

Lecture 5

Consumption and Saving

Macroeconomics (Quantitative)
Econ 101B

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Where Do We Stand?

- We have studied:
 - Production (labor demand and capital demand)
 - Household Labor-Leisure (labor supply)
 - Efficiency of private property/free market setting
- Next topic:
 - Household Consumption-Savings decision

The Consumption-Savings Model

- Suppose Robinson Crusoe lives for two periods
- Let's simplify: Ignore both production and labor-leisure decision
- Instead: Robinson Crusoe gets a set **endowment** of coconuts in each of the two periods: Y_1 and Y_2
- For simplicity, we assume that Robinson Crusoe knows values of both Y_1 and Y_2 in period 1

A Savings Technology

- Robinson Crusoe has access to a “savings technology” (safe investment opportunity)
- He can choose to save some of Y_1
- If he saves B coconuts at time 1, the savings technology yields $(1 + R)B$ coconuts at time 2
- We say that the savings technology has a **gross return** of $(1 + R)$ and a **net return** of R
 - R is the “interest rate” in the economy

Saving and Borrowing

- The “savings technology” is symmetric: Robinson Crusoe can either “save” coconuts or “borrow” coconuts
- We can think of the savings technology as borrowing and lending with a neighboring islander (i.e., Friday)
- If Robinson Crusoe borrows, $B < 0$
- In both cases, the return is R (for simplicity)

The Consumption-Savings Model

- Robinson Crusoe's budget constraints are:
- In period 1?

$$C_1 + B = Y_1$$

- In period 2?

$$C_2 = Y_2 + (1 + R)B$$

Limits on Saving and Borrowing

- What is the most Robinson Crusoe can save?

$$B \leq Y_1$$

- What is the most Robinson Crusoe can borrow?
 - As much as he can pay off in period 2:

$$(1 + R)B \geq -Y_2 \text{ which implies } B \geq -\frac{Y_2}{(1+R)}$$

Utility Function

- In general, Robinson Crusoe's utility function can be written
$$U(C_1, C_2)$$
- We will, however, specialize and consider the utility function
$$U(C_1) + \beta U(C_2)$$
- Here, β is called Robinson Crusoe's **subjective discount factor**

Interpretation of β

$$U(C_1) + \beta U(C_2)$$

- What is the interpretation of β ?
 - β is the weight Robinson puts on future consumption relative to current consumption
 - β governs Robinson Crusoe's degree of patience
 - If $\beta < 1$, Robinson Crusoe down-weights (discounts) future consumption
 - If period length is one year, a typical value used is something like $\beta = 0.98$ (but lower numbers may be reasonable)

Consumption-Savings Model

- Robinson Crusoe maximizes

$$U(C_1) + \beta U(C_2)$$

- Subject to:

$$\begin{aligned}C_1 + B &= Y_1 \\C_2 &= Y_2 + (1 + R)B\end{aligned}$$

(and limits on borrowing and saving)

- What are the endogenous variables? C_1, C_2, B
- Exogenous variables: Y_1, Y_2, R
- Parameters: β

Solving Model: Plug and Chug

1. Combine budget constraints into one “intertemporal” budget constraint:

$$C_1 + \frac{1}{1+R} C_2 = Y_1 + \frac{1}{1+R} Y_2$$

- This eliminates B from the household’s problem
- Intertemporal budget constraint says:
 - “Present value” of consumption must equal “present value” of income
- What does “present value” mean?

Present Value

- How much does it cost at time 1 to purchase the right to one coconut in period 2?
- Price must be equal to the number of coconuts you need to store in period 1 to get one coconut in period 2. Which is?

$$\frac{1}{1 + R}$$

- We therefore say that the present value in period 1 of the right to one coconut in period 2 is $1/(1 + R)$ coconuts

Solving Model

2. Plug intertemporal budget constraint into objective function:

$$U(C_1) + \beta U(C_2)$$

$$U(C_1) + \beta U((1 + R)Y_1 + Y_2 - (1 + R)C_1)$$

- This eliminates C_2 from the household's problem
- Now we have a simple optimization problem with one choice variable
- How is this solved?

