NEOCLASSICAL LABOR SUPPLY

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Basic Macro Building Blocks

- Consumption-Savings Decision
- Labor-Leisure Decision
- Capital Accumulation
- Factor Demand
- Price and Wage Setting (Phillips Curve)
- Etc.
Plausible (likely) that “frictions” are important in the labor market:

- Jobs and workers are very heterogeneous, suggesting that search frictions may be important
- Monopsony power may be important
- Monopoly power may be important (unions)
- Unemployment (the market doesn’t clear)

Nevertheless, useful to understand neoclassical labor market theory (i.e., perfectly competitive labor market) as one benchmark

- Neoclassical labor market theory may make sense for “big” questions
Labor Demand:

\[ W_t = F_L(L_t, \cdot) \]

- Ignores hiring and firing costs
- Views labor market as a spot market

Steinsson (UC Berkeley)

Neoclassical Labor Supply
Neoclassical Labor Economics

- **Labor Demand:**
  \[ W_t = F_L(L_t, \cdot) \]
  - Ignores hiring and firing costs
  - Views labor market as a spot market

- **Labor Supply:**
  - Household’s intratemporal labor-leisure choice
  \[ \max U(C_t, L_t) \]
  subject to: \[ C_t = W_t L_t \]
  - First order condition:
    \[ \frac{U_{Lt}}{U_{Ct}} = W_t \]
  - Ignores participation margin for simplicity
Let’s assume for simplicity that

\[ U(C_t, L_t) = U(C_t) - V(L_t) \]

What properties should \( U \) and \( V \) have?
Let’s assume for simplicity that

\[ U(C_t, L_t) = U(C_t) - V(L_t) \]

What properties should \( U \) and \( V \) have?

- \( U \) should be upward sloping and concave
- \( V \) should be upward sloping and convex

\( V \) sometimes formulated in terms of leisure: \( V(1 - L_t) \)

Labor supply becomes

\[ \frac{V'(L_t)}{U'(C_t)} = W_t \]
Effect of Wage on Labor Supply

\[ \frac{V'(L_t)}{U'(C_t)} = W_t \]

How does an increase in the wage affect labor supply?
Effect of Wage on Labor Supply

$$\frac{V'(L_t)}{U'(C_t)} = W_t$$

- How does an increase in the wage affect labor supply?
- Two effects!!
- Substitution effect:
  - Higher wage makes working more attractive. Increases labor supply.
  - Holding $C_t$ fixed, if $W_t$ goes up on RHS, $L_t$ has to go up on LHS
- Income effect:
  - But increase in $W_t$ affects $C_t$ since $C_t = W_tL_t$
  - (one period model for simplicity)
  - Holding RHS fixed, increase in $C_t$ reduces $U'$, so $L_t$ must got down to leave LHS fixed.
How Strong Are Income Effects?
Are income effects something we need to take seriously from a quantitative perspective?

Keynes thought so!

- Economic Possibilities for Our Grandchildren (1930)
- “Suppose that a hundred years hence we are all of us, on the average, eight times better off in the economic sense than we are to-day.” (i.e., 2% annual growth)
- “Absolute needs ... satisfied.”
- “Prefer to devote our further energies to non-economic purposes.”
- Main worry “general ‘nervous breakdown’”
- “need to do some work ... to be contented”
- “Three hour shifts of fifteen hour week.”
REAL WAGES IN ENGLAND

Source: Clark (2005, 2010)
We now go over the hours data from various perspectives: across time and space.

2.1 Hours over time

Figure 2 is the main justification for the assumption of constant hours worked maintained in the macroeconomic literature. At least in postwar U.S. data this seems to

Notes: Source: GGDC Total Economy Database for total hours worked and OECD for the data on population aged 15–64. The figure is comparable to the ones in Rogerson (2006). Regressing the logarithm of hours worked on time gives an insignificant slope coefficient.

