

# CONSUMPTION-BASED ASSET PRICING

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# BIG ASSET PRICING QUESTIONS

- Why is the return on the stock market so high?  
(Relative to the "risk-free rate")
- Why is the stock market so volatile?
- What does this tell us about the risk and risk aversion?

# CONSUMPTION-BASED ASSET PRICING

- Consumption-based asset pricing starts from the Consumption Euler equation:

$$U'(C_t) = E_t[\beta U'(C_{t+1})R_{i,t+1}]$$

- Where does this equation come from?
  - Consume \$1 less today
  - Invest in asset  $i$
  - Use proceeds to consume \$  $R_{it+1}$  tomorrow
- Two perspectives:
  - Consumption Theory: Conditional on  $R_{it+1}$ , determine path for  $C_t$
  - Asset Pricing: Conditional on path for  $C_t$ , determine  $R_{it+1}$

- Return is defined as payoff divided by price:

$$R_{i,t+1} = \frac{X_{i,t+1}}{P_{i,t}}$$

where

- $X_{i,t+1}$  is (state contingent) payoff from asset  $i$  in period  $t + 1$
- $P_{i,t}$  is price of asset  $i$  at time  $t$

# CONSUMPTION-BASED ASSET PRICING

$$U'(C_t) = \beta E_t[U'(C_{t+1})R_{i,t+1}]$$

- A little manipulation yields:

$$1 = E_t \left[ \frac{\beta U'(C_{t+1})}{U'(C_t)} R_{i,t+1} \right]$$

- and using  $R_{i,t+1} = X_{i,t+1}/P_{i,t}$ :

$$P_{i,t} = E_t \left[ \beta \frac{U'(C_{t+1})}{U'(C_t)} X_{i,t+1} \right]$$

- Fundamental equation of consumption-based asset pricing

# STOCHASTIC DISCOUNT FACTOR

$$1 = E_t \left[ \frac{\beta U'(C_{t+1})}{U'(C_t)} R_{i,t+1} \right] \quad P_{i,t} = E_t \left[ \beta \frac{U'(C_{t+1})}{U'(C_t)} X_{i,t+1} \right]$$

- Stochastic discount factor:

$$M_{t+1} = \beta \frac{U'(C_{t+1})}{U'(C_t)}$$

- Yields:

$$1 = E_t [M_{t+1} R_{i,t+1}] \quad P_{i,t} = E_t [M_{t+1} X_{i,t+1}]$$

# STOCHASTIC DISCOUNT FACTOR

$$P_{i,t} = E_t [M_{t+1} X_{i,t+1}]$$

- Stochastic discount factor generalizes standard notion of discount factor
- With no uncertainty, standard present value formula gives

$$P_{i,t} = \frac{1}{R_{f,t}} X_{i,t+1}$$

- Since gross interest rates are usually above one, the payoff sells “at a discount”
- In this case,  $1/R_{f,t}$  is the discount factor
- $M_{t+1}$  is the appropriate discount factor when there is risk

# STOCHASTIC DISCOUNT FACTOR

$$1 = E_t [M_{t+1} R_{i,t+1}] \quad P_{i,t} = E_t [M_{t+1} X_{i,t+1}]$$

- Stochastic discount factor prices all assets!!
- All risk correlations for any asset can be incorporate by defining a single (random) variable  $M_{t+1}$  to discount payoffs with
- This (conceptually) simple view holds under the rather strong assumption that there exists a complete set of competitive markets

(Sometimes also called: pricing kernel, marginal rate of substitution, change of measure, or state-price density)



- Assets can have payoffs in multiple periods:

$$P_{i,t} = E_t[M_{t+1}(D_{i,t+1} + P_{i,t+1})]$$

where  $D_{i,t+1}$  is the dividend, and  $P_{i,t+1}$  is (ex dividend) price

- Works for stocks, bonds, options, everything.

- Suppose  $P_{s,t,t+1}$  is the price at time  $t$  of the Arrow security that pays \$ 1 if state  $s$  occurs at time  $t + 1$  and zero otherwise
- The price of any security can be written two ways:

$$P_{i,t} = \sum_s P_{s,t,t+1} X_{s,t+1}, \quad P_{i,t} = E_t[M_{t+1} X_{t+1}]$$

which implies

$$M_{s,t+1} = \frac{P_{s,t,t+1}}{\pi_{s,t+1}}$$

where  $\pi_{s,t+1}$  is the probability of state  $s$  in period  $t + 1$ .

- This is why you sometimes see  $E_t[M_{t+1} X_{t+1}]$  type terms in budget constraints

# MODIGLIANI-MILLER THEOREM

- Value of an asset is the sum of its parts:

$$P_{i,t} = \sum_s P_{s,t,t+1} X_{s,t+1}$$

- Why? Arbitrage!
- Doesn't matter how the asset is sliced up!  
(as long as the total payoff is not changed)
  - Capital structure of a firm doesn't matter for its value!
  - Dividend policy of a firm doesn't matter for its value!
  - Whether a firm buys insurance (hedges a risk) doesn't matter!
- Holds when:
  - Complete set of competitive markets exists  
(no bankruptcy costs, no agency costs, etc.)
  - No taxes

