Ideas and Economic Growth

Jón Steinsson

University of California, Berkeley

Fall 2024

Big Picture Questions about Growth

- What sustains growth at the frontier?
  (Will it continue in the future?)
- Why are some countries so far behind the frontier?
  (What might help them close the gap?)

This lecture focuses on the first of these questions

Knowledge versus Capital

- Solow model: Capital accumulation not a source of long-run growth
    - Reason: Diminishing returns
- What about knowledge?
- If knowledge succeeds where capital fails, there must be something fundamentally different about knowledge than capital

The AK Model

- To drive home the importance of diminishing returns, let's consider a model without diminishing returns
- Suppose

$$Y(t) = AK(t)$$

and

$$\dot{K}(t) = sY(t) - \delta K(t)$$

where

  - s is the exogenous savings rate (as in Solow model)
  - Labor is assumed constant and normalized to one (which implies that $Y(t)$ is output per person)

The AK Model

- Combining these two equations yields

$$\dot{Y}(t) = sAY(t) - \delta Y(t)$$

$$g_Y = \frac{\dot{Y}_t}{Y_t} = sA - \delta$$

- We get long-run growth from capital accumulation
- The long-run growth rate of output (per person) is governed by s, A, and δ
- Long-run growth is endogenous to the extent that s, A, and δ can be influenced by policy / behavior

Growth from Externalities

- But why might we think $Y = AK$ makes sense?
- One "micro-foundation" is learning-by-doing externalities
  - Productivity gains coming from investment and production
  - Empirical evidence from airframe manufacturing, shipbuilding, etc. (Wright 36, Searle 46, Asher 56, Rapping 65)
- Several early endogenous growth models followed this path (e.g., Frankel 62, Griliches 79, Romer 86, Lucas 88)
- We consider Romer (1986) version here (see Romer 19, p. 119-121; Barro and Sala-i-Martin 04, sec. 4.3; Acemoglu 09, sec. 11.4)

Romer (1986): Knowledge Spillovers

- Suppose there is a continuum of firms with production function

$$Y_i(t) = F(K_i(t), A_i(t)L_i(t))$$

- Two assumptions:
  - Strong learning-by-doing (investing):
    Knowledge grows proportionally with firm's capital stock
  - Knowledge spillovers are perfect across firms
    (all firms benefit from each firm's learning)
- These assumptions imply:

$$A_i(t) = BK(t)$$

Romer (1986): Knowledge Spillovers

- Combining prior two equations:

$$Y_i(t) = F(K_i(t), BK(t)L_i(t))$$

- Suppose further that all firms are identical:

$$Y(t) = F(K(t), BK(t)L(t))$$

- If F is homogeneous of degree one, we have

$$Y(t) = F(1, BL(t))K(t)$$

- This model therefore yields a production function of the $Y = AK$ form

Growth and Knowledge Spillovers

- Romer (1986) model yields endogenous growth
- But arguably makes unrealistic assumptions:
  - Assumes very large amounts of learning-by-doing
  - Doesn't work if knowledge grows less than proportionally with K
- Lucas (1988) builds similar model with human capital externalities.
  - Arguably also makes unrealistic assumptions (see Jones 21, section 2.2)
- Doesn't seem to capture what is "special" about knowledge

Why Is Knowledge Special?

- Knowledge is non-rival
- This is the fundamental difference versus capital
- Implies that knowledge can be a source of long-run growth

Ideas vs. Objects

- Ideas: a design, a blueprint, or a set of instructions
  - How to make fire using sticks, calculus, the design of the incandescent light bulb, oral rehydration therapy, Beethoven's 3th symphony, etc.
- Objects: Goods, capital, labor, land, highways, barrels of oil, etc.
  - A particular incandescent light bulb, a particular oral rehydration pill, etc.

Ideas vs. Objects

- Objects are rival:
  - If I use a particular lawn mower, you can't use that same lawn mower at the same time
- Ideas are non-rival:
  - My use of calculus, does not negatively affect your ability to use calculus at the same time
  - Once invented, calculus can be used by any number of people simultaneously (ideas are "infinitely usable")

Non-Rivalry and Returns to Scale

- Consider production function

$$Y = F(A, X)$$

  - A is index of the stock of knowledge
  - X is all rival inputs (vector)

- Replication implies constant returns to objects:

$$\lambda Y = F(A, \lambda X)$$

  - This argument implicitly uses non-rivalry of ideas
  - We can use same A to build second factory as first factory.

