CAPITAL ACCUMULATION AND GROWTH: 
THE SOLOW MODEL

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The paper is divided broadly into two parts. First, I present the facts related to the growth of the “frontier” over time: what are the growth patterns exhibited by the richest countries in the world? Second, I focus on the spread of economic growth throughout the world. To what extent are countries behind the frontier catching up, falling behind, or staying in place? And what characteristics do countries in these various groups share?

1. GROWTH AT THE FRONTIER

We begin by discussing economic growth at the “frontier.” By this I mean growth among the richest set of countries in any given time period. For much of the last century, the United States has served as a stand in for the frontier, and we will follow this tradition.

1.1 Modern Economic Growth

Fig. 1 shows one of the key stylized facts of frontier growth: For nearly 150 years, GDP per person in the US economy has grown at a remarkably steady average rate of around 2% per year. Starting at around $3,000 in 1870, per capita GDP rose to more than $50,000 by 2014, a nearly 17-fold increase.

Beyond the large, sustained growth in living standards, several other features of this graph stand out. One is the significant decline in income associated with the Great Depression of the early 1930s. The recovery in the late 1930s and 1940s was associated with an explosive growth in productivity. The sudden increase in productivity growth is associated with the growth of the Industrial Revolution. These improvements in productivity were the result of technological advances and the development of new factories. The growth rate of productivity is an index of the rate of technological innovation.

**Fig. 1** GDP per person in the United States. Source: Data for 1929–2014 are from the U.S. Bureau of Economic Analysis, NIPA table 7.1. Data before 1929 are spliced from Maddison, A. 2008. Statistics on world population, GDP and per capita GDP, 1-2006 AD. Downloaded on December 4, 2008 from http://www.ggdc.net/maddison/.

Source: Jones (2016)
UNEVEN GROWTH ACROSS THE WORLD

GDP per person (ratio scale, 2017 dollars)

Source: The Maddison-Project, www.ggdc.net/maddison/. Observations are presented every decade after 1950 and less frequently before that as a way of smoothing the series.

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Source: Jones (2021)
UNEVEN GROWTH ACROSS THE WORLD

1978 raises China’s living standards to more than a factor of 25 over the benchmark level of $300 per year.

Fig. 22 shows the spread of growth since 1870 in an alternative way, by plotting incomes relative to the US level. A key fact that stands out when the data are viewed this way is the heterogeneity of experiences. Some countries like the United Kingdom, Argentina, and South Africa experience significant declines in their incomes relative to the United States, revealing the fact that their growth rates over long periods of time fell short of the 2% growth rate of the frontier. Other countries like Japan and China see large increases in relative incomes.

4.2 The Spread of Growth in Recent Decades

Fig. 23 focuses in on the last 30 years using the Penn World Tables 8.0 data, again showing GDP per person relative to the US. Several facts then stand out. First, incomes in the countries of Western Europe have been roughly stable, around 75% of the US level. It is perhaps surprising that countries like France, Germany, and the United Kingdom are this far behind the United States. Prescott (2004) observes that a large part of the difference is in hours worked: GDP per hour is much more similar in these countries, and it is the fact that work hours per adult are substantially lower in Western Europe that explains their lower GDP per person. Jones and Klenow (2015) note that in addition to the higher leisure, Western Europeans tend to have higher life expectancy and lower consumption inequality. Taking all of these factors into account in constructing a consumption-equivalent welfare measure, the Western European countries look


Source: Jones (2016)
Growth is a recent phenomenon!

Source: Clark (2010)
Figures like these often plotted on linear scale to make them more dramatic (hockey stick).

This is misleading.

Fluctuations before 1800 were large!

(Also Maddison data back thousands of years are “guestimates”)

Importance of a Log Scale
Big Picture Questions about Growth

- What sustains growth at the frontier?
  (Will it continue in the future?)

- Why are some countries so far behind the frontier?
  (What might help them close the gap?)

- Why did growth begin?

- Why was there no growth before Industrial Revolution?