Solving Model

$$U(C_1) + \beta U((1 + R)Y_1 + Y_2 - (1 + R)C_1)$$

- Differentiate and set to zero:

$$U'(C_1) - \beta(1 + R)U'(C_2) = 0$$

- Rearrange:

$$U'(C_1) = \beta(1 + R)U'(C_2)$$

- This equation is called the “consumption Euler equation”

Euler Equation by Variational Argument

- We can alternatively derive the consumption Euler equation using a “variational argument” (as usual this yields more insight)
- Starting from some candidate optimal choice (C_1, C_2) suppose Robinson Crusoe contemplates saving a little bit more
- Marginal cost: $U'(C_1)$
- Marginal benefit: $\beta U'(C_2)(1 + R)$
- Robinson Crusoe must set these equal to each other:

$$U'(C_1) = \beta(1 + R)U'(C_2)$$

Log Utility

- To gain further insight, we assume

$$U(C) = \log C$$

- This implies

$$U'(C) = \frac{1}{C}$$

- Consumption Euler equation becomes

$$\frac{C_2}{C_1} = \beta(1 + R)$$

Consumption Growth

$$\frac{C_2}{C_1} = \beta(1 + R)$$

- How does growth in income (Y_2/Y_1) affect Robinson Crusoe's consumption growth?
- It doesn't!!!
- Consumption growth is the same whether bulk of income is received in period 1 or period 2

Consumption-Savings

- Suppose most of income is in period 2
- Robinson Crusoe could have consumption track income
- Why is this not the “optimal” thing to do?
- Because of diminishing marginal utility
 - Marginal utility will be much lower in period 2 than in period 1
 - Last coconut in period 2 yields much less utility than next coconut in period 1
 - So, Robinson Crusoe should transfer consumption to period 1 and increase utility

Consumption Smoothing

$$\frac{C_2}{C_1} = \beta(1 + R)$$

- Suppose for simplicity that $\beta(1 + R) = 1$
- This implies that:

$$C_1 = C_2$$

- Irrespective of relative size of Y_1 and Y_2
- Robinson Crusoe wants to **smooth** his consumption over time because marginal utility is diminishing

