Lecture 7 Capital Accumulation and Growth

Macroeconomics (Quantitative) Econ 101B

Jón Steinsson University of California, Berkeley

The Rise of Capital

- Pre-Industrial times:
 - Land important factor of production
 - Quantity of land fixed
 - Major impediment to growth
- Industrial Revolution: Land intensity falls ...
 - Fossil fuels replace firewood / animal power
 - Food share of consumption falls
- ... importance of capital rises
 - Wave of machines are invented

- Capital is fundamentally different from land
 - It can be accumulated through investment!
- 1800 1950:
 - Industrial nations (UK, US, France,
 Germany, etc.) accumulated a great
 deal of capital and grew rapidly
 - Stalinist Soviet Union: Five year plans emphasize capital accumulation and growth takes off
 - Many other regions lag behind

Capital Accumulation and Growth

- Widely held belief in 1950s:
 - Capital accumulation is key to growth (the "elixir" in Easterly's words)
 - Krugman and Easterly readings discuss this history
- Common development advice:
 - To grow faster, poor countries must increase investment rate
- Does this make sense theoretically?
- Robert Solow addressed this question in a seminal 1956 article

Production

Firm's production function:

$$Y_t = \bar{A} K_t^{1/3} L_t^{2/3}$$

- Firms hire labor and rent capital in competitive markets (same as in lecture 2)
- Households:
 - Supply a fixed amount of labor: \overline{L}
 - Own capital stock. Decide how much to consume and how much to invest in new capital

Consumption-Savings Decision

- Two ways to go:
 - Maximize utility → consumption Euler equation
 - Rule of thumb (shortcut to simplify model)
- Solow model assumes rule of thumb behavior for household savings:
 - Households save a constant fraction of their income $\bar{s}Y_t$

(Notice that income is equal to output since households "own" both labor and capital and there are no profits)

Savings, Investment and Consumption

- Savings: $\bar{s}Y_t$
- Investment?
 - Well, it has to equal savings!

$$I_t = \bar{s}Y_t$$

- Consumption?
 - Whatever is not saved is consumed

$$C_t = Y_t - I_t$$

Capital Accumulation

$$K_{t+1} = K_t + I_t - \bar{d}K_t$$

- Capital tomorrow is equal to:
 - Capital today
 - Plus investment
 - Minus depreciation: Assume constant faction of capital stock "wears out" each period.
- Assume an initial capital stock: K_0

Time, t	Capital, <i>K_t</i>	Investment, I _t	Depreciation, $\overline{d}K_t$	Change in capital, ΔK_t
0	1,000	200	100	100
1	1,100	200	110	90
2	1,190	200	119	81
3	1,271	200	127	73
4	1,344	200	134	66
5	1,410	200	141	59

The last column is found by applying the capital accumulation equation: $\Delta K_t = I_t - \overline{d}K_t$. That is, it is computed by taking the difference between the two prior columns. The next period's capital stock is then the sum of K_t and ΔK_t .

Dynamic Model

- The Solow model is a dynamic model (like Malthus model)
- Describes evolution of economy over time (not just at one moment)
- Five equations for each point in time

The Solow Model

- 1. Firm's production function:
- 2. Labor supply exogenously given:
- 3. Investment equal to savings:
- 4. Consumption:
- 5. Capital accumulation:

$$Y_t = \bar{A} K_t^{1/3} L_t^{2/3}$$

$$L_t = \overline{L}$$

$$I_t = \bar{s}Y_t$$

$$C_t = Y_t - I_t$$

$$K_{t+1} = K_t + I_t - \bar{d}K_t$$

What about Labor and Capital Demand?

- We didn't talk about:
 - Wage
 - Rental rate on capital
- Think of them as being in the background
- Not needed to solve rest of model
- Two more equations and two more endogenous variables (labor demand and capital demand)

Solving the Solow Model

1. Combine:

$$I_t = \bar{s}Y_t$$
 and $K_{t+1} = K_t + I_t - dK_t$

to get:

$$K_{t+1} - K_t = \bar{s}Y_t - \bar{d}K_t$$

- Interpretation:
 - LHS: Change in capital stock
 - RHS: "Net" investment (investment minus depreciations)

Solving the Solow Model

2. Combine:

$$K_{t+1} - K_t = \bar{s}Y_t - \bar{d}K_t$$

and

$$Y_t = \bar{A}K_t^{1/3}L_t^{2/3}$$
 as well as $L_t = \bar{L}$

to get:

$$K_{t+1} - K_t = \bar{s}\bar{A}\bar{L}^{2/3}K_t^{1/3} - \bar{d}K_t$$

Evolution of Capital in Solow Model

$$K_{t+1} - K_t = \bar{s}\bar{A}\bar{L}^{2/3}K_t^{1/3} - \bar{d}K_t$$

- This is the key equation in the Solow Model
- Describes the evolution of the capital stock
- We solve this equation graphically
 - Very similar to Malthus model

Drawing the Solow Graph

Investment:

$$I_t = \bar{s}Y_t = \bar{s}\bar{A}\bar{L}^{2/3}K_t^{1/3}$$

• Depreciation:

$$\bar{d}K_t$$

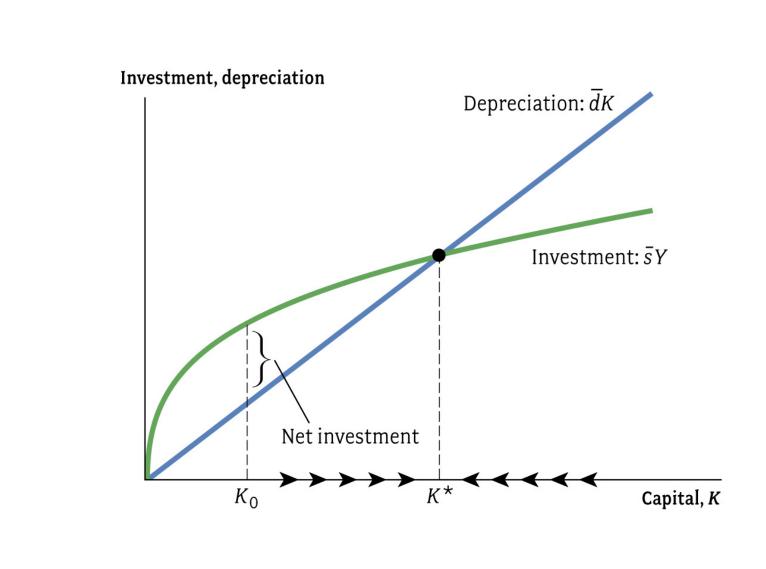
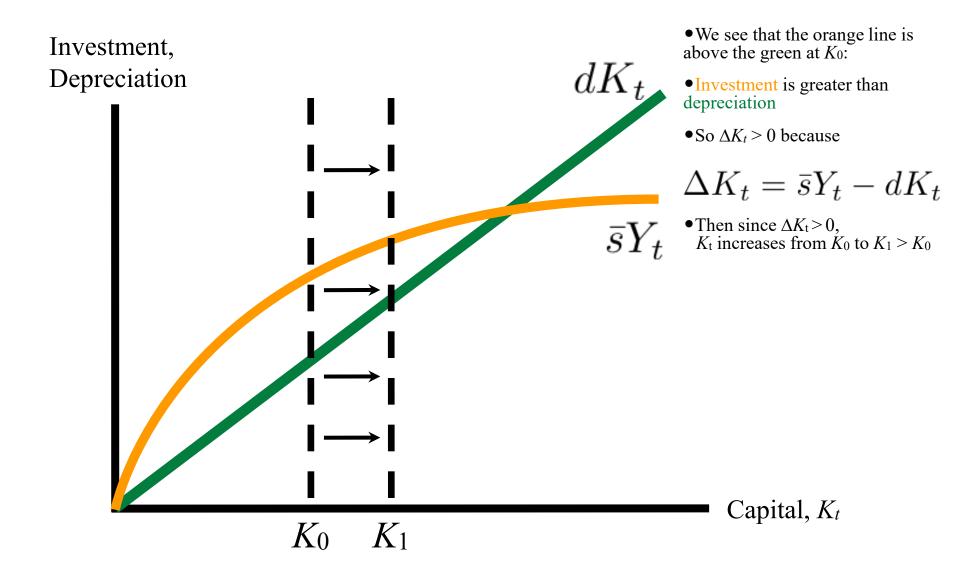