We will focus on first two questions. (210A in the spring covers later two.)
Steinsson, J. (2021): “Malthus and Pre-Industrial Stagnation,”
draft textbook chapter.
https://eml.berkeley.edu/~jsteinsson/teaching/malthus.pdf

Revolution and Its Antecedents,” draft textbook chapter.
https://eml.berkeley.edu/~jsteinsson/teaching/
originsofgrowth.pdf
THREE TEXTBOOKS


The Solow Model
Is Capital Accumulation Key to Growth?

- Seems plausible!
- Conventional wisdom in 1950s: Yes!
- See discussion in Easterly (2002)
- Solow (1956) tackled this question
The Production Function

\[ Y(t) = F[K(t), A(t)L(t)] \]

- \( Y(t) \): Output at time \( t \)
- \( K(t) \): Capital stock at time \( t \)
- \( L(t) \): Labor supply at time \( t \)
- \( A(t) \): “effectiveness of labor” at time \( t \) (aka “productivity”)
Production Function

\[ Y(t) = F[K(t), A(t)L(t)] \]

- The model is dynamics
- Time is continuous
- Time only enters production function through inputs
- Productivity is “labor augmenting” (Harrod neutral)
- This last point is important for getting “balanced growth”
Kaldor (1963): As per capita income has risen

- The capital-output ratio has been roughly constant
- Real interest rates have no trend
- The labor and capital share of production have been roughly constant
Roughly Constant Capital-Output Ratio

Fig. 3 The ratio of physical capital to GDP. Source: Bureau of Economic Analysis Fixed Assets tables 1.1 and 1.2. The numerator in each case is a different measure of the real stock of physical capital, while the denominator is real GDP.

Source: Jones (2016)
Source: FRED. 3 month T-bill rate minus 12-month CPI inflation.
Roughly Constant Labor and Capital Shares

**Fig. 6** Capital and labor shares of factor payments, United States. Source: *The series starting in 1975 are from Karabarbounis, L., Neiman, B. 2014. The global decline of the labor share. Q. J. Econ. 129 (1), 61–103. [http://ideas.repec.org/a/oup/ajecon/v129y2014i1p61-103.html](http://ideas.repec.org/a/oup/ajecon/v129y2014i1p61-103.html)* and *measure the factor shares for the corporate sector, which the authors argue is helpful in eliminating issues related to self-employment. The series starting in 1948 is from the Bureau of Labor Statistics Multifactor Productivity Trends, August 21, 2014, for the private business sector. The factor shares add to 100%.*

Source: Jones (2016)
FORMS OF TECHNICAL PROGRESS

- Hicks Neutral:
  \[ A(t)F[K(t), L(t)] \]
  (Ratio of marginal products remains constant for a given \( K/L \) ratio)

- Harrod Neutral / Labor-Augmenting:
  \[ F[K(t), A(t)L(t)] \]
  (Ratio of input shares \( (F_K K / F_L L) \) remain constant for a given \( K/Y \) ratio)

- Solow Neutral / Capital-Augmenting:
  \[ F[A(t)K(t), L(t)] \]
  (Ratio of input shares \( (F_K K / F_L L) \) remain constant for a given \( L/Y \) ratio)

- In general: Some combination of all three
UZAWA’S (1961) THEOREM

Roughly speaking:

- Balanced growth in the long run is only possible if all technical progress is labor augmenting

  (See Acemoglu (2009, sec. 2.7) and Barro-Sala-I-Martin (2004, sec. 1.2.12) for details)

Why balanced growth:

- Empirically: We see a stable $K/Y$ ratio and relatively stable factor shares
- Theoretically: Very convenient because model will have a steady state when technical progress is constant
Acemoglu (2009, p. 59):

This result is very surprising and troubling, since there are no compelling reasons for why technological progress should take this form. [i.e., be labor augmenting]
Much technology seems capital-embodied

- Textiles: spinning jenny, water frame, mule, mechanized weaving
- Power: windmill, water wheel, steam engine, electric motor
- Transportation: trains, cars, trucks, airplanes
- Agriculture: tractors, combine harvester, fertilizer
- Computing: abacus, transistor, microprocessor