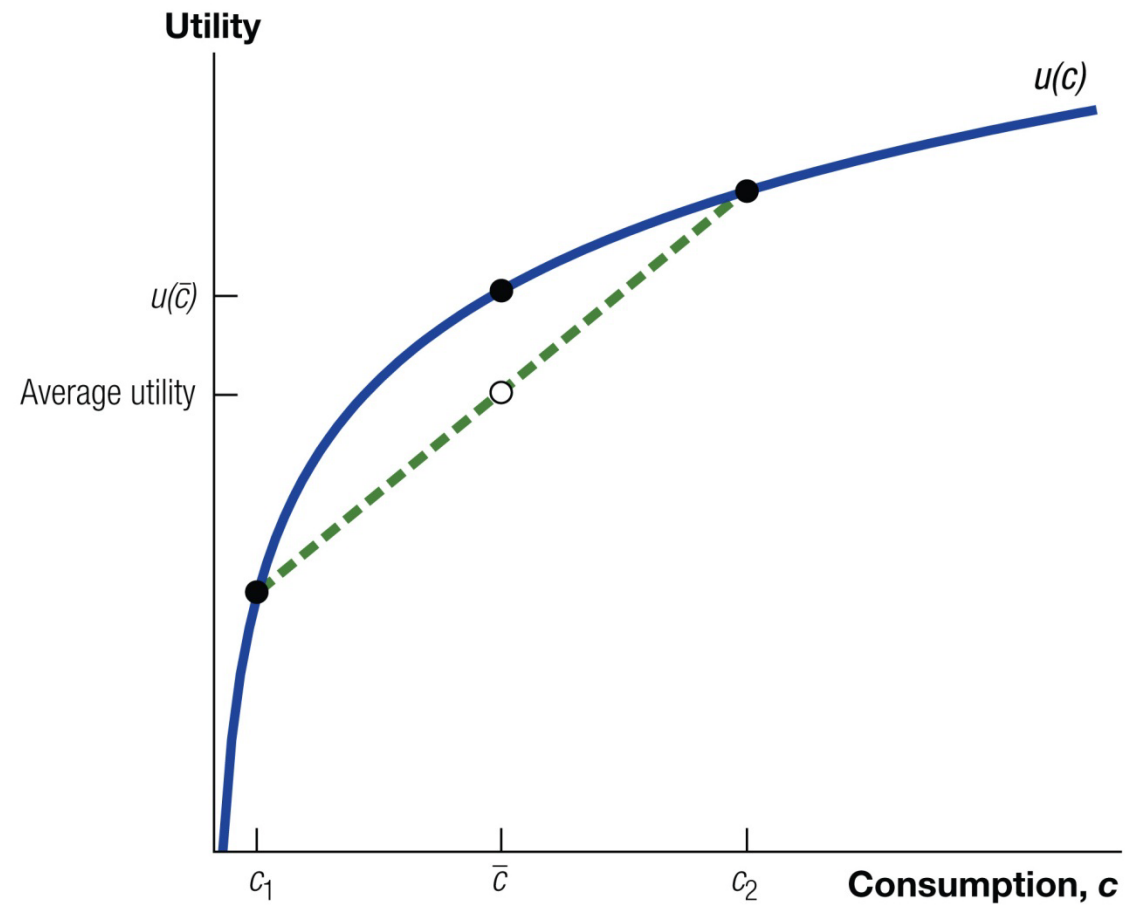


FIGURE 15.2 The Desire to Smooth Consumption

Macroeconomics, 2nd Ed
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Consumption Growth

$$\frac{C_2}{C_1} = \beta(1 + R)$$

- Holding β fixed, how does consumptions growth vary with R ?
- The higher is the return on saving, the higher is the growth rate of consumption
- If return to saving is high, Robinson Crusoe decides to save a lot
- This implies that he consumes little in period 1 and a lot in period 2. So, C_2/C_1 is big.

Solving for Level of Consumption

- With log utility we have:

$$\frac{C_2}{C_1} = \beta(1 + R)$$

$$C_1 + \frac{1}{1 + R} C_2 = Y_1 + \frac{1}{1 + R} Y_2$$

- This is two equations in two unknown.
- What are the unknowns? C_1 and C_2
- We can therefore solve for C_1 and C_2

Level of Consumption

- Solving the two equations on the previous slide yields:

$$C_1 = \frac{1}{1 + \beta} \left(Y_1 + \frac{1}{1 + R} Y_2 \right)$$

- Consumption at time 1 is a fraction of the present value of life-time income
- Consumption at time 1 only depends on current income in so far as it affects the present value of life-time income
- Current income is not special when it comes to consumption

45 Year Working Life

- We can extend our model to a setting in which Robinson Crusoe works for 45 years (say from age 22 to 67)
- Let's assume for simplicity that $\beta = 1$
- Then we have:

$$C_1 = \frac{1}{45} \sum_{t=1}^{45} \left(\frac{1}{1+R} \right)^{t-1} Y_t$$

- For simplicity: No initial wealth/debt and no retirement period

Consumption and Income

$$C_1 = \frac{1}{45} \sum_{t=1}^{45} \left(\frac{1}{1+R} \right)^{t-1} Y_t$$

- Again, consumption is a fraction of present value of life-time income
- Does not depend on shape of income profile
- This idea is often called the **permanent income hypothesis**