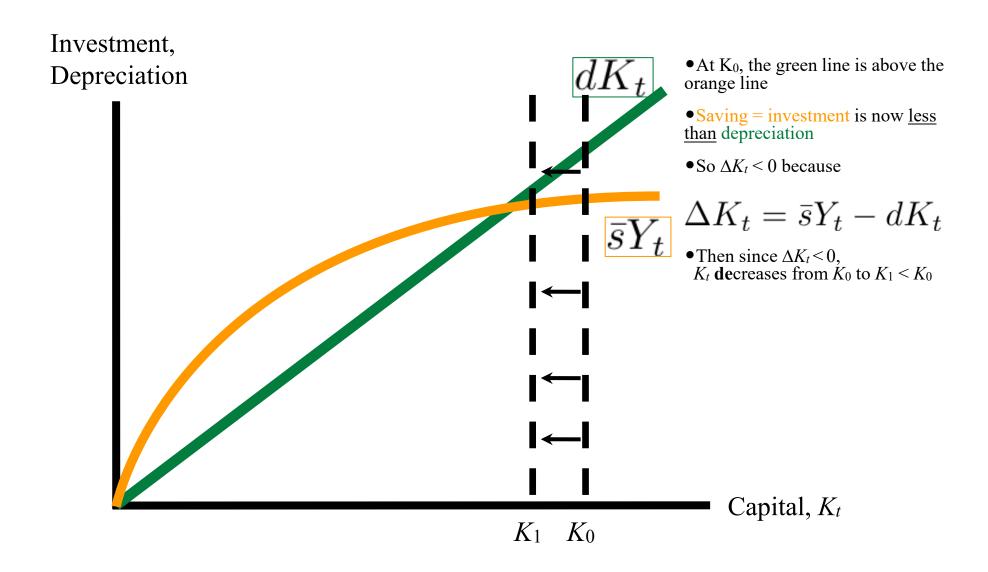
FIGURE 5.1 The Solow Diagram

Macroeconomics, Charles I. Jones Copyright © 2008 W. W. Norton & Company

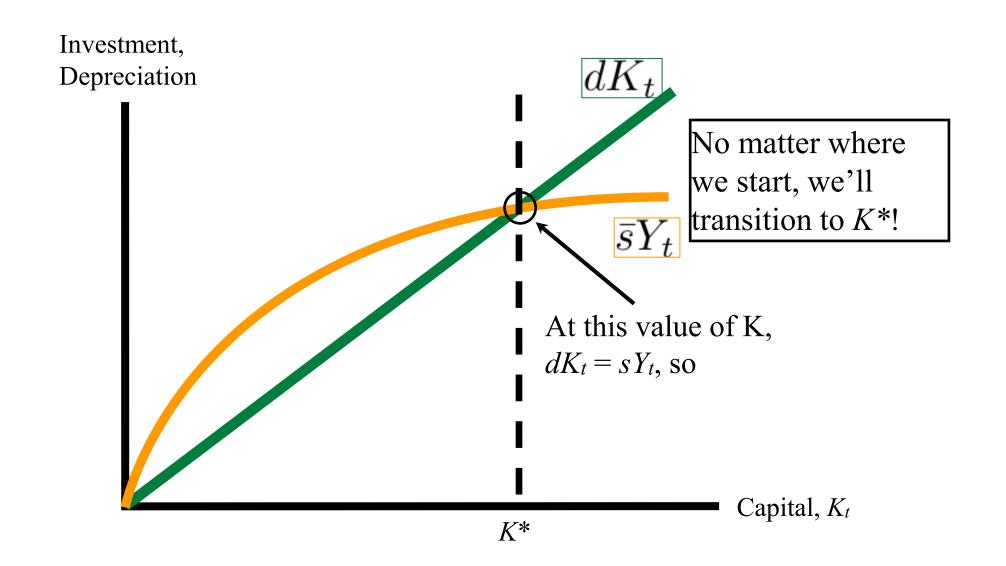
Economy starts at K₀:



Now imagine if we start at a K_0 here:



Transition Dynamics and Steady State



Evolution of Capital

- If investment is greater than depreciation, capital stock rises.
- Capital continues to rise until investment and depreciation are equal. At this point capital stock stops growing
- Growth process is referred to as transition dynamics
- Point at which capital stops growing is called the steady state

Solving for Steady State Capital

$$K_{t+1} - K_t = \bar{s}\bar{A}\bar{L}^{2/3}K_t^{1/3} - \bar{d}K_t$$

Definition of steady state: Point were

$$K_{t+1} = K_t = \overline{K}$$

• Key idea: Replace all K_{t+1} and K_t with \overline{K} $\bar{s} \bar{A} \bar{L}^{2/3} \bar{K}^{1/3} = \bar{d} \bar{K}$

