Growth Accounting

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Growth Accounting: Basics

$$Y = F(K, AL)$$

- How much of growth is "due to":
  - Growth in inputs (capital, labor, etc.)
  - Growth in technology (A)
- First step in understanding determinants of growth since it does not attempt to explain growth in inputs
- Exercise that goes back to Abramovitz (1956) and Solow (1957)

Growth Accounting: Basics

- Starting point:

$$Y(t) = F[K(t), A(t)L(t)]$$

- Differentiate with respect to time

$$\dot{Y}(t) = \frac{\partial Y(t)}{\partial K(t)}\dot{K}(t) + \frac{\partial Y(t)}{\partial L(t)}\dot{L}(t) + \frac{\partial Y(t)}{\partial A(t)}\dot{A}(t)$$

where $\frac{\partial Y}{\partial L}$ denotes $\frac{\partial Y}{\partial AL}A$ and $\frac{\partial Y}{\partial A}$ denotes $\frac{\partial Y}{\partial AL}L$

- Divide both sides by Y(t):

$$\frac{\dot{Y}(t)}{Y(t)} = \frac{K(t)}{Y(t)}\frac{\partial Y(t)}{\partial K(t)}\frac{\dot{K}(t)}{K(t)} + \frac{L(t)}{Y(t)}\frac{\partial Y(t)}{\partial L(t)}\frac{\dot{L}(t)}{L(t)} + \frac{A(t)}{Y(t)}\frac{\partial Y(t)}{\partial A(t)}\frac{\dot{A}(t)}{A(t)}$$

Growth Accounting: Basics

$$\frac{\dot{Y}(t)}{Y(t)} = \frac{K(t)}{Y(t)}\frac{\partial Y(t)}{\partial K(t)}\frac{\dot{K}(t)}{K(t)} + \frac{L(t)}{Y(t)}\frac{\partial Y(t)}{\partial L(t)}\frac{\dot{L}(t)}{L(t)} + \frac{A(t)}{Y(t)}\frac{\partial Y(t)}{\partial A(t)}\frac{\dot{A}(t)}{A(t)}$$

- Elasticity of output with respect to capital and labor

$$\alpha_K(t) = \frac{K(t)}{Y(t)}\frac{\partial Y(t)}{\partial K(t)} \qquad \alpha_L(t) = \frac{L(t)}{Y(t)}\frac{\partial Y(t)}{\partial L(t)}$$

- We get:

$$\frac{\dot{Y}(t)}{Y(t)} = \alpha_K(t)\frac{\dot{K}(t)}{K(t)} + \alpha_L(t)\frac{\dot{L}(t)}{L(t)} + R(t)$$

where

$$R(t) = \frac{A(t)}{Y(t)}\frac{\partial Y(t)}{\partial A(t)}\frac{\dot{A}(t)}{A(t)}$$

is referred to as the Solow Residual

Growth Accounting: Basics

$$\frac{\dot{Y}(t)}{Y(t)} = \alpha_K(t)\frac{\dot{K}(t)}{K(t)} + \alpha_L(t)\frac{\dot{L}(t)}{L(t)} + R(t)$$

- In principle measurable:
  - Growth in output: $\dot{Y}(t)/Y(t)$
  - Growth in capital: $\dot{K}(t)/K(t)$
  - Growth in labor: $\dot{L}(t)/L(t)$
  - Elasticity of output with respect to capital: $\alpha_K(t)$
  - Elasticity of output with respect to labor: $\alpha_L(t)$
- Yields $R(t)$ as a residual (hence "Solow residual" name)
  - One perspective: Measure of our ignorance

Measurement: Capital

- Ideally we could measure flow of services from capital
- In practice: Measure stock and assume flow is proportional to stock
- Perpetual inventory method:

$$K(t+1) = K(t) + I(t) - \delta K(t)$$

  - Start with some $K(0)$
  - Measure $I(t)$ from National Income and Product Accounts
  - Use estimates of δ

Measurement: Quality of Inputs

- Simple measure of labor input: hours worked
- But workers differ, e.g., in education and health
- Increase in output may be due to increases in labor quality
- Jorgenson and Griliches (1967):
    - Disaggregate inputs by schooling, etc.
    - Weight each category by average wage
- Growth in overall labor input weighted average of categories
- Can also be done for capital