At the micro level, much technology seems capital-embodied
Present a model where all technology is
- Purely capital-embodied at the micro level
- Purely labor-augmenting at the macro level

Task-based production function (Zeira 98, Acemoglu-Restrepo 18)

Two kinds of innovation:
- More tasks performed by capital (increases capital share)
- Innovation on already automated tasks (decreases capital share)
  - Innovation reduced price of that task
  - If tasks are complements, this reduces spending on that task

Combination of the two can yield stable capital share
The Cobb-Douglas production function satisfies all three properties

- **Hicks Neutral:**
  \[ A(t)K(t)^\alpha L(t)^{1-\alpha} \]

- **Harrod Neutral:**
  \[ K(t)^\alpha [\tilde{A}(t)L(t)]^{1-\alpha} \text{ where } \tilde{A}(t) = A(t)^{1/(1-\alpha)} \]

- **Solow Neutral:**
  \[ [\tilde{A}(t)K(t)]^\alpha L(t)^{1-\alpha} \text{ where } \tilde{A}(t) = A(t)^{1/\alpha} \]
**Definition:** A function $f$ is homogeneous of degree $m$ in $x$ and $y$ if

$$f(\lambda x, \lambda y, z) = \lambda^m f(x, y, z)$$

- $m < 1$: decreasing returns to scale
- $m = 1$: constant returns to scale
- $m > 1$: increasing returns to scale
**Euler’s Theorem**

*Euler’s Theorem*: If $f$ is homogeneous of degree $m$ in $x$ and $y$:

$$mf(x, y, z) = \frac{\partial}{\partial x} f(x, y, z)x + \frac{\partial}{\partial y} f(x, y, z)y$$

(See Acemoglu (2009, p. 29) for a more careful statement of this theorem.)
We assume that the production function is constant returns to scale:

\[ F(cK, cAL) = cF(k, AL) \]

Why?
We assume that the production function is constant returns to scale:

\[ F(cK, cAL) = cF(k, AL) \]

Why?

- Economy large enough that each establishment has reached efficient size (micro returns to scale and gains from specialization exhausted)
- Fixed factors (e.g., land) unimportant
- Positive and negative externalities between establishments unimportant
- \( A(t) \) non-rival (can be used many times)
- Replication argument: Can build a second identical establishment with double the inputs
Since

\[ F(cK, cAL) = cF(K, AL) \]

we can write production function in intensive form:

\[ \frac{Y}{AL} = \frac{1}{AL} F(K, AL) = F \left( \frac{K}{AL}, 1 \right) \]

Define:

- \( k = K/AL \): Capital per effective worker
- \( y = Y/AL \): Output per effective worker

Also define: \( f(k) = F(k, 1) \)

Then we have:

\[ y = f(k) \]

(Why do this? ... Will become clear in a few slides.)
What do we want to assume about returns to capital?
What do we want to assume about returns to capital?

Returns to capital are ...

- Positive: \( f'(k) > 0 \)
- Diminishing: \( f''(k) < 0 \)

Also ...

- \( f(0) = 0 \)
- Inada conditions:
  
  \[
  \lim_{k \to 0} f'(k) = \infty \quad \text{and} \quad \lim_{k \to \infty} f'(k) = 0
  \]
The intensive-form production function, $f(k)$, is assumed to satisfy $f(0) = 0$, $f'(k) > 0$, and $f''(k) < 0$. Since $F(K, AL)$ equals $ALf(K/AL)$, it follows that the marginal product of capital, $aF(K, AL)/\partial K$, equals $ALf''(K/AL(1/AL)$, which is just $f''(k)$. Thus the assumptions that $f(k)$ is positive and $f''(k)$ is negative imply that the marginal product of capital is positive, but that it declines as capital (per unit of effective labor) rises. In addition, $f()$ is assumed to satisfy the Inada conditions (Inada, 1964): 

$$\lim_{k \to 0} f'(k) = \infty, \quad \lim_{k \to \infty} f'(k) = 0.$$ 

These conditions (which are stronger than needed for the model's central results) state that the marginal product of capital is very large when the capital stock is sufficiently small and that it becomes very small as the capital stock becomes large; their role is to ensure that the path of the economy does not diverge. A production function satisfying $f''(k) < 0$, and the Inada conditions is shown in Figure 1.1.