- Implies that if we increase A as well we get increasing returns:

$$F(\lambda A, \lambda X) > F(A, \lambda X) = \lambda Y$$

Non-Rivalry and Growth

- Since ideas are non-rival, per capita output depends on the overall stock of knowledge, NOT knowledge **per capita**

$$Y(t) = A(t)^{\sigma} K(t)^{\alpha} L(t)^{1-\alpha}$$

$$y(t) = A(t)^{\sigma} k(t)^{\alpha}$$

- Output per person depends on:
  - Total stock of knowledge ($A(t)^{\sigma}$)
  - Capital **per capita** ($k(t)^{\alpha}$)
- Solow model: Capital **per capita** can't grow forever (if A is constant)
- If stock of knowledge can grow forever, $y(t)$ can growth forever

Romer (1990)

- Romer (1990) is the paper that crystallized these ideas
- See Jones (2019) for role of this paper in relation to earlier and subsequent literature
- But Romer (1990) made some extreme assumptions that we will want to move away from

Knowledge Production

- Key new feature: Knowledge is produced
- Workers do one of two things:
  - Produce goods and services
  - Produce knowledge (R&D)
- Key choice: How are workers allocated between these activities?
- Simplifying assumption: A fraction s of workers work on R&D
  - Similar to Solow assumption about savings rate
  - Workers choose optimally in Romer (1990)
  - We will consider a model where workers choose optimally later on
  - For now:

$$L_A(t) = sL(t) \qquad\qquad L_Y(t) = (1-s)L(t)$$

Knowledge Production in Romer (1990)

- Knowledge production function in Romer (1990):

$$\dot{A}(t) = \theta L_A(t) A(t)$$

- Knowledge production depends on two inputs:
  - Research effort: $L_A(t)$ denotes labor devoted to research
  - Existing knowledge: $A(t)$

- Importantly, exponent on $A(t)$ is one

- Implies that

$$g_A(t) = \frac{\dot{A}(t)}{A(t)} = \theta L_A(t)$$

Knowledge Production in Romer (1990)

- Suppose for simplicity that $L_A(t) = L_A$ (i.e., a constant)
- Then growth rate of knowledge is constant

$$g_A = \frac{\dot{A}(t)}{A(t)} = \theta L_A$$

- Suppose for simplicity that goods production function is

$$Y(t) = A(t)^{\sigma} L_Y \qquad => \qquad y(t) = A(t)^{\sigma}(1 - s)$$

  where $1 - s$ is (constant) share of pop. working on goods production, σ is importance of ideas for production (degree of increasing returns)

- This implies

$$g_y = \sigma g_A = \sigma \theta L_A$$

Knowledge Production

- But why would knowledge production be linear in A(t) and L(t)?
- More generally:

$$\dot{A}(t) = \theta L_A(t)^{\lambda} A(t)^{\phi}$$

- Not necessarily constant returns to objects ($\lambda = 1$):
  - Twice as much research effort may not generate twice as much knowledge
  - There may be congestion / duplication / diminishing returns
  - This would yield $\lambda < 1$
  - We assume however that $\lambda > 0$

Knowledge Production

$$\dot{A}(t) = \theta L_A(t)^{\lambda} A(t)^{\phi}$$

- Not necessarily constant returns to existing knowledge ($\phi = 1$)
- $\phi > 0$: Standing on the shoulders of giants
    - Having more knowledge lets a researcher create knowledge faster
    - E.g., printed books, internet, computers, microscopes, etc.
- $\phi < 0$: No more low hanging fruit
    - Suppose you are fishing in a pond with 100 fish
    - As you catch more, harder to catch the rest
- Nothing particularly natural about $\phi = 1$

Simple Endogenous Growth Model

1. Goods production: $Y(t) = A(t)^{\sigma} L_Y(t)$
2. Ideas production: $\dot{A}(t) = \theta L_A(t)^{\lambda} A(t)^{\phi}$
3. Allocation: $L_A(t) = sL(t)$
4. Resource constraint: $L(t) = L_A(t) + L_Y(t)$
5. Population growth: $L(t) = L(0)e^{nt}$

Simple Endogenous Growth Model

Notable features:

- Constant fraction of labor force s conducts research
  - Simple short cut
  - Similar to constant savings rate in Solow model
  - We will endogenize later
- Constant population growth at rate n
- σ captures degree to which increase in knowledge increases productivity in production of goods and services
  - How much does 1% increase in knowledge increase productivity?
  - But what is a 1% increase in knowledge? How is this measured?