Source: Boppart and Krusell (2016)
We propose a choice- and technology-based theory for the long-run behavior of the main macroeconomic aggregates. Such a theory—standard balanced-growth theory, specifying preferences and production possibilities along with a market mechanism to be consistent with the data—already exists, but we argue that it needs to be changed. A change is required because of data on hours worked that we document here: over a longer perspective—going back a hundred years and more—and across many countries, hours worked are falling at a remarkably steady rate: at roughly half a percentage point per year. Figure 1 illustrates this fact for a set of countries and for hours on the intensive margin (the extensive margin is rather stationary; we discuss this and other data sources extensively in the paper). This finding contrasts the postwar U.S., where hours per capita are well described as stationary, but this period is an exception to earlier U.S. history and to postwar data from other countries.

The persistent fall in hours worked is not consistent with the preferences and technology used in the standard macroeconomic framework. Our proposed alteration...

**Figure 1: Hours worked per worker**

*Notes:* The figure shows data for the following countries: Belgium, Denmark, France, Germany, Ireland, Italy, Netherlands, Spain, Sweden, Switzerland, the U.K., Australia, Canada, and the U.S. The scale is logarithmic which suggests that hours fall at roughly 0.57 percent per year. Source: Huberman and Minns (2007). Maddison (2001) shows a similar systematic decline in hours per capita.

Figure 3: Selected countries average annual hours per capita aged 15–64, 1950–2015

Notes: Source: GGDC Total Economy Database for total hours worked and OECD for the data on population aged 15–64. The figure is comparable to the ones in Rogerson (2006). Regressing the logarithm of hours worked on time gives a slope coefficient of -0.00393.

Source: Boppart and Krusell (2018)
Balanced Growth

- Long-run trends on real wages and hours suggest that income effects are (a little bit) stronger than substitution effects.

- More traditional view: Labor supply constant as wages rise.

- “Balanced growth preferences” (King, Plosser, and Rebelo, 1988):

\[
U(C_t, L_t) = \begin{cases} 
\frac{(C_t v(L_t))^{1-\sigma} - 1}{1-\sigma} & \text{if } \sigma \neq 1 \\
\log(C_t) + \log v(L_t) & \text{if } \sigma = 1 
\end{cases}
\]

Imply that income and substitution effects exactly cancel out in response to permanent increase in wage.
**Balanced Growth**

- Common choice for preferences:

\[ \log(C_t) - \psi \frac{L_t^{1+1/\nu}}{1 + 1/\nu} \]

- Falls into balanced growth preference set with \( v(L_t) = \exp(-\psi \frac{L_t^{1+1/\nu}}{1+1/\nu}) \)

- Implied labor supply:

\[ \psi L_t^{1/\nu} C_t = W_t \]
Suppose constant (gross) growth rates for:

- Consumption: $g_C$
- Labor: $g_L$
- Wages: $g_W$

Labor supply curve $\psi L_t^{1/\nu} C_t = W_t$ implies $g_L^{1/\nu} g_C = g_W$

Resource constraint $W_t L_t = C_t$ implies $g_L g_W = g_C$

Solving this system yields:

$$g_C = g_W \quad \text{and} \quad g_L = 1$$
MacCurdy (1981) assumed

\[
\frac{C_t^{1-\sigma} - 1}{1 - \sigma} - \psi \frac{L_t^{1+1/\nu}}{1 + 1/\nu}
\]

- Consumption term generalization of $\log C_t$
- When $\sigma \neq 1$, growth not balanced
- Labor supply becomes

\[
\psi L_t^{1/\nu} C_t^\sigma = W_t
\]
In growth rates we have:

\[ g^{1/\nu} g^\sigma = g_w \quad \text{and} \quad g_L g_w = g_C \]

Which implies:

\[ g_L = g_w^{\nu (1-\sigma)/(1+\nu \sigma)} \quad \text{and} \quad g_C = g_w^{1+\nu/(1+\nu \sigma)} \]

So \( g_L < 1 \) (i.e., falling hours) if \( \sigma > 1 \)

\( \sigma \) governs strength of the income effect

In other contexts, \( \sigma \) is:

- Coefficient of relative risk aversion
- Reciprocal of elasticity of intertemporal substitution
Generalized “balanced growth” preferences:

\[ U(C_t, L_t) = \begin{cases} 
\left( \frac{C_t v \left( \frac{L_tC_t^{1-\theta}}{L_tC_t^{1-\theta}} \right)^{1-\sigma}}{1-\sigma} \right)^{-1} & \text{if } \sigma \neq 1 \\
\log(C_t) + \log v \left( \frac{L_tC_t^{1-\theta}}{L_tC_t^{1-\theta}} \right) & \text{if } \sigma = 1
\end{cases} \]