# MODIGLIANI-MILLER THEOREM

- Does hedging a risk raise the value of a firm?
- Let's adopt vector notation:
  - $S$  state of the world in the future
  - $X_{t+1}$  is an  $S \times 1$  vector of payoffs in these states
  - $P_t$  is an  $S \times 1$  vector of state prices
- Value of Firm A before hedging risk:

$$P_t^A = P_t \cdot X_{t+1}^A$$

where  $X_{t+1}^A$  denotes the payoffs of firm A over future states

# MODIGLIANI-MILLER THEOREM

- Consider some other cashflow  $X_{t+1}^B$
- Price of that cashflow:

$$P_t^B = P_t \cdot X_{t+1}^B$$

- Suppose the firm were to purchase this cashflow
- At that point the firm's value would be the value of the combined cashflow minus the price of the cashflow:

$$P_t \cdot [X_{t+1}^A + X_{t+1}^B] - P_t^B = P_t \cdot X_{t+1}^A + P_t \cdot X_{t+1}^B - P_t^B = P_t^A + P_t^B - P_t^B = P_t^A$$

- True of any cashflow!! (Hedge, Bond, Dividend, etc.)
- Flows from the linearity of the pricing: By arbitrage, assets are worth the sum of their parts

- Larry Summers (1985) critique of (then) finance
- Two kinds of research on ketchup market
- General Economists:
  - Ask what the fundamental determinants of the price of ketchup is
  - Analyze messy data on supply and demand
  - Tough question, modest progress
- Ketchup Economists:
  - Analyze hard data on transactions
  - Two quart sized ketchup bottles invariably sell for twice as much as one
  - No bargains from storing ketchup or mixing ketchup, etc.
  - Conclude from this that ketchup market is efficient

- Now suppose that

$$U(C_t) = \frac{C_t^{1-\gamma} - 1}{1-\gamma}$$

- This utility function is sometimes called CRRA utility for “constant relative risk aversion”
- Relative risk aversion:

$$-\frac{U''(C)C}{U'(C)} = \gamma$$

- Why do we think that this utility function is reasonable?

- Consider agent with CRRA utility and wealth  $W$  facing portfolio choice between risky and risk-free asset. Fraction allocated to risky asset is independent of wealth.  
(CARA utility: Dollar amount invested in risky asset is independent of wealth)
- Consistent with stable interest rate and risk premia in the presence of long-run growth



- Consider the following gamble: I flip a coin and ...
  - If it comes up heads, I multiply your lifetime income by 1 million
  - If it comes up tails, I reduce your lifetime income by XX%

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- What about 20% reduction? If yes,  $CRRA < 5$

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- What about 30% reduction? If yes, CRRA < 3

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- What about 20% reduction? If yes, CRRA < 5
- What about 30% reduction? If yes, CRRA < 3
- What about 50% reduction? If yes, CRRA < 2

- What fraction of your lifetime wealth would you be willing to pay to avoid a 50/50 risk of gaining or losing a share  $\alpha$  of your lifetime wealth
  - $\alpha = 0.10$
  - $\alpha = 0.30$

**Table 2.1**Relative risk premium  $\hat{\pi}$  associated to the risk of gaining or losing ( $\alpha\%$  of wealth)

| RRA            | $\alpha = 10\%$ | $\alpha = 30\%$ |
|----------------|-----------------|-----------------|
| $\gamma = 0.5$ | 0.3             | 2.3             |
| $\gamma = 1$   | 0.5             | 4.6             |
| $\gamma = 4$   | 2.0             | 16.0            |
| $\gamma = 10$  | 4.4             | 24.4            |
| $\gamma = 40$  | 8.4             | 28.7            |

Source: Gollier (2001)

$$U(C_t) = \frac{C_t^{1-\gamma} - 1}{1-\gamma}$$

- With time separable power utility,  $\gamma$  is also the inverse of the intertemporal elasticity of substitution

$$\frac{d \log(C_{t+1}/C_t)}{d \log(P_{t+1}/P_t)} = \frac{d \log(C_{t+1}/C_t)}{d \log R_{ft}} = \frac{1}{\gamma}$$

- Only one parameter. So, it plays many roles.  
(Also governs strength of wealth effect on labor supply)



$$U(C_t) = \frac{C_t^{1-\gamma} - 1}{1-\gamma}$$

- Implies:

$$U'(C_t) = C_t^{-\gamma}$$

$$M_{t+1} = \frac{\beta U'(C_{t+1})}{U'(C_t)} = \beta \left( \frac{C_{t+1}}{C_t} \right)^{-\gamma}$$

- For risk-free bonds we have:

$$1 = E_t[M_{t+1}R_{f,t}] \Rightarrow 1 = E_t[M_{t+1}]R_{f,t} \Rightarrow R_{f,t} = \frac{1}{E_t M_{t+1}}$$

- Since risk free return is risk free, it is determined at time  $t$
- With power utility

$$R_{f,t} = 1/E_t \left[ \beta \left( \frac{C_{t+1}}{C_t} \right)^{-\gamma} \right]$$

- If  $X_{t+1}$  is log-normal, then

$$\log E_t X_{t+1} = E_t \log X_{t+1} + \frac{1}{2} \text{Var}_t \log X_{t+1}$$

- If we assume consumption growth is log-normal, we get:

$$r_{f,t} = \delta + \gamma E_t[\Delta \log C_{t+1}] - \frac{\gamma^2}{2} \sigma_t^2(\Delta \log C_{t+1})$$

where  $\beta = e^{-\delta}$ ,  $r_{f,t} = \log R_{f,t}$

- Risk-free rate is determined by
  - Discount rate  $\delta$
  - Expected consumption growth
  - Precautionary savings ( $\frac{\gamma^2}{2} \sigma_t^2(\Delta \log C_{t+1})$ )

$$P_{i,t} = E_t [M_{t+1} X_{i,t+1}]$$

- The definition of covariance implies

$$\text{cov}_t(M_{t+1}, X_{i,t+1}) = E_t[M_{t+1} X_{i,t+1}] - E[M_{i,t+1}]E[X_{i,t+1}]$$

