Create Destruction: The Quality Ladder Model

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Creative Destruction

- Expanding variety model generates growth from new goods
- Much innovation improves existing goods
- Improved products often **replace** older products
- Schumpeter (1934) labeled this "creative destruction"
- Quality ladder model captures this
- Original version due to Aghion and Howitt (1992)

Quality Ladder Model

- Basic structure the same as expanding variety model
- Model has three classes of agents:
  - Households
  - Final-goods producing firms
  - Intermediate-goods producing / R&D firms
- For simplicity, we do the "lab-equipment" version of knowledge production function (Acemoglu, 2009, ch. 14.1)

HOUSEHOLDS

- Constant population of households that consume and supply labor
- Households supply an aggregate quantity L of labor inelastically
- Households own all firms in equal proportions
- Household utility

$$U = \int_0^\infty \exp(-\rho t) \frac{C(t)^{1-\theta}}{1-\theta} dt$$

- As in Ramsey model, household optimization yields:

$$\frac{\dot{C}(t)}{C(t)} = \frac{1}{\theta}(r(t) - \rho)$$

Final Goods Producing Firms

- Final goods are produced in a perfectly competitive market with the production function

$$Y(t) = \frac{1}{1-\beta} L^{\beta} \int_0^1 q(i,t) x(i,t)^{1-\beta} di$$

- Differences versus expanding variety model:
  - Measure 1 of intermediate inputs as opposed to $N(t)$
  - All labor L used for final goods (knowledge produced with final goods)
  - Each intermediate input has a quality level $q(i,t)$
    (More on functional form assumption later)

Quality Ladder

- Quality of good i evolves according to:

$$q(i,t) = \lambda^{n(i,t)} q(i,0)$$

- $n(i,t)$ is number of improvements of product i between time 0 and time t
- $\lambda > 1$ is the size of each quality improvement
- There is a "quality ladder"
- Each improvement raises quality by "one rung" on the ladder
- Growth is the result of these quality improvements

Quality Ladder

- Different versions of same good are perfect substitutes
- In equilibrium, only leading-edge version will be used

  (more on this later / this was implicitly assumed in our notation)
- Higher quality versions replace ("destroy") previous vintages

Quality Ladder

- Timing of quality improvements in each product line is random (but influenced by resources devoted to innovation)
- $n(i,t)$ is therefore a random variable
- $q(i,t)$ is a also a random variable
- This randomness of innovation in each product line washes out in the aggregate due to the law of large numbers
- Aggregate output will thus not be stochastic

Final Goods Production Function

- Notice that production function can also be written

$$Y(t) = \frac{1}{1-\beta} L^{\beta} \mathbf{X}(t)^{1-\beta}$$

where

$$\mathbf{X}(t) = \left[\int_0^1 q(i,t) x(i,t)^{\frac{\phi-1}{\phi}} di \right]^{\frac{\phi}{\phi-1}}$$

and $\phi = 1/\beta$

- So, intermediate input part of production function takes Dixit-Stiglitz form (with a quality twist)

Final Goods Producing Firms

- Final goods firms maximize profits

$$\Pi = \frac{1}{1-\beta} L^{\beta} \int_0^1 q(i,t) x(i,t)^{1-\beta} di - \int_0^1 p(i,t) x(i,t) di - w(t)L$$

where $p(i,t)$ is the price of intermediate input $x(i,t)$

(the highest quality version)

- Intermediate input demand:

$$L^{\beta} q(i,t) x(i,t)^{-\beta} - p(i,t) = 0$$

and rearranging:

$$x(i,t) = \left(\frac{q(i,t)}{p(i,t)} \right)^{1/\beta} L$$

- Labor demand:

$$\beta \frac{Y(t)}{L} = w(t)$$

Intermediate Goods Producers / R&D Firms

- Free entry into developing improved version of each product line
- Both incumbent firm and new firms can innovate (more on this later)
- Once a firm develops a new version, it has a monopoly on producing **that version**, but must potentially compete with older (and eventually newer) versions
- Marginal cost of producing version of quality $q(i, t)$ is $\psi q(i, t)$
- Let's start by considering the pricing decision of leading-edge version

Pricing of Leading-Edge Firm

Two cases:

- Large innovation:
  - Leading-edge firm can set monopoly price without facing competition from lower quality competitors
- Modest innovation:
  - Leading-edge firm must take account of potential competition from second highest quality firm
  - Leading-edge firm will “limit price”: Set highest price that is still too low for second highest quality firm to produce profitably

Monopoly Pricing by Leading-Edge Firm

- Let's start by calculating the monopoly price
- Flow profits

$$\Pi(i,t) = p(i,t)x(i,t) - q(i,t)\psi x(i,t)$$

- Let's plug in the demand curve: (and drop the (i,t)'s)

$$\Pi = p\left(\frac{q}{p}\right)^{1/\beta} L - q\psi\left(\frac{q}{p}\right)^{1/\beta} L$$

- Differentiating and setting to zero:

$$\left(1-\frac{1}{\beta}\right)p^{-1/\beta} + \frac{1}{\beta}q\psi p^{-1/\beta-1} = 0$$

- Rearranging:

$$p = (1-\beta)^{-1}\psi q$$

Can Leading-Edge Firm Set Monopoly Price?

