Ideas and Economic Growth

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Fall 2023
Big Picture Questions about Growth

- What sustains growth at the frontier? (Will it continue in the future?)
- Why are some countries so far behind the frontier? (What might help them close the gap?)

This lecture focuses on the first of these questions.
Knowledge versus Capital

- Solow model: Capital accumulation not a source of long-run growth
  - Reason: Diminishing returns

- What about knowledge?

- If knowledge succeeds where capital fails, there must be something fundamentally different about knowledge than capital
To drive home the importance of diminishing returns, let’s consider a model without diminishing returns.

Suppose

\[ Y(t) = AK(t) \]

and

\[ \dot{K}(t) = sY(t) - \delta K(t) \]

where

- \( s \) is the exogenous savings rate (as in Solow model)
- Labor is assumed constant and normalized to one (which implies that \( Y(t) \) is output per person)
Combining these two equations yields

\[ \dot{Y}(t) = sAY(t) - \delta Y(t) \]

\[ g_Y = \frac{\dot{Y}_t}{Y_t} = sA - \delta \]

We get long-run growth from capital accumulation.

The long-run growth rate of output (per person) is governed by \( s, A, \) and \( \delta \).

Long-run growth is endogenous to the extent that \( s, A, \) and \( \delta \) can be influenced by policy / behavior.
But why might we think $Y = AK$ makes sense?

One “micro-foundation” is learning-by-doing externalities
- Productivity gains coming from investment and production
- Empirical evidence from airframe manufacturing, shipbuilding, etc.
  (Wright 36, Searle 46, Asher 56, Rapping 65)

Several early endogenous growth models followed this path
(e.g., Frankel 62, Griliches 79, Romer 86, Lucas 88)

We consider Romer (1986) version here
(see Romer 19, p. 119-121; Barro and Sala-i-Martin 04, sec. 4.3;
Acemoglu 09, sec. 11.4)
Suppose there is a continuum of firms with production function

\[ Y_i(t) = F(K_i(t), A_i(t)L_i(t)) \]

Two assumptions:
- Strong learning-by-doing (investing):
  Knowledge grows proportionally with firm’s capital stock
- Knowledge spillovers are perfect across firms
  (all firms benefit from each firm’s learning)

These assumptions imply:

\[ A_i(t) = BK(t) \]
Combining prior two equations:

\[ Y_i(t) = F(K_i(t), BK(t)L_i(t)) \]

Suppose further that all firms are identical:

\[ Y(t) = F(K(t), BK(t)L(t)) \]

If \( F \) is homogeneous of degree one, we have

\[ Y(t) = F(1, BL(t))K(t) \]

This model therefore yields a production function of the \( Y = AK \) form
Romer (1986) model yields endogenous growth

But arguably makes unrealistic assumptions:
  - Assumes very large amounts of learning-by-doing
  - Doesn’t work if knowledge grows less than proportionally with $K$

  - Arguably also makes unrealistic assumptions
    (see Jones 21, section 2.2)

Doesn’t seem to capture what is “special” about knowledge
Why Is Knowledge Special?

- Knowledge is non-rival
- This is the fundamental difference versus capital
- Implies that knowledge can be a source of long-run growth
Ideas vs. Objects

- **Ideas:** a design, a blueprint, or a set of instructions
  - How to make fire using sticks, calculus, the design of the incandescent light bulb, oral rehydration therapy, Beethoven’s 3th symphony, etc.

- **Objects:** Goods, capital, labor, land, highways, barrels of oil, etc.
  - A particular incandescent light bulb, a particular oral rehydration pill, etc.
Objects are rival:
  - If I use a particular lawn mower, you can’t use that same lawn mower at the same time

Ideas are non-rival:
  - My use of calculus, does not negatively affect your ability to use calculus at the same time
  - Once invented, calculus can be used by any number of people simultaneously (ideas are “infinitely usable”)
Consider production function

\[ Y = F(A, X) \]

- \( A \) is index of the stock of knowledge
- \( X \) is all rival inputs (vector)

Replication implies constant returns to objects:

\[ \lambda Y = F(A, \lambda X) \]

- This argument implicitly uses non-rivalry of ideas
- We can use same \( A \) to build second factory as first factory.