- Using this yields

$$P_{i,t} = E[M_{i,t+1}]E[X_{i,t+1}] + \text{cov}_t(M_{t+1}, X_{i,t+1})$$

- Using  $R_{f,t} = 1/E_t[M_{t+1}]$  yields

$$P_{i,t} = \frac{E[X_{i,t+1}]}{R_{f,t}} + \text{cov}_t(M_{t+1}, X_{i,t+1})$$

$$P_{i,t} = \frac{E[X_{i,t+1}]}{R_{f,t}} + \text{cov}_t(M_{t+1}, X_{i,t+1})$$

- Second term is a risk adjustment
  - Price of asset is higher if payoff covaries positively with SDF
  - In this case, asset is a hedge
- With power utility:

$$P_{i,t} = \frac{E[X_{i,t+1}]}{R_{f,t}} + \beta \frac{\text{cov}_t(U'(C_{t+1}), X_{i,t+1})}{U'(C_t)}$$

- Asset is a hedge if:
  - Payoff covaries positively with marginal utility
  - Payoff covaries negatively with consumption

# RISK ADJUSTED RETURNS

- Similar manipulations starting with  $1 = E_t[M_{t+1}R_{i,t+1}]$  yield:

$$E_t[R_{i,t+1}] - R_{f,t} = -R_{f,t}\text{cov}_t(M_{t+1}, R_{i,t+1})$$

and

$$E_t[R_{i,t+1}] - R_{f,t} = -\frac{\text{cov}_t(U'(C_{t+1}), R_{i,t+1})}{E_t[U'(C_{t+1})]}$$

- The return premium of asset  $i$  is higher if:
  - The return on asset  $i$  is negatively correlated with the  $M_{t+1}$
  - The return on asset  $i$  is negatively correlated with the  $U'(C_{t+1})$
  - The return on asset  $i$  is positively correlated with the  $C_{t+1}$

# RISK ADJUSTMENT WITH POWER UTILITY

$$1 = E_t \left[ (1 + R_{i,t+1}) \beta \left( \frac{C_{t+1}}{C_t} \right)^{-\gamma} \right]$$

- Taking logs and assuming log-normality:

$$E_t r_{i,t+1} = \delta + \gamma E_t [\Delta \log C_{t+1}]$$

$$-\frac{1}{2} \sigma_t^2 (\log R_{i,t+1}) - \frac{\gamma^2}{2} \sigma_t^2 (\Delta \log C_{t+1}) + \gamma \text{cov}(\log R_{i,t+1}, \Delta \log C_{t+1})$$

- Combining this with expression for risk-free rate yields

$$E_t r_{i,t+1} - r_{f,t} + \frac{1}{2} \sigma_t^2 (\log R_{i,t+1}) = \gamma \text{cov}(\log R_{i,t+1}, \Delta \log C_{t+1})$$

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- Equity premium is risk aversion times covariance between consumption growth and return on equity
- But what is with this  $\frac{1}{2} \sigma_t^2(\log R_{i,t+1})$  term?



$$E_t r_{i,t+1} - r_{f,t} + \frac{1}{2} \sigma_i^2 (\log R_{i,t+1}) = \gamma \text{cov}(\log R_{i,t+1}, \Delta \log C_{t+1})$$

- Comes from difference between geometric and arithmetic returns:

$$\log E_t R_{i,t+1} - \log R_{f,t} = E_t r_{i,t+1} - r_{f,t} + \frac{1}{2} \sigma_i^2 (\log R_{i,t+1})$$

- Geometric mean:  $E_t r_{i,t+1}$
- (Log of) Arithmetic mean:  $\log E_t R_{i,t+1}$

$$\log E_t R_{i,t+1} = E_t r_{i,t+1} + \frac{1}{2} \sigma_i^2 (\log R_{i,t+1})$$

- Standard deviation annual real return on stocks is roughly 18%

$$\frac{1}{2} \text{Var}_t \log R_{i,t+1} = \frac{1}{2} \sigma_i^2 = 1.5\%$$

- Two ways to write equity premium equation:

$$E_t r_{i,t+1} - r_{f,t} + \frac{1}{2} \sigma_t^2 (\log R_{i,t+1}) = \gamma \text{cov}(\log R_{i,t+1}, \Delta \log C_{t+1})$$

$$\log E_t R_{i,t+1} - \log R_{f,t} = \gamma \text{cov}(\log R_{i,t+1}, \Delta \log C_{t+1})$$

- Also recall that log of expected gross return is approximately equal to the expected net return:

$$\log(1 + x) \approx x$$

for small  $x$

- Complete markets
- Representative agent with CRRA preferences:

$$C_t^{-\gamma} = E_t[\beta C_{t+1}^{-\gamma} R_{i,t+1}]$$

- Endowment economy (“Lucas tree”):

$$\log C_{t+1} = \mu + \log C_t + \epsilon_{t+1}$$

$$\epsilon_{t+1} \sim N(0, \sigma^2)$$

(Original consumption process is a little different from this.)