Balanced Growth in Simple Model

- Combining (1), (3) and (4) and dividing by L(t) we get:

$$y(t) = A(t)^{\sigma}(1 - s)$$

- Taking logs and time derivatives yields

$$g_y(t) = \sigma g_A(t)$$

- Suppose there is a balanced growth path with constant growth:

$$g_y(t) = g_y \qquad \text{and} \qquad g_A(t) = g_A$$

- Then we have

$$g_y = \sigma g_A$$

Balanced Growth in Simple Model

- Combining (2) and (3) and dividing by $A(t)$:

$$g_A(t) = \theta s^{\lambda} L(t)^{\lambda} A(t)^{\phi-1}$$

- Taking logs and time derivatives yields

$$0 = \lambda g_L + (\phi - 1) g_A$$

  where we use $g_A(t) = g_A$ on BGP

- Rearranging and using $g_L = n$ we get

$$g_y = \sigma g_A = \frac{\sigma\lambda}{1-\phi} n$$

Output Growth and Population Growth

$$g_y = \sigma g_A = \frac{\sigma\lambda}{1-\phi} n$$

- Long-run growth proportional to population growth rate
- If $L_A(t)$ were constant at L_A (which implies $n = 0$):

$$\frac{\dot{A}(t)}{A(t)} = \theta L_A^{\lambda} A(t)^{\phi-1} = \frac{\theta L_A^{\lambda}}{A(t)^{1-\phi}}$$

- If $1 - \phi > 0$, or equivalently $\phi < 1$:

$$g_A(t) = \frac{\dot{A}(t)}{A(t)} \rightarrow 0$$

- Growth can't keep up with the level and thus goes to zero

Research Effort Must Grow Exponentially

$$g_y = \sigma g_A = \frac{\sigma\lambda}{1-\phi} n$$

- With $\phi < 1$, research effort must grow exponentially for knowledge to grow exponentially
- Exponential population growth and constant share of labor force working on research (s) does the trick

Something Must Have Linear Differential Eq.

Three ways to get sustained growth:

1. AK Model: Capital accumulation linear differential eq.

$$\dot{K}(t) = sAK(t) - \delta K(t) \qquad => \qquad \dot{K}(t) = (sA - \delta)K(t)$$

2. Romer (1990) / $\phi = 1$: Knowledge prod. linear differential eq.

$$\dot{A}(t) = \theta L_A(t)A(t)$$

   - "Fully-endogenous" growth model
   - Also true of Aghion-Howitt 92, Grossman-Helpman 91

3. Jones (1995) / $\phi < 1$: Pop. growth linear differential eq.

$$\dot{A}(t) = \theta L_A(t)A(t)^{\phi} \qquad \dot{L}(t) = nL(t)$$

   - "Semi-endogenous" growth model

Evolution of Growth in Simple Model

- Growth of knowledge is generally (even outside BGP):

$$g_A(t) = \theta s^{\lambda} L(t)^{\lambda} A(t)^{\phi-1}$$

- Taking logs and differentiating by time yields

$$\frac{\dot{g}_A(t)}{g_A(t)} = \lambda n - (1-\phi) g_A(t)$$

- Multiplying through by $g_A(t)$ yields

$$\dot{g}_A(t) = \lambda n g_A(t) - (1-\phi) g_A(t)^2$$

Evolution of Growth in Simple Model

$$g_A(t) = \theta s^{\lambda} L(t)^{\lambda} A(t)^{\phi-1} \tag{1}$$

$$\dot{g}_A(t) = \lambda n g_A(t) - (1-\phi) g_A(t)^2 \tag{2}$$

- Equation (1) determines initial level of $g_A(t)$
  - Depends, e.g., on s (and therefore innovation policy)
- Equation (2) determines subsequent evolution of $g_A(t)$
  - Independent of s
- With $\phi < 1$ a change in s only has a “level effect”, not a “growth effect”

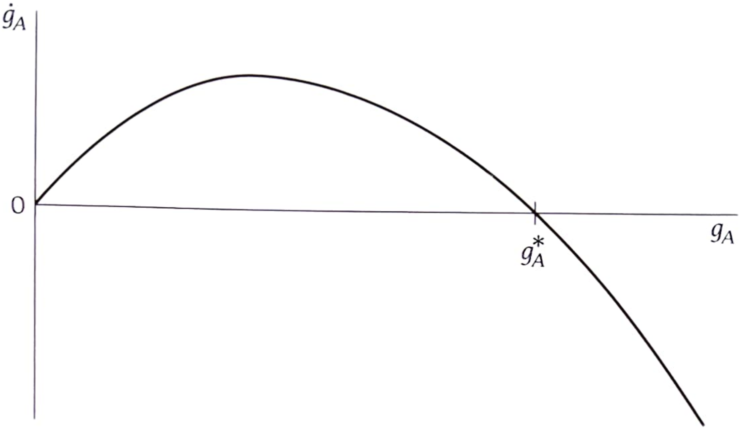


FIGURE 3.1 The dynamics of the growth rate of knowledge when $\theta < 1$

Source: Romer (2019). In Romer's notation $\theta < 1$ is what I have called $\phi < 1$.

