# ESTIMATION OF THE INTERTEMPORAL ELASTICITY OF SUBSTITUTION

Emi Nakamura and Jon Steinsson

UC Berkeley

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- 1. Traditional estimation strategies for IES in macro (e.g., Hall, 1988; Campbell and Mankiw, 1989)
  - Very structural approach (although it doesn't look it)
  - Example of a common type of reasoning in empirical macro
- 2. Critique of traditional strategy
  - Identification challenges are broader and more challenging than sometimes acknowledged
- 3. Example of different sort of structural approach (Best-Cloyne-Ilzetzki-Kleven 2019)

• Consumption Euler equation with power utility and log-normality:

$$E_t \Delta \log C_{t+1} = \psi E_t r_{i,t+1} + \psi \log \beta + \frac{1}{2} [\psi \sigma_i^2 + \psi^{-1} \sigma_c^2 - 2\sigma_{ic}]$$

• Can be rewritten as:

$$\Delta \log C_{t+1} = \mu_i + \psi r_{i,t+1} + \epsilon_{i,t+1}$$

#### where

$$\epsilon_{i,t+1} = \psi(E_t r_{i,t+1} - r_{i,t+1}) - (E_t \Delta \log C_{t+1} - \Delta \log C_{t+1})$$

$$\Delta \log C_{t+1} = \mu_i + \psi r_{i,t+1} + \epsilon_{i,t+1}$$

$$\epsilon_{i,t+1} = \psi(\boldsymbol{E}_t \boldsymbol{r}_{i,t+1} - \boldsymbol{r}_{i,t+1}) - (\boldsymbol{E}_t \Delta \log \boldsymbol{C}_{t+1} - \Delta \log \boldsymbol{C}_{t+1})$$

• Can we estimate this using OLS?

$$\Delta \log C_{t+1} = \mu_i + \psi r_{i,t+1} + \epsilon_{i,t+1}$$

$$\epsilon_{i,t+1} = \psi(E_t r_{i,t+1} - r_{i,t+1}) - (E_t \Delta \log C_{t+1} - \Delta \log C_{t+1})$$

- Suppose there is a "good shock" that leads to a high realization of r<sub>i,t+1</sub>
- This shock will be correlated with the error term (consumption (and return) will rise relative to expectation)

$$\Delta \log C_{t+1} = \mu_i + \psi r_{i,t+1} + \epsilon_{i,t+1}$$

$$\epsilon_{i,t+1} = \psi(E_t r_{i,t+1} - r_{i,t+1}) - (E_t \Delta \log C_{t+1} - \Delta \log C_{t+1})$$

• Can we think of instruments that will work in this case? (Hint: Error term is an expectation error)

$$\Delta \log C_{t+1} = \mu_i + \psi r_{i,t+1} + \epsilon_{i,t+1}$$

$$\epsilon_{i,t+1} = \psi(E_t r_{i,t+1} - r_{i,t+1}) - (E_t \Delta \log C_{t+1} - \Delta \log C_{t+1})$$

- Can we think of instruments that will work in this case? (Hint: Error term is an expectation error)
- Any variable know at time t works as an instrument
- Since *ϵ<sub>i,t+1</sub>* is an expectation error, it is orthogonal to all variables known at time *t* or earlier
- So, we can use lags of anything as instruments (Wow, lots of possible instruments)

- If  $r_{i,t+1}$  is the risk-free rate  $(r_{f,t})$  it is known at time t
- Then we have:

$$\Delta \log C_{t+1} = \mu_i + \psi r_{f,t} + \epsilon_{i,t+1}$$

$$\epsilon_{i,t+1} = \Delta \log C_{t+1} - E_t \Delta \log C_{t+1}$$

- In this case, OLS would work!
- In practice, the real return on even Tbills is uncertain due to inflation
- Could estimate by OLS using TIPS (Treasury Inflation Protected Securities) although sample would be short (TIPS started trading in 1997)

• Campbell and Mankiw (1989) estimate:

$$\Delta \log C_{t+1} = \mu_i + \psi r_{i,t+1} + \epsilon_{i,t+1}$$

using lags of real rates, consumption growth, and nominal rates as instruments (see also Hall (1988))