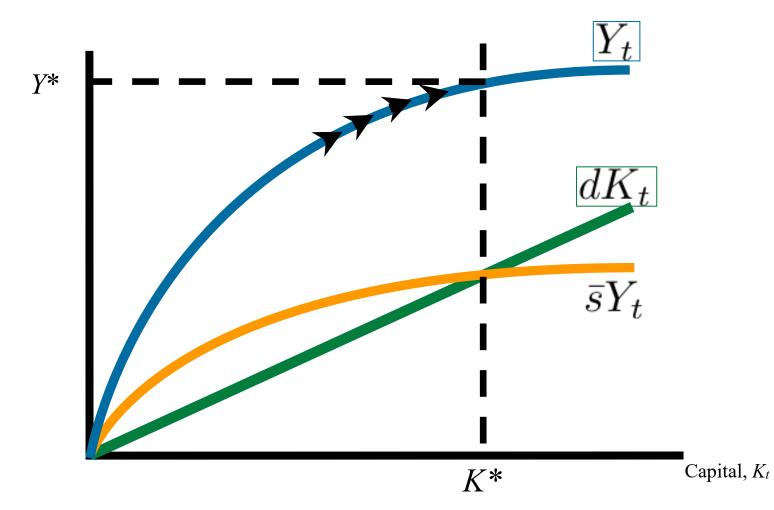
$$\overline{K} = \left(\frac{\overline{s}\overline{A}}{\overline{d}}\right)^{3/2} \overline{L}$$

Output in the Solow Model

Output is given by:

$$Y_t = \bar{A} K_t^{1/3} L_t^{2/3}$$

When capital grows, so does output



Consumption in the Solow Model

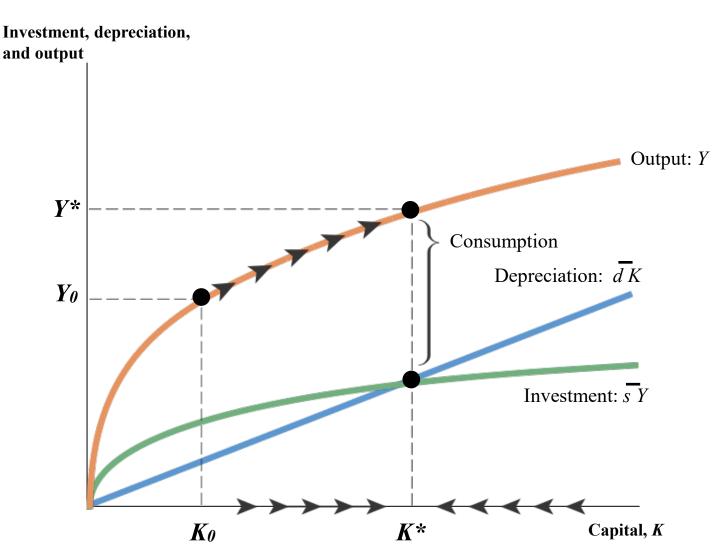
Consumption is given by:

$$C_t = Y_t - I_t$$

$$C_t = (1 - \bar{s})Y_t$$

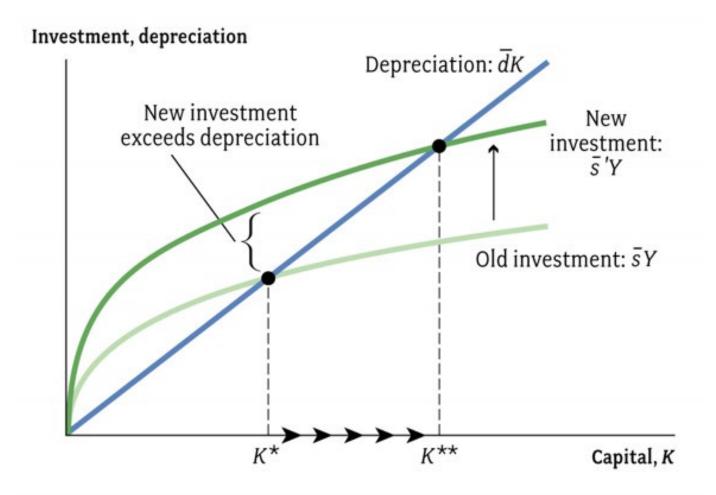
= $(1 - \bar{s}) \bar{A} \bar{L}^{2/3} K_t^{1/3}$

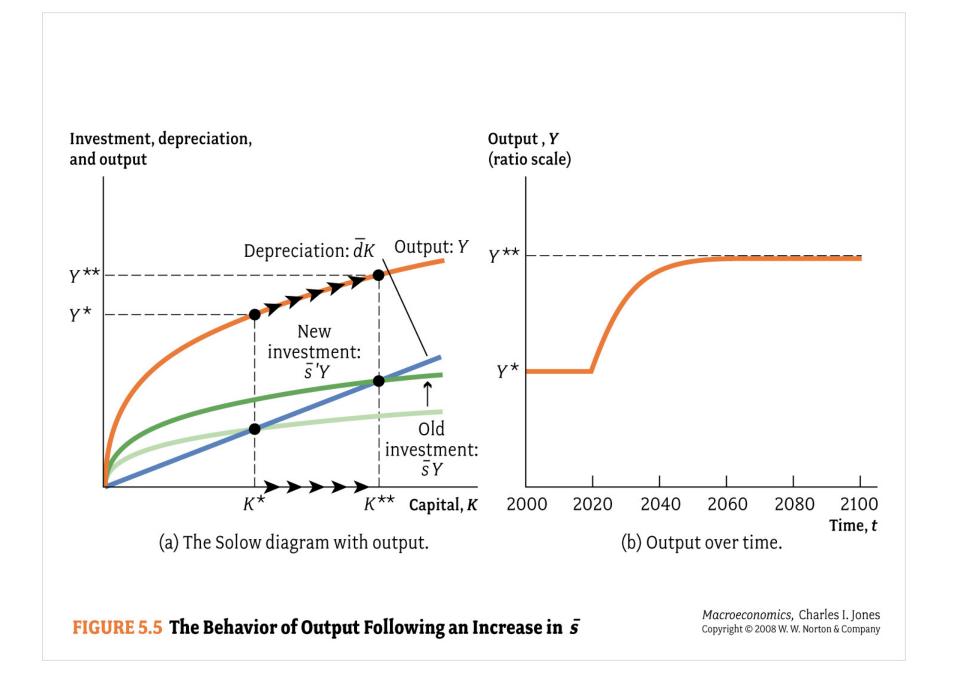
Also grows with capital stock



Increase in the Savings Rate

- Start in a steady state
- Suppose savings rate increases
 - Perhaps due to change in tax policy or government exploitation
- How does the economy respond to such a "shock"?





Optimal Capital Accumulation

- Higher savings rate → higher steady state output
- Does that mean that a high savings rate is good?
- Is a 100% savings rate good?
 - Maximizes capital stock and output
 - But no consumption!