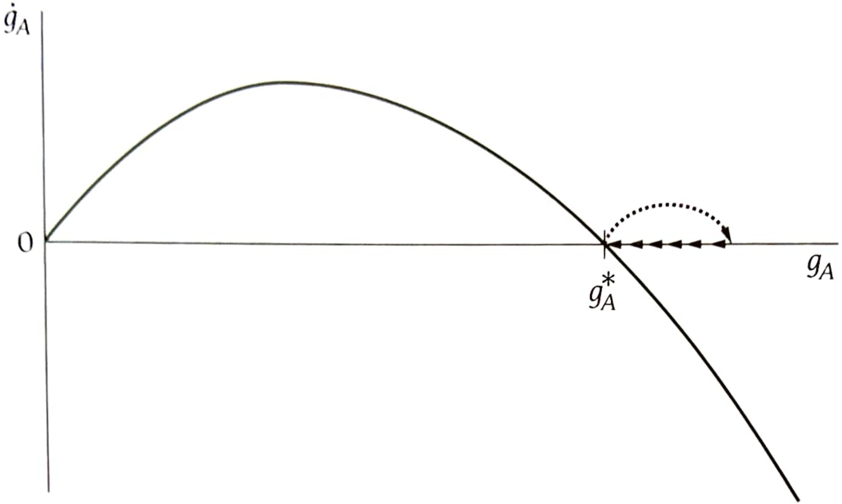


FIGURE 3.2 The effects of an increase in a_L when $\theta < 1$

Source: Romer (2019). In Romer's notation $\theta < 1$ is what I have called $\phi < 1$ and a_L is what I have called s

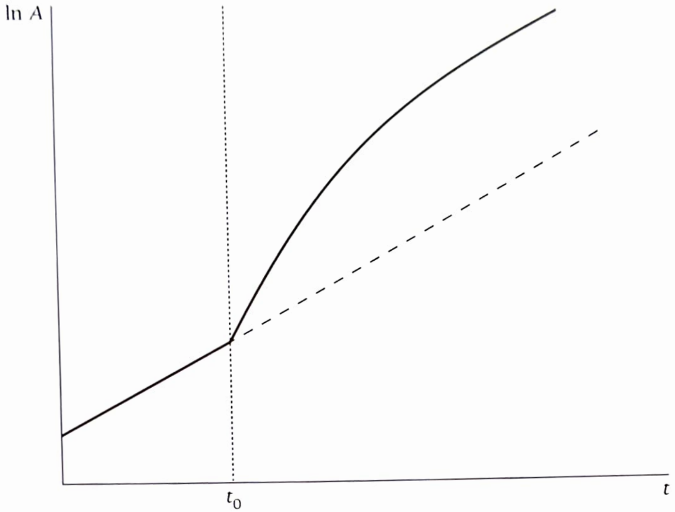


FIGURE 3.3 The impact of an increase in a_L on the path of A when $\theta < 1$

Source: Romer (2019). In Romer's notation $\theta < 1$ is what I have called $\phi < 1$ and a_L is what I have called s

Effect of s on Growth

$$g_A(t) = \frac{\dot{A}(t)}{A(t)} = \theta L_A^{\lambda} A(t)^{\phi-1} = \frac{\theta s^{\lambda} L(t)^{\lambda}}{A(t)^{1-\phi}}$$

- Models with $\phi = 1$: s affects long run growth rate

$$g_A(t) = \frac{\dot{A}(t)}{A(t)} = \theta s^{\lambda} L(t)^{\lambda}$$

- Models with $\phi < 1$: s does not affect long run growth rate

$$g_y = \sigma g_A = \frac{\sigma\lambda}{1-\phi} n$$

Scale Effects

- Models with $\phi = 1$ have "strong" scale effects
  - Growth rate is increasing in **level** of population:

$$g_A(t) = \frac{\dot{A}(t)}{A(t)} = \theta s^{\lambda} L(t)^{\lambda}$$

- Models with $\phi < 1$ have "weak" scale effects
  - Growth rate is increasing in **growth rate** of population:

$$g_y = \sigma g_A = \frac{\sigma\lambda}{1-\phi} n$$

- These are interesting testable implications of these model classes

Do Scale Effects Apply at Country Level?