- Complication:  $C_t$  is a time average over a quarter
  - Even if *C<sub>t</sub>* were a random walk, time averaging would imply serial correlation of changes (Working, 1960)
  - Campbell and Mankiw (1989) lag instruments by 2 periods to avoid this

		First-stage 1	regressions	a estimate	Test of restrictions	
Row	Instruments	$\Delta c$ equation	r equation	(s.e.)		
1	None (OLS)		—	0.276 (0.079)	-	
2	$r_{t-2}, \ldots, r_{t-4}$	0.063 (0.009)	0.431 (0.000)	0.270 (0.118)	0.031 (0.029)	
3	$r_{t-2}, \ldots, r_{t-6}$	0.067 (0.014)	0.426 (0.000)	0.281 (0.118)	0.034 (0.050)	
4	$\Delta c_{t-2}, \ldots, \Delta c_{t-4}$	0.024 (0.101)	-0.021 (0.966)	-0.707 (2.586)	0.000 (0.215)	
5	$\Delta c_{t-2}, \ldots, \Delta c_{t-6}$	0.018 (0.007)	0.007 (0.316)	0.992 (0.478)	0.008 (0.189)	
6	$\Delta i_{t-2},\ldots,\Delta i_{t-4}$	0.061 (0.010)	0.024 (0.105)	1.263 (0.545)	-0.021 (0.918)	
7	$\Delta i_{t-2},\ldots,\Delta i_{t-6}$	0.102 (0.002)	0.028 (0.119)	1.213 (0.445)	-0.022 (0.700)	
8	$r_{t-2}, \ldots, r_{t-4}, \Delta c_{t-2}, \ldots, \Delta c_{t+4},$	0.062 (0.026)	0.455 (0.000)	0.204 (0.114)	0.047 (0.033)	
9	$r_{t-2}, \ldots, r_{t-4}, \\ \Delta c_{t-2}, \ldots, \Delta c_{t-4}, \\ \Delta i_{t-2}, \ldots, \Delta i_{t-4}$	0.103 (0.006)	0.476 (0.000)	0.150 (0.111)	0.100 (0.005)	

Table 3 UNITED STATES, 1953–1986

 $\Delta c_t = \mu + \sigma r_t$ 

Note: The columns labeled "First-stage regressions" report the adjusted  $R^2$  for the OLS regressions of the two variables on the instruments; in parentheses is the p-value for the null that all the coefficients except the constant are zero. The column labeled "A estimate" reports the V estimate of  $\lambda$  and, in parentheses, its standard error. The column labeled "Test of restrictions" reports the adjusted  $R^2$  of the OLS regression of the residual on the instruments; in parenthesis is the p-value for the null that all the coefficients are zero.

Source: Campbell and Mankiw (1989)

- Hall (1988) ran similar specifications. He favored estimates close to zero and interpreted them as estimates of the IES.
- Campbell and Mankiw (1989) worry about misspecification due to hand-to-mouth consumers
  - 1. Consumption growth predictable. Should not be true if  $\psi = 0$
  - 2. Over-identifying restrictions rejected
  - 3. Estimates very unstable
  - 4. Reverse regression not consistent with  $\psi = 0$

Just as the consumption Euler equation implies that

$$\Delta \log C_{t+1} = \mu_i + \psi r_{i,t+1} + \epsilon_{i,t+1}$$

it also implies that

$$r_{i,t+1} = \alpha_i + \frac{1}{\psi} \Delta \log C_{t+1} + \eta_{i,t+1}$$

- If  $\psi =$  0, this "reverse regression" should yield a large estimate for 1/ $\psi$
- Under the maintained assumptions above, this "reverse regression" can be estimated using IV with the same set of instruments
- This is the specification used by Hansen and Singleton (1983)