Implies that if we increase \( A \) as well we get increasing returns:

\[ F(\lambda A, \lambda X) > F(A, \lambda X) = \lambda Y \]
Since ideas are non-rival, per capita output depends on the overall stock of knowledge, NOT knowledge per capita

\[ Y(t) = A(t)^\sigma K(t)^\alpha L(t)^{1-\alpha} \]

\[ y(t) = A(t)^\sigma k(t)^\alpha \]

Output per person depends on:
- Total stock of knowledge \((A(t)^\sigma)\)
- Capital per capita \((k(t)^\alpha)\)

Solow model: Capital per capita can’t grow forever (if \(A\) is constant)

If stock of knowledge can grow forever, \(y(t)\) can growth forever
Romer (1990) is the paper that crystallized these ideas.

See Jones (2019) for role of this paper in relation to earlier and subsequent literature.

But Romer (1990) made some extreme assumptions that we will want to move away from.
Knowledge Production

- Key new feature: Knowledge is produced

- Workers do one of two things:
  - Produce goods and services
  - Produce knowledge (R&D)

- Key choice: How are workers allocated between these activities?

- Simplifying assumption: A fraction $s$ of workers work on R&D
  - Similar to Solow assumption about savings rate
  - Workers choose optimally in Romer (1990)
  - We will consider a model where workers choose optimally later on
  - For now:
    \[
    L_A(t) = sL(t) \quad \quad L_Y(t) = (1 - s)L(t)
    \]
Knowledge Production in Romer (1990):

- Knowledge production function in Romer (1990):
  \[
  \dot{A}(t) = \theta L_A(t)A(t)
  \]

- Knowledge production depends on two inputs:
  - Research effort: \( L_A(t) \) denotes labor devoted to research
  - Existing knowledge: \( A(t) \)

- Importantly, exponent on \( A(t) \) is one

- Implies that
  \[
  g_A(t) = \frac{\dot{A}(t)}{A(t)} = \theta L_A(t)
  \]
Suppose for simplicity that $L_A(t) = L_A$ (i.e., a constant).

Then growth rate of knowledge is constant

$$g_A = \frac{\dot{A}(t)}{A(t)} = \theta L_A$$

Suppose for simplicity that goods production function is

$$Y(t) = A(t)^{\sigma} L_Y$$

$$y(t) = A(t)^{\sigma} (1 - s)$$

where $1 - s$ is (constant) share of pop. working on goods production

This implies

$$g_y = \sigma g_A = \sigma \theta L_A$$
But why would knowledge production be linear in $A(t)$?

More generally:

$$\dot{A}(t) = \theta L_A(t) \lambda A(t)^\phi$$

Not necessarily constant returns to objects:

- Twice as much research effort may not generate twice as much knowledge
- There may be congestion / duplication / diminishing returns
- This would yield $\lambda < 1$
- We assume however that $\lambda > 0$
Knowledge Production

\[ \dot{A}(t) = \theta L_A(t)^\lambda A(t)^\phi \]

- Not necessarily constant returns to existing knowledge \((\phi = 1)\)

- \(\phi > 0\): Standing on the shoulders of giants
  - Having more knowledge lets a researcher create knowledge faster
  - E.g., printed books, internet, computers, microscopes, etc.

- \(\phi < 0\): No more low hanging fruit
  - Suppose you are fishing in a pond with 100 fish
  - As you catch more, harder to catch the rest

- Nothing particularly natural about \(\phi = 1\)
**Simple Endogenous Growth Model**

1. Goods production: \( Y(t) = A(t)^{\sigma} L_Y(t) \)

2. Ideas production: \( \dot{A}(t) = \theta L_A(t)^{\lambda} A(t)^{\phi} \)

3. Allocation: \( L_A(t) = sL(t) \)

4. Resource constraint: \( L(t) = L_A(t) + L_Y(t) \)

5. Population growth: \( L(t) = L(0)e^{nt} \)
Simple Endogenous Growth Model

Notable features:

- Constant fraction of labor force $s$ conducts research
  - Simple short cut
  - Similar to constant savings rate in Solow model
  - We will endogenize later

- Constant population growth at rate $n$

- $\sigma$ captures degree to which increase in knowledge increases productivity in production of goods and services
Balanced Growth in Simple Model

- Combining (1), (3) and (4) and dividing by L(t) we get:

\[ y(t) = A(t)^\sigma (1 - s) \]

- Taking logs and time derivatives yields

\[ g_y(t) = \sigma g_A(t) \]

- Suppose there is a balanced growth path with constant growth:

\[ g_y(t) = g_y \quad \text{and} \quad g_A(t) = g_A \]

- Then we have

\[ g_y = \sigma g_A \]
Combining (2) and (3) and dividing by \( A(t) \):

\[
g_A(t) = \theta s^\lambda L(t)^\lambda A(t)^{\phi - 1}
\]

Taking logs and time derivatives yields

\[
0 = \lambda g_L + (\phi - 1)g_A
\]

where we use \( g_A(t) = g_A \) on BGP

Rearranging and using \( g_L = n \) we get

\[
g_y = \sigma g_A = \sigma \lambda \frac{n}{1 - \phi}
\]
Output Growth and Population Growth

\[ g_y = \sigma g_A = \frac{\sigma \lambda}{1 - \phi} n \]

- Long-run growth proportional to population growth rate
- If \( L_A(t) \) were constant at \( L_A \) (which implies \( n = 0 \)):
  \[ \frac{\dot{A}(t)}{A(t)} = \theta L_A^\lambda A(t)^{\phi - 1} = \frac{\theta L_A^\lambda}{A(t)^{1-\phi}} \]
- If \( 1 - \phi > 0 \), or equivalently \( \phi < 1 \):
  \[ g_A(t) = \frac{\dot{A}(t)}{A(t)} \to 0 \]
- Growth can’t keep up with the level and thus goes to zero
With $\phi < 1$, research effort must grow exponentially for knowledge to grow exponentially.

Exponential population growth and constant share of labor force working on research ($s$) does the trick.

$$g_y = \sigma g_A = \frac{\sigma \lambda}{1 - \phi} n$$
Three ways to get sustained growth:

1. AK Model: Capital accumulation linear differential eq.
   \[
   \dot{K}(t) = sAK(t) - \delta K(t) \quad \Rightarrow \quad \dot{K}(t) = (sA - \delta)K(t)
   \]

2. Romer (1990) / $\phi = 1$: Knowledge prod. linear differential eq.
   \[
   \dot{A}(t) = \theta L_A(t)A(t)
   \]
   - “Fully-endogenous” growth model
   - Also true of Aghion-Howitt 92, Grossman-Helpman 91

   \[
   \dot{A}(t) = \theta L_A(t)A(t)^\phi \quad \dot{L}(t) = nL(t)
   \]
   - “Semi-endogenous” growth model
Growth of knowledge is generally (even outside BGP):

\[ g_A(t) = \theta s^\lambda L(t)^\lambda A(t)^{\phi-1} \]

Taking logs and differentiating by time yields

\[ \frac{\dot{g}_A(t)}{g_A(t)} = \lambda n - (1 - \phi)g_A(t) \]

Multiplying through by \( g_A(t) \) yields

\[ \dot{g}_A(t) = \lambda ng_A(t) - (1 - \phi)g_A(t)^2 \]
Evolution of Growth in Simple Model

\[ g_A(t) = \theta s^\lambda L(t)^\lambda A(t)^{\phi - 1} \]  
\[ \dot{g}_A(t) = \lambda n g_A(t) - (1 - \phi) g_A(t)^2 \]

- Equation (1) determines initial level of \( g_A(t) \)
  - Depends, e.g., on \( s \) (and therefore innovation policy)

- Equation (2) determines subsequent evolution of \( g_A(t) \)
  - Independent of \( s \)