- One reading of scale effects is that large countries or countries with fast population growth should have high TFP growth
- Obviously counterfactual (Luxembourg, Iceland, Singapore)
- But ideas flow between countries
- Scale effects likely to operate largely at the world level (although flow of ideas is not perfect or instantaneous)

Strong Scale Effects

- There is arguably very strong evidence against strong scale effects:
  - Frontier growth has been quite stable for a long time
  - Research effort has increased very substantially
- With strong scale effects, increased research effort should increase TFP growth at frontier

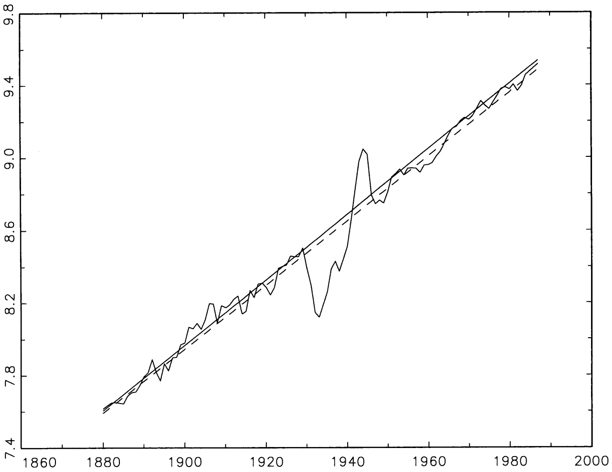


FIGURE I
Per Capita GDP in the United States, 1880–1987 (Natural logarithm)

Source. The data are from Maddison [1982, 1989] as compiled by Bernard [1991]. The solid trend line represents the time trend calculated using data only from 1880 to 1929. The dashed line is the trend for the entire sample.

Source: Jones (1995).

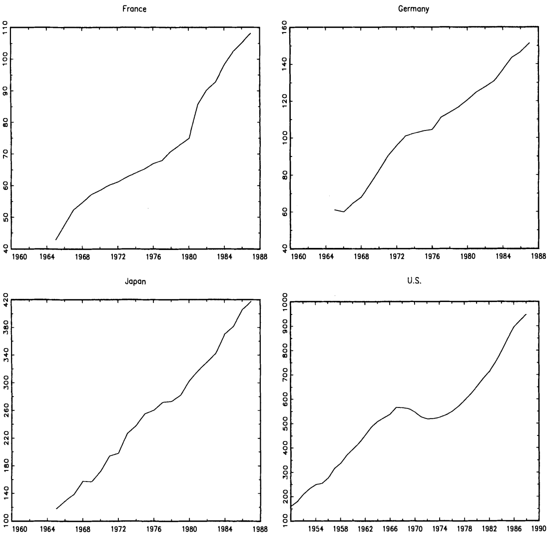


FIGURE IV
Scientists and Engineers Engaged in R&D (1000s)

Source. NSF Science and Engineering Indicators 1989 and Bureau of the Census (various).

Source: Jones (1995).

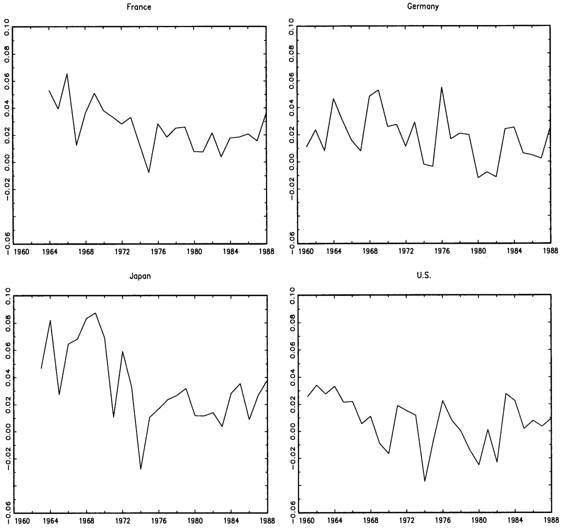


FIGURE V
Aggregate Total Factor Productivity Growth
Source. OECD Department of Economics and Statistics Analytic Database. Data provided by Steven Englander.

Source: Jones (1995).

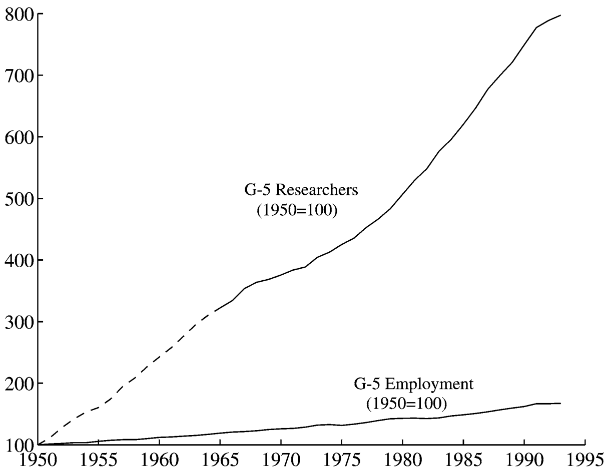


Figure 2. Researchers and employment in the G-5 countries (index). *Note*. From calculations in Jones (2002b). Data on researchers before 1950 in countries other than the United States is backcasted using the 1965 research share of employment. The G-5 countries are France, Germany, Japan, the United Kingdom and the United States.