- What matters for buyer (final goods firm) is not price but **price per unit quality**
- We need to know both price and "marginal product" of each version
- This is a bit tricky given the way Acemoglu sets up the model
  (We are following ch. 14.1 of Acemoglu (2009))

Can Leading-Edge Firm Set Monopoly Price?

- How do we compare marginal product of different versions of good i that have different levels of quality?
- Production Function:

$$Y(t) = \frac{1}{1-\beta} L^{\beta} \int_0^1 q(i,t) x(i,t)^{1-\beta} di$$

- Since they are perfect substitutes, they should enter linearly
- Let's rewrite the production function:

$$Y(t) = \frac{1}{1-\beta} L^{\beta} \int_0^1 \left(q(i,t)^{\frac{1}{1-\beta}} x(i,t) \right)^{1-\beta} di$$

- Written this way, the different versions enter linearly

Can Leading-Edge Firm Set Monopoly Price?

- If leading-edge version has quality q, then second-best version has quality $\lambda^{-1} q$ (one rung lower)
- If both are being produced they enter production function as:

$$\lambda^{\frac{-1}{1-\beta}} q^{\frac{1}{1-\beta}} x_2 + q^{\frac{1}{1-\beta}} x_1$$

  where x_2 is quantity of second-best version and x_1 is quantity of leading-edge version

- The marginal product of the second best firm is lower by a factor $\lambda^{\frac{-1}{1-\beta}}$

Can Leading-Edge Firm Set Monopoly Price?

- Lowest price second-best firm can offer is its marginal cost $\lambda^{-1}q\psi$
- Leading-edge firm can set monopoly price if **ratio of this price to marginal product** is lower than ratio of marginal cost to marginal product for second best firm:

$$(1-\beta)^{-1}q\psi < \frac{\lambda^{-1}q\psi}{\lambda^{\frac{-1}{1-\beta}}}$$

  denominator on RHS is adjustment for difference in marginal product

- Simplifying then yields:

$$\lambda > \left(\frac{1}{1-\beta}\right)^{\frac{1-\beta}{\beta}}$$

Can Leading-Edge Firm Set Monopoly Price?

Summary:

- Leading-edge firm can set monopoly price if

$$\lambda > \left(\frac{1}{1-\beta}\right)^{\frac{1-\beta}{\beta}}$$

  In this case, quality difference is big enough that second-best firm can't compete even when leading-edge firm sets monopoly price

- Otherwise leading-edge firm must set a lower price (low enough to drive second-best firm out of the market)

Who Innovates?

- Both current leading-edge firms and others can undertake R&D to invent higher quality products
- They however face different incentives to do so
- Incumbent will "cannibalize" prior profits
  - Change in profits for incumbent is new profit level less old profit level
- Change in profits for new leader is entire new profit level
- If both have same cost of innovating, incumbents will not innovate

Who Innovates?

- In reality, incumbents do a lot of innovation
- Incumbents may have a cost advantage

  (i.e., it may be easier to incumbents to improve products)
- Perhaps incumbents can act as Stackelberg leaders
  - Commit to a certain amount of innovation
  - Thereby discourage innovation by others
  - See Barro and Sala-i-Martin (2004, ch. 7.1)
- See also models in Acemoglu (2009, ch. 14.3-14.4)

Aggregate Output

- Two maintained assumptions:
  - Leading-edge firm sets monopoly price (rung size large enough)
  - All innovation by new firms
- Normalize $\psi = 1 - \beta$
- Monopoly price then becomes

$$p(i,t) = (1-\beta)^{-1}\psi q(i,t) = q(i,t)$$

- Output for good i becomes

$$x(i,t) = \left(\frac{q(i,t)}{p(i,t)}\right)^{1/\beta} L = L$$

Aggregate Output

- Aggregate output becomes

$$Y(t) = \frac{1}{1-\beta} L^{\beta} \int_0^1 q(i,t) x(i,t)^{1-\beta} di$$

which simplifies to

$$Y(t) = \frac{1}{1-\beta} Q(t) L$$

where

$$Q(t) = \int_0^1 q(i,t) di$$

- Economic growth comes from growth in quality of intermediate inputs
- $Q(t)$ plays same role here as $N(t)$ in expanding variety model

How Much Innovation Is There?