		First-stage 1	regressions	1/σ estimate	Test of	
Row	Instruments	$\Delta c$ equation	r equation	(s.e.)	restrictions	
1	None (OLS)	_	_	0.304 (0.087)	_	
2	$r_{t-2}, \ldots, r_{t-4}$	0.063 (0.009)	0.431 (0.000)	1.581 (0.486)	0.086 (0.001)	
3	$r_{t-2}, \ldots, r_{t-6}$	0.067 (0.014)	0.426 (0.000)	1.347 (0.390)	0.113 (0.001)	
4	$\Delta c_{t-2}, \ldots, \Delta c_{t-4}$	0.024 (0.101)	-0.021 (0.966)	-0.342 (0.428)	-0.021 (0.878)	
5	$\Delta c_{t-2}, \ldots, \Delta c_{t-6}$	0.018 (0.007)	0.007 (0.316)	0.419 (0.258)	-0.010 (0.440)	
6	$\Delta i_{t-2},\ldots,\Delta i_{t-4}$	0.061 (0.010)	0.024 (0.105)	0.768 (0.334)	-0.021 (0.919)	
7	$\Delta i_{t-2},\ldots,\Delta i_{t-6}$	0.102 (0.002)	0.028 (0.119)	0.638 (0.249)	-0.024 (0.747)	
8	$r_{t-2}, \ldots, r_{t-4}, \Delta c_{t-2}, \ldots, \Delta c_{t-4}$	0.062 (0.026)	0.455 (0.000)	1.034 (0.333)	0.236 (0.000)	
9	$r_{t-2'}, \ldots, r_{t-4'}$ $\Delta c_{t-2'}, \ldots, \Delta c_{t-4},$ $\Delta i_{t-2'}, \ldots, \Delta i_{t-4}$	0.103 (0.006)	0.476 (0.000)	0.521 (0.220)	0.455 (0.000)	

Table 4 UNITED STATES, 1953–1986

 $r_t = \mu + 1/\sigma \, \Delta c_t$ 

Note: The columns labeled "First-stage regressions" report the adjusted  $R^2$  for the OLS regressions of the two variables on the instruments; in parentheses is the p-value for the null that all the coefficients except the constant are zero. The column labeled "A estimate" reports the IV estimate of A and, in parentheses, its standard error. The column labeled "Test of restrictions" reports the adjusted  $R^2$  of the OLS regression of the residual on the instruments; in parenthesis is the p-value for the null that all the coefficients are zero.

Source: Campbell and Mankiw (1989)

# IS IES BIG OR SMALL??

- What is going on!!
- It can't both be true that:
  - $\psi$  is close to zero
  - $1/\psi$  is relatively small

- What is going on!!
- It can't both be true that:
  - $\psi$  is close to zero
  - $1/\psi$  is relatively small
- Yogo (2004): Puzzle due to weak instruments
  - Consumption growth notoriously hard to predict!!
  - Employs first-stage F-stat for weak instruments developed by Stock and Yogo (2003)
  - Concludes that reverse regression is unreliable due to weak instruments, but regression with real Tbill rate as regressor is reliable

					p-Value		
Country	Sample Period	Variable	F	TSLS Bias	TSLS Size	Fuller-k	LIML
USA	1947.3-1998.4	$\Delta c$ $r_f$ $r_e$	2.93 15.53 2.88	0.93 0.00 0.93	1.00 0.66 1.00	0.53 0.00 0.54	0.37 0.00 0.39
AUL	1970.3-1998.4	$\Delta c$ $r_f$ $r_e$	1.79 21.81 1.82	0.99 0.00 0.99	1.00 0.14 1.00	0.81 0.00 0.80	0.69 0.00 0.68
CAN	1970.3-1999.1	$\Delta c$ $r_f$ $r_e$	3.03 15.37 2.51	0.92 0.00 0.96	1.00 0.67 1.00	0.50 0.00 0.64	0.35 0.00 0.48
FR	1970.3-1998.3	$\Delta c$ $r_f$ $r_e$	0.17 38.43 3.09	1.00 0.00 0.91	1.00 0.00 1.00	1.00 0.00 0.49	1.00 0.00 0.34
GER	1979.1–1998.3	$\frac{\Delta c}{r_f}$ $r_e$	0.83 17.66 0.69	1.00 0.00 1.00	1.00 0.45 1.00	0.97 0.00 0.98	0.93 0.00 0.95
USA	1970.3-1998.4	$\Delta c$ $r_f$ $r_e$	3.53 11.92 2.16	0.86 0.02 0.97	1.00 0.92 1.00	0.39 0.00 0.72	0.25 0.00 0.58
SWD	1921–1994	$\Delta c$ $r_f$ $r_e$	1.02 5.50 1.67	1.00 0.55 0.99	1.00 1.00 1.00	0.95 0.10 0.84	0.89 0.05 0.72
UK	1921–1994	$\Delta c$ $r_f$ $r_e$	1.93 4.87 4.18	0.98 0.66 0.77	1.00 1.00 1.00	0.78 0.16 0.26	0.65 0.08 0.15
USA	1891–1995	$\Delta c$ $r_f$ $r_e$	1.55 2.87 1.00	0.99 0.93 1.00	1.00 1.00 1.00	0.86 0.54 0.95	0.76 0.39 0.90