Source: Jones (2005).

Evidence Against Strong Scale Effects

$$g_A(t) = \frac{\dot{A}(t)}{A(t)} = \theta s L(t)$$

- Research effort has risen by a factor of 8
- Models with $\phi = 1$ imply that growth should have increased by a factor of 8
- Clearly way off!

Ideas Harder to Find

- This evidence suggests that ideas are harder to find
- By ideas, we mean “proportional increases in productivity”
- Research productivity is falling. It takes more research effort to produce the same growth rate
- This means $\phi < 1$ ($\beta > 0$ using Jones (2021) notation)
- But by how much?
  - If $\phi = 0.95$ growth effects of change in *s* on transition path would last for a long time

Bloom, Jones, Van Reenen, Webb (2020)

- Estimate extent to which ideas are getting harder to find at both macro and micro level
- Ideas production function

$$\frac{\dot{A}(t)}{A(t)} = \alpha A(t)^{-\beta} S(t)$$

  - $S(t)$ denotes "scientists" (i.e., research effort)
  - Notice that $\beta = 1 - \phi$
- If g_A is constant:

$$\beta = \frac{g_S}{g_A}$$

- Define:

$$\text{Research Productivity} = \frac{\dot{A}(t)/A(t)}{S(t)}$$

Aggregate Evidence

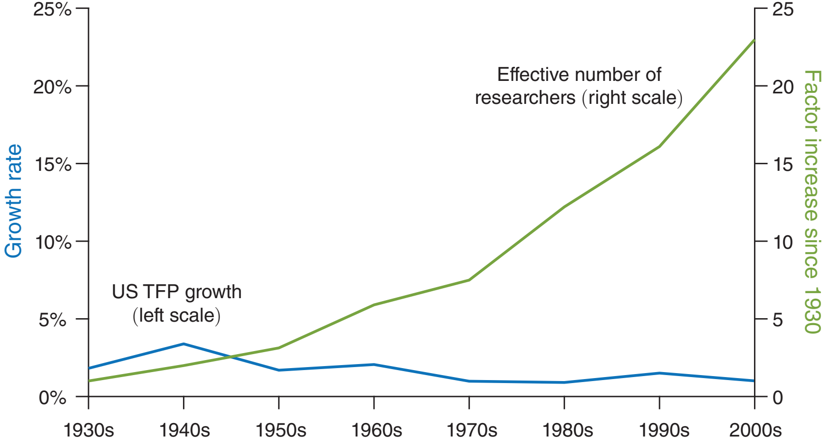


Figure 1. Aggregate Data on Growth and Research Effort

Source: Bloom, Jones, Van Reenen, Webb (2020).

MOORE'S LAW

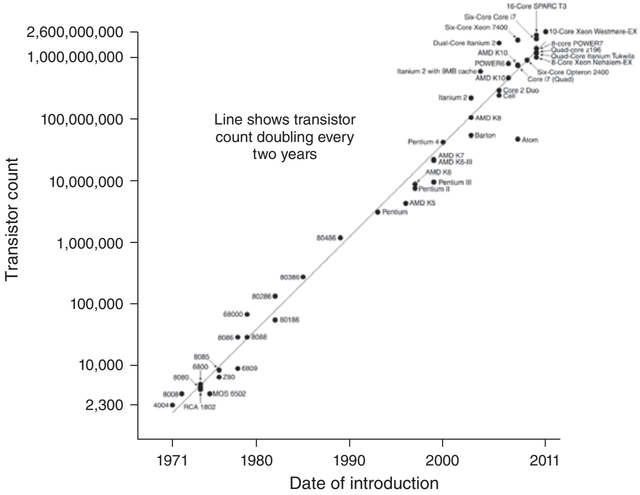


FIGURE 3. THE STEADY EXPONENTIAL GROWTH OF MOORE'S LAW

Source: Bloom, Jones, Van Reenen, Webb (2020).

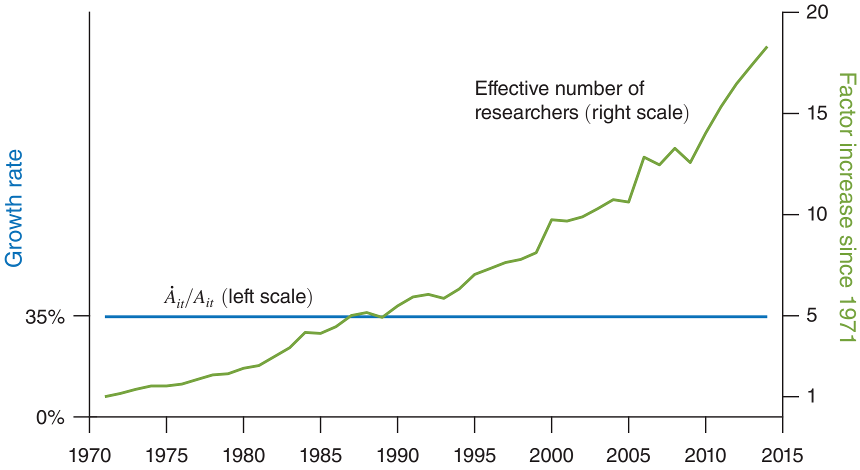


FIGURE 4. DATA ON MOORE'S LAW

Source: Bloom, Jones, Van Reenen, Webb (2020).