We must compare:

- Cost of making an innovation
- Value of an innovation once made

We use the fact that on a balanced growth path:

- Interest rate r will be constant
- Rate of innovations in each product line z^* is constant

Knowledge Production Function

- If firm spends $Z(i,t)$ on R&D it generates innovations at at flow rate:

$$\frac{\eta Z(i,t)}{q(i,t)}$$

- Implicitly uses existing know-how (researches an improvement)
- Innovating gets more costly the larger is $q(i,t)$

  (but each rung is larger since they are proportional)
- Cost is final output not labor (“lab-equipment model”)

Value of Innovation Once Made

- Flow profits:

$$\begin{aligned} \Pi(i,t) &= p(i,t)x(i,t) - q(i,t)\psi x(i,t) \\ &= q(i,t)L - q(i,t)(1-\beta)L \\ &= \beta q(i,t)L \end{aligned}$$

- Present value of profits:

$$V(i,t) = \frac{\beta q(i,t)L}{r+z^*}$$

- "Effective discount rate" of profits $r + z^*$

Free Entry into Innovation

- Free entry into innovation implies that marginal value of innovation must equal marginal cost
- Consider spending one more unit of final good on innovation
- Marginal cost: 1
- Marginal value: $\eta V/(\lambda^{-1} q)$
  - If successful: V
  - Flow rate of success per unit spent: $\eta/(\lambda^{-1} q)$
- Setting marginal value equal to marginal cost:

$$V(i,t)\frac{\eta}{\lambda^{-1} q(i,t)} = 1$$

(I am not quite sure about the λ^{-1} factor. But I am following Acemoglu on this point.)

Solving for Growth Rate

- Present value of profits:

$$V(i,t) = \frac{\beta q(i,t)L}{r+z^*}$$

- Free entry implies:

$$V(i,t)\frac{\eta}{\lambda^{-1}q(i,t)} = 1$$

- Combining these yields:

$$r + z^* = \lambda\eta\beta L$$

Solving for Growth Rate

- Consumption Euler equation:

$$\frac{\dot{C}(t)}{C(t)} = \frac{1}{\theta}(r(t) - \rho)$$

- Interest rate is constant on BGP
- Consumption growth must equal output growth on BGP
- Consumption Euler equation thus implies:

$$g = \frac{1}{\theta}(r - \rho)$$

Solving for Growth Rate

- We need equation relating g to z^*

$$Y(t) = \frac{1}{1-\beta} Q(t) L \qquad \text{implies} \qquad \frac{\dot{Y}(t)}{Y(t)} = \frac{\dot{Q}(t)}{Q(t)}$$

- z^* is rate of innovation on each product line
- Over interval Δt a fraction $z^* \Delta t$ of sectors experience innovation
- This implies (up to first order):

$$Q(t+\Delta t) = \lambda Q(t) z^* \Delta t + Q(t)(1 - z^* \Delta t)$$

Solving for Growth Rate

$$Q(t + \Delta t) = \lambda Q(t) z^* \Delta t + Q(t)(1 - z^* \Delta t)$$

- Subtracting $Q(t)$ from both sides, dividing by Δt, and taking limit $\Delta t \leftarrow 0$ yields

$$\dot{Q}(t) = (\lambda - 1) z^* Q(t)$$

which in turn implies that

$$g = (\lambda - 1) z^*$$

Solving for Growth Rate

- So we have:

$$r + z^* = \lambda\eta\beta L$$

$$g = \frac{1}{\theta}(r - \rho)$$

$$g = (\lambda - 1)z^*$$

- Combining these equations yields

$$g = \frac{\lambda\eta\beta L - \rho}{\theta + (\lambda - 1)^{-1}}$$

- Qualitatively similar to expanding variety model

  (i.e., model has strong scale effects)

Is Growth Rate Optimal?

- No!
- **Appropriability**: Monopolist cannot appropriate full social value of its invention. Therefore innovates too little
- **R&D Externality**: Inventor doesn't take into account that new knowledge (higher $N(t)$) raises the productivity of future invention. Therefore innovates too little
- **Business Stealing**: Part of profits from innovation are "stolen" from exiting incumbent. Private value of innovation larger than social value
- Growth rate can be either too low or too high
  (See Acemoglu (2009, ch 14.1.4 for derivations)

Political Economy with Creative Destruction

- Consider a tax on R&D
- The tax will reduce R&D and thus reduce innovation and growth
- The tax will therefore benefit incumbents!

  (Longer until they loose their leadership position and profit flow)
- Incumbents have an incentive to lobby for growth-retarding policy