TABLE 1.—TEST FOR WEAK INSTRUMENTS

The table reports the first-stage F-statistic from a regression of the endogenous variable onto the instruments. The endogenous variables are consumption growth ( $\Delta c$ ), real interest rate ( $r_j$ ), and real stock return ( $r_j$ ). The instruments are twice lagged nominal interest rate, inflation, consumption growth, and log dividend-price ratio. The table last properts the p-value of the test for weak instruments. The null hypotheses are: (1) the TSLS relative bias is greater than 10%, (2) the size of 8% TSLS r-test can be greater than 10%, (3) the fillest relative bias is greater than 10%, exist of 9% LSLM r-test can be greater than 10%, (3) the fillest relative bias is greater than 10%, (3) the fillest relative bias is greater than 0% of 9% LSLM r-test can be greater than 10%, (3) the fillest relative bias is greater than 0% of 9% LSL r-test can be greater than 10%, (3) the fillest relative bias is greater than 0% of 9% LSLM r-test can be greater than 10%, (3) the fillest relative bias is greater than 0% of 9% LSL relative bias ( $r_j = 10\%$ ).

#### Source: Yogo (2004)

			1/ψ			ψ	
Country	Sample Period	TSLS	Fuller-k	LIML	TSLS	Fuller-k	LIML
USA	1947.3-1998.4	0.68 (0.48)	3.30 (3.20)	34.11 (112.50)	0.06 (0.09)	0.03 (0.10)	0.03 (0.10)
AUL	1970.3-1998.4	0.50 (0.48)	2.37 (2.45)	30.03 (107.71)	0.05 (0.11)	0.04 (0.12)	0.03 (0.12)
CAN	1970.3-1999.1	-1.04 (0.39)	-2.40 (1.13)	-2.98 (1.54)	-0.30 (0.16)	-0.33 (0.17)	-0.34 (0.17)
FR	1970.3-1998.3	-3.12 (3.75)	-1.83 (1.72)	-12.38 (29.61)	-0.08 (0.19)	-0.08 (0.19)	-0.08 (0.19)
GER	1979.1–1998.3	-1.05 (0.62)	-1.38 (0.90)	-2.29 (1.87)	-0.42 (0.35)	-0.43 (0.35)	-0.44 (0.36)
ITA	1971.4-1998.1	-3.34 (1.98)	-5.82 (4.47)	-14.81 (18.55)	-0.07 (0.08)	-0.07 (0.08)	-0.07 (0.08)
JAP	1970.3-1998.4	-0.18 (0.43)	-0.86 (1.23)	-21.56 (106.53)	-0.04 (0.21)	-0.04 (0.23)	-0.05 (0.23)
NTH	1977.3-1998.4	-0.53 (0.41)	-1.41 (1.33)	-6.94 (13.96)	-0.15 (0.28)	-0.15 (0.29)	-0.14 (0.29)
SWD	1970.3-1999.2	-0.10 (1.10)	-0.21 (1.54)	-399.86 (16075.06)	0.00 (0.10)	0.00 (0.10)	0.00 (0.10)
SWT	1976.2-1998.4	-1.56 (0.83)	-1.51 (0.79)	-2.00 (1.18)	-0.49 (0.29)	-0.49 (0.29)	-0.50 (0.29)
UK	1970.3-1999.1	1.06 (0.45)	3.76 (2.42)	6.21 (5.17)	0.17 (0.13)	0.16 (0.13)	0.16 (0.13)
USA	1970.3-1998.4	0.53 (0.50)	2.19 (2.60)	47.66 (249.47)	0.06 (0.09)	0.02 (0.11)	0.02 (0.11)
SWD	1921–1994	1.17 (1.13)	3.30 (3.34)	17.77 (38.67)	0.06 (0.11)	0.06 (0.12)	0.06 (0.12)
UK	1921–1994	2.40 (1.01)	2.99 (1.33)	3.52 (1.65)	0.26 (0.12)	0.27 (0.13)	0.28 (0.13)
USA	1891–1995	-0.38 (1.12)	-1.17 (2.90)	-39.71 (257.54)	-0.03 (0.11)	-0.03 (0.15)	-0.03 (0.16)