Measurement: Output Elasticities

$$\alpha_K(t) = \frac{K(t)}{Y(t)} \frac{\partial Y(t)}{\partial K(t)} \qquad \alpha_L(t) = \frac{L(t)}{Y(t)} \frac{\partial Y(t)}{\partial L(t)}$$

- If labor and capital earn their marginal product:

$$r(t) = \frac{\partial Y(t)}{\partial K(t)} \qquad w(t) = \frac{\partial Y(t)}{\partial L(t)}$$

- In this case output elasticities become factor shares:

$$\alpha_K(t) = \frac{r(t)K(t)}{Y(t)} = s_K(t) \qquad \alpha_L(t) = \frac{w(t)L(t)}{Y(t)} = s_L(t)$$

- Data on factor shares usually used to estimate $\alpha_K(t)$ and $\alpha_L(t)$.
- But this is only valid under idealized assumptions

  (i.e., perfect competition)

Why Not Estimate?

$$\frac{\dot{Y}(t)}{Y(t)} = \alpha_K(t)\frac{\dot{K}(t)}{K(t)} + \alpha_L(t)\frac{\dot{L}(t)}{L(t)} + R(t)$$

- Alternative approach: Estimate this equation using data on $\dot{Y}(t)/Y(t)$, $\dot{K}(t)/K(t)$, $\dot{L}(t)/L(t)$
  - Recover α_K and α_L as parameters
  - Recover $R(t)$ as a residual
- Why not do this instead?

Why Not Estimate?

$$\frac{\dot{Y}(t)}{Y(t)} = \alpha_K(t)\frac{\dot{K}(t)}{K(t)} + \alpha_L(t)\frac{\dot{L}(t)}{L(t)} + R(t)$$

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  - Recover α_K and α_L as parameters
  - Recover $R(t)$ as a residual
- Why not do this instead?
- Would be hard since productivity affects inputs (i.e., labor and capital are endogenous)

Table 10.1
Growth Accounting for a Sample of Countries

Country	(1) Growth Rate of GDP	(2) Contribution from Capital	(3) Contribution from Labor	(4) TFP Growth Rate
Panel A: OECD Countries, 1947–73				
Canada	0.0517	0.0254	0.0088	0.0175
($\alpha = 0.44$)		(49%)	(17%)	(34%)
France[a]	0.0542	0.0225	0.0021	0.0296
($\alpha = 0.40$)		(42%)	(4%)	(54%)
Germany[b]	0.0661	0.0269	0.0018	0.0374
($\alpha = 0.39$)		(41%)	(3%)	(56%)
Italy[b]	0.0527	0.0180	0.0011	0.0337
($\alpha = 0.39$)		(34%)	(2%)	(64%)
Japan[b]	0.0951	0.0328	0.0221	0.0402
($\alpha = 0.39$)		(35%)	(23%)	(42%)
Netherlands[c]	0.0536	0.0247	0.0042	0.0248
($\alpha = 0.45$)		(46%)	(8%)	(46%)
U.K.[d]	0.0373	0.0176	0.0003	0.0193
($\alpha = 0.38$)		(47%)	(1%)	(52%)
U.S.	0.0402	0.0171	0.0095	0.0135
($\alpha = 0.40$)		(43%)	(24%)	(34%)
Panel B: OECD Countries, 1960–95				
Canada	0.0369	0.0186	0.0123	0.0057
($\alpha = 0.42$)		(51%)	(33%)	(16%)
France	0.0358	0.0180	0.0033	0.0130
($\alpha = 0.41$)		(53%)	(10%)	(38%)
Germany	0.0312	0.0177	0.0014	0.0132
($\alpha = 0.39$)		(56%)	(4%)	(42%)
Italy	0.0357	0.0182	0.0035	0.0153
($\alpha = 0.34$)		(51%)	(9%)	(42%)
Japan	0.0566	0.0178	0.0125	0.0265
($\alpha = 0.43$)		(31%)	(22%)	(47%)
U.K.	0.0221	0.0124	0.0017	0.0080
($\alpha = 0.37$)		(56%)	(8%)	(36%)
U.S.	0.0318	0.0117	0.0127	0.0076
($\alpha = 0.39$)		(37%)	(40%)	(24%)

Table continued

Source: Barro and Sala-I-Martin (2004) who report results from Christensen, Cummings, Jorgenson (1980) in panel A and Jorgenson and Yip (2001) in panel B. Not per capita.