Table 7—Summary of the Evidence on Research Productivity

Scope	Time period	Average annual growth rate (%)	Half-life (years)	Dynamic diminishing returns, β
Aggregate economy	1930–2015	−5.1	14	3.1
Moore's Law	1971–2014	−6.8	10	0.2
Semiconductor TFP growth	1975–2011	−5.6	12	0.4
Agriculture, US R&D	1970–2007	−3.7	19	2.2
Agriculture, global R&D	1980–2010	−5.5	13	3.3
Corn, version 1	1969–2009	−9.9	7	7.2
Corn, version 2	1969–2009	−6.2	11	4.5
Soybeans, version 1	1969–2009	−7.3	9	6.3
Soybeans, version 2	1969–2009	−4.4	16	3.8
Cotton, version 1	1969–2009	−3.4	21	2.5
Cotton, version 2	1969–2009	+1.3	−55	−0.9
Wheat, version 1	1969–2009	−6.1	11	6.8
Wheat, version 2	1969–2009	−3.3	21	3.7
New molecular entities	1970–2015	−3.5	20	…
Cancer (all), publications	1975–2006	−0.6	116	…
Cancer (all), trials	1975–2006	−5.7	12	…
Breast cancer, publications	1975–2006	−6.1	11	…
Breast cancer, trials	1975–2006	−10.1	7	…
Heart disease, publications	1968–2011	−3.7	19	…
Heart disease, trials	1968–2011	−7.2	10	…
Compustat, sales	3 decades	−11.1	6	1.1
Compustat, market cap	3 decades	−9.2	8	0.9
Compustat, employment	3 decades	−14.5	5	1.8
Compustat, sales/employment	3 decades	−4.5	15	1.1
Census of Manufacturing	1992–2012	−7.8	9	…

Source: Bloom, Jones, Van Reenen, Webb (2020).

Growth in the Past and Future

- Semi-endogenous growth model imply that long-run growth is governed by population growth
- Many other factors have "level effects"
  (e.g., increases in education, R&D share, misallocation)
- But level effects can be large
- How much of recent growth is due to such level effects?
- What does this suggest about the future of growth?

- Goods production:

$$Y_t = K_t^{\alpha}(Z_t h_t L_{Yt})^{1-\alpha}$$

  - h_t is human capital per person

- Productivity:

$$Z_t = A_t M_t$$

  - A_t is knowledge
  - M_t is misallocation

- Some manipulation:

$$y_t = \left(\frac{K_t}{Y_t}\right)^{\alpha/(1-\alpha)} A_t M_t h_t l_t (1 - s_t)$$

- Ideas Production function:

$$\dot{A}(t) = \theta L_A(t)^{\lambda} A(t)^{\phi}$$

$$\frac{\dot{A}(t)}{A(t)} = \theta s(t)^{\lambda} L(t)^{\lambda} A(t)^{\phi-1}$$

- With constant growth of $A(t)$:

$$0 = \lambda g_s + \lambda g_L - (1-\phi) g_A$$

$$g_A = \frac{\lambda}{1-\phi}(g_s + g_L)$$

- Jones (2021) assumes $\lambda/(1-\phi) = \lambda/\beta = \gamma = 1/3$

  (Results that follow are sensitive to this!)