TABLE 2.- ESTIMATES OF THE EIS USING THE INTEREST RATE

The reciprocal of the EIS is estimated from  $r_{j,j+1} = \mu_j + (1/\psi)\Delta c_{j+1} + \eta_{j,j+1}$ , and the EIS is estimated from  $\Delta c_{j+1} = \tau_j + \psi r_{j,j+1} + \xi_{j,j+1}$ . The instruments are the twice lagged nominal interest rate, inflation, consumption growth, and log dividend-price ratio. Standard errors in parentheses.

#### Source: Yogo (2004)

- Weak instruments is not the only empirical challenge!
- Above approach relies heavily on

$$\Delta \log C_{t+1} = \mu_i + \psi r_{i,t+1} + \epsilon_{i,t+1} \tag{1}$$

being a structural equation

- In particular, heavy reliance on  $\epsilon_{i,t+1}$  being **only** an expectation error
- But what if equation (1) is misspecified?

- Suppose  $U'(C_t, \eta_t)$  and  $\eta_t$  is persistent
- $\eta_t$  can be:
  - Preference shocks (e.g., sentiment, preference for borrowing)
  - Labor supply (i.e., non-separable utility)
- If  $\eta_t$  is persistent, then  $\eta_{t-j}$  will affect both
  - Lagged variables being used as instruments
  - Current  $\eta_t$
- This will lead IV with lagged variables to be biased

• Suppose we are estimating

$$\Delta \log C_{t+1} = \mu_i + \psi r_{i,t+1} + \epsilon_{i,t+1}$$

by OLS in the presence of preference shocks

- Increased desire to save drives down interest rates, and raises consumption growth
- Implies current interest rates negatively correlated with  $\epsilon_{i,t+1}$
- Downward bias in OLS estimate of  $\psi$

• Suppose we are estimating

$$\Delta \log C_{t+1} = \mu_i + \psi r_{i,t+1} + \epsilon_{i,t+1}$$

by IV with lagged instruments and persistent preference shocks

- Increase in desired savings in period *t* − *j* will affect instruments at time *t* − *j* and also increase desire to save in period *t*
- Part of correlation between instruments and r<sub>i,t+1</sub> due to lagged preference shock
- Lagged preference shock lowers r<sub>i,t+1</sub> and raises Δ log C<sub>t+1</sub> due to affect on current preference shock
- Same downward bias as OLS

### OTHER SOURCES OF MISSPECIFICATION

- Hand-to-mouth consumers (more generally: liquidity constraints) (see Werning (2015))
- 2. Time-varying volatility:

$$E_t \Delta \log C_{t+1} = \psi E_t r_{i,t+1} + \psi \log \beta + \frac{1}{2} [\psi \sigma_i^2 + \psi^{-1} \sigma_c^2 - 2\sigma_{ic}]$$

- We have been assuming that all the σ terms are constant
- What if they are not?
- Bansal and Yaron (2004):  $\psi$  will be downward biased
- Persistent increase in  $\sigma_c$  lowers  $E_t r_{i,t+1}$  and is part of error term (see also Carrol, 1997; Blundell et al., 1994; Guvenen, 2000)
- 3. Consumption commitments (housing, cars) lead to more complicated consumption Euler equation (Chetty and Szeidl, 2016)