Table 10.1
(Continued)

Country	(1) Growth Rate of GDP	(2) Contribution from Capital	(3) Contribution from Labor	(4) TFP Growth Rate
Panel C: Latin American Countries, 1940–90				
Argentina	0.0279	0.0128	0.0097	0.0054
($\alpha = 0.54$)		(46%)	(35%)	(19%)
Brazil	0.0558	0.0294	0.0150	0.0114
($\alpha = 0.45$)		(53%)	(27%)	(20%)
Chile	0.0362	0.0120	0.0103	0.0138
($\alpha = 0.52$)		(33%)	(28%)	(38%)
Colombia	0.0454	0.0219	0.0152	0.0084
($\alpha = 0.63$)		(48%)	(33%)	(19%)
Mexico	0.0522	0.0259	0.0150	0.0113
($\alpha = 0.69$)		(50%)	(29%)	(22%)
Peru	0.0323	0.0252	0.0134	−0.0062
($\alpha = 0.66$)		(78%)	(41%)	(−19%)
Venezuela	0.0443	0.0254	0.0179	0.0011
($\alpha = 0.55$)		(57%)	(40%)	(2%)
Panel D: East Asian Countries, 1966–90				
Hong Kong[e]	0.073	0.030	0.020	0.023
($\alpha = 0.37$)		(41%)	(28%)	(32%)
Singapore	0.087	0.056	0.029	0.002
($\alpha = 0.49$)		(65%)	(33%)	(2%)
South Korea	0.103	0.041	0.045	0.017
($\alpha = 0.30$)		(40%)	(44%)	(16%)
Taiwan	0.094	0.032	0.036	0.026
($\alpha = 0.26$)		(34%)	(39%)	(28%)

Source: Barro and Sala-I-Martin (2004) who report results from Elias (1990) in panel C and Young (1995) in panel D.

East Asian Growth Miracle

- Young's (1995) results were surprising to many
- Could it really be that high growth rates were not associated with large changes in TFP?
- Some considered them less miraculous due to this. (But why?)
- Hsieh (2002) took a different approach

Dual Growth Accounting: Hsieh (2002)

- We start with the accounting identity:

$$Y = rK + wL$$

- Take logarithms and differentiate with respect to time:

$$\frac{\dot{Y}}{Y} = s_K \left(\frac{\dot{r}}{r} + \frac{\dot{K}}{K} \right) + s_L \left(\frac{\dot{w}}{w} + \frac{\dot{L}}{L} \right)$$

- Rearrange

$$\frac{\dot{Y}}{Y} - s_K \frac{\dot{K}}{K} - s_L \frac{\dot{L}}{L} = s_K \frac{\dot{r}}{r} + s_L \frac{\dot{w}}{w}$$

  - LHS: “primal” measure of Solow residual (what we had before)
  - RHS: “dual” measure of Solow residual

Dual Growth Accounting: Hsieh (2002)

$$\frac{\dot{Y}}{Y} - s_K \frac{\dot{K}}{K} - s_L \frac{\dot{L}}{L} = s_K \frac{\dot{r}}{r} + s_L \frac{\dot{w}}{w}$$

- Primal and dual approach should yield the same answer
- If one is (in)valid, the other is (in)valid
- Hsieh (2002) applied dual approach to East Asian "Tigers"

East Asian Growth: Primal vs. Dual

Table 10.2
Primal and Dual Estimates of TFP Growth Rates

Country	Primal Estimate	Dual Estimate
Hong Kong, 1966–91	0.023	0.027
Singapore, 1972–90	−0.007	0.022
South Korea, 1966–90	0.017	0.015
Taiwan, 1966–90	0.021	0.037

Notes: These estimates are from Hsieh (2002, table 1). The primal estimates are computed from data on growth rates of quantities of factor inputs, using factor income shares as weights. The dual estimates are computed from data on growth rates of prices of factor inputs, using the same factor income shares as weights. The lack of coincidence for the primal and dual estimates of TFP growth rates reflects the use of different data, as described in the text.