- Equity modeled as a claim to the consumption process :

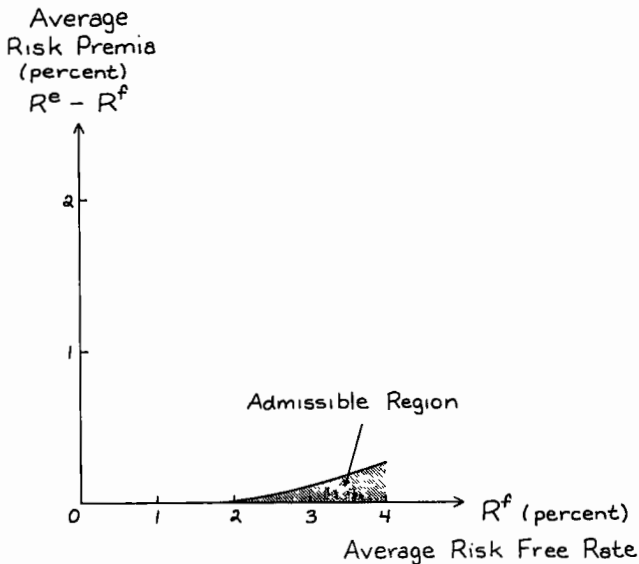
$$R_{i,t+1} = R_{C,t+1}$$

- In this case, equity premium and risk-free rate:

$$\log E_t R_{C,t+1} - \log R_{f,t} = \gamma \text{var}_t(\Delta \log C_{t+1})$$

$$\log R_{f,t} = \delta + \gamma E_t[\Delta \log C_{t+1}] - \frac{\gamma^2}{2} \text{var}_t(\Delta \log C_{t+1})$$

- Does this model fit the data?
- We need data on:
  - Average returns on stocks and risk-free asset
  - Mean and variance of consumption growth
- We need a view as to what values are “reasonable” for  $\gamma$ 
  - Mehra-Prescott: Values of  $\gamma < 10$  “admissible”



**Fig. 4. Set of admissible average equity risk premia and real returns.**

Source: Mehra and Prescott (1985). Values of equity premium and risk-free rate consistent with model given measured mean and variance of consumption growth.

Table 1

| Time periods | % growth rate of per capita real consumption |                    | % real return on a relatively riskless security |                    | % risk premium             |                    | % real return on S&P 500   |                    |
|--------------|--|--------------------|---|--------------------|----------------------------|--------------------|----------------------------|--------------------|
|              | Mean   | Standard deviation | Mean  | Standard deviation | Mean                       | Standard deviation | Mean                       | Standard deviation |
| 1889–1978    | 1.83<br>(Std error = 0.38)                   | 3.57               | 0.80<br>(Std error = 0.60)                      | 5.67               | 6.18<br>(Std error = 1.76) | 16.67              | 6.98<br>(Std error = 1.74) | 16.54              |
| 1889–1898    | 2.30   | 4.90               | 5.80  | 3.23               | 1.78                       | 11.57              | 7.58                       | 10.02              |
| 1899–1908    | 2.55   | 5.31               | 2.62  | 2.59               | 5.08                       | 16.86              | 7.71                       | 17.21              |
| 1909–1918    | 0.44   | 3.07               | -1.63   | 9.02               | 1.49                       | 9.18               | -0.14                      | 12.81              |
| 1919–1928    | 3.00   | 3.97               | 4.30  | 6.61               | 14.64                      | 15.94              | 18.94                      | 16.18              |
| 1929–1938    | -0.25  | 5.28               | 2.39  | 6.50               | 0.18                       | 31.63              | 2.56                       | 27.90              |
| 1939–1948    | 2.19   | 2.52               | -5.82   | 4.05               | 8.89                       | 14.23              | 3.07                       | 14.67              |
| 1949–1958    | 1.48   | 1.00               | -0.81   | 1.89               | 18.30                      | 13.20              | 17.49                      | 13.08              |
| 1959–1968    | 2.37   | 1.00               | 1.07  | 0.64               | 4.50                       | 10.17              | 5.58                       | 10.59              |
| 1969–1978    | 2.41   | 1.40               | -0.72   | 2.06               | 0.75                       | 11.64              | 0.03                       | 13.11              |

Source: Mehra and Prescott (1985)

- Mehra-Prescott 85 made “extra” assumptions:
  - Endowment economy
  - Specific process for consumption growth
  - Equity is a consumption claim
- Equity premium equation can be evaluated independent of these assumptions:

$$E_t r_{i,t+1} - r_{f,t} + \frac{1}{2} \sigma_t^2(\log R_{i,t+1}) = \gamma \text{cov}(\log R_{i,t+1}, \Delta \log C_{t+1})$$

Table 5  
The equity premium puzzle<sup>a</sup>

| Country | Sample period | $\overline{aer}_e$ | $\sigma(er_e)$ | $\sigma(m)$ | $\sigma(\Delta c)$ | $\rho(er_e, \Delta c)$ | $\text{Cov}(er_e, \Delta c)$ | RRA(1)   | RRA(2) |
|---------|---------------|--------------------|----------------|-------------|--------------------|------------------------|------------------------------|----------|--------|
| USA     | 1947.2–1996.3 | 7.852              | 15.218         | 51.597      | 1.084              | 0.193                  | 3.185                        | 246.556  | 47.600 |
| AUL     | 1970.1–1996.2 | 3.531              | 23.194         | 15.221      | 2.142              | 0.156                  | 7.725                        | 45.704   | 7.107  |
| CAN     | 1970.1–1996.2 | 3.040              | 16.673         | 18.233      | 2.034              | 0.159                  | 5.387                        | 56.434   | 8.965  |
| FR      | 1973.2–1996.2 | 7.122              | 22.844         | 31.175      | 2.130              | -0.047                 | -2.295                       | < 0      | 14.634 |
| GER     | 1978.4–1996.2 | 6.774              | 20.373         | 33.251      | 2.495              | 0.039                  | 1.974                        | 343.133  | 13.327 |
| ITA     | 1971.2–1995.2 | 2.166              | 27.346         | 7.920       | 1.684              | 0.002                  | 0.088                        | 2465.323 | 4.703  |
| JPN     | 1970.2–1996.2 | 6.831              | 21.603         | 31.621      | 2.353              | 0.100                  | 5.093                        | 134.118  | 13.440 |
| NTH     | 1977.2–1996.1 | 9.943              | 15.632         | 63.607      | 2.654              | 0.023                  | 0.946                        | 1050.925 | 23.970 |
| SWD     | 1970.1–1994.4 | 9.343              | 23.541         | 39.688      | 1.917              | 0.003                  | 0.129                        | 7215.176 | 20.705 |
| SWT     | 1982.2–1996.2 | 12.393             | 20.466         | 60.553      | 2.261              | -0.129                 | -5.978                       | < 0      | 26.785 |
| UK      | 1970.1–1996.2 | 8.306              | 21.589         | 38.473      | 2.589              | 0.095                  | 5.314                        | 156.308  | 14.858 |
| USA     | 1970.1–1996.3 | 5.817              | 16.995         | 34.228      | 0.919              | 0.248                  | 3.875                        | 150.136  | 37.255 |
| SWD     | 1920–1993     | 6.000              | 18.906         | 31.737      | 2.862              | 0.169                  | 9.141                        | 65.642   | 11.091 |
| UK      | 1919–1993     | 8.677              | 21.706         | 39.974      | 2.820              | 0.355                  | 21.738                       | 39.914   | 14.174 |
| USA     | 1891–1994     | 6.258              | 18.534         | 33.767      | 3.257              | 0.497                  | 30.001                       | 20.861   | 10.366 |