Permanent Income Hypothesis

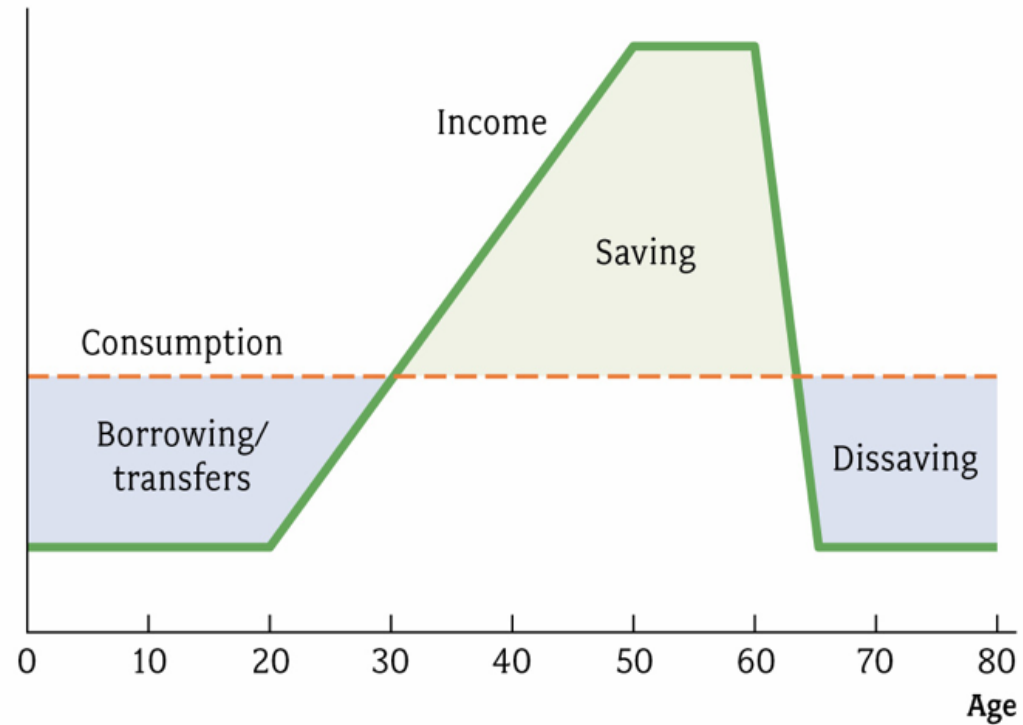


FIGURE 10.5 The Life-Cycle Model of Consumption

Poll

- Suppose you receive a surprise one-time \$1,000 scholarship.
How much of the funds would you spend within a year?
(paying down debts is not spending)
- a) 0-20%
- b) 20-40%
- c) 40-60%
- d) 60-80%
- e) 80-100%

Consumption and Income

$$C_1 = \frac{1}{45} \sum_{t=1}^{45} \left(\frac{1}{1+R} \right)^{t-1} Y_t$$

- Say you unexpectedly get a large bonus in your first year of working life (very high Y_1)
- Suppose this doesn't change your beliefs about income in future years
- How large a fraction should you consume of this windfall within a year and how much should you save?

Marginal Propensity to Consume

$$C_1 = \frac{1}{45} \sum_{t=1}^{45} \left(\frac{1}{1+R} \right)^{t-1} Y_t$$

- An extra dollar of income today only raises consumption today by about $\approx 2\%$ ($1/45$)
- **Marginal propensity to consume** (MPC) is said to be 2%
- MPC should be higher later in life

Forward-Looking Behavior

- Consumption-savings model embodies **forward-looking behavior** on the part of Robinson Crusoe
 - Nothing special about current period income
 - Current consumption depends on current and future income, NOT past consumption
- Forward-looking behavior unique to economics (nothing like this in physics)

Forward-Looking Behavior

- Suppose you are hiking down a mountain.
 - You don't take the easiest first step, the easiest second step, ..., and then fall off a cliff
 - You look ahead and plot a route that overall is best.
 - This is forward-looking behavior
- Water does not look down the hillside and decide on the “best” way down
- Water is not forward-looking

Forward-Looking Behavior

- 60 years ago, economists modeled people as behaving much like water (today's consumption depending on yesterday's consumption and current income)
- The idea that people display forward-looking behavior is one of the most important ideas of 20th century macroeconomics
 - Pioneered by Milton Friedman and Robert Lucas among others

Rational Expectations

- Today, most economists model people as forming “**rational expectations**” about the future and acting optimally given these expectations.
- Rational expectations: People should form optimal forecasts given their information
 - They should not make systematic mistakes
(mistakes they should have been able to avoid given their information)
- Very strong form of forward-looking behavior

Rational Expectations

- Do people have rational expectations?
 - Think of a wide-receiver in football. Where does he run?
 - In contrast, think of a squirrel (or an infant).
- Needless to say, rational expectations is very controversial
- Real world is probably somewhere in between:
 - People are forward-looking
 - But perhaps not fully rational
- Much current research focuses on this

Consumption-Savings: Review

- Consumption Euler Equation:

$$U'(C_1) = \beta(1 + R)U'(C_2)$$

- Marginal benefit of saving an extra dollar equal to marginal cost

- With $U(C) = \log C$:

$$\frac{C_2}{C_1} = \beta(1 + R)$$

- Consumption growth unrelated to income growth
 - Determined by interest rate and discount factor

Consumption-Savings: Review

- Level of consumption if Robinson Crusoe lives for two periods:

$$C_1 = \frac{1}{1 + \beta} \left(Y_1 + \frac{1}{1 + R} Y_2 \right)$$

- Consumption at time t is a fraction of the present value of life-time income
- Consumption at time t only depends on current income in so far as it affects the present value of life-time income

Does the Model Fit the Data?