Source: Barro and Sala-I-Martin (2004)

East Asian Growth: Primal vs. Dual

Hsieh (2002) argues:

- NIPA data implies that capital/output ratio rose sharply
- Since factor shares are roughly constant, this implies that rate of return on capital should have fallen sharply
- True for Korea but not for Singapore
- Singapore's NIPA overstate investment

Return on Capital: Korea

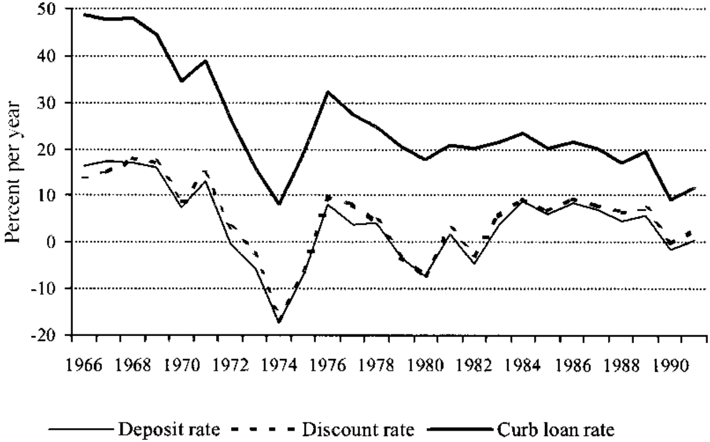


Figure 1. Return to Capital in Korea

Source: Hsieh (2002)

Return on Capital: Singapore

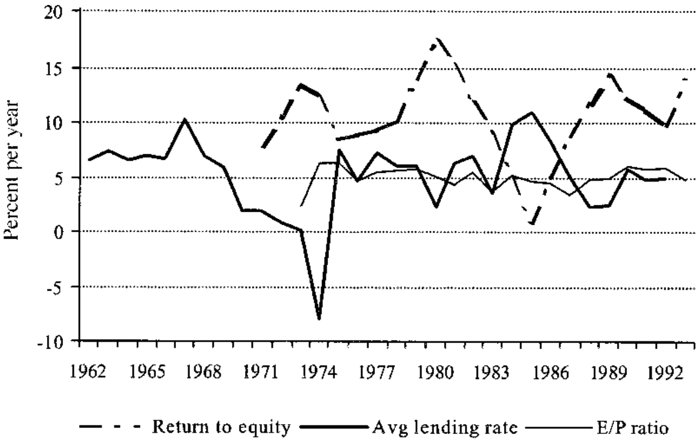


Figure 2. Return to Capital in Singapore

Source: Hsieh (2002)

Who is Right? Young or Hsieh?

- Subsequent research has tended to favor Young
- Fernald and Nieman (2011):
  - Economy with two sectors: favored and unfavored
  - Distortions mean both primal and dual measures of TFP differ form true productivity growth
  - Bottom-up measurement for Singapore indicates low growth in aggregate technology
  - Hsieh's user cost estimates are from unfavored sector
  - Falling pure profits also missed by Hsieh's approach

Growth Accounting and Sources of Growth

- Growth accounting is just accounting, not causal analysis
- Example:

$$Y = AK^{\alpha}(Le^{xt})^{1-\alpha}$$

  - Suppose A and L are constant
  - x is labor-augmented growth in technology
- Take logarithms and differentiate with respect to time:

$$\frac{\dot{Y}}{Y} = \alpha \frac{\dot{K}}{K} + (1-\alpha)x$$

Growth Accounting and Sources of Growth

$$\frac{\dot{Y}}{Y} = \alpha\frac{\dot{K}}{K} + (1-\alpha)x$$

- In Solow and Ramsey models: capital-output ratio will be constant along a balanced growth path

$$\frac{\dot{Y}}{Y} = \frac{\dot{K}}{K} = x$$

- αx of growth attributed to growth of capital
- TFP growth measured to be $\hat{g} = (1-\alpha)x$
- But growth in capital stock is consequence of growth in technology
- To attribute to technology both direct and indirect effects on GDP we need to divide measured TFP growth by $(1-\alpha)$