<sup>a</sup>  $\overline{aer}_e$  is the average excess log return on stock over a money market instrument, plus one half the variance of this excess return:  $\overline{aer}_e = \overline{r_e - r_f} + \sigma^2(r_e - r_f)/2$ . It is multiplied by 400 in quarterly data and 100 in annual data to express in annualized percentage points.  $\sigma(er_e)$  and  $\sigma(\Delta c)$  are the standard deviations of the excess log return  $er_e = r_e - r_f$  and consumption growth  $\Delta c$ , respectively, multiplied by 200 in quarterly data and 100 in annual data to express in annualized percentage points.  $\sigma(m) = 100\overline{aer}_e/\sigma(er_e)$  is calculated from equation (12) as a lower bound on the standard deviation of the log stochastic discount factor, expressed in annualized percentage points.  $\rho(er_e, \Delta c)$  is the correlation of  $er_e$  and  $\Delta c$ .  $\text{Cov}(er_e, \Delta c)$  is the product  $\sigma(er_e)\sigma(\Delta c)\rho(er_e, \Delta c)$ . RRA(1) is  $100\overline{aer}_e/\text{Cov}(er_e, \Delta c)$ , a measure of risk aversion calculated from equation (16) using the empirical covariance of excess stock returns with consumption growth. RRA(2) is  $100\overline{aer}_e/(\sigma(er_e)\sigma(\Delta c))$ , a measure of risk aversion calculated using the empirical standard deviations of excess stock returns and consumption growth, but assuming perfect correlation between these series.

Abbreviations: AUL, Australia; CAN, Canada; FR, France; GER, Germany; ITA, Italy; JPN, Japan; NTH, Netherlands; SWD, Sweden; SWT, Switzerland; UK, United Kingdom; USA, United States of America.

Source: Campbell (1999)



**Table 5 Long-Period Averages of Rates of Return**

| Country                           | Start | Stocks             | Bills              | Start | Bonds             | Bills             |
|-----------------------------------|-------|--------------------|--------------------|-------|-------------------|-------------------|
| <b>Part 1: OECD countries</b>     |       |                    |                    |       |                   |                   |
| Australia                         | 1876  | 0.1027 (0.1616)    | 0.0126 (0.0566)    | 1870  | 0.0352 (0.1157)   | 0.0125 (0.0569)   |
| Belgium                           | --    | --                 | --                 | 1870  | 0.0291 (0.1584)** | 0.0179 (0.1447)** |
| Canada                            | 1916  | 0.0781 (0.1754)    | --                 | 1916  | 0.0392 (0.1199)   | --                |
| Denmark                           | 1915  | 0.0750 (0.2300)    | 0.0265 (0.0652)    | 1870  | 0.0392 (0.1137)   | 0.0317 (0.0588)   |
| Finland                           | 1923  | 0.1268 (0.3155)    | 0.0128 (0.0935)    | --    | --                | --                |
| France                            | 1870  | 0.0543 (0.2078)*   | -0.0061 (0.0996)*  | 1870  | 0.0066 (0.1368)   | -0.0079 (0.1000)  |
| Germany                           | 1870  | 0.0758 (0.2976)    | -0.0153 (0.1788)   | 1924  | 0.0402 (0.1465)   | 0.0158 (0.1173)   |
| Italy                             | 1906  | 0.0510 (0.2760)    | -0.0112 (0.1328)   | 1870  | 0.0173 (0.1879)   | 0.0046 (0.1191)   |
| Japan                             | 1894  | 0.0928 (0.3017)    | -0.0052 (0.1370)   | 1883  | 0.0192 (0.1820)   | 0.0043 (0.1475)   |
| Netherlands                       | 1920  | 0.0901 (0.2116)**  | 0.0114 (0.0474)**  | 1881  | 0.0308 (0.1067)   | 0.0118 (0.0512)   |
| New Zealand                       | 1927  | 0.0762 (0.2226)    | 0.0234 (0.0529)    | 1926  | 0.0276 (0.1209)   | 0.0240 (0.0529)   |
| Norway                            | 1915  | 0.0716 (0.2842)    | 0.0098 (0.0782)    | 1877  | 0.0280 (0.1130)   | 0.0204 (0.0709)   |
| Spain                             | 1883  | 0.0610 (0.2075)†   | 0.0173 (0.0573)†   | --    | --                | --                |
| Sweden                            | 1902  | 0.0923 (0.2347)    | 0.0180 (0.0719)    | 1922  | 0.0292 (0.0941)   | 0.0176 (0.0448)   |
| Switzerland                       | 1911  | 0.0726 (0.2107)††  | 0.0083 (0.0531)††  | 1916  | 0.0218 (0.0717)   | 0.0065 (0.0545)   |
| U.K.                              | 1870  | 0.0641 (0.1765)    | 0.0179 (0.0624)    | 1870  | 0.0280 (0.1049)   | 0.0179 (0.0624)   |
| U.S.                              | 1870  | 0.0827 (0.1866)    | 0.0199 (0.0482)    | 1870  | 0.0271 (0.0842)   | 0.0199 (0.0482)   |
| <b>Part 2: Non-OECD countries</b> |       |                    |                    |       |                   |                   |
| Chile                             | 1895  | 0.1430 (0.4049)    | -0.0094 (0.1776)   | --    | --                | --                |
| India                             | 1921  | 0.0514 (0.2341)*** | 0.0133 (0.0835)*** | 1874  | 0.0191 (0.1147)   | 0.0240 (0.0785)   |
| South Africa                      | 1911  | 0.0890 (0.2006)    | --                 | 1911  | 0.0248 (0.1165)   | --                |
| Overall means†††                  | --    | 0.0814 (0.2449)    | 0.0085 (0.0880)    | --    | 0.0266 (0.1234)   | 0.0147 (0.0805)   |