- This lagged instrument strategy has been common in macro
  - E.g., Phillips curve estimation
- Stems from taking simple structural model extremely literally
- Applied micro approach very different:
  - Error term contains all sorts of things
  - We don't know the true model
  - Want conclusions to be robust to many structural stories
- Lagged instrument strategy becoming less common in macro

- Most work estimates IES using time-series variation, but hard to find exogenous variation in the time series
- Gruber (2013) exploits variation in rates of return in the cross-section to identify the IES
- After-tax rates of return are influenced by capital tax rates
- Exploits exogenous variation in capital tax rates

- Most variation in tax rates potentially endogenous (e.g., due to variation in income)
- Constructs "simulated" tax rates based on predicted income from exogenous characteristics (e.g., education, age, and sex)
- Controls flexibly for these characteristics
- Identification comes only from changes in the tax system over the sample period

### Specification:

$$GC_{i,t+1} = \alpha + \beta \mathsf{ATRATE}_{it} + X_{it}\delta + \Delta Z_{it,t+1}\eta + \epsilon$$

- GC: Non-durable consumption growth for household *i* (from CEX)
- ATRATE: Income specific after tax rate of return for household i (SCF portfolio shares and NBER TAXSIM tax rates)
- X: vector of baseline demographic characteristics
- $\Delta Z$ : vector of demographic changes
- Includes time and state fixed effects
- IV: Instrument for ATRATE using tax rate based on predicted income for a given demographic group

Table 2. Base case estimates.

	After-Tax T-Bill Rate	After-Tax Rate of Return
OLS, no year dummies	-0.551	0.105
	(0.116)	(0.032)
Lag IV, no year dummies	2.616	0.328
	(0.490)	(0.130)
Tax IV	2.032	2.239
	(0.796)	(0.894)
Number Obs.	66,314	66,208

*Notes*: Estimates from models such as Eq. (1) in text. Each cell represents the estimated EIS from a separate model: first column uses after-tax T-bill rate, while second column uses weighted average after-tax rate of return. Standard errors in parentheses.

Source: Gruber (2013)

- No consensus in the literature!!
- Macro people often use IES < 1, influenced by Hall (1988)
- Asset pricing people often use IES > 1 because values < 1 yield counter-intuitive responses of asset prices to shocks
  - With IES < 1, bad news about future growth increases stock prices because of strong desire to save

- Simplest form of inference:
  - Run regression in which one of the coefficients may be interpreted as direct causal evidence of parameter in question (e.g., IES)
- Often what we can measure is not directly what we are interested in estimating
- What we can measure, however, often yield powerful inference about what we are interested in if viewed through the lens of a structural model (provide tell-tale signs about parameter of interest)

(See Nakamura-Steinsson (2018) for more discussion of this idea.)

# BEST-CLOYNE-ILZETZKI-KLEVEN (2019)

- Use "mortgage notches" in the UK to shed light on IES
- Statistics they calculate do not provide direct estimates of IES
- But viewed through sensible structural models, they (arguably) provide powerful inference about IES
- How to make convincing inference through the lens of a structural model is a complicated art
  - Which model to use?
  - How robust are the conclusions?
- Best et al. paper is a good example of this

- UK mortgages have low fixed rates for 2, 3, or 5 years then much higher flexible rates
- High penalty for refinancing early
- Most people refinance at the time of interest rate reset
- Authors focus on refinancers so as to abstract from the effect of mortgage size on home size

#### FIGURE A.1: REFINANCING HAPPENS WHEN THE RESET RATE KICKS IN



Notes: The figure shows the distribution of the time to refinance, excluding individuals where the date on which the reset rate kicks in is unobserved. The figure shows individuals individuals who refinance more than 6 months after their reset rate kicks in in black, individuals who refinance more than 2 months before their reset rate kicks in in white, and the remainder who refinance around their reset date in gray.