- With \( \phi < 1 \) a change in \( s \) only has a “level effect”, not a “growth effect”
We therefore focus on the dynamics of $A$, which are given by (3.6). This equation implies that the growth rate of $A$, $g_A(t)$, with respect to time gives us an expression for the growth rate of $g_A$ (that is, for the growth rate of the growth rate of $A$):

$$g_A(t) = \frac{\partial g_A}{\partial t} = \gamma A + (\theta - 1)g_A(t).$$

Multiplying both sides of this expression by $g_A(t)$ yields

$$g_A(t) = \gamma g_A(t) + (\theta - 1)g_A(t).$$

The behavior of output per worker, $y$, must distinguish among the cases $\theta < 1$, $\theta > 1$, and $\theta = 1$. We discuss each in turn.

**Case 1: $\theta < 1$**

Figure 3.1 shows the phase diagram for $g_A$ when $\theta$ is less than 1. That is, it plots $g_A$ as a function of $A$ for positive values of $g_A$. As the diagram shows, equation (3.9) implies $g_A(t)$.

Source: Romer (2019). In Romer's notation $\theta < 1$ is what I have called $\phi < 1$. 

**FIGURE 3.1 The dynamics of the growth rate of knowledge when $\theta < 1$**
FIGURE 3.2 The effects of an increase in $a_L$ when $\theta < 1$

Source: Romer (2019). In Romer’s notation $\theta < 1$ is what I have called $\phi < 1$ and $a_L$ is what I have called $s$. 

Steinsson

Ideas and Growth
3.2 The Model without Capital

In Figure 3.2, the impact of an increase in $a_L$ on the path of $A$ when $\theta < 1$ is shown. The permanent increase in $a_L$ starting from a situation where $A$ is growing at rate $g_A$ is analyzed. $a_L$ does not enter expression (3.9) for $\dot{g}_A$: $\dot{g}_A(t) = y\eta g_A(t) + (1-\theta)[A(t)]^e$. Thus, the rise in $a_L$ does not affect the curve showing $\dot{g}_A$ as a function of $g_A$. However, $a_L$ does enter expression (3.7) for $g_A$: $g_A(t) = B[\theta L(t)]^\gamma A(t)$. The increase in $a_L$ therefore causes an immediate increase in $g_A$ but no change in $\dot{g}_A$ as a function of $g_A$. This is shown by the dotted arrow in Figure 3.2.

As the phase diagram shows, the increase in the growth rate of knowledge is not sustained. When $g_A$ is above $g_A$, $g_A$ is negative. $g_A$ therefore returns gradually to $g_A$ and then remains there. This is shown by the solid arrows in the figure. Intuitively, the fact that $\theta$ is less than 1 means that the contribution of additional knowledge to the production of new knowledge is not strong enough to be self-sustaining.

This analysis implies that, paralleling the impact of a rise in the saving rate on the path of output in the Solow model, the increase in $a_L$ results in a rise in $g_A$ followed by a gradual return to its initial level. That is, it has a level effect but not a growth effect on the path of $A$. This information is summarized in Figure 3.3.

See Problem 3.1 for an analysis of how the change in $a_L$ affects the path of output.

Source: Romer (2019). In Romer's notation $\theta < 1$ is what I have called $\phi < 1$ and $a_L$ is what I have called $s$.
**Effect of s on Growth**

\[ g_A(t) = \frac{\dot{A}(t)}{A(t)} = \theta L_A^\lambda A(t)^{\phi - 1} = \frac{\theta s^\lambda L(t)^\lambda}{A(t)^{1-\phi}} \]

- Models with \( \phi = 1 \): s affects long run growth rate
  \[ g_A(t) = \frac{\dot{A}(t)}{A(t)} = \theta s^\lambda L(t)^\lambda \]

- Models with \( \phi < 1 \): s does not affect long run growth rate
  \[ g_y = \sigma g_A = \frac{\sigma \lambda}{1-\phi} n \]
**Scale Effects**

- Models with \( \phi = 1 \) have “strong” scale effects
  - Growth rate is increasing in **level** of population:
    \[
g_A(t) = \frac{\dot{A}(t)}{A(t)} = \theta sL(t)
    \]

- Models with \( \phi < 1 \) have “weak” scale effects
  - Growth rate is increasing in **growth rate** of population:
    \[
g_y = \sigma g_A = \frac{\sigma \lambda}{1 - \phi} n
    \]

- These are interesting testable implications of these model classes
One reading of scale effects is that large countries or countries with fast population growth should have high TFP growth.