\*missing 1940-41, \*\*missing 1945-46, †missing 1936-40, ††missing 1914-16, \*\*\*missing 1926-27

†††Averages of means and standard deviations for 17 countries with stock and bill data and 15 countries with bond and bill data

Source: Barro and Ursua (2008)

- Volatility of consumption seems to be relatively modest
- World seems to be a relatively safe place
- People must be very risk averse to not want to bid up prices of stocks
- High equity premium implies that stocks are cheap!!

# EQUITY PREMIUM IS VERY BIG

- Suppose we invest \$ 1 in:
  - Equity with 8% real return
  - Tbills with 1% real return

| Horizon | Equity  | Tbills |
|---------|---------|--------|
| 1       | 1.08    | 1.01   |
| 5       | 1.50    | 1.05   |
| 10      | 2.15    | 1.10   |
| 25      | 6.85    | 1.28   |
| 50      | 46.90   | 1.64   |
| 100     | 2199.76 | 2.70   |

- Dutch (supposedly) bought Manhattan from natives for \$24 in 1626
- Suppose natives invested this in the stock market:

$$\$24 \times 1.08^{(2021-1626)} = \$3.82 \times 10^{14} = \$382 \text{ Trillion}$$

# EQUITY PREMIUM IS VERY BIG ... OR IS IT?

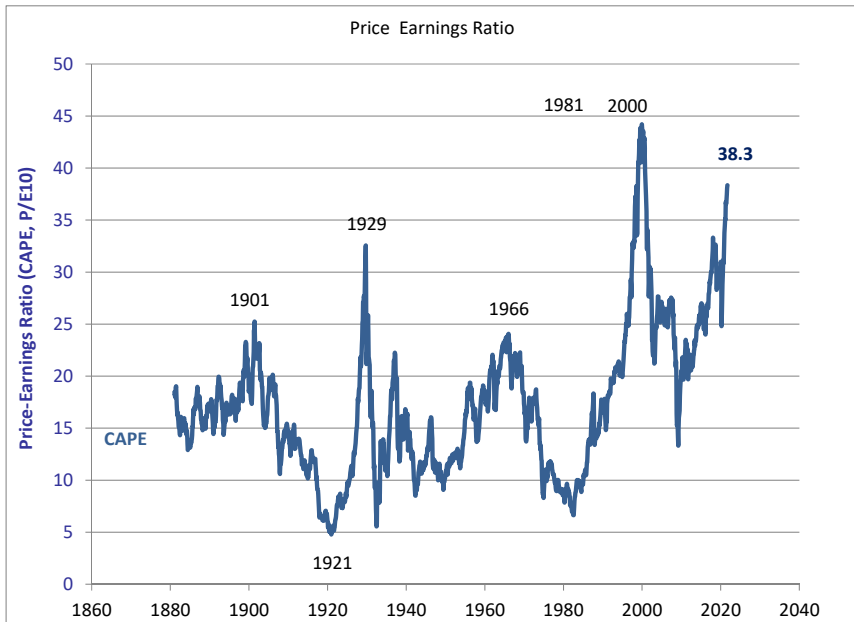
- Mean equity premium:  $\approx 6.5\%$
- Standard deviation of equity premium:  $\approx 18\%$
- Standard error on equity premium:  $\sigma/\sqrt{T} = 2.1\%$  (post-WWII)  
 $\sigma/\sqrt{T} = 1.5\%$  (post-1870)
- Using post-WWII standard error:
  - 95% confidence interval for equity premium: [2.3%, 10.7%]
- Perhaps last 100 years have been unusually good

# EQUITY PREMIUM IS VERY BIG ... OR IS IT?

- Relative to prior history, 20th century was good for growth and stocks
- Simple Gordon growth model:

$$\frac{P}{D} = \frac{1}{r - g}$$

- Maybe expectations about future growth have risen
- Maybe equity premium has fallen
  
- Would yield an unusually high return not to be repeated in the future



Source: Robert Shiller's website.