A specific example of a production function is the Cobb-Douglas production function:

$$F(K, AL) = K^a(1/AL)^{1-a}, \quad 0 < a < 1.$$ 

This production function is easy to analyze, and it appears to be a good approximation to actual production functions. As a result, the notation $f'(k)$ denotes the first derivative of $f(k)$, and $f''(k)$ denotes the second derivative.
If the production function is Cobb-Douglas, we have

\[ y = \frac{Y}{AL} = \frac{1}{AL}K^\alpha (AL)^{1-\alpha} = \left( \frac{K}{AL} \right)^\alpha = k^\alpha. \]

So, we have:

\[ y = k^\alpha \]

This function satisfies all the conditions we have specified on previous slides.
CAMBRIDGE CAPITAL CONTROVERSY

- Early post-WWII debate between (mostly) British and (mostly) US economists
- Does it make sense to talk about aggregate capital?
- Do lower interest rates lead to higher capital/labor ratios?
- Outcome:
  - Various pathologies possible
  - Similar to Giffen goods in consumption theory
  - Not clear any of this is practically important
Cambridge Capital Controversy

Cambridge, U.K.:


Cambridge, U.S.:

Output is divided between consumption and investment:

\[ Y(t) = C(t) + I(t) \]

How much is invested?

Simplifying assumption: Constant savings rate

\[ I(t) = sY(t) \]

(We will introduce optimizing households in Ramsey model)
Evolution of Capital

\[ \dot{K}(t) = I(t) - \delta K(t) \]
\[ = sY(t) - \delta K(t) \]

\((\dot{K}(t) = dK(t)/dt)\)

- Each instant:
  - New investment adds to capital stock
  - Existing capital depreciates by some fraction (per unit time)

- Change in capital stock is the difference between these two
Labor and productivity grow at constant rates:

\[ \dot{L}(t) = nL(t) \]
\[ \dot{A}(t) = gA(t) \]

Notice that

\[ \frac{d \log X(t)}{dt} = \frac{d \log X(t)}{dX(t)} \frac{dX(t)}{dt} = \frac{\dot{X}(t)}{X(t)} \]

where \( \log \) denotes the natural log

\[ \frac{d \log L(t)}{dt} = \frac{\dot{L}(t)}{L(t)} = n \]
\[ \log L(t) = \log L(0) + nt \]
\[ L(t) = L(0)e^{nt} \]

and similarly for \( A(t) \).
**Full Solow Model**

\[ Y(t) = F[K(t), A(t)L(t)] \]

\[ Y(t) = C(t) + I(t) \]

\[ I(t) = sY(t) \]

\[ \dot{K}(t) = I(t) - \delta K(t) \]

\[ \dot{L}(t) = nL(t) \]

\[ \dot{A}(t) = gA(t) \]

- Initial Conditions: \( K(0), A(0), L(0) \) given
- Goal: Solve for evolution of \( K(t), Y(t), C(t), I(t), L(t), A(t) \)
Solow model is a gross simplification

Not necessarily a defect

Real world is fully realistic, but too complicated to understand

Simple models can provide insight about specific issues

But may cause “theory-induced blindness”

Kahneman: “Once you have accepted a theory, it is extraordinarily difficult to notice its flaws.”