Optimal Saving Rate

- Suppose a policymaker can choose the savings rate.
- What savings rate should they choose?
- The rate that maximized output?
- No. The rate that maximizes consumption!
- Steady state value of capital that maximizes consumption:
 Golden rule level of capital

Golden Rule Level of Capital

Consumption is output minus investment:

$$\bar{C} = \bar{Y} - \bar{I}$$

Steady state output:

$$\bar{Y} = \bar{A}\bar{L}^{2/3}\bar{K}^{1/3}$$

• Steady state investment:

$$\bar{I} = \bar{d}\bar{K}$$

Deriving the Golden Rule

Consumption

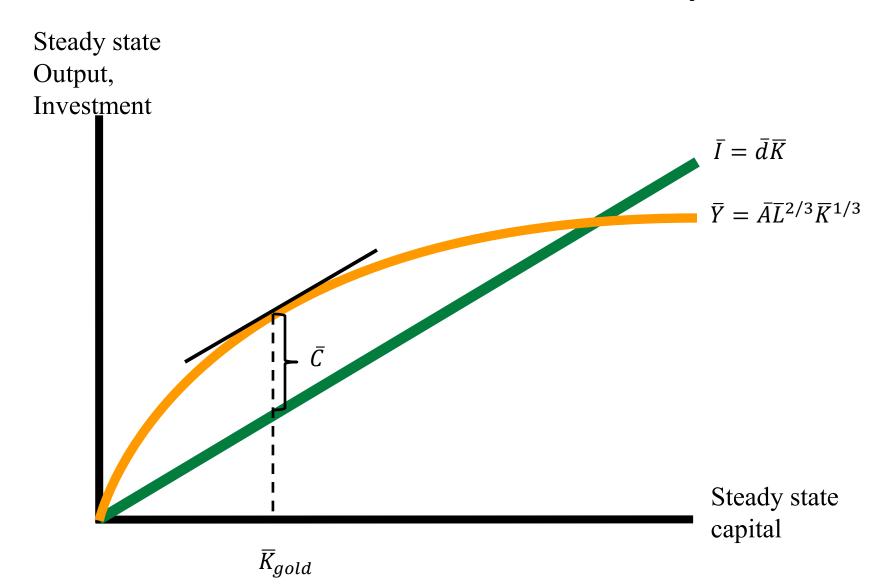
$$\bar{C} = \bar{Y} - \bar{I}$$

$$\bar{C} = \bar{A}\bar{L}^{2/3}\bar{K}^{1/3} - \bar{d}\bar{K}$$

- To maximize consumption:
 - Differentiate with respect to \overline{K} and set to zero

$$\frac{1}{3}\bar{A}\bar{L}^{2/3}\bar{K}^{-2/3} = \bar{d}$$
$$MPK = \bar{d}$$

Golden Rule Level of Capital



Oversaving?

- Many countries worry that savings are too low
- Some countries have very high savings rates:
 - China for last 15 years
 - Soviet Union in mid 20th century
- China and Soviet Union may be / have been "on the wrong side of the Golden Rule"

Moving to the Golden Rule

- Consider a situation in which an economy starts off in a steady state with too much capital
 - Derive transition
- Consider a situation in which an economy starts off in a steady state with too little capital
 - Derive transition

- Key difference:
 - When economy begins above the Golden Rule, reaching Golden rule produces higher consumption at all points in time.
 - When economy begins below the Golden Rule, reaching Golden rule requires initially reducing consumption to increase consumption later on.

Growth in the Solow Model

- What does the Solow model teach us about long run growth?
- There is no long run growth in the Solow Model!!
- Capital, output and consumption settle down to constant steady state values
- A central lesson from the Solow Model:
 - Capital accumulation cannot serve as an engine for long run growth

Why Is There a Steady State?

- Capital and output converge to a constant level in the Solow model
- What feature of the model generates this result?

$$K_{t+1} - K_t = \bar{s}\bar{A}\bar{L}^{2/3}K_t^{1/3} - \bar{d}K_t$$

Diminishing Returns to Capital

- Diminishing returns to investment in capital
 - Depreciation is linear (constant returns)
 - Eventually investment returns can't keep up with depreciation
- This results in growth converging to zero
 (i.e. a steady state for capital, output etc.)

Investment and Growth

Development economics of the 1950's:

- Arthur Lewis: Surplus labor model
 - Army of surplus labor in country side
 - Capital only constraint on output
 - Output proportional to growth since "surplus labor" can be soaked up from country side
 - Aka Harrod-Domar model (Domar never endorsed this)

Harrod-Domar/Surplus Labor Model

- Hugely influential at World Bank etc.
- Policy implication:
 - To double growth one just needs to double investment
- Common belief: Poor can't save/borrow enough
- Policy implication:
 - "Financing gap": Difference between national savings and "required investment." Foreign aid can fill this gap.
- "Empirical evidence": Soviet takeoff
 - But Soviet Union differed in many other ways from poor countries

Investment: Elixir of Growth? Or Not?

Easterly:

- Harrod-Domar/Surplus Labor model doesn't fit the facts.
- Investment neither sufficient nor necessary for growth
- Surplus labor idea not reasonable
 - If there is lots of surplus labor, wages will fall and firms will use more labor intensive production methods

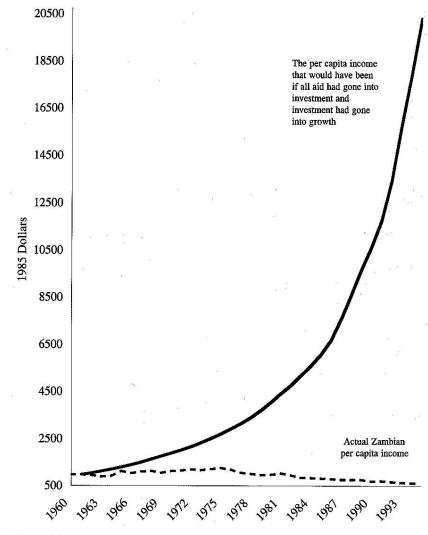


Figure 2.1

The gap between the financing gap model and the actual outcome in Zambia

Population Growth in Solow Model

What if we have population growth?

$$L_{t+1} = L_t(1+\bar{n})$$

Then we have:

$$K_{t+1} - K_t = \bar{s}Y_t - \bar{d}K_t$$
$$Y_t = \bar{A}K_t^{1/3}L_t^{2/3}$$

(same as before except L_t as opposed to \overline{L})

- How do we solve the model in this case?
 - No longer a steady state in K_t
 - $-\bar{s}Y_t$ is constantly shifting up since L_t is growing

Solving the Model with Pop Growth

- Trick: Divide though by L_t
- Write entire model in terms of per capita variables
- Let's do this for the production function:

$$Y_t = \bar{A} K_t^{1/3} L_t^{2/3}$$

It becomes:

$$y_t = \bar{A}k_t^{1/3}$$

where lower case letters denote per capita (e.g., $y_t = Y_t/L_t$)

Solving the Model with Pop Growth

Capital accumulation:

$$K_{t+1} - K_t = \bar{s}Y_t - \bar{d}K_t$$

• Divide by L_t :

$$\frac{K_{t+1}}{L_t} - \frac{K_t}{L_t} = \bar{s} \frac{Y_t}{L_t} - \bar{d} \frac{K_t}{L_t}$$

• Problem: What do we do about K_{t+1}/L_t ?