# HANSEN-JAGANATHAN BOUND

$$E_t r_{i,t+1} - r_{f,t} + \frac{1}{2} \sigma_t^2 (\log R_{i,t+1}) = -\text{cov}(\log R_{i,t+1}, \Delta \log M_{t+1})$$

- Let's adopt the notation:  $E_t r_{i,t+1} - r_{f,t} + \frac{1}{2} \sigma_i^2 = -\sigma_{im}$
- Definition of correlation coefficient:

$$\rho_{im} = \frac{\sigma_{im}}{\sigma_i \sigma_m}$$

$$-1 \leq \rho_{im}$$

$$\sigma_m \geq \frac{-\sigma_{im}}{\sigma_i}$$

$$\sigma_m \geq \frac{E_t r_{i,t+1} - r_{f,t} + \frac{1}{2} \sigma_i^2}{\sigma_i}$$

- Ratio on right-hand-side called “Sharpe ratio”

# HANSEN-JAGANATHAN BOUND

$$\sigma_m \geq \frac{E_t r_{i,t+1} - r_{f,t} + \frac{1}{2}\sigma_i^2}{\sigma_i}$$

- Sharp ratio for stocks: 0.4
- Sharp ratio for other assets: >1
- Hansen-Jaganathan bound implies that volatility of stochastic discount factor is enormous
- Seems implausible



# RISK-FREE RATE PUZZLE

$$\log R_{f,t} = \delta + \gamma E_t[\Delta \log C_{t+1}] - \frac{\gamma^2}{2} \text{var}_t(\Delta \log C_{t+1})$$

- $\text{var}_t(\Delta \log C_{t+1}) \ll E_t[\Delta \log C_{t+1}]$
- High value of  $\gamma$  therefore implies high risk free rate
- What is the intuition for this?

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  - Consumers must be compensated a lot to allow their consumption profile to be upward sloping
  - This is  $\gamma$  acting in its incarnation as 1/IES

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- High value of  $\gamma$  therefore implies high risk free rate
- What is the intuition for this?
  - Consumers must be compensated a lot to allow their consumption profile to be upward sloping
  - This is  $\gamma$  acting in its incarnation as  $1/\text{IES}$
- To get a low risk-free rate,  $\beta > 1$

Table 6  
The riskfree rate puzzle<sup>a</sup>

| Country | Sample period | $\bar{r}_f$ | $\bar{\Delta c}$ | $\sigma(\Delta c)$ | RRA(1)  | TPR(1)   | RRA(2) | TPR(2)  |
|---------|---------------|-------------|------------------|--------------------|---------|----------|--------|---------|
| USA     | 1947.2–1996.3 | 0.794       | 1.908            | 1.084              | 246.556 | -112.474 | 47.600 | -76.710 |
| AUL     | 1970.1–1996.2 | 1.820       | 1.854            | 2.142              | 45.704  | -34.995  | 7.107  | -10.196 |
| CAN     | 1970.1–1996.2 | 2.738       | 1.948            | 2.034              | 56.434  | -41.346  | 8.965  | -13.066 |
| FR      | 1973.2–1996.2 | 2.736       | 1.581            | 2.130              | < 0     | N/A      | 14.634 | -15.536 |
| GER     | 1978.4–1996.2 | 3.338       | 1.576            | 2.495              | 343.133 | >1000    | 13.327 | -12.142 |
| ITA     | 1971.2–1995.2 | 2.064       | 2.424            | 1.684              | >1000   | >1000    | 4.703  | -9.021  |
| JPN     | 1970.2–1996.2 | 1.538       | 3.416            | 2.353              | 134.118 | 41.222   | 13.440 | -39.375 |
| NTH     | 1977.2–1996.1 | 3.705       | 1.466            | 2.654              | >1000   | >1000    | 23.970 | -11.201 |
| SWD     | 1970.1–1994.4 | 1.520       | 0.750            | 1.917              | >1000   | >1000    | 20.705 | -6.126  |
| SWT     | 1982.2–1996.2 | 1.466       | 0.414            | 2.261              | < 0     | N/A      | 26.785 | 8.698   |
| UK      | 1970.1–1996.2 | 1.081       | 2.025            | 2.589              | 156.308 | 503.692  | 14.858 | -21.600 |
| USA     | 1970.1–1996.3 | 1.350       | 1.710            | 0.919              | 150.136 | -160.275 | 37.255 | -56.505 |
| SWD     | 1920–1993     | 2.073       | 1.748            | 2.862              | 65.642  | 63.778   | 11.091 | -12.274 |
| UK      | 1919–1993     | 1.198       | 1.358            | 2.820              | 39.914  | 10.364   | 14.174 | -10.057 |
| USA     | 1891–1994     | 1.955       | 1.742            | 3.257              | 20.861  | -11.305  | 10.366 | -10.406 |

<sup>a</sup>  $\bar{r}_f$  is the mean money market return from Table 2, in annualized percentage points.  $\bar{\Delta c}$  and  $\sigma(\Delta c)$  are the mean and standard deviation of consumption growth from Table 3, in annualized percentage points. RRA(1) and RRA(2) are the risk aversion coefficients from Table 5.  $TPR(1) = \bar{r}_f - RRA(1)\bar{\Delta c} + RRA(1)^2\sigma^2(\Delta c)/200$ , and  $TPR(2) = \bar{r}_f - RRA(2)\bar{\Delta c} + RRA(2)^2\sigma^2(\Delta c)/200$ . From Equation (17), these time preference rates give the real interest rate, in annualized percentage points, that would prevail if consumption growth had zero mean and zero standard deviation and risk aversion were RRA(1) or RRA(2), respectively.