Obviously counterfactual (Luxembourg, Iceland, Singapore).

But ideas flow between countries.

Scale effects likely to operate largely at the world level (although flow of ideas is not perfect or instantaneous).
There is arguably very strong evidence against strong scale effects:

- Frontier growth has been quite stable for a long time
- Research effort has increased very substantially

With strong scale effects, increased research effort should increase TFP growth at frontier
Per Capita GDP in the United States, 1880–1987 (Natural logarithm)

Source. The data are from Maddison [1982, 1989] as compiled by Bernard [1991]. The solid trend line represents the time trend calculated using data only from 1880 to 1929. The dashed line is the trend for the entire sample.

instance, the number of scientists and engineers engaged in R&D in the United States has grown from less than 200,000 to almost one million, a more than five-fold increase. For Japan the growth has been even more striking: from about 120,000 in 1965 to over 400,000 by 1987, an increase of more than 300 percent in just over two decades. If instead the resources devoted to R&D are measured as real R&D expenditure, the figure looks very similar.

Figure V completes the analysis of the R&D equation by plotting total factor productivity growth rates for France, Germany, Japan, and the United States. Negative trends are visible for TFP growth in France and Japan, while no distinct trend is evident for Germany and the United States. 

FIGURE V
Aggregate Total Factor Productivity Growth

Source. OECD Department of Economics and Statistics Analytic Database. Data provided by Steven Englander.

This stylized fact represents an important benchmark that any growth model must match. Whatever the engine driving long run growth, it must (a) be able to produce relatively stable growth rates for a century or more, and (b) must not predict that growth rates in the United States over this period of time should depart from such a pattern. To see this force of this argument, consider first a theory like Lucas (1988) that predicts that investment in human capital is the key to growth. In this model, the growth rate of the economy is proportional to the investment rate in human capital. But if investment rates in human capital have risen significantly in the 20th century in the United States, as data on educational attainment suggests, this is a problem for the theory. It could be rescued if investment rates in human capital in the form of on-the-job training have fallen to offset the rise in formal education, but there is little evidence suggesting that this is the case.

This stylized fact is even more problematic for the first-generation idea-based growth models of Romer (1990), Aghion and Howitt (1992) and Grossman and Helpman (1991) (R/AH/GH). These models predict that growth is an increasing function of research effort, but research effort has apparently grown tremendously over time. As one example of this fact, consider Figure 2. This figure plots an index of the number of scientists and engineers engaged in research in the G-5 countries. Between 1950 and 1993, this index of research effort rose by more than a factor of eight. In part this is because of the general growth in employment in these countries, but as the figure shows, it also reflects a large increase in the fraction of employment devoted to research. A similar fact can be seen in Figure 2.

Figure 2. Researchers and employment in the G-5 countries (index). Note. From calculations in Jones (2002b). Data on researchers before 1950 in countries other than the United States is backcasted using the 1965 research share of employment. The G-5 countries are France, Germany, Japan, the United Kingdom and the United States.

Source: Jones (2005).
Evidence Against Strong Scale Effects

\[ g_A(t) = \frac{\dot{A}(t)}{A(t)} = \theta s_L(t) \]

- Research effort has risen by a factor of 8
- Models with \( \phi = 1 \) imply that growth should have increased by a factor of 8
- Clearly way off!
This evidence suggests that ideas are harder to find.

By ideas, we mean “proportional increases in productivity.”

Research productivity is falling. It takes more research effort to produce the same growth rate.

This means $\phi < 1$ ($\beta > 0$ using Jones (2021) notation).

But by how much?