Solving the Model with Pop Growth

• Multiply and divide by L_{t+1}

$$\frac{K_{t+1}L_{t+1}}{L_{t+1}} - \frac{K_t}{L_t} = \bar{s} \frac{Y_t}{L_t} - \bar{d} \frac{K_t}{L_t}$$

Use population growth equation:

$$k_{t+1}(1+\bar{n}) - k_t = \bar{s}y_t - \bar{d}k_t$$

• Subtract $\bar{n}k_t$ from both sides

$$(1 + \bar{n})(k_{t+1} - k_t) = \bar{s}y_t - (\bar{d} + \bar{n})k_t$$

Per Capita Steady State

• Finally, divide through by $(1 + \bar{n})$

$$(k_{t+1} - k_t) = \frac{1}{1 + \bar{n}} [\bar{s}y_t - (\bar{d} + \bar{n})k_t]$$

- This yields a steady state for per capita variables
- In steady state:
 - Per capita output, capital and consumption constant
- What will growth rate of the level of output, capital and consumption be?

Steady State with Population Growth

$$(k_{t+1} - k_t) = \frac{1}{1 + \bar{n}} \left[\bar{s} y_t - (\bar{d} + \bar{n}) k_t \right]$$

$$0 = \frac{1}{1 + \bar{n}} \left[\bar{s} \bar{y} - (\bar{d} + \bar{n}) \bar{k} \right]$$

$$0 = \left[\bar{s} \bar{A} \bar{k}^{1/3} - (\bar{d} + \bar{n}) \bar{k} \right]$$

$$\bar{k} = \left(\frac{\bar{s} \bar{A}}{\bar{d} + \bar{n}} \right)^{3/2}$$

$$y_t = \bar{A}k_t^{1/3}$$
$$\bar{y} = \bar{A}\bar{k}^{1/3}$$

$$\bar{y} = \left(\frac{\bar{s}}{\bar{d} + \bar{n}}\right)^{1/2} \bar{A}^{3/2}$$

• How does population growth (\bar{n}) affect steady state output and capital per capita? (Why?)

Growth in TFP

Let's add TFP growth as well:

$$A_{t+1} = A_t(1 + \bar{g})$$

One more little change:

$$Y_t = A_t^{2/3} K_t^{1/3} L_t^{2/3}$$

- Think of this as a change of units for A_t (simplifies algebra)
- We also have:

$$K_{t+1} - K_t = \bar{s}Y_t - \bar{d}K_t$$
$$L_{t+1} = L_t(1 + \bar{n})$$

How Do We Solve This Model?

- Two things growing!
- Same trick as before:
 - Divide by variables that are growing: $L_t A_t$
- Capital and output per "effective" worker:

$$\hat{k}_t = \frac{K_t}{L_t A_t} \qquad \hat{y}_t = \frac{Y_t}{L_t A_t}$$

Write Model in Per Effective Worker Variables

Similar algebra as before yields:

$$\hat{y}_t = \hat{k}_t^{1/3}$$

$$\hat{k}_{t+1} - \hat{k}_t = \frac{1}{(1+\bar{n})(1+\bar{g})} \left[\bar{s}\hat{y}_t - \left(\bar{d} - 1 + (1+\bar{n})(1+\bar{g})\right) \hat{k}_t \right]$$

Steady State per Effective Worker

$$\hat{k}_{t+1} - \hat{k}_t = \frac{1}{(1+\bar{n})(1+\bar{g})} \left[\bar{s}\hat{y}_t - \left(\bar{d} - 1 + (1+\bar{n})(1+\bar{g})\right) \hat{k}_t \right]$$

- Model has a steady state for capital and output "per effective workers"
- In steady state:
 - Output, capital and consumption per effective worker are constant
 - What about output per person?

Growth in Per Capita Income

$$\hat{y}_t = \frac{Y_t}{A_t L_t}$$

• Implies:

$$g_{\hat{y}_t} = g_{Y/L} - g_A$$

$$0 = g_{Y/L} - g_A$$

$$g_{Y/L} = g_A$$

Growth Rates

On the last slide we used the following type of growth rate rule:

$$y_t = a_t k_t^a l_t^{1-a}$$

$$\ln y_t = \ln a_t + a \ln k_t + (1 - a) \ln l_t$$

$$\ln y_t - \ln y_{t-1} = \ln a_t - \ln a_{t-1} + a(\ln k_t - \ln k_{t-1}) + (1-a)(\ln l_t - \ln l_{t-1})$$

$$g_y = g_a + ag_k + (1 - a)g_l$$

• Where last step uses first order approx.: $\ln x = \ln x_0 + \frac{x - x_0}{x_0}$

Exogenous Growth

- Technical progress yields growth in per capita output in the Solow model
- So, we know what types of things can yield long-run growth (anything that yields sustained growth in A_t)
- But technical progress is given exogenously
- Our theory of growth is thus limited
- We need a theory for growth in A_t

Growth in the Solow Model

- Capital accumulation cannot serve as engine for long run growth (due to diminishing return to capital)
- But perhaps the "long run" is really far off from a practical point of view
- Perhaps capital accumulation can play a big role for growth for quite some time
- Let's look at "growth miracles" and see if they are due to capital accumulation or TFP growth

Growth "Miracle" of Asian "Tigers"

• Growth of Asian "Tigers" 1966-1990:

Korea 10.3%

Taiwan 9.4%

Singapore 8.7%

Hong Kong 7.3%

- Is their growth driven by TFP growth or growth in factor inputs?
- Young (1995): Tyranny of Numbers

Growth Accounting

$$Y_t = A_t K_t^{1/3} L_t^{2/3}$$

Take logs and a first order approximation:

$$g_Y = g_A + \frac{1}{3}g_K + \frac{2}{3}g_L$$

- Where g's denote growth rates
- We can measure g_{v} , g_{K} , and g_{L}
- This gives us g_A as a residual ("Total Factor Productivity growth" aka "Solow residual")

TABLE VII
TOTAL FACTOR PRODUCTIVITY GROWTH: SOUTH KOREA

	Annual growth of:						
Time period	Output	Raw capital	Weighted capital	Raw labor	Weighted labor	TFP	Labor share
Econom	y—excludi	ing agricu	lture:				
60–66	0.077	0.069	0.070	0.062	$\boldsymbol{0.072}$	0.005	0.690
66–70	0.144	0.167	0.194	0.095	0.103	0.013	0.690
70–75	0.095	0.121	0.118	0.052	0.055	0.019	0.661
75–80	0.093	0.158	0.178	0.040	0.052	0.002	0.694
80-85	0.085	0.102	0.099	0.031	0.047	0.024	0.729
85–90	0.107	0.105	0.108	0.061	0.072	0.026	0.739
66–90	0.103	0.129	0.137	0.054	0.064	0.017	0.703

TABLE VI
TOTAL FACTOR PRODUCTIVITY GROWTH: SINGAPORE

Annual growth of:							
Time period	Output	Raw capital	Weighted capital	Raw labor	Weighted labor	TFP	Labor share
Econom	ıy:						
66-70	0.130	0.119	0.134	0.054	0.033	0.046	0.503
70-80	0.088	0.122	0.140	0.050	0.058	-0.009	0.517
80-90	0.069	0.091	0.084	0.036	0.066	-0.005	0.506
66–90	0.087	0.108	0.115	0.045	0.057	$\boldsymbol{0.002}$	0.509
Manufa	cturing:*						
70–80	0.103	0.123	0.130	0.086	0.089	-0.009	0.423
80-90	0.067	0.090	0.094	0.021	0.051	-0.011	0.385
70–90	0.085	0.107	0.112	0.054	0.070	-0.010	0.404

^{*}Only covering firms recorded in the Census of Industrial Production.