Fully realistic model not insightful but would allow for calculation of counterfactuals and the analysis of policy experiments
Two uses of models:

- Provide insight about mechanisms
  - Such models must be (relatively) simple
  - Unlikely to be good guides to real-world counterfactuals

- Provide a basis for policy evaluation
  - Such models need not be insightful
  - But they must be “realistic”

Important to keep this distinction clear
Finding a Steady State

- When solving a dynamic system of equations, often useful to find a steady state
- A **stable** steady state is a point the system stays at if unperturbed and returns to if perturbed by a small amount
- Since $L(t)$ and $A(t)$ are growing, no steady state in the original variables
- Key to finding a steady state to work with transformed variables:

$$y(t) = \frac{Y(t)}{A(t)L(t)} \quad k(t) = \frac{K(t)}{A(t)L(t)}$$
Dynamics of $k(t)$

Using the chain rule we have that

$$
\dot{k}(t) = \frac{\dot{K}(t)}{A(t)L(t)} - \frac{K(t)}{[A(t)L(t)]^2} [\dot{A}(t)L(t) + A(t)\dot{L}(t)]
$$

$$
= \frac{\dot{K}(t)}{A(t)L(t)} - \frac{K(t)}{A(t)L(t)} \frac{\dot{L}(t)}{L(t)} - \frac{K(t)}{A(t)L(t)} \frac{\dot{A}(t)}{A(t)}
$$

Using $\dot{L}/L = n$, $\dot{A}/A = g$, and $\dot{K} = sY - \delta K$ we have that

$$
\dot{k}(t) = \frac{sY(t) - \delta K(t)}{A(t)L(t)} - nk(t) - gk(t)
$$

Using $y = f(k)$ we have that

$$
\dot{k}(t) = sf(k(t)) - (n + g + \delta)k(t)
$$
Dynamics of $k(t)$

$$\dot{k}(t) = sf(k(t)) - (n + g + \delta)k(t)$$

- Rate of change of $k(t)$ difference between:
  - Actual investment: $sf(k(t))$
  - Break-even investment: $(n + g + \delta)k(t)$

- Notice that break-even investment determined by:
  - Population growth: $n$
  - Productivity growth: $g$
  - Depreciation: $\delta$

- Intuition: capital per effective worker must keep up with amount of effective labor (which is growing due to $n$ and $g$)
The Dynamics of the Model

Actual investment: \( k \)

Break-even investment: \( (n+g+\delta)k \)

FIGURE 1.2 Actual and break-even investment

- \( k^* \) is the value of \( k \) (other than zero) where actual investment and break-even investment are equal.
- Regardless of where \( k \) starts, it converges to \( k^* \) and remains there.

The Balanced Growth Path

Since \( k \) converges to \( k^* \), it is natural to ask how the variables of the model behave when \( k \) equals \( k^* \). By assumption, labor and knowledge are growing at rates \( n \) and \( g \), respectively. The capital stock, \( K \), equals \( ALk \); since \( k \) is constant at \( k^* \), \( K \) is growing at rate \( n+g \) (that is, \( K/K \) equals \( n+g \)). With both capital and effective labor growing at rate \( n+g \), the assumption of Constant returns implies that output, \( Y \), is also growing at that rate. Finally, capital per worker, \( K/L \), and output per worker, \( Y/L \), are growing at rate \( g \).

Source: Romer (2019)
Thus the Solow model implies that, regardless of its starting point, the economy converges to a balanced growth path—a situation where each variable of the model is growing at a constant rate. On the balanced growth path, the growth rate of output per worker is determined solely by the rate of technological progress.

1.4 The Impact of a Change in the Saving Rate

The parameter of the Solow model that policy is most likely to affect is the saving rate. The division of the government's purchases between consumption and investment goods, the borrowing, and its tax revenues between taxes and...
Economy Converges to a Steady State $k^*$

- Inada conditions and $f''(k) < 0$ imply that actual investment and break-even investment lines cross once (with actual investment crossing from above)
- This point is denoted $k^*$

- $k^*$ is a steady state for $k(t)$

- Economy converges to $k^*$ globally (i.e., from any (positive) starting point)
At steady state $k(t)$ is constant.

This implies that $K = ALk$ grows at a rate $n + g$.

Since both $K$ and $AL$ grow at $n + g$, $Y$ also grows at rate $n + g$.