Thousands of 1987 dollars

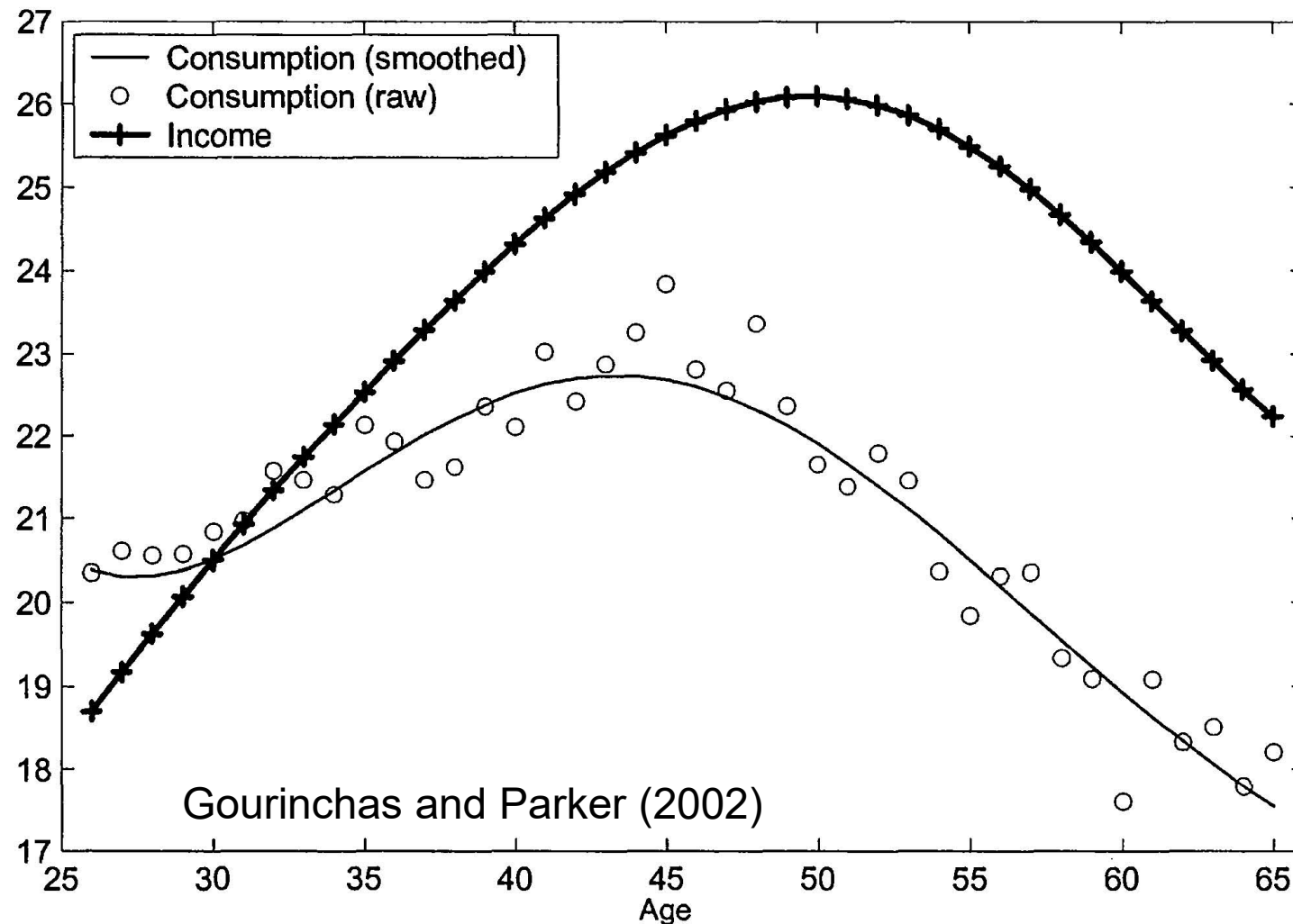


FIGURE 2.—Household consumption and income over the life cycle.