Was it a Miracle?

- Young (1995):
 - Growth driven primarily by growth in factor inputs
 - TFP growth unremarkable
- Young's study came as a shock
 - Many people's reaction was that it was bad for some reason that growth was due to factor input growth
- Krugman (1994) questions sustainability based on the logic of the Solow model

Was It a Miracle?

- Is it any less of an achievement?
- Asian Tigers were able to increase capital stock and labor supply by huge amounts
- Many other countries weren't
- Why were they able to do this while other weren't?

TABLE 6.2

Growth Accounting for the United States

	1948-2017	1948-1973	1973-1995	1995-2003	2003-2017
Output per hour, Y/L	2.3	3.3	1.5	3.2	1.4
Contribution of K/L	0.9	1.0	0.8	1.4	0.5
Contribution of labor composition	0.2	0.2	0.2	0.3	0.2
Contribution of TFP, A	1.2	2.1	0.5	1.5	0.7

The table shows the average annual growth rate (in percent) for different variables.

Source: Bureau of Labor Statistics, Multifactor Productivity Trends.

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TFP Growth in the U.S.

- TFP growth accounts for more than 50% of growth in U.S. GDP per capita since WWII
- Most of the rest comes from K/L

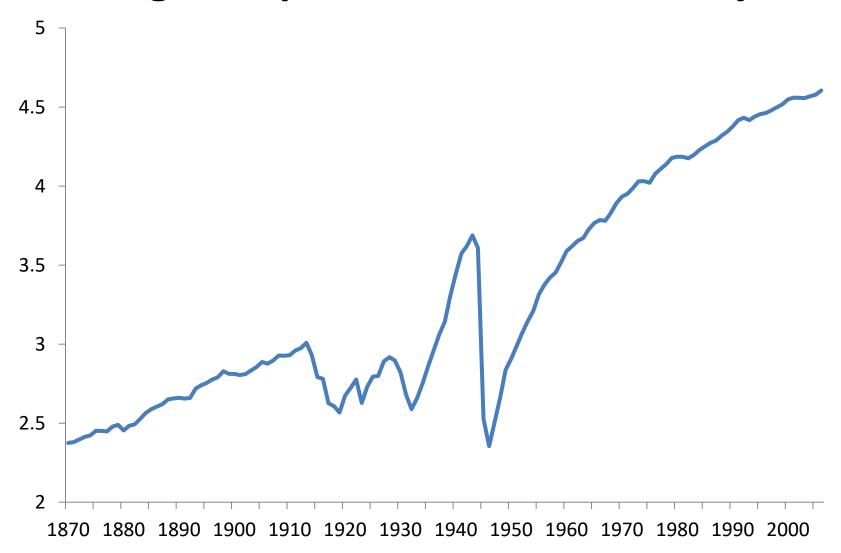
 TFP growth much more important proportionally in U.S. than in case of Asian Tigers

TFP Growth in the U.S.

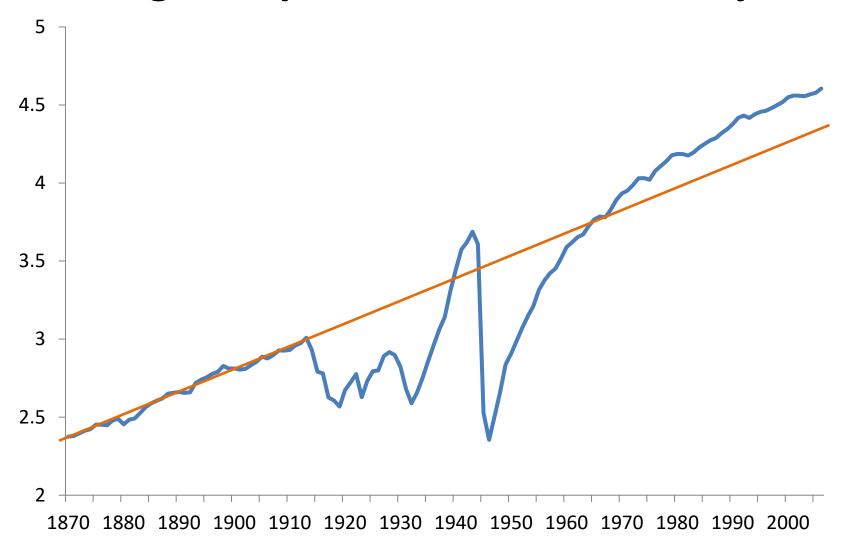
- 1948-1973: Rapid TFP growth
- Post-1973: Productivity slowdown
- Late 1980's and early 1990's:
 "Productivity Paradox"
 - Major innovations in computing, fiber optics, etc.
 - Continued slow growth in TFP
 - Solow: "You can see the computer age every-where these days, except in the productivity statistics."

- Productivity speedup in mid 1990's
 - "New Economy"(internet, email made computers useful)
 - Paul David (1990): Delays between major innovation and productivity growth common.
 E.g. steam engine and electric dynamo.
- 2003-present: Second productivity slowdown