If $\phi = 0.95$ growth effects of change in $s$ on transition path would last for a long time.
Estimate extent to which ideas are getting harder to find at both macro and micro level.

Ideas production function

\[
\frac{\dot{A}(t)}{A(t)} = \alpha A(t)^{-\beta} S(t)
\]

- \(S(t)\) denotes “scientists” (i.e., research effort)
- Recall that \(\beta = 1 - \phi\)

If \(g_A\) is constant:

\[
\beta = \frac{g_S}{g_A}
\]

Define:

Research Productivity = \(\frac{\dot{A}(t)/A(t)}{S(t)}\)
elsewhere. It is for this reason that the literature, and this paper, turns to the micro side of economic growth.

II. Refining the Conceptual Framework

In this section, we further develop the conceptual framework. First, we explain why the aggregate evidence just presented can be misleading, motivating our focus on microdata. Second, we consider the measurement of research productivity when

Figure 1. Aggregate Data on Growth and Research Effort

Notes: The idea output measure is TFP growth, by decade (and for 2000–2014 for the latest observation). For the years since 1950, this measure is the Bureau of Labor Statistics (2017) Private Business Sector multifactor productivity growth series, adding back in the contributions from R&D and IPP. For the 1930s and 1940s, we use the measure from Gordon (2016). The idea input measure, Effective number of researchers, is gross domestic investment in intellectual property products from the National Income and Product Accounts (Bureau of Economic Analysis 2017), deflated by a measure of the nominal wage for high-skilled workers.

Figure 2. Aggregate Evidence on Research Productivity

Notes: Research productivity is the ratio of idea output, measured as TFP growth, to the effective number of researchers. See Notes to Figure 1 and the online Appendix. Both research productivity and research effort are normalized to the value of 1 in the 1930s.

Source: Bloom, Jones, Van Reenen, Webb (2020).
main firms by the nominal wage of high-skilled workers, as discussed above. Our semiconductor R&D series includes research spending by Intel, Fairchild, National Semiconductor, Motorola, Texas Instruments, Samsung, and more than two dozen other semiconductor firms and equipment manufacturers. More details are provided in the notes to Table 1 and in the online Appendix.

The striking fact, shown in Figure 4, is that research effort has risen by a factor of 18 since 1971. This increase occurs while the growth rate of chip density is more or less stable: the constant exponential growth implied by Moore’s Law has been achieved only by a massive increase in the amount of resources devoted to pushing the frontier forward.

Assuming a constant growth rate for Moore’s Law, the implication is that research productivity has fallen by this same factor of 18, an average rate of 6.8 percent per year. If the null hypothesis of constant research productivity were correct, the growth rate underlying Moore’s Law should have increased by a factor of 18 as well. Instead, it was remarkably stable. Put differently, because of declining research productivity, it is around 18 times harder today to generate the exponential growth behind Moore’s Law than it was in 1971.

The top panel of Table 1 reports the robustness of this result to various assumptions about which R&D expenditures should be counted. No matter how we measure R&D spending, we see a large increase in effective research and a corresponding large decline in research productivity. Even by the most conservative measure in the table, research productivity falls by a factor of 8 between 1971 and 2014.

The bottom panel of Table 1 considers an alternative to Moore’s Law as the “idea output” measure, focusing instead on TFP growth in the “semiconductor and related device manufacturing” industry (NAICS 334413) from the

Figure 3. The Steady Exponential Growth of Moore’s Law

Source: Bloom, Jones, Van Reenen, Webb (2020).
science may lead to a new idea that improves computer chips. Such positive spill-
overs are not a problem for our analysis; instead, they are one possible factor that
our research productivity measure captures. Of course, other things equal, positive
spillovers would show up as an increase in research productivity rather than as the
declines that we document in this paper. Alternatively, if such spillovers were larger
at the start of our time period than at the end, then this would be one possible story
for why research productivity has declined.13

A type of measurement error that could cause our findings to be misleading is if
we systematically understate R&D in early years and this bias gets corrected over
time. In the case of Moore's Law, we are careful to include research spending by
firms that are no longer household names, like Fairchild Camera and Instrument
(later Fairchild Semiconductor) and National Semiconductor so as to minimize this
bias: for example, in 1971, Intel's R&D was just 0.4 percent of our estimate for total
semiconductor R&D in that year. Throughout the paper, we try to be as careful as
we can with measurement issues, but this type of problem must be acknowledged.