Yield balanced growth with trending hours

Balanced growth:

- Output, consumption, hours, investment, and capital grow at a constant rate in response to constant growth in productivity
Why Do Americans Work So Much More Than Europeans?
Fig. 1.—Aggregate hours in the United States and Europe

output was probably significantly larger than normal and there may have been associated problems with the market hours statistics. The earlier period was selected because it is the earliest one for which sufficiently good data are available to carry out the analysis. The relative numbers after 2000 are pretty much the same as they were in the pretechnology boom period 1993–96.

I emphasize that my labor supply measure is hours worked per person aged 15–64 in the taxed market sector. The two principal margins of work effort are hours actually worked by employees and the fraction of the working-age population that works. Paid vacations, sick leave, and holidays are hours of nonworking time. Time spent working in the underground economy or in the home sector is not counted. Other things equal, a country with more weeks of vacation and more holidays will have a lower labor supply in the sense that I am using the term. I focus only on that part of working time for which the resulting labor income is taxed.

Table 1 reports the G-7 countries' output, labor supply, and productivity statistics relative to the United States for 1993–96 and 1970–74. The important observation for the 1993–96 period is that labor supply (hours per person) is much higher in Japan and the United States than it is in Germany, France, and Italy. Canada and the United Kingdom are in the intermediate range. Another observation is that U.S. output per person is about 40 percent higher than in the European countries, with most of the differences in output accounted for by differences in hours worked per person and not by differences in productivity, that is, in output per hour worked. Indeed, the OECD statistics indicate that French productivity is 10 percent higher than U.S. productivity. In Japan, the output per person difference is accounted for by lower productivity and not by lower labor supply.

Table 1 shows a very different picture in the 1970–74 period. The difference is not in output per person. Then, European output per person was about 70 percent of the U.S. level, as it was in 1993–96 and is today. However, the reason for the lower output in Europe is not fewer market hours worked, as is the case in the 1993–96 period, but rather lower output per hour. In 1970–74, Europeans worked more than Americans. The exception is Italy. What caused these changes in labor supply?

Theory Used


This theory has a stand-in household that faces a labor-leisure decision and a consumption-savings decision. The preferences of this stand-in household are ordered by

\[ V(t) = \frac{1}{\gamma} \log(\frac{C(t)}{h(t)}) + \int_0^\infty \frac{1}{\gamma} \log(\frac{C(s)}{h(s)}) e^{-\lambda s} ds \]

\[ \frac{\partial V(t)}{\partial h(t)} = \frac{C(t)}{h(t)} - \frac{\partial V(t)}{\partial C(t)} \]

\[ \frac{\partial V(t)}{\partial C(t)} = \gamma \frac{\partial C(t)}{\partial C(t)} \]

Variables

\[ c(t) \text{ denotes consumption, and } h(t) \text{ denotes hours of labor supplied to the market sector per person per week. Time is indexed by } t. \]

The discount factor \( \gamma \) is less than one.