Furthermore, $K/L$ and $Y/L$ grow at rate $g$.

Economy converges to a balanced growth path.

These conclusions flow from the fact that the growth rate of the product of two variables is the sum of their growth rates. See, Problem 1.1 in Romer (2019).
Capital accumulation cannot serve as a source of long-run growth in living standards
  - If $g = 0$, growth in $Y/L$ is zero

Why?
First Lesson from Solow Model

- Capital accumulation cannot serve as a source of long-run growth in living standards
  - If $g = 0$, growth in $Y/L$ is zero
- Why? Because of diminishing returns to capital.
  - Diminishing returns mean actual investment eventually cannot keep up with break-even investment
  - This gives rise to a steady state with property listed above
- Long-run growth must come from $A(t)$
One can use the Solow model to think about changes in:

- The savings rate $s$
- The population growth rate $n$
- The growth rate of technology $g$
- The depreciation rate $\delta$

Such exercises are “other things equal” type exercises.

Let’s consider a permanent increase in the savings rate.

How does this affect actual and break-even investment curves?
The increase in the saving rate shifts the actual investment line upward, and so $k^*$ rises. This is shown in Figure 1.4. But $k$ does not immediately jump to the new value of $k^*$. Initially, $k$ is equal to the old value of $k^*$. At this level, actual investment now exceeds break-even investment—more resources are being devoted to investment than are needed to hold $k$ constant—and so $k$ is positive. Thus $k$ begins to rise. It continues to rise until it reaches the new value of $k^*$, at which point it remains constant. These results are summarized in the first three panels of Figure 1.5.

To denote the time of the increase in the saving rate. By assumption, $s$ jumps up at time $t_0$ and remains constant thereafter. Since the jump in $s$ causes actual investment to exceed break-even investment by a strictly positive amount, $k$ jumps from zero to a strictly positive amount. $k$ rises gradually from the old value of $k$ to the new value, and $k$ falls gradually back to zero.2

For a sufficiently large rise in the saving rate, $k$ can rise for a while after $t_0$ before starting to fall back to zero.

Source: Romer (2019)
We are likely to be particularly interested in the behavior of output per worker, $Y/L$. $Y/L$ equals $A(f(k))$. When $k$ is increasing, $Y/L$ grows both because $A$ is increasing and because $k$ is increasing. Thus its growth rate exceeds $g$.

Source: Romer (2019)
We are likely to be particularly interested in the behavior of output per worker, $Y/L$. $Y/L$ equals $A(t)k(t)$. When $k(t)$ is increasing, $Y/L$ grows both because $A(t)$ is increasing and because $k(t)$ is increasing. Thus its growth rate exceeds $g$.

Source: Romer (2019)
Increase in Savings Rate

- Increase in savings rate has a “level effect” on per capita output
- It does not have a “growth effect”
Suppose half the capital stock of a country is destroyed

What does Solow model predict about output

In the short run?
In the long run?
DESTRUCTION OF CAPITAL

Suppose half the capital stock of a country is destroyed

What does Solow model predict about output
  - In the short run?
  - In the long run?

When has this happened in the real world?
Log GDP per Person for Germany
Log GDP per Person for Germany
Transition Dynamics
Our focus has been on long run effects

Solow model also has interesting implications about “short run”
Transition Dynamics in the Solow Model

- Start with

\[ \dot{k}(t) = sf(k(t)) - (n + g + \delta)k(t) \]

- Divide by \( k(t) \):

\[ \frac{\dot{k}(t)}{k(t)} = \frac{sf(k(t))}{k(t)} - (n + g + \delta) \]

- Left-hand-side is growth rate of capital

- \((n + g + \delta)\) is constant as a function of \( k(t) \)

- While

\[ \lim_{k \to 0} \frac{sf(k(t))}{k(t)} = \infty \quad \lim_{k \to \infty} \frac{sf(k(t))}{k(t)} = 0 \]

\[ \frac{d}{dk} \frac{sf(k)}{k} = -s \frac{f(k)/k - f'(k)}{k} < 0 \]