IV. Agricultural Crop Yields

Our next application for measuring research productivity is agriculture. Due partly
to the sector's historical importance, crop yields and agricultural R&D spending are
relatively well measured. We begin in Figure 5 by showing research productivity for
the agriculture sector as a whole. As our “idea output” measure, we use

(3)

13 Lucking, Bloom, and Van Reenen (2017) provides an analysis of R&D spillovers using US firm-level data
over the last three decades. They find evidence that knowledge spillovers are substantial, but have been broadly
stable over time.

Figure 4. Data on Moore's Law

Notes: The effective number of researchers is measured by deflating the nominal semiconductor R&D expenditures
of key firms by the average wage of high-skilled workers and is normalized to 1 in 1970. The R&D data include
research by Intel, Fairchild, National Semiconductor, Texas Instruments, Motorola, and more than two dozen other
semiconductor firms and equipment manufacturers; see Table 1 for more details.

Source: Bloom, Jones, Van Reenen, Webb (2020).
quantify the magnitude of the declines in research productivity by reporting the half-life in each case. Taking the aggregate economy number as a representative example, research productivity declines at an average rate of 5.3 percent per year, meaning that it takes around 13 years for research productivity to fall by half. Or put another way, the economy has to double its research efforts every 13 years just to maintain the same overall rate of economic growth.

A natural question is whether these empirical patterns can be reproduced in a general equilibrium model of growth. One class of models that is broadly consistent with this evidence is the semi-endogenous growth approach of Jones (1995), Kortum (1997), and Segerstrom (1998). These models propose that the idea production function takes the form

\[ A_t = \left( \frac{A_t}{\beta} \right) \cdot S_t. \]

Table 7—Summary of the Evidence on Research Productivity

<table>
<thead>
<tr>
<th>Scope</th>
<th>Time period</th>
<th>Average annual growth rate (%)</th>
<th>Half-life (years)</th>
<th>Dynamic diminishing returns, ( \beta )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Aggregate economy</td>
<td>1930–2015</td>
<td>-5.1</td>
<td>14</td>
<td>3.1</td>
</tr>
<tr>
<td>Moore’s Law</td>
<td>1971–2014</td>
<td>-6.8</td>
<td>10</td>
<td>0.2</td>
</tr>
<tr>
<td>Semiconductor TFP growth</td>
<td>1975–2011</td>
<td>-5.6</td>
<td>12</td>
<td>0.4</td>
</tr>
<tr>
<td>Agriculture, US R&amp;D</td>
<td>1970–2007</td>
<td>-3.7</td>
<td>19</td>
<td>2.2</td>
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<tr>
<td>Agriculture, global R&amp;D</td>
<td>1980–2010</td>
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<td>13</td>
<td>3.3</td>
</tr>
<tr>
<td>Corn, version 1</td>
<td>1969–2009</td>
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<td>7.2</td>
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<tr>
<td>Corn, version 2</td>
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<td>9</td>
<td>6.3</td>
</tr>
<tr>
<td>Soybeans, version 2</td>
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<td>-4.4</td>
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</table>

Source: Bloom, Jones, Van Reenen, Webb (2020).
Semi-endogenous growth model imply that long-run growth is governed by population growth.

Many other factors have “level effects” (e.g., increases in education, R&D share, misallocation).

But level effects can be large.

How much of recent growth is due to such level effects?