<table>
<thead>
<tr>
<th>Period</th>
<th>Country</th>
<th>Output per Person*</th>
<th>Hours Worked per Person*</th>
<th>Output per Hour Worked</th>
</tr>
</thead>
<tbody>
<tr>
<td>1993–96</td>
<td>Germany</td>
<td>74</td>
<td>75</td>
<td>99</td>
</tr>
<tr>
<td></td>
<td>France</td>
<td>74</td>
<td>68</td>
<td>110</td>
</tr>
<tr>
<td></td>
<td>Italy</td>
<td>57</td>
<td>64</td>
<td>90</td>
</tr>
<tr>
<td></td>
<td>Canada</td>
<td>79</td>
<td>88</td>
<td>89</td>
</tr>
<tr>
<td></td>
<td>United Kingdom</td>
<td>67</td>
<td>88</td>
<td>76</td>
</tr>
<tr>
<td></td>
<td>Japan</td>
<td>78</td>
<td>104</td>
<td>74</td>
</tr>
<tr>
<td></td>
<td>United States</td>
<td>100</td>
<td>100</td>
<td>100</td>
</tr>
<tr>
<td>1970–74</td>
<td>Germany</td>
<td>75</td>
<td>105</td>
<td>72</td>
</tr>
<tr>
<td></td>
<td>France</td>
<td>77</td>
<td>105</td>
<td>74</td>
</tr>
<tr>
<td></td>
<td>Italy</td>
<td>53</td>
<td>82</td>
<td>65</td>
</tr>
<tr>
<td></td>
<td>Canada</td>
<td>86</td>
<td>94</td>
<td>91</td>
</tr>
<tr>
<td></td>
<td>United Kingdom</td>
<td>68</td>
<td>110</td>
<td>62</td>
</tr>
<tr>
<td></td>
<td>Japan</td>
<td>62</td>
<td>127</td>
<td>49</td>
</tr>
<tr>
<td></td>
<td>United States</td>
<td>100</td>
<td>100</td>
<td>100</td>
</tr>
</tbody>
</table>

*These data are for persons aged 15–64.

Fig. 13. Hours gap explained by unemployment: France.  

Source: Rogerson (RED 2006).
To address these questions, it is useful to decompose the change in hours worked per capita into its different components:

$$\Delta(HN/P) = \Delta\ln H + \Delta\ln (N/L) + \Delta\ln (L/P_A) + \Delta\ln (P_A/P).$$

The change in hours worked per capita, $HN/P$, can be written as the change in hours worked per worker, $H$, plus the change in the employment rate—the ratio of employment, $N$, to the labor force, $L$—plus the change in the participation rate—the ratio of the labor force $L$ to the population of working age, $P_A$, plus the change in the ratio of the population of working age to total population, $P$. The decomposition of the change in hours worked into these components is given in Table 2 for France and the United States, for the period 1970 to 2000.

**Table 2**

**A Decomposition of the Change in Hours Worked Per Capita in France and the United States from 1970 to 2000** *(percentage)*

<table>
<thead>
<tr>
<th>Percentage change in:</th>
<th>$HN/P$</th>
<th>$H$</th>
<th>$N/L$</th>
<th>$L/P_A$</th>
<th>$P_A/P$</th>
</tr>
</thead>
<tbody>
<tr>
<td>France</td>
<td>-21</td>
<td>-21</td>
<td>-7</td>
<td>3</td>
<td>4</td>
</tr>
<tr>
<td>United States</td>
<td>21</td>
<td>-4</td>
<td>1</td>
<td>10</td>
<td>14</td>
</tr>
<tr>
<td>Difference</td>
<td>-42</td>
<td>-17</td>
<td>-8</td>
<td>-7</td>
<td>-10</td>
</tr>
</tbody>
</table>

*Source: OECD Economic Outlook database.*

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*Source: Blanchard (JEP 2004).*
7.3. What are individuals doing if they are not working?

While there are several alternative uses of time that are potentially of interest, my discussion here will focus entirely on the dimension of home work versus market work. Time spent on educational activities would also appear to be an important candidate for analysis, but I do not consider it here. As noted earlier, cross-country time series data on time use does not exist, though recent developments imply that this situation will be different in the future. Eurostat has initiated a Harmonized Time Use Survey, and the US has now added time use questions into the regular CPS survey.

My goal here is simply to summarize some existing studies that document cross-country patterns of home versus market work. I begin with the study of Freeman and Schettkat (2002) who compare time use across German and American married couples using data from the 1990s. Their analysis yields a stark finding: they find that total work (home plus market) is roughly the same in the two countries, with Americans spending much more time in market work and much less time in home work than their German counterparts. They also looked at consumer expenditure survey data and found that German families spend less money on eating out at restaurants, consistent with the finding in the time use data the Germans spend more time preparing meals at home. In a more recent paper, Freeman and Schettkat (2005) examine the issue for a larger set of countries and again find that Europeans devote much more time to home work than do Americans.

Olovsson (2004) contrasts time use in the US and Sweden, and finds a similar result, though not quite as strong. In particular, he finds that Swedes engage in more home work than Americans.