Table 10.3
TFP Growth Adjusted for Endogenous Responses of Capital

Country	(1) GDP Growth Rate	(2) TFP Growth Rate	(3) TFP Growth Adjusted for Physical Capital	(4) TFP Growth Adjusted for Broad Capital
Hong Kong	0.073	0.027 (37%)	0.043 (59%)	0.090 (123%)
Singapore	0.087	0.022 (25%)	0.043 (49%)	0.073 (84%)
South Korea	0.103	0.015 (14%)	0.021 (20%)	0.050 (49%)
Taiwan	0.094	0.037 (39%)	0.050 (53%)	0.123 (131%)

Notes: Column 1 shows the growth rate of GDP as given in table 10.1, panel D. Column 2 shows the TFP growth rate indicated for the dual column in table 10.2. Column 3 adjusts for responses of physical capital by multiplying the TFP growth rate by $1/(1-\alpha)$, where α is the capital share shown in table 10.1, panel D. Column 4 adjusts for responses of physical and human capital by multiplying the TFP growth rate by $1/0.3$, that is, by assuming a broad capital share of $\alpha = 0.7$. The numbers in parentheses show the percentages of the growth rate of GDP accounted for by each measure of TFP growth.

Source: Barro and Sala-I-Martin (2004)

A small positive number for $\hat{g}$ is, in principle, consistent with a situation in which technological progress is ultimately responsible for a small part of GDP growth, but it is also consistent with a situation in which it is ultimately responsible for all of GDP growth.

Barro and Sala-I-Martin (2004, p. 460)

Alternative Growth Accounting Approach

- Start with a Cobb-Douglas production function:

$$Y_t = A_t K_t^{\alpha} H_t^{1-\alpha}$$

  - Here H_t denotes human capital

- Divide both sides by Y_t^{α} and raise to power $1/(1-\alpha)$:

$$Y_t = \left(\frac{K_t}{Y_t}\right)^{\frac{\alpha}{1-\alpha}} H_t Z_t$$

where $Z_t = A_t^{\frac{1}{1-\alpha}}$

- Divide through by L_t:

$$\frac{Y_t}{L_t} = \left(\frac{K_t}{Y_t}\right)^{\frac{\alpha}{1-\alpha}} \frac{H_t}{L_t} Z_t$$

Alternative Growth Accounting Approach

$$\frac{Y_t}{L_t} = \left(\frac{K_t}{Y_t}\right)^{\frac{\alpha}{1-\alpha}} \frac{H_t}{L_t} Z_t$$

- Decomposes per capita (or per hour) growth into:
  - Capital deepening: K_t/Y_t
  - Growth in human capital per hour: H_t/L_t
  - Total factor productivity: Z_t
- Importantly Solow and Ramsey model imply that K_t/Y_t is constant along a balanced growth path
- Take logarithms and differentiate with respect to time to get a growth accounting equation
- This approach popularized by Klenow and Rodriguez-Clare (1997) (Goes back at least to David (1977))

Table 3 Growth accounting for the United States

Period	Output per hour	Contributions from *K/Y*	Labor composition	Labor-Aug. TFP
1948–2013	**2.5**	**0.1**	**0.3**	**2.0**
1948–1973	3.3	−0.2	0.3	3.2
1973–1990	1.6	0.5	0.3	0.8
1990–1995	1.6	0.2	0.7	0.7
1995–2000	3.0	0.3	0.3	2.3
2000–2007	2.7	0.2	0.3	2.2
2007–2013	1.7	0.1	0.5	1.1

Note: Average annual growth rates (in percent) for output per hour and its components for the private business sector, following Eq. (3).
Source: Authors calculations using Bureau of Labor Statistics, *Multifactor Productivity Trends*, August 21, 2014.

Source: Jones (2016)

Philippon (2023): Additive Growth?