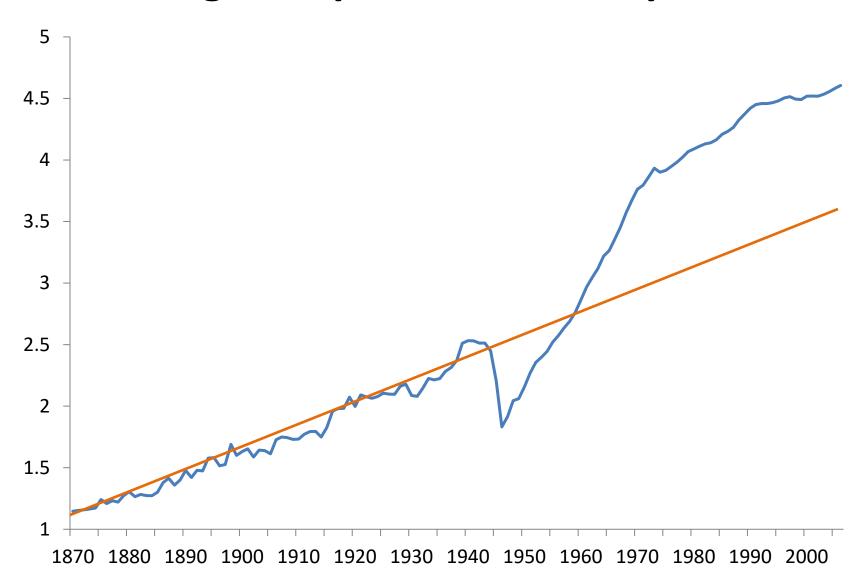
Log GDP per Person for Germany



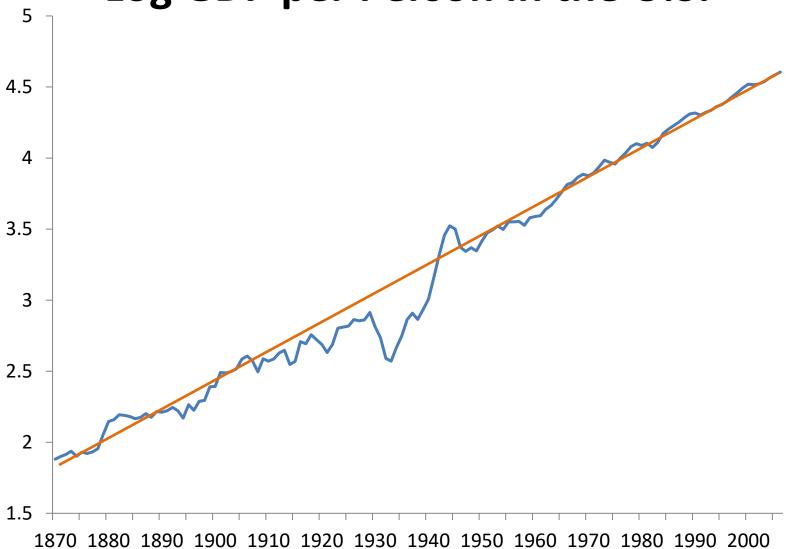
Log GDP per Person for Germany



Log GDP per Person in Japan

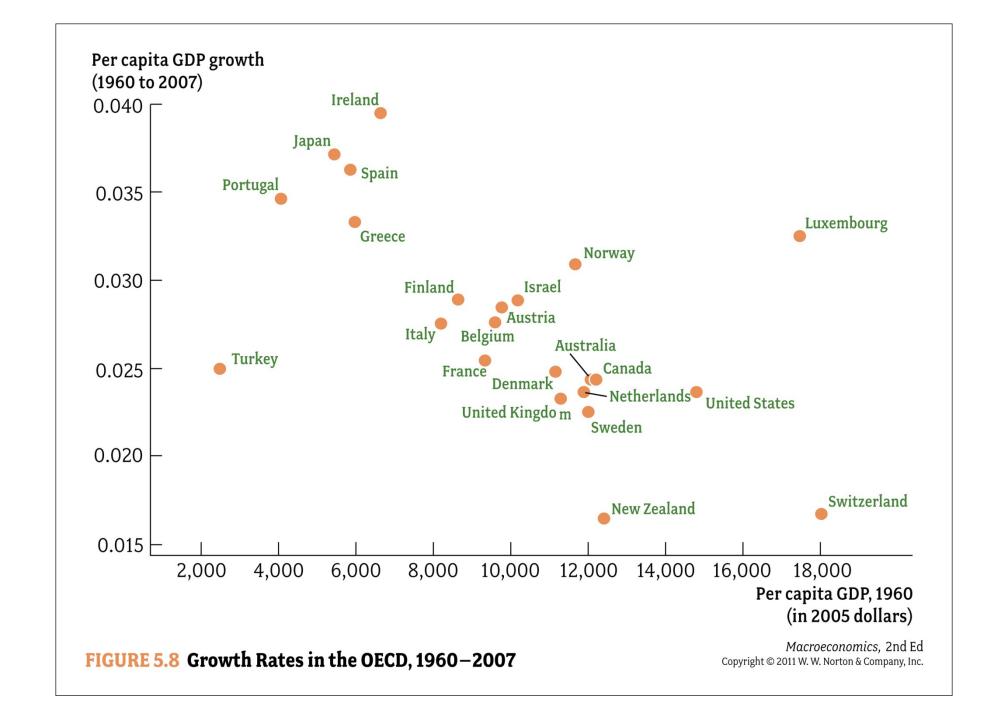


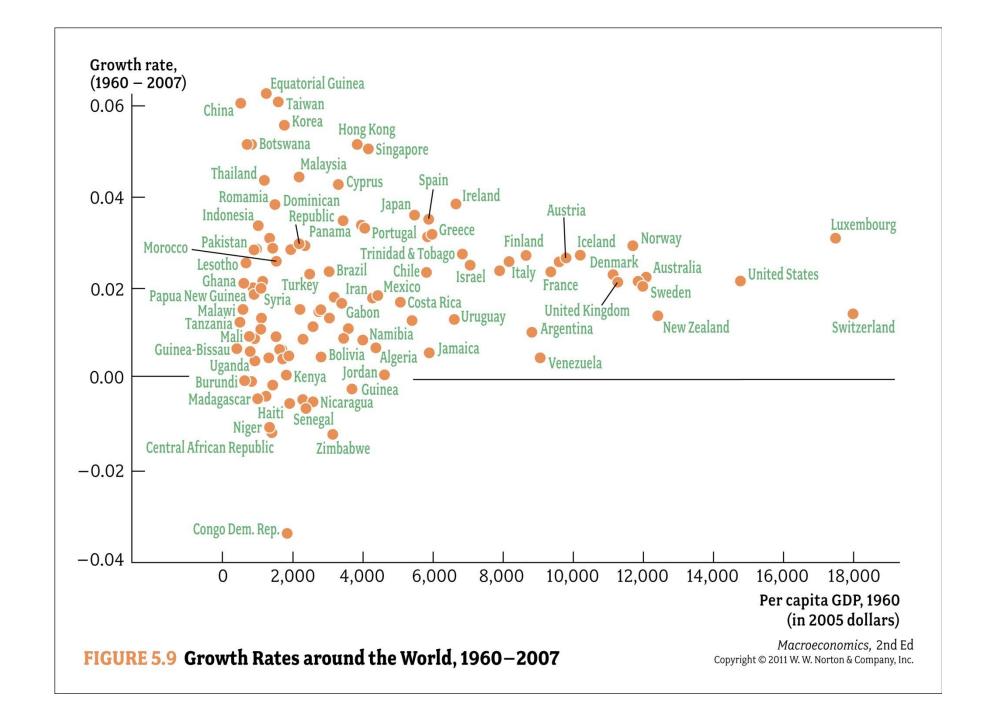
Log GDP per Person in the U.S.



