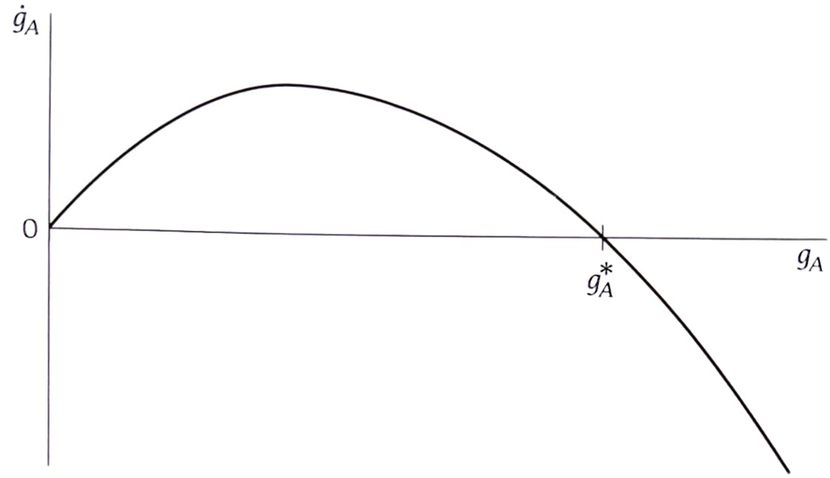

FIGURE 3.1 The dynamics of the growth rate of knowledge when $\theta < 1$

We therefore focus on the dynamics of A, which are given by (3.6). This equation implies that the growth rate of A, denoted g_A, is

$$\begin{aligned} g_A(t) &\equiv \frac{\dot{A}(t)}{A(t)} \\ &= Ba_L^{\gamma} L(t)^{\gamma} A(t)^{\theta-1}. \end{aligned} \tag{3.7}$$

Taking logs of both sides of (3.7) and differentiating the two sides with respect to time gives us an expression for the *growth rate* of g_A (that is, for the growth rate of the growth rate of A):

$$\frac{\dot{g}_A(t)}{g_A(t)} = \gamma n + (\theta - 1)g_A(t). \tag{3.8}$$

Multiplying both sides of this expression by $g_A(t)$ yields

$$\dot{g}_A(t) = \gamma n g_A(t) + (\theta - 1)[g_A(t)]^2. \tag{3.9}$$

The initial values of L and A and the parameters of the model determine the initial value of g_A (by [3.7]). Equation (3.9) then determines the subsequent behavior of g_A.

To describe further how the growth rate of A behaves (and thus to characterize the behavior of output per worker), we must distinguish among the cases $\theta < 1$, $\theta > 1$, and $\theta = 1$. We discuss each in turn.

Case 1: $\theta < 1$

Figure 3.1 shows the phase diagram for g_A when θ is less than 1. That is, it plots $\dot{g}_A$ as a function of A for this case. Because the production function for knowledge, (3.6), implies that g_A is always positive, the diagram considers only positive values of g_A. As the diagram shows, equation (3.9) implies

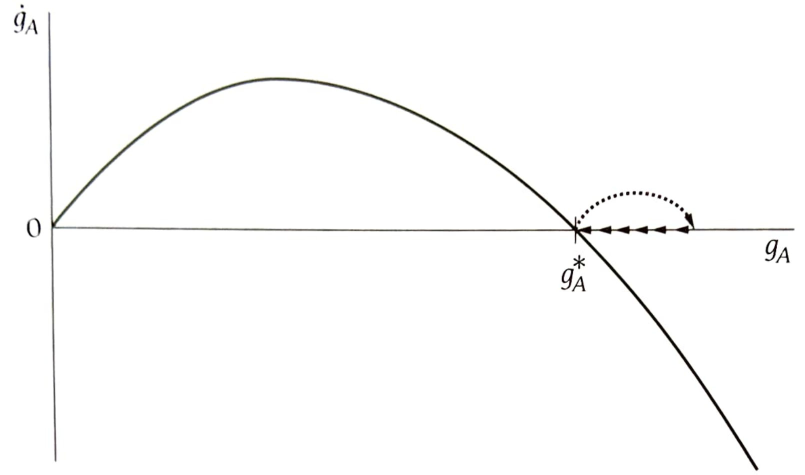

FIGURE 3.2 The effects of an increase in a_L when $\theta < 1$

that for the case of θ less than 1, $\dot{g}_A$ is positive for small positive values of g_A and negative for large values. We will use g_A^* to denote the unique positive value of g_A that implies that $\dot{g}_A$ is zero. From (3.9), g_A^* is defined by $\gamma n + (\theta - 1)g_A^* = 0$. Solving this for g_A^* yields

$$g_A^* = \frac{\gamma}{1-\theta} n. \tag{3.10}$$

This analysis implies that regardless of the economy's initial conditions, g_A converges to g_A^*. If the parameter values and the initial values of L and A imply $g_A(0) < g_A^*$, for example, $\dot{g}_A$ is positive; that is, g_A is rising. It continues to rise until it reaches g_A^*. Similarly, if $g_A(0) > g_A^*$, then g_A falls until it reaches g_A^*. Once g_A reaches g_A^*, both A and Y/L grow steadily at rate g_A^*. Thus the economy is on a balanced growth path.

This model is our first example of a model of *endogenous growth*. In this model, in contrast to the Solow, Ramsey, and Diamond models, the long-run growth rate of output per worker is determined within the model rather than by an exogenous rate of technological progress.

The model implies that the long-run growth rate of output per worker, g_A^*, is an increasing function of the rate of population growth, n. Indeed, positive population growth is necessary for sustained growth of output per worker. This may seem troubling; for example, the growth rate of output per worker is not on average higher in countries with faster population growth. We will return to this issue after we consider the other cases of the model.

Equation (3.10) also implies that the fraction of the labor force engaged in R&D does not affect long-run growth. This too may seem surprising: since growth is driven by technological progress and technological progress is endogenous, it is natural to expect an increase in the fraction of the ec