- Very new paper! No peer review, back-and-forth, etc.
- Central claim:
  - We usually assume growth is exponential:

$$A_{t+\tau} = A_t(1+g)^\tau$$

  - In fact, growth is linear:

$$A_{t+\tau} = A_t + b\tau$$

- Data:
  - U.S. TFP 1947-2019 from Fernald (2012)
  - TFP in 23 countries 1890-2019 from Bergeaud, Cette, Lecat (2016)

Exponential versus Linear

- Exponential growth:

$$A_{t+\tau} = A_t(1+g)^\tau \quad => \quad \log A_{t+\tau} = \log A_t + \tau \log(1+g)$$
$$=> \quad \log A_{t+\tau} \approx \log A_t + g\tau$$

- Linear growth

$$A_{t+\tau} = A_t + b\tau$$

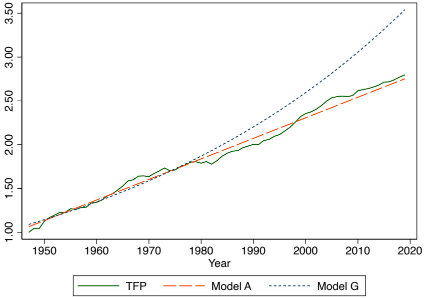
Philippon (2023): Additive Growth?

- Consider the U.S. post-WWII sample
- Use the first half of sample to predict $A(t)$ for the second half
- Compare prediction of:
  - Person who believes in exponential growth (Model G)
  - Person who believes in linear growth (Model A)

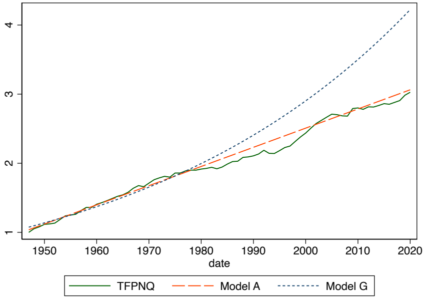
Linear Growth Fits Better Out of Sample

Figure 1: Out-of-Sample TFP Forecasts

(a) BCL Data



(b) Fernald Data



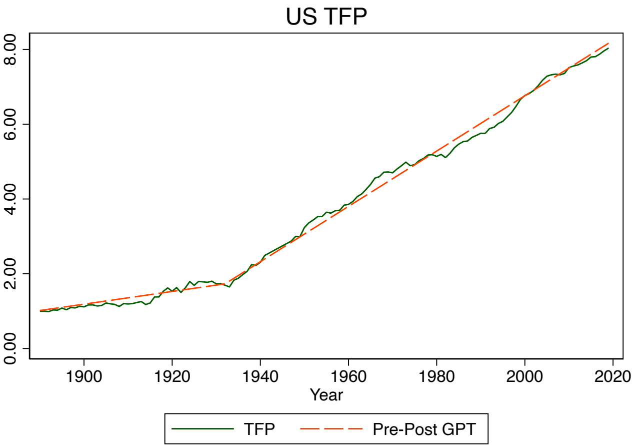
Source: Philippon (2023)

Linear Growth in the Long-Run

- Growth cannot have been linear forever
- If so, $A(t)$ would be negative at some point in the past
- Philippon proposes that "General Purpose Technologies" cause breaks
  - Enlightenment / Glorious Revolution?
  - Steam Engine / Industrial Revolution
  - Electrification / Second Industrial Revolution

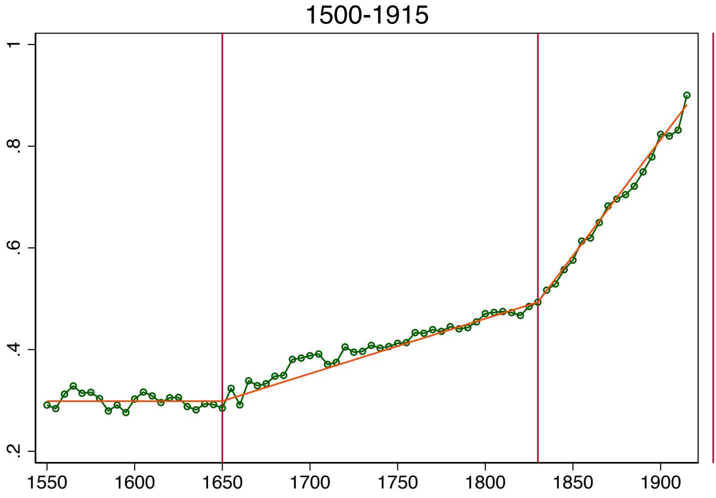
Occasional Breaks over Longer Sample

Figure 10: Linear US TFP with One Break



Source: Philippon (2023)

Occasional Breaks over Longer Sample



Source: Philippon (2023). U.K. Pseudo-TFP (GDP per capita to the power 2/3).