Fig. 37. Employment relative to the US by age.

Source: Rogerson (RED 2006).
POSSIBLE EXPLANATIONS

- Differences in preferences/culture
- Differences in social norms (coordination problem/unions)
- More generous social safety net
- Higher minimum wage
- Hiring and firing costs
- Higher taxes
POSSIBLE EXPLANATIONS

- Differences in preferences/culture
- Differences in social norms (coordination problem/unions)
- More generous social safety net
- Higher minimum wage
- Hiring and firing costs
- Higher taxes
Representative household maximizes:

$$E \left\{ \sum_{t=0}^{\infty} \beta^t (\log c_t + \alpha \log (1 - h_t)) \right\}$$

Household owns capital and rents to firms

Law of motion for capital:

$$k_{t+1} = (1 - \delta)k_t + x_t$$

Representative firm produces using Cobb-Douglas technology:

$$y_t = c_t + x_t + g_t \leq A_t k_t^\theta h_t^{1-\theta}$$

Household budget constraint:

$$(1 + \tau_c) c_t + (1 + \tau_x) x_t = (1 - \tau_h) w_t h_t + (1 - \tau_k)(r_t - \delta) k_t + \delta k_t + T_t$$
Labor supply:

\[ \frac{\alpha}{(1 - h)} \cdot \frac{1}{c} = (1 - \tau)w \]

where

\[ (1 - \tau) = \left( 1 - \frac{\tau h + \tau c}{1 + \tau c} \right) = \frac{1 - \tau h}{1 + \tau c} \]

Labor demand:

\[ (1 - \theta)\frac{y}{h} = w \]
Combining labor supply and labor demand yields:

\[ h_{it} = \frac{1 - \theta}{1 - \theta + \frac{c_{it}}{y_{it}} \frac{\alpha}{1 - \tau_{it}}} \]

Hours worked governed by:
- consumption-output ratio \( (c_{it}/y_{it}) \)
- taxes \( (\tau_{it}) \)

Tax revenue rebated lump sum to households (no income effect)
- Without this, effect of taxes on hours would be zero since Prescott uses KPR preferences
Calibrates:

- $\theta = 0.3224$ to match capital cost share
- $\alpha = 1.54$ to roughly match hours worked

Estimates tax rates based on national accounts data
(I didn’t understand all the details of this part)
Table 2

Actual and Predicted Labor Supply
In Selected Countries in 1993–96 and 1970–74

<table>
<thead>
<tr>
<th>Period</th>
<th>Country</th>
<th>Labor Supply*</th>
<th>Differences (Predicted Less Actual)</th>
<th>Prediction Factors</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Actual</td>
<td>Predicted</td>
<td>Tax Rate</td>
</tr>
<tr>
<td>1993–96</td>
<td>Germany</td>
<td>19.3</td>
<td>19.5</td>
<td>.59</td>
</tr>
<tr>
<td></td>
<td>France</td>
<td>17.5</td>
<td>19.5</td>
<td>.59</td>
</tr>
<tr>
<td></td>
<td>Italy</td>
<td>16.5</td>
<td>18.8</td>
<td>.64</td>
</tr>
<tr>
<td></td>
<td>Canada</td>
<td>22.9</td>
<td>21.3</td>
<td>.52</td>
</tr>
<tr>
<td></td>
<td>United Kingdom</td>
<td>22.8</td>
<td>22.8</td>
<td>.44</td>
</tr>
<tr>
<td></td>
<td>Japan</td>
<td>27.0</td>
<td>29.0</td>
<td>.37</td>
</tr>
<tr>
<td></td>
<td>United States</td>
<td>25.9</td>
<td>24.6</td>
<td>.40</td>
</tr>
<tr>
<td>1970–74</td>
<td>Germany</td>
<td>24.6</td>
<td>24.6</td>
<td>.52</td>
</tr>
<tr>
<td></td>
<td>France</td>
<td>24.4</td>
<td>25.4</td>
<td>.49</td>
</tr>
<tr>
<td></td>
<td>Italy</td>
<td>19.2</td>
<td>28.3</td>
<td>.41</td>
</tr>
<tr>
<td></td>
<td>Canada</td>
<td>22.2</td>
<td>25.6</td>
<td>.44</td>
</tr>
<tr>
<td></td>
<td>United Kingdom</td>
<td>25.9</td>
<td>24.0</td>
<td>.45</td>
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<td></td>
<td>Japan</td>
<td>29.8</td>
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<td>.25</td>
</tr>
<tr>
<td></td>
<td>United States</td>
<td>23.5</td>
<td>26.4</td>
<td>.40</td>
</tr>
</tbody>
</table>

*Labor supply is measured in hours worked per person aged 15–64 per week.