# IS THE EQUITY PREMIUM A LIQUIDITY PREMIUM?

- Perhaps low return on short term bonds is a liquidity premium for “money-like” features
- Campbell argues against this based on the term premium:
  - Long-term bonds don't have this type of liquidity premium
  - Yet their returns are only slightly higher than those of short-term bonds

Table 7  
International yield spreads and bond excess returns<sup>a</sup>

| Country | Sample period | $\bar{s}$ | $\sigma(s)$ | $\rho(s)$ | $\overline{er_b}$ | $\sigma(er_b)$ | $\rho(er_b)$ |
|---------|---------------|-----------|-------------|-----------|-------------------|----------------|--------------|
| USA     | 1947.2–1996.4 | 1.199     | 0.999       | 0.783     | 0.011             | 8.923          | 0.070        |
| AUL     | 1970.1–1996.3 | 0.938     | 1.669       | 0.750     | 0.156             | 8.602          | 0.162        |
| CAN     | 1970.1–1996.3 | 1.057     | 1.651       | 0.819     | 0.950             | 9.334          | -0.009       |
| FR      | 1973.2–1996.3 | 0.917     | 1.547       | 0.733     | 1.440             | 8.158          | 0.298        |
| GER     | 1978.4–1996.3 | 0.991     | 1.502       | 0.869     | 0.899             | 7.434          | 0.117        |
| ITA     | 1971.2–1995.3 | -0.200    | 2.025       | 0.759     | -1.386            | 9.493          | 0.335        |
| JPN     | 1970.2–1996.3 | 0.593     | 1.488       | 0.843     | 1.687             | 9.165          | -0.058       |
| NTH     | 1977.2–1996.2 | 1.212     | 1.789       | 0.574     | 1.549             | 7.996          | 0.032        |
| SWD     | 1970.1–1995.1 | 0.930     | 2.046       | 0.724     | -0.212            | 7.575          | 0.244        |
| SWT     | 1982.2–1996.3 | 0.471     | 1.655       | 0.755     | 1.071             | 6.572          | 0.268        |
| UK      | 1970.1–1996.3 | 1.202     | 2.106       | 0.893     | 0.959             | 11.611         | -0.057       |
| USA     | 1970.1–1996.4 | 1.562     | 1.190       | 0.737     | 1.504             | 10.703         | 0.033        |
| SWD     | 1920–1994     | 0.284     | 1.140       | 0.280     | -0.075            | 6.974          | -0.185       |
| UK      | 1919–1994     | 1.272     | 1.505       | 0.694     | 0.318             | 8.812          | -0.098       |
| USA     | 1891–1995     | 0.720     | 1.550       | 0.592     | 0.172             | 6.499          | 0.153        |

<sup>a</sup>  $\bar{s}$  is the mean of the log yield spread, the difference between the log yield on long-term bonds and the log 3-month money market return, expressed in annualized percentage points.  $\sigma(s)$  is the standard deviation of the log yield spread and  $\rho(s)$  is its first-order autocorrelation.  $\overline{er_b}$ ,  $\sigma(er_b)$ , and  $\rho(er_b)$  are defined in the same way for the excess 3-month return on long-term bonds over money market instruments, where the bond return is calculated from the bond yield using the par-bond approximation given in Campbell, Lo and MacKinlay (1997), Chapter 10, equation (10.1.19). Full details of this calculation are given in the Data Appendix.

Abbreviations: AUL, Australia; CAN, Canada; FR, France; GER, Germany; ITA, Italy; JPN, Japan; NTH, Netherlands; SWD, Sweden; SWT, Switzerland; UK, United Kingdom; USA, United States of America.

## Restatement of Problem:

- To fit equity premium evidence, need high risk aversion
- High risk aversion implies low IES (with CRRA utility)
- Low IES implies high risk-free interest rate

“Obvious” solution:

- Consider preferences where IES may differ from  $1/\text{CRRA}$
- Make IES **and** CRRA high
  
- Epstein-Zin-Weil preferences deliver this



- Epstein-Zin (1989, 1991) and Weil (1989) propose:

$$U_t = \left\{ (1 - \delta) C_t^{\frac{1-\gamma}{\theta}} + \delta \left( E_t U_{t+1}^{1-\gamma} \right)^{\frac{1}{\theta}} \right\}^{\frac{\theta}{1-\gamma}}$$

- Parameters:

$$\theta = \frac{1 - \gamma}{1 - 1/\psi}$$

- $\gamma$ : Coefficient of relative risk aversion
- $\psi$ : Intertemporal elasticity of substitution
- Falls outside expected utility framework
- Large literature about “weird” properties

- Consumption Euler equation with Epstein-Zin-Weil preferences:

$$1 = E_t \left[ \beta^\theta \left( \frac{C_{t+1}}{C_t} \right)^{-\theta/\psi} (1 + R_{W,t+1})^{-(1-\theta)} (1 + R_{i,t+1}) \right]$$

- $R_{W,t+1}$  return on wealth

# CRRA OR IES??

- With power utility case, it is not clear whether  $\gamma$  appears in a particular equation because it is the CRRA or because it is 1/IES
- This is clarified in EZW case:

$$E_t r_{i,t+1} - r_{f,t} + \frac{1}{2}\sigma_i^2 = \theta \frac{\sigma_{ic}}{\psi} + (1 - \theta)\sigma_{iw}$$

$$r_{f,t} = -\log \beta + \frac{1}{\psi} E_t \Delta \log C_{t+1} + \frac{1}{2}(\theta - 1)\sigma_w^2 - \frac{1}{2} \frac{\theta}{\psi^2} \sigma_c^2$$

- Since both  $\gamma$  and  $\psi$  can be big at the same time, EP and RF puzzles can be resolved
- But are large values of  $\gamma$  and  $\psi$  “reasonable”