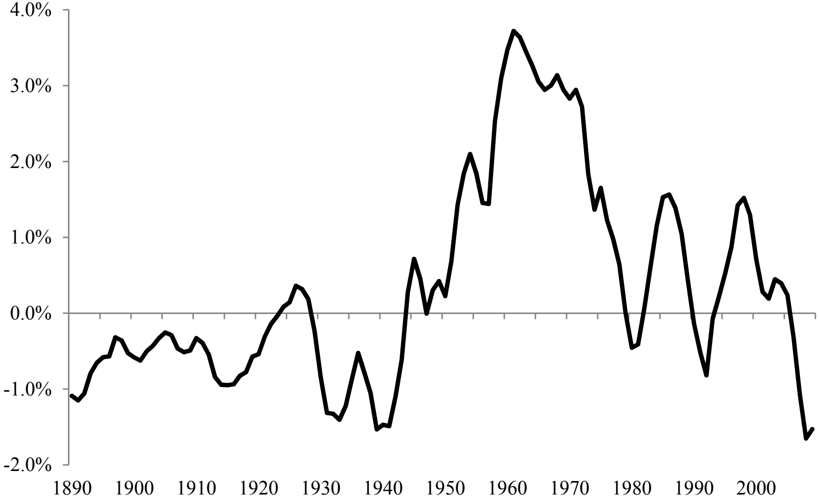

FIGURE II

The World Growth-Rate Process

The figure plots the posterior mean value of $x_{w,t}$ for each year in our sample.

Source: Nakamura, Sergeyev, Steinsson (2017)

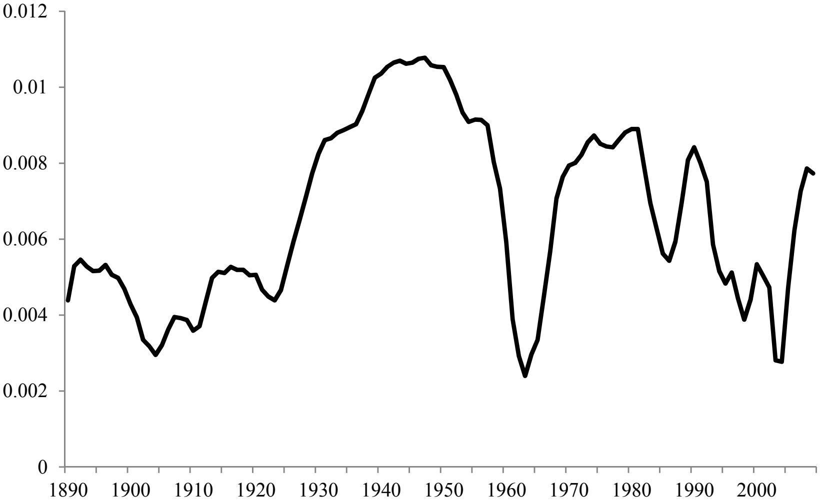

FIGURE III
World Stochastic Volatility
The figure plots the posterior mean value of $\sigma_{w,t}$ for each year in our sample.

Source: Nakamura, Sergeyev, Steinsson (2017)

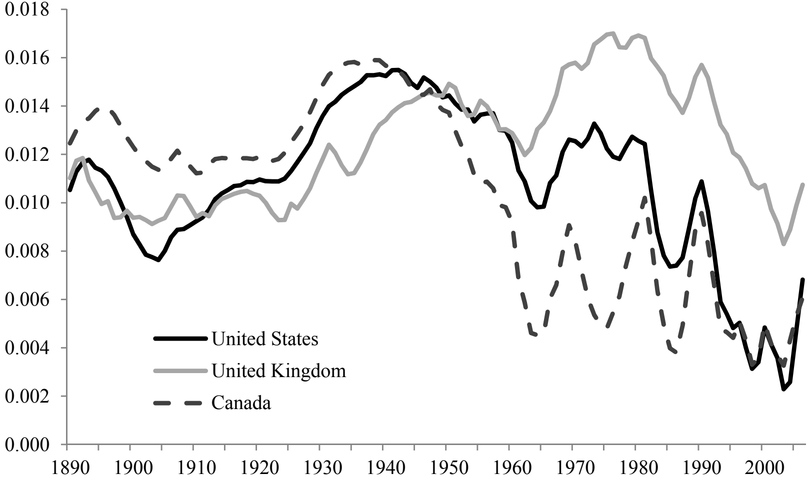

FIGURE IV
Stochastic Volatility for the United States, the United Kingdom and Canada

Source: Nakamura, Sergeyev, Steinsson (2017)

Correlations between Growth-Rate and Uncertainty Shocks

	Baseline
Country-Specific (λ)	-0.47
	(0.17)
World (λ_W)	-0.42
	(0.24)

Source: Nakamura, Sergeyev, Steinsson (2017)

Properties of Consumption Growth

	Median Country		
	Data	Model	
		Median	*[2.5%, 97.5%]*
AC(1)	0.13	-0.01	*[0.17,0.17]*
AC(2)	0.14	0.13	*[0.03,0.27]*
AC(3)	0.04	0.10	*[0.01,0.25]*
AC(4)	0.07	0.07	*[-0.01,0.22]*
AC(5)	0.00	0.06	*[-0.02,0.20]*
AC(10)	0.12	0.02	*[-0.05,0.13]*

Source: Nakamura, Sergeyev, Steinsson (2017)

TABLE V
Asset Pricing Summary Statistics

	Data		Model	
	Median	U.S.	Median	U.S.
$E(R_m - R_f)$	6.87	7.10	6.60	6.90
$\sigma(R_m - R_f)$	21.82	17.37	13.85	13.91
$E(R_m - R_f)/\sigma(R_m - R_f)$	0.32	0.41	0.48	0.50
$E(R_m)$	9.10	8.23	7.74	8.03
$\sigma(R_m)$	21.99	17.89	13.84	13.88
$E(R_f)$	1.43	1.13	0.92	1.13
$\sigma(R_f)$	4.57	3.33	1.55	1.55
$E(p-d)$	3.30	3.30	2.94	2.92
$\sigma(p-d)$	0.41	0.40	0.27	0.27
$AC1(p-d)$	0.85	0.90	0.90	0.90

Source: Nakamura, Sergeyev, Steinsson (2017)