Does the Model Fit the Data?

- No
- What does it miss?
 - Model implies a constant growth rate of consumption (Euler equation)
 - Data show a hump shaped pattern of consumption over the life cycle
 - Consumption profile seems to mimic income profile, while model says it shouldn't

What Is Wrong with the Model?

- People are not forward looking?
 - Well, they do save for retirement
- People's preferences change over time?
 - Need more consumption when have young children
(Gourinchas-Parker try to take this into account)
- People can't borrow against future income?
- Young people want to build up buffer stock of savings ("rainy day fund")?

How to Make Sense of Evidence?

Gourinchas and Parker (2002):

- People are impatient
 $\beta < 1 + R$
- Want downward sloping consumption profile
- But are borrowing constrained earlier in life
- Engage in precautionary savings early in life
- Save for retirement late in life

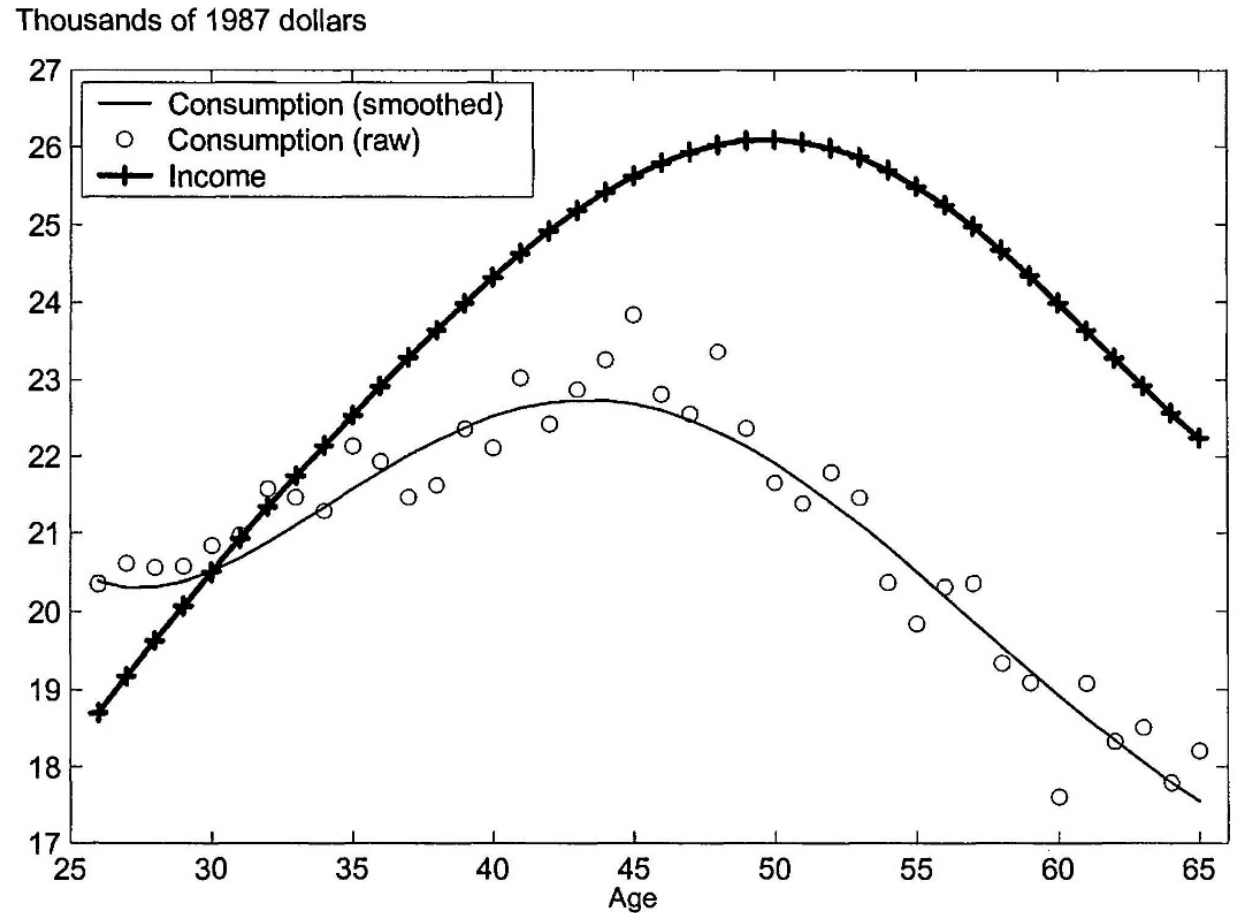


FIGURE 2.—Household consumption and income over the life cycle.
Gourinchas and Parker (2002)

Consumption-Saving

- Three important ideas:
 1. Consumption smoothing
 - Diminishing marginal utility implies that households should borrow when income is low and save when income is high
 2. Borrowing constraints
 - Prevent households from smoothing consumption as much as they would like
 3. Precautionary savings
 - Even when income is low, might want to build up buffer stock of savings as a precaution against future setbacks

History of Thought

- John Maynard Keynes introduced idea of the **consumption function** in the 1930's
 - Consumption a function of current income
$$C_t = a + bY_t$$
 - b denotes marginal propensity to consume
 - $0 < b < 1$, MPC between zero and one
 - Argued that interest rate was unimportant
- Tryggve Haavelmo and Paul Samuelson provided early estimates of $MPC > 2/3$
 - Correlation vs. causation??

History of Thought

- Keynes' consumption function implied:
 - Savings rate should rise as country gets richer
 - Some economists worried that there wouldn't be enough profitable investment to absorb increase in savings. This would lead to stagnation.
- In 1940's Simon Kuznetz gathered data on GDP and consumption back to 1860's.
 - Consumption turned out to be stable as a share of GDP. Savings rate did not seem to rise as economy grew