#### Source: Best-Cloyne-Ilzetzki-Kleven (2019)

- Interest rate jumps by discrete amounts (features notches) at certain loan-to-value (LTV) thresholds
- Very salient: daily menu in newspapers, on bank websites, etc.
- Estimate rate function:

 $r_i = f(LTV_i) + \beta_1 \text{bank}_i + \beta_2 \text{variability}_i \otimes \text{duration}_i \otimes \text{month}_i + \beta_3 \text{repayment}_i + \beta_4 \text{term}_i + \nu_i$ 

- No individual characteristics because UK mortgage market is like a supermarket (no individual negotiation)
- But adding age, income and family status has no effect on results



Interest rate jumps at notches

*Notes:* This figure shows the conditional interest rate as a function of the LTV ratio from the non-parametric regression (2.1). In each LTV bin, we plot the coefficient on the LTV bin dummy plus a constant given by the mean predicted value  $E[\hat{r}_i]$  from all the other covariates (*i.e.* omitting the contribution of the LTV bin dummies). The figure shows that the mortgage interest rate evolves as a step function with sharp notches at LTV ratios of 60%, 70%, 80%, and 85%.

#### Source: Best-Cloyne-Ilzetzki-Kleven (2019)

Nakamura-Steinsson (UC Berkeley)	Estimation of IES	
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Observed LTV distribution among U.K. refinancers

*Notes*: This figure shows the observed distribution of LTV ratios among refinancers in the U.K. between 2008–14. There are interest rate notches at LTV ratios of 60%, 70%, 75%, 80%, 85%, and 90% (depicted by vertical lines).

#### Source: Best-Cloyne-Ilzetzki-Kleven (2019)

- Large amount of bunching below interest rate notches
- Intuitively, this is informative about IES:
  - Households must cut consumption to get below notch
  - How willing are households to cut consumption now to raise life-time consumption?

Two Challenges:

- 1. Need to translate bunching into IES estimate
  - What is counterfactual?
  - Is observed bunching a lot or a little?
- 2. Many other features of reality affect bunching
  - Patience
  - Demand for buffer stock savings (i.e., income risk and risk aversion)
  - Frictions to household optimization (inattention, inertia, myopia)

- Authors write down a structural model
- Ask: For what parameter values model can match bunching in the data?
- Bunching highly sensitive to the IES
- Relatively insensitive to reasonable variation in other parameters

- This is a good example of the use of structural estimation
- The moments being used are shown to be highly informative about something specific in the model and are used to estimate that thing
- Often structural estimation is a big black box with lots of moments estimating lots of parameters without a clear sense of what identifies what





Constructing the counterfactual LTV distribution.

Notes: This figure shows the two steps in the construction of the counterfactual LTV distribution among refinancers. Each panel shows the actual LTV distribution with dots (as in Figure 1). Panel A shows the distribution of passive LTVs with crosses, calculated based on the LTV of the previous mortgage, amortization, and the house value at the time of refinancing. Panel B shows the distribution of counterfactual LTVs with crosses, which adjusts passive LTVs for the average equity extraction of non-bunchers in the actual distribution.

Source: Best-Cloyne-Ilzetzki-Kleven (2019).

Based on "passive behavior", i.e., what would LTV have been if no refinancing.

## SIMPLE STRUCTURAL MODEL

- Two periods (0 and 1)
- Households have decided to stay in current home
- Face a refinancing decision
- Utility:

$$\frac{\sigma}{\sigma-1} \left( \textbf{C}_{0}^{(\sigma-1)/\sigma} + \delta \textbf{C}_{1}^{(\sigma-1)/\sigma} \right)$$

where  $\delta$  is discount factor and  $\sigma$  is IES

Budget constraints:

$$c_0 = y_0 + W_0 - (1 - \lambda)P_0H$$

$$c_1 = y_1 - R\lambda P_0 H + (1-d)P_1 H$$

where  $\lambda$  is LTV on new mortgage, *d* is depreciation rate of houses, and *R* is mortgate interest rate

### SIMULATION OF SIMPLE MODEL

- Authors simulate model for different values of IES
- Distribution of W<sub>0</sub> is calibrated to replicate counterfactual LTV distribution when R is constant
- Other parameters are calibrated to "reasonable" values:

 $\delta = 0.96$ d = 0.025 $\frac{P_1}{P_0} = 1.026$  $y_1 = y_0$ 



Source: Best-Cloyne-Ilzetzki-Kleven (2019)

- But could it be that other values of the other parameters could justify a large IES?
- Authors set IES = 1 and then vary other parameters to maximize fit at notches
- Varying other parameters cannot give good fit with IES = 1 even allowing for very unreasonable values for other parameters