The labor factor is important in accounting for depressions. In some cases, a low labor factor can be accounted for by a high marginal tax rate on labor income and consumption. In other cases, as I will show, other policies that distort labor markets must be the cause of the low labor input. The labor input might also be low because the economy’s capital stock is above its constant growth path associated with its current policies. If the economy were near its constant growth path and an unexpected change in policy lowered the constant growth path, the labor input would fall below its new constant growth level and then converge up to this new level.

A. The Cause of the Current French Depression: Taxes

France is currently depressed by about 30 percent relative to the United States with the labor factor accounting for nearly all of the depression. The capital factor and the productivity factor are essentially equal in the two countries, whereas market time is about 30 percent lower in France than it is in the United States. Some suggest that the French can make more productive use of their non-market time. But why did they work 10 percent more than the U.S. workers in the 1970’s? My analysis finds that French and U.S. preferences are similar and that the large difference in labor supply is the result of differences in policy that result in different intratemporal tax wedges.

For France and the United Kingdom, I now determine how much of the difference in labor supply is due to differences in the intertemporal tax wedge. I need an estimate of the consumption tax rate and the marginal tax rate on labor income to calculate the intratemporal tax wedge. These tax rates are estimated as follows.

My estimate of the consumption tax rate is the ratio of indirect taxes divided by private consumption net of indirect taxes. The motivation for this procedure is as follows. Most of indirect taxes, including sales and value-added taxes, are consumption taxes. A property tax on an owner-occupied house is equivalent to a consumption tax on the consumption services that the house provides to the owner. The small part of indirect taxes on investment and public consumption will be ignored. Given that the same procedure is used for each country, this will not affect my conclusions.

The procedure for calculating the marginal tax rate on labor income is more complicated. First I calculate the average social-security tax rate on labor income by dividing social-security taxes by an estimate of labor income. The estimate of labor income is the labor-share parameter times output, where output is GDP less indirect taxes. The labor-share parameter used is 0.70. Next I calculate the average tax rate on factor income and assume that the average tax rate on factor income is equal to the average tax rate on labor income. The estimated average tax rates on labor income are direct taxes paid by households divided by GDP less the sum of indirect taxes and depreciation. Given the progressivity of the tax systems, these average tax rates are multiplied by 1.6 to obtain estimates of marginal income tax rates on labor income not including the social-security tax.

A summary of the tax rates for France, the United Kingdom, and the United States is reported in Table 4, which shows that the intratemporal tax wedge is 2.60 for France, 1.82 for the United Kingdom, and 1.66 for the United States.

<table>
<thead>
<tr>
<th></th>
<th>France</th>
<th>United Kingdom</th>
<th>United States</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\tau_c$</td>
<td>0.33</td>
<td>0.26</td>
<td>0.13</td>
</tr>
<tr>
<td>$\tau_h$</td>
<td>0.49</td>
<td>0.31</td>
<td>0.32</td>
</tr>
<tr>
<td>Social-security tax</td>
<td>0.33</td>
<td>0.10</td>
<td>0.12</td>
</tr>
<tr>
<td>Marginal income tax</td>
<td>0.15</td>
<td>0.21</td>
<td>0.20</td>
</tr>
<tr>
<td>Intratemporal tax wedge</td>
<td>2.60</td>
<td>1.82</td>
<td>1.66</td>
</tr>
<tr>
<td>Hours, $h$</td>
<td>0.183</td>
<td>0.235</td>
<td>0.268</td>
</tr>
<tr>
<td>Predicted $h$</td>
<td>0.189</td>
<td>0.250</td>
<td>0.268</td>
</tr>
</tbody>
</table>