(numerator is average product of capital minus marginal product of capital)
**Transition Dynamics**

Growth rate $> 0$
- $n + \delta$

Growth rate $< 0$
- $s \cdot f(k)/k$

$k(0)_{poor}$ \quad $k(0)_{rich}$ \quad $k^*$

Source: Barro and Sala-I-Martin (2004). Figure is for $g = 0$. Adding $g > 0$ would just shift up horizontal line.
Differentiate \( y(t) = f(k(t)) \) with respect to \( t \)

\[
\dot{y}(t) = f'(k(t)) \dot{k}(t)
\]

Divide through by \( y(t) \):

\[
\frac{\dot{y}(t)}{y(t)} = \frac{f'(k(t))k(t)}{f(k(t))} \frac{\dot{k}(t)}{k(t)}
\]

Let \( g_x \) denote the growth rate of \( x_t \) and \( \alpha_K(k(t)) = f'(k(t))k(t)/f(k(t)) \)

\[
g_y = \alpha_K(k(t)) g_k
\]

(\( \alpha_K(k(t)) \) is the elasticity of output with respect to capital.)

Growth rate of output is proportional to growth rate of capital
Second Lesson from Solow Model

- Countries that are below their steady state level of capital/output should grow faster than countries that are above their steady state.

- If countries share same fundamentals, Solow model predicts *absolute convergence* 

- More generally, Solow model predicts *conditional convergence*
Analyzed data for 16 industrialized countries for which long historical data were available

Estimated:

$$\log \tilde{y}_{i,1979} - \log \tilde{y}_{i,1870} = a + b \log \tilde{y}_{i,1870} + \epsilon_i$$

where $\tilde{y}_{i,t}$ denotes output per person in country $i$ at time $t$

Negative $b$ indicates convergence (initial poor grow faster)
FIGURE 1.7 Initial income and subsequent growth in Baumol's sample (from DeLong, 1988; used with permission)

Source: Romer (2019)
De Long (1988) presented two important critiques of Baumol (1986)

- **Sample selection:**
  - Baumol chose countries that were ex post rich
  - Any difference in initial conditions will yield convergence
  - Data more likely to be available for ex post successful countries
  - De Long selects countries based on initial GDP per capita

- **Measurement error:**
  - Initial income shows up both on LHS and RHS
  - Measurement error in initial income creates bias toward convergence
FIGURE 1.8  Initial income and subsequent growth in the expanded sample (from DeLong, 1988; used with permission)

Source: Romer (2019)
Fig. 25 shows one of the more famous graphs from the empirical growth literature, illustrating the "catch-up" behavior of OECD countries since 1960. Among OECD countries, those that were relatively poor in 1960—like Japan, Portugal, and Greece—grew rapidly, while those that were relatively rich in 1960—like Switzerland, Norway, and United States—grew more slowly.

GDP per person (US = 1) in 1960

Growth rate, 1960 – 2011

**Fig. 25** Convergence in the OECD. Source: *The Penn World Tables 8.0. Countries in the OECD as of 1970 are shown.*

Source: Jones (2016)
Figure 1. Convergence of Personal Income across U.S. States: 1880 Income and Income Growth from 1880 to 1988

Annual growth rate, 1880–1988 (percent)

Sources: Bureau of Economic Analysis (1984), Easterlin (1960a, 1960b), and Survey of Current Business, various issues. The postal abbreviation for each state is used to plot the figure. Oklahoma, Alaska, and Hawaii are excluded from the analysis.

Source: Barro and Sala-I-Martin (1991)
All Countries Post-1960

Fig. 26 The lack of convergence worldwide. Source: The Penn World Tables 8.0.