**Panel B:**  $\sigma = 1$ ; Calibrated  $\delta, y, P$ 



Source: Best-Cloyne-Ilzetzki-Kleven (2019). Parameter values:  $\delta = 0.24$ ,  $P_1/P_0 = 0.88$ ,  $y_1 = 0.58y_0$ 

### PRECAUTIONARY SAVINGS

- Adjustment to interest rate notches affected by demand for buffer stock savings
- Simple structural model doesn't capture this (no risk)
- But simple structural model provides an upper bound:
  - Makes extreme assumption of no liquid wealth
  - All adjustment borne by consumption
- Addition of liquid wealth and precautionary savings would add an adjustment margin
- Even lower IES needed to justify small amount of bunching

- Most important "treat to identification" is frictions to household optimization (inattention, inertia, myopia)
- Estimate fraction of non-optimizers as those in dominated region (right above notch) relative to counterfactual
- Redoes model simulations assuming this fraction of non-optimizers



![](_page_47_Figure_1.jpeg)

Constructing the counterfactual LTV distribution.

Notes: This figure shows the two steps in the construction of the counterfactual LTV distribution among refinancers. Each panel shows the actual LTV distribution with dots (as in Figure 1). Panel A shows the distribution of passive LTVs with crosses, calculated based on the LTV of the previous mortgage, amortization, and the house value at the time of refinancing. Panel B shows the distribution of counterfactual LTVs with crosses, which adjusts passive LTVs for the average equity extraction of non-bunchers in the actual distribution.

#### Source: Best-Cloyne-Ilzetzki-Kleven (2019)

![](_page_48_Figure_0.jpeg)

#### FIGURE A.7: OBSERVED VS SIMULATED LTV DISTRIBUTIONS WITH FRICTION ADJUSTMENT

Source: Best-Cloyne-Ilzetzki-Kleven (2019)

			Notch		
	70	75	80	85	Pooled
Panel A: Adjustment factor a					
(1) Notched banks only	0.11 (0.02)	0.15 (0.02)	0.15 (0.03)	0.03 (0.01)	0.12 (0.01)
(2) Dominated region	0.21 (0.02)	0.30 (0.03)	0.15 (0.02)	0.08 (0.03)	0.22 (0.01)
(3) Entire hole	0.67 (0.05)	0.60 (0.02)	0.57 (0.04)	0.40 (0.09)	0.61 (0.02)
Panel B: Elasticity of intertemporal substitution	nσ				
(4) Unadjusted	0.02 (0.00)	0.08 (0.01)	0.06 (0.01)	0.11 (0.04)	0.05 (0.01)
(5) Dominated region: notched banks only	0.02 (0.00)	0.11 (0.01)	0.08 (0.02)	0.11 (0.04)	0.07 (0.01)
(6) Dominated region: all banks	0.03 (0.00)	0.17 (0.02)	0.08 (0.02)	0.13 (0.05)	0.09 (0.01)
(7) All mass in the hole is friction	0.16 (0.05)	0.50 (0.07)	0.31 (0.08)	0.30 (8.53)	0.30 (0.03)

TABLE 3 Bounding optimization frictions and the EIS

*Notes:* The table shows how the estimated EIS is affected by assumptions on optimization frictions. The top panel of the table shows the friction adjustment factor *a* estimated in three different cases. Row (1) shows the friction adjustment based on mass in the dominated region using only notched banks, row (2) shows the friction adjustment based on mass the dominated region using all banks (our baseline estimates), while row (3) shows the friction adjustment assuming that all mass in the hole is due to friction. The bottom panel of the table shows the estimated EIS when not adjusting for optimization friction (in row (4)), and when adjusting for friction using each of the three measures provided in the top panel (in rows (5)–(7)). As explained in the main text of the article, the EIS estimates provided in rows (4) or (5) are in general lower bounds, whereas the EIS estimate provided in row (7) is an upper bound. The upper bound is based on the explained by friction rather than by heterogeneity in true preferences (*i.e.* true preferences are assumed to be homogeneous in the population).

#### Source: Best-Cloyne-Ilzetzki-Kleven (2019)