Growth Miracles in the Solow Model

- What does the Solow model suggest as an explanation for high growth rates of e.g. Asian Tigers?
- In the Solow Model:
 - Countries below their steady state grow fast
 - Further below, faster you grow
 - Countries above their steady state grow slow (contract)
- Perhaps this may explain differences in growth rates across countries (e.g., fast growing East Asian Tigers)





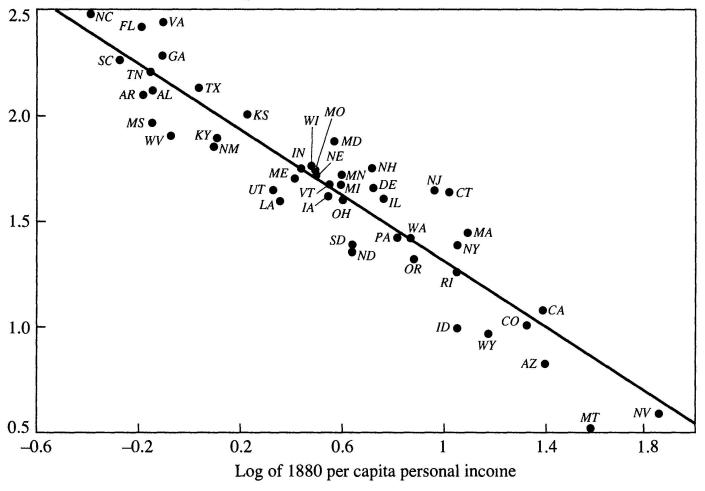
Convergence in the Solow Model

$$\bar{y} = \left(\frac{\bar{s}}{\bar{d} + \bar{n}}\right)^{1/2} \bar{A}^{3/2}$$

- Output should converge if \bar{s} , \bar{d} , \bar{n} , \bar{A} are the same
- Perhaps this is more true for OECD countries than countries more generally
- Where is it even more likely to be true?

Figure 1. Convergence of Personal Income across U.S. States: 1880 Income and Income Growth from 1880 to 1988

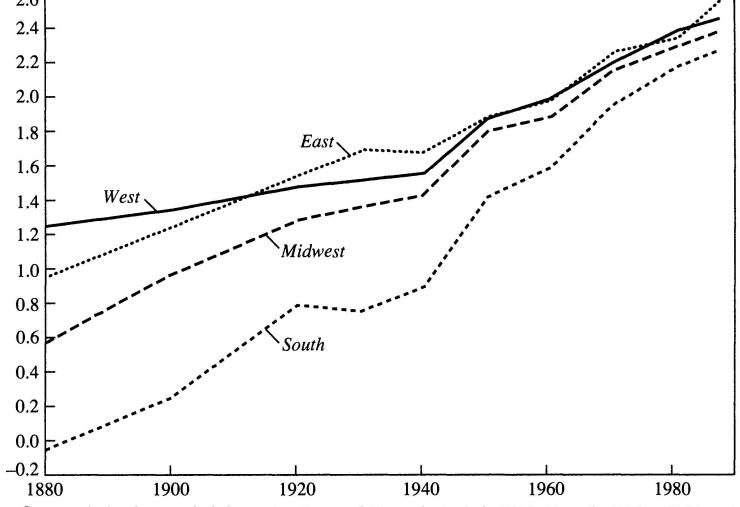
Annual growth rate, 1880–1988 (percent)



Sources: Bureau of Economic Analysis (1984), Easterlin (1960a, 1960b), and Survey of Current Business, various issues. The postal abbreviation for each state is used to plot the figure. Oklahoma, Alaska, and Hawaii are excluded from the analysis.

Figure 5. Personal Income of U.S. Regions, 1880–1988

Log of real per capita income 2.6



Sources: Authors' own calculations using Bureau of Economic Analysis (1984), Easterlin (1960a, 1960b), and Survey of Current Business, various issues. The data are plotted for 1880, 1900, 1920, every ten-year interval that follows, and 1988.

Barro and Sala-I-Martin (1991): Convergence Across States and Regions

Convergence

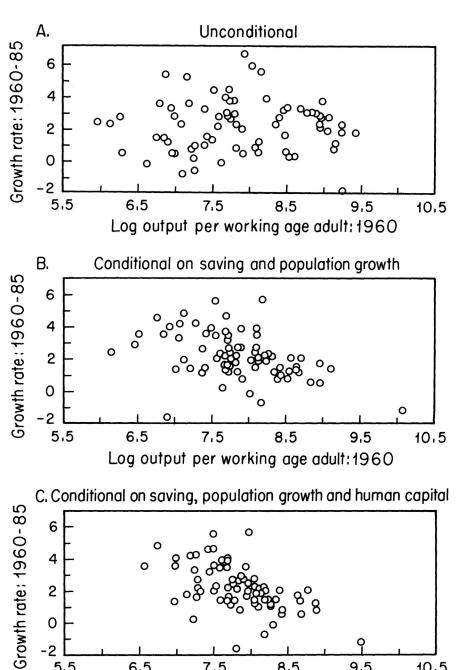
- Strong convergence across
 - OECD countries 1960-2000
 - U.S. states 1880-1990
 - European regions 1950-1990
 - Japanese prefectures 1955-1990

(Barro and Sala-I-Martin, 1999, Economic Growth)

Conditional Convergence

$$\bar{y} = \left(\frac{\bar{s}}{\bar{d} + \bar{n}}\right)^{1/2} \bar{A}^{3/2}$$

- What if \bar{s} , \bar{d} , \bar{n} , \bar{A} are not the same?
- We can control for differences in \bar{s} , \bar{d} , \bar{n} , \bar{A}
- Countries that are further from their steady state should grow faster



8.5

9.5

10.5

5.5

6.5

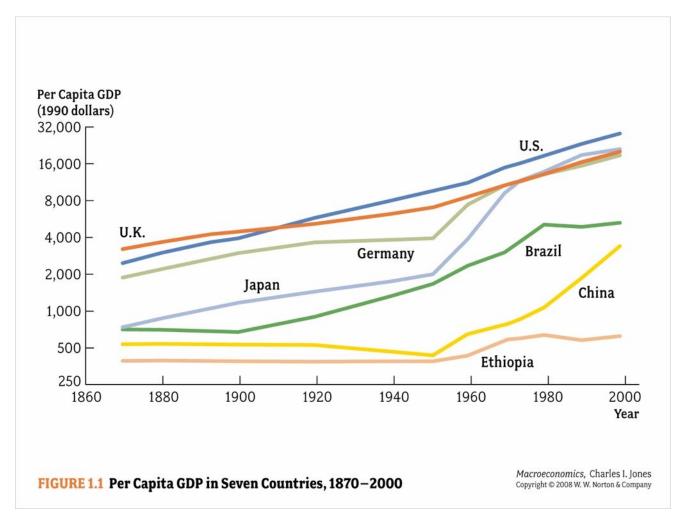
7.5

Log output per working age adult: 1960

Mankiw, Romer and Weil (1992): A Contribution to the Empirics of Economic Growth

Solow Model and Key Questions

- What determines the growth of the "frontier"?
 (Solow model?)
- Why are some countries so far behind the frontier? (Solow model?)



Solow Model and Key Questions

1. Growth in the frontier

- Capital accumulation not a source of long run growth
- Growth in the frontier due to changes in productivity

2. Behind the frontier

- Temporarily below due to history but converging
- Lower steady state (Why?)