Growth Accounting

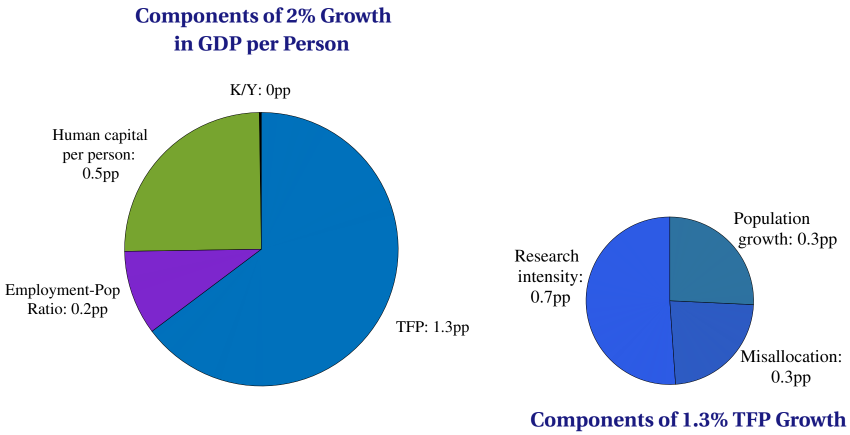
$$\underbrace{d\log y_t}_{\text{GDP per person}} = \underbrace{\frac{\alpha}{1-\alpha} d\log \frac{K_t}{Y_t}}_{\text{Capital-Output ratio}} + \underbrace{d\log h_t}_{\text{Educational att.}} + \underbrace{d\log \ell_t}_{\text{Emp-Pop ratio}} + \underbrace{d\log(1-s_t)}_{\text{Goods intensity}}$$
$$+ \underbrace{d\log M_t + d\log A_t}_{\text{TFP growth}} \tag{15}$$

where

$$\text{TFP growth} \equiv \underbrace{d\log M_t}_{\text{Misallocation}} + \underbrace{d\log A_t}_{\text{Ideas}} = \underbrace{d\log M_t}_{\text{Misallocation}} + \underbrace{\gamma\, d\log s_t}_{\text{Research intensity}} + \underbrace{\gamma\, d\log L_t}_{\text{LF growth}} \tag{16}$$

Source: Jones (2021).

Figure 2: Historical Growth Accounting



Note: The figure shows a growth accounting exercise for the United States since the 1950s using equations (15) and (16). See the main text for details.

Source: Jones (2021).

Growth in Past and Future

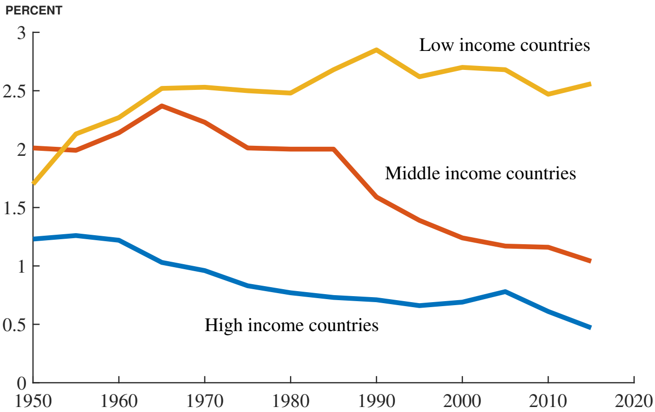
- In the long run:
  - All terms are zero except population growth
  - 100% of growth due to population growth
- Historically:
  - 80% of growth due to other factors
  - Only 20% of growth due to population growth

    (Sensitive to assumption on γ.)

Will Growth Slow?

- Many sources of growth are temporary:
  - Increased education
  - Higher Emp-Pop ratio
  - Falling misallocation
  - Rising research intensity
- But some of these might continue for a very long time (e.g., increased research intensity)
- Population growth is slowing
  (Population likely to start shrinking soon!)

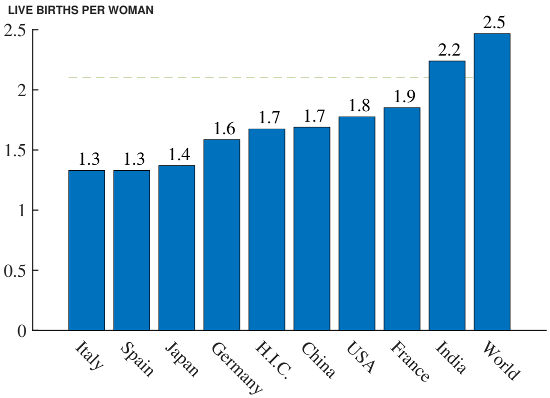
Figure 4: Population Growth around the World



Note: Average annual rates of population growth for countries classified according to their 2018 World Bank income grouping. Each data point corresponds to a five-year period. Source: United Nations (2019).

Source: Jones (2021).

Figure 5: The Total Fertility Rate around the World



Note: The total fertility rate is the average number of live births a hypothetical cohort of women would have over their reproductive life if they were subject during their whole lives to the fertility rates of a given period and if they were not subject to mortality. Each data point corresponds to the five-year period 2015–2020. Source: United Nations (2019).

Source: Jones (2021).

Might Growth Speed Up?

- Finding Einsteins
  - Traditionally most people not able to reach their potential as producers of ideas/knowledge
  - Extreme poverty, cast/class restrictions, discrimination
  - How many Einsteins and Doudnas have we missed
- Automation and Artificial Intelligence
  - Interesting discussion in Jones (2021, sec. 6)
  - Automation of ideas production could even imply a “singularity” (explosive growth driven by AGI)