Source: Prescott (2002). Intratemporal tax wedge: \((1 + \tau_c)/(1 - \tau_h)\)
Main Conclusion:

“The important observation is that the low labor supplies in Germany, France, and Italy are due to high tax rates.”
Simpler Model

- Household preferences:

\[
\frac{C_t^{1-\sigma} - 1}{1 - \sigma} - \psi \frac{L_t^{1+1/\nu}}{1 + 1/\nu}
\]

- No capital

- Production function:

\[
Y_t = A_t L_t
\]

- Resource constraint:

\[
Y_t = C_t
\]

- Household budget constraint:

\[
(1 + \tau_c)C_t = (1 - \tau_L)W_t L_t + T_t
\]
**SIMPLER MODEL**

- **Labor Supply:**
  \[
  \psi L_t^{1/\nu} C_t^\sigma = \frac{1 - \tau_L}{1 + \tau_C} W_t
  \]

- **Labor Demand:**
  \[
  W_t = A_t = \frac{Y_t}{L_t}
  \]

- **Combining these yields:**
  \[
  \psi L_t^{\sigma + 1/\nu} = (1 - \tau) A^{1-\sigma}
  \]
Taking logs and differences yields:

$$\log L_{it} - \log L_{jt} = \frac{\nu}{1 + \sigma \nu} (\log(1 - \tau_{it}) - \log(1 - \tau_{jt}))$$

With balanced growth preferences $\sigma = 1$:

$$\log L_{it} - \log L_{jt} = \frac{\nu}{1 + \nu} (\log(1 - \tau_{it}) - \log(1 - \tau_{jt}))$$

Two key assumptions regarding effect of taxes on labor supply:

1. Parameter $\nu$
2. Absence of income effects
The Parameter $\nu$

- Labor supply in logs:

$$\log L_t = \nu \log W_t + \nu \log (1 - \tau) - \nu \log C_t - \log \psi$$

- From this we see that $\nu$ is the “Frisch” elasticity of labor supply

- Frisch elasticity: elasticity of labor supply with respect to the wage holding marginal utility (i.e., consumption) constant
Prescott and the Labor Supply Elasticity

Prescott assumed:

$$\log c_t + \alpha \log(1 - h_t)$$

What does this imply about Frisch elasticity?
If $V(L_t) = \alpha \log(1 - L_t)$ then Frisch elasticity is

$$\frac{1}{\nu} = \frac{V''(L)L}{V'(L)}$$

$$\nu = \frac{1 - L}{L}$$

If we assume that steady state labor is $1/4$ of available time (this is ballpark what Prescott assumed)

$$\nu = \frac{3/4}{1/4} = 3$$
\[
\log L_{it} - \log L_{jt} = \frac{\nu}{\nu + 1} (\log(1 - \tau_{it}) - \log(1 - \tau_{jt}))
\]

- With \( \nu = 3 \), we have that \( \frac{\nu}{\nu + 1} = 3/4 \) and
  \[
  \frac{\nu}{\nu + 1} (\log(1 - \tau_{FR}) - \log(1 - \tau_{US})) = \frac{3}{4} (\log(1 - 0.59) - \log(1 - 0.4)) = -0.29
  \]

- With \( \nu = 0.5 \), we have that \( \frac{\nu}{\nu + 1} = 1/3 \) and
  \[
  \frac{\nu}{\nu + 1} (\log(1 - \tau_{FR}) - \log(1 - \tau_{US})) = \frac{1}{3} (\log(1 - 0.59) - \log(1 - 0.4)) = -0.13
  \]

- With \( \nu = 0.1 \), we have that \( \frac{\nu}{\nu + 1} = 1/11 \) and
  \[
  \frac{\nu}{\nu + 1} (\log(1 - \tau_{FR}) - \log(1 - \tau_{US})) = \frac{1}{11} (\log(1 - 0.59) - \log(1 - 0.4)) = -0.03
  \]
The Many Elasticities of Labor Supply
Labor Supply Elasticities

- **Marshallian:**
  - Total or “uncompensated” elasticity
  - Holds non-labor income constant
  - Includes both income and substitutions effects