What does this suggest about the future of growth?
Goods production:

\[ Y_t = K_t^\alpha (Z_t h_t L Y_t)^{1-\alpha} \]

- \( h_t \) is human capital per person

Productivity:

\[ Z_t = A_t M_t \]

- \( A_t \) is knowledge
- \( M_t \) is misallocation

Some manipulation:

\[ y_t = \left( \frac{K_t}{Y_t} \right)^{\alpha/(1-\alpha)} A_t M_t h_t l_t (1 - s_t) \]
Growth Accounting

- Ideas Production function:
  \[ \dot{A}(t) = \theta L_A(t) \lambda A(t) \phi \]
  \[ \frac{\dot{A}(t)}{A(t)} = \theta s(t) L(t) \lambda A(t) \phi^{-1} \]

- With constant growth of \(A(t)\):
  \[ 0 = \lambda g_s + \lambda g_L - (1 - \phi) g_A \]
  \[ g_A = \frac{\lambda}{1 - \phi} (g_s + g_L) \]

- Jones (2021) assumes \(\lambda / (1 - \phi) = \lambda / \beta = \gamma = 1/3\)
  (Results that follow are sensitive to this!)
Growth Accounting

\[
\begin{align*}
\frac{d \log y_t}{d \log K_t} &= \frac{\alpha}{1 - \alpha} \frac{d \log K_t}{Y_t} + \frac{d \log h_t}{d \log h_t} + \frac{d \log \ell_t}{d \log \ell_t} + \frac{d \log (1 - s_t)}{d \log (1 - s_t)} \\
&+ \frac{d \log M_t + d \log A_t}{d \log M_t + d \log A_t} \\
&= \text{Capital-Output ratio} + \text{Educational att.} + \text{Emp-Pop ratio} + \text{Goods intensity} + \text{TFP growth}
\end{align*}
\]

where

\[
\text{TFP growth} \equiv \frac{d \log M_t + d \log A_t}{d \log M_t + d \log A_t} = \frac{d \log M_t}{d \log M_t} + \frac{\gamma d \log s_t}{d \log s_t} + \frac{\gamma d \log L_t}{d \log L_t}
\]

Source: Jones (2021).
Figure 2: Historical Growth Accounting

Components of 2% Growth in GDP per Person

- Human capital per person: 0.5pp
- Employment-Pop Ratio: 0.2pp
- TFP: 1.3pp
- K/Y: 0pp

Components of 1.3% TFP Growth

- Research intensity: 0.7pp
- Population growth: 0.3pp
- Misallocation: 0.3pp

Note: The figure shows a growth accounting exercise for the United States since the 1950s using equations (15) and (16). See the main text for details.

Source: Jones (2021).
In the long run:
- All terms are zero except population growth
- 100% of growth due to population growth

Historically:
- 80% of growth due to other factors
- Only 20% of growth due to population growth
(Sensitive to assumption on $\gamma$.)
**Will Growth Slow?**

- Many sources of growth are temporary:
  - Increased education
  - Higher Emp-Pop ratio
  - Falling misallocation
  - Rising research intensity

- But some of these might continue for a very long time (e.g., increased research intensity)

- Population growth is slowing
  (Population likely to start shrinking soon!)
Figure 4: Population Growth around the World

Note: Average annual rates of population growth for countries classified according to their 2018 World Bank income grouping. Each data point corresponds to a five-year period. Source: United Nations (2019).

Source: Jones (2021).
Figure 4: Population Growth around the World

Note: Average annual rates of population growth for countries classified according to their 2018 World Bank income grouping. Each data point corresponds to a five-year period. Source: United Nations (2019).

Figure 5: The Total Fertility Rate around the World

Note: The total fertility rate is the average number of live births a hypothetical cohort of women would have over their reproductive life if they were subject during their whole lives to the fertility rates of a given period and if they were not subject to mortality. Each data point corresponds to the five-year period 2015–2020. Source: United Nations (2019).

Source: Jones (2021).
MIGHT GROWTH SPEED UP?

- Finding Einsteins
  - Traditionally most people not able to reach their potential as producers of ideas/knowledge
  - Extreme poverty, cast/class restrictions, discrimination
  - How many Einsteins and Doudnas have we missed

- Automation and Artificial Intelligence
  - Interesting discussion in Jones (2021, sec. 6)
  - Automation of ideas production could even imply a “singularity” (explosive growth driven by AGI)