History of Thought

- Dorothy Brady and Rose D. Friedman (1947):
 - Re-analyzed budget study data
 - Consumption function shifts up over time as average income increases
- Margret Reid (unpublished):
 - Re-analyzed budget study data
 - Introduced the concept of “permanent component of income”

“dealt a fatal blow to this extraordinarily simple view of the savings process” (i.e. the Keynesian consumption function)

--- Modigliani (1986)

History of Thought

- In the 1950's Franco Modigliano (and co-authors) proposed the **life-cycle hypothesis**:
 - Consumers smooth consumption relative to income (e.g., save for retirement)
 - Consumers are forward-looking
 - Savings rate not related to level of income but rather current income relative to average income over life
 - Savings rate doesn't rise with life-time income

History of Thought

- In 1957, Milton Friedman proposed the permanent income effect (developed with Brady, Rose Friedman, and Reid)
 - Consumption not related to current income but rather “permanent income”
 - MPC out of “windfall” much lower than what Keynesians say, perhaps about $1/3$
 - Why $1/3$ as opposed to $1/45$?
(Today we use the term permanent income hypothesis for a model that implies $MPC = 1/45$)

History of Thought

- Empirical Evidence in 1950's and 1960's:
 - Spending out of unanticipated payments in 1950 to U.S. veterans holding National Service Life Insurance was between 0.3 and 0.5
 - Spending out of reparations payments by Germany to Israelis in 1957-58 was about 0.2

History of Thought

- In 1978, Robert Hall writes down model much like ours and argues for a MPC of 0.05.
 - Also, consumption should be a **random walk** (with trend because of interest rate)
 - Only unanticipated changes income should affect consumption (anticipated changes should already be factored into consumption since people are forward-looking)
- In 1982, Hall and Mishkin (1982) estimate MPC to consume of 0.2 and conclude model is wrong because MPC is too high.

History of Thought

- Since 1980:
 - Economists incorporate **borrowing constraints** and **precautionary savings** due to uncertainty about the future into model
 - This raises MPC in the model back up to 0.2-0.4
 - Key contributors: Stephen Zeldes, Christopher Carrol, Pierre-Olivier Gourinchas and Jonathan Parker.