- **Hicksian:**
  - “Compensated”
  - Holds utility constant
  - No income effect

- **Frisch:**
  - Holds marginal utility of consumption constant
    (i.e., holds consumption constant)
  - Also no income effect (but slightly different from Hicksian)
  - Intertemporal elasticity of labor supply


**Labor Supply Elasticities**

- **Marshallian:**
  \[ \nu_M = \frac{1 - \sigma S}{1/\nu + \sigma S} \]

- **Hicksian:**
  \[ \nu_H = \frac{1}{1/\nu + \sigma S} \]

- **Frisch:**

Where \( S \) is the labor income as a fraction of total income

See Keane (JEL 2011) for derivations
**Labor Supply Elasticities**

\[
\frac{1 - \sigma S}{1/\nu + \sigma S} \leq \frac{1}{1/\nu + \sigma S} \leq \nu
\]

\[\nu_M \leq \nu_H \leq \nu\]

- With quasi-linear preference (i.e., linear in consumption \((\sigma = 0)\):\[
\nu_M = \nu_H = \nu
\]

- Common assumption in applied micro
- For long-run general equilibrium analysis, this assumption is suspect
- For \(\sigma = 1\) and \(S = 1\), \(\nu_M = 0\). For \(\sigma > 1\) and \(S = 1\), \(\nu_M < 0\).
Consider change in a flat tax with revenue rebated back lump sum

We calculated this before:

$$\log L_{it} - \log L_{jt} = \frac{\nu}{1+\sigma\nu} (\log(1 - \tau_{it}) - \log(1 - \tau_{jt}))$$

$$= \frac{1}{1/\nu + \sigma} (\log(1 - \tau_{it}) - \log(1 - \tau_{jt}))$$

Relevant elasticity is the Hicksian elasticity (with $S = 1$)

The fact that revenue is rebated kills the income effect
Change in a flat tax with revenue used for $G_t$ that enter utility separately (e.g., military spending, infrastructure, or wasteful spending)

- **Labor Supply:**
  \[
  \psi L_t^{1/\nu} C_t^\sigma = (1 - \tau) W_t
  \]

- **Labor Demand:**
  \[
  W_t = A_t
  \]

- **Consumption:**
  \[
  C_t = Y_t - \tau Y_t = (1 - \tau) A_t L_t
  \]

- **Combining these yields:**
  \[
  \psi L_t^{\sigma+1/\nu} = (1 - \tau)^{1-\sigma} A^{1-\sigma}
  \]
Change in Flat Tax, No Rebate

\[ \psi L_t^{\sigma+1/\nu} = (1 - \tau)^{1-\sigma} A^{1-\sigma} \]

- Taking log yields:

\[ \log \psi + \left( \sigma + \frac{1}{\nu} \right) \log L = (1 - \sigma) \log(1 - \tau) + (1 - \sigma) \log A \]

- Taking differences yields:

\[ \log L_t - \log L_s = \left( \frac{1 - \sigma}{1/\nu + \sigma} \right) (\log(1 - \tau_t) - \log(1 - \tau_s)) \]

- Marshallian elasticity (with \( S = 1 \)) governs effects of this tax change
- Potentially much smaller than with rebate due to income effect
- With balanced growth preferences (\( \sigma = 1 \)) effect is zero
- Arguably the relevant result for fighting a war
TEMPORARY TAX CHANGE

- Consider a temporary tax change in a dynamic setting
- Take log of labor supply
  \[ \log \psi + \frac{1}{\nu} \log L_t + \sigma \log C_t = \log(1 - \tau) + \log W_t \]
- Take differences:
  \[ \Delta \log L_t = \nu \Delta \log (1 - \tau) + \nu \Delta \log W_t - \nu \Delta \sigma \log C_t \]
- If tax change has no effect on wages and consumption, Frisch elasticity governs effect
- In general equilibrium, wages and consumption may change
All our derivations have been done in general equilibrium

Most empirical analysis makes use of diff-in-diff strategies which difference out general equilibrium

Empirical analysis therefore estimates partial equilibrium effects

Important to keep in mind when going from empirical estimates to policy advice