Source: Jones (2016)
Conditional Convergence

- Solow model implies:

\[ \dot{k}(t) = sf(k(t)) - (n + g + \delta)k(t) \]

- If \( f(k(t)) = k(t)^\alpha \), steady state:

\[ k^* = \left( \frac{s}{n + g + \delta} \right)^{1/(1-\alpha)} \]
But $k = K/A L$ is not observable ($A$ is not observable)

Let’s rewrite the steady state in terms of $K/L$

$\left( \frac{K}{L} \right)^* = A \left( \frac{s}{n + g + \delta} \right)^{1/(1 - \alpha)}$

Model implies convergence conditional on: $A, s, n, g, \delta$
### TABLE III

**Tests for Unconditional Convergence**

Dependent variable: log difference GDP per working-age person 1960–1985

<table>
<thead>
<tr>
<th>Sample:</th>
<th>Non-oil</th>
<th>Intermediate</th>
<th>OECD</th>
</tr>
</thead>
<tbody>
<tr>
<td>Observations:</td>
<td>98</td>
<td>75</td>
<td>22</td>
</tr>
<tr>
<td>CONSTANT</td>
<td>$-0.266$</td>
<td>$0.587$</td>
<td>$3.69$</td>
</tr>
<tr>
<td>($0.380$)</td>
<td>($0.433$)</td>
<td>($0.68$)</td>
<td></td>
</tr>
<tr>
<td>ln(Y60)</td>
<td>$0.0943$</td>
<td>$-0.00423$</td>
<td>$-0.341$</td>
</tr>
<tr>
<td>($0.0496$)</td>
<td>($0.05484$)</td>
<td>($0.079$)</td>
<td></td>
</tr>
<tr>
<td>$\bar{R}^2$</td>
<td>$0.03$</td>
<td>$-0.01$</td>
<td>$0.46$</td>
</tr>
<tr>
<td>s.e.e.</td>
<td>$0.44$</td>
<td>$0.41$</td>
<td>$0.18$</td>
</tr>
<tr>
<td>Implied $\lambda$</td>
<td>$-0.00360$</td>
<td>$0.00017$</td>
<td>$0.0167$</td>
</tr>
<tr>
<td>($0.00219$)</td>
<td>($0.00218$)</td>
<td>($0.0023$)</td>
<td></td>
</tr>
</tbody>
</table>

*Note. Standard errors are in parentheses. Y60 is GDP per working-age person in 1960.*

Source: Mankiw, Romer, Weil (1992)
### TABLE IV

**Tests for Conditional Convergence**

<table>
<thead>
<tr>
<th>Sample:</th>
<th>Non-oil</th>
<th>Intermediate</th>
<th>OECD</th>
</tr>
</thead>
<tbody>
<tr>
<td>Observations:</td>
<td>98</td>
<td>75</td>
<td>22</td>
</tr>
<tr>
<td>$\text{CONSTANT}$</td>
<td>1.93 (0.83)</td>
<td>2.23 (0.86)</td>
<td>2.19 (1.17)</td>
</tr>
<tr>
<td>$\ln(Y_{60})$</td>
<td>$-0.141$ (0.052)</td>
<td>$-0.228$ (0.057)</td>
<td>$-0.351$ (0.066)</td>
</tr>
<tr>
<td>$\ln(I/GDP)$</td>
<td>0.647 (0.087)</td>
<td>0.644 (0.104)</td>
<td>0.392 (0.176)</td>
</tr>
<tr>
<td>$\ln(n + g + \delta)$</td>
<td>$-0.299$ (0.304)</td>
<td>$-0.464$ (0.307)</td>
<td>$-0.753$ (0.341)</td>
</tr>
<tr>
<td>$R^2$</td>
<td>0.38</td>
<td>0.35</td>
<td>0.62</td>
</tr>
<tr>
<td>$s.e.e.$</td>
<td>0.35</td>
<td>0.33</td>
<td>0.15</td>
</tr>
<tr>
<td>Implied $\lambda$</td>
<td>0.00606 (0.00182)</td>
<td>0.0104 (0.0019)</td>
<td>0.0173 (0.0019)</td>
</tr>
</tbody>
</table>

*Note. Standard errors are in parentheses. $Y_{60}$ is GDP per working-age person in 1960. The investment and population growth rates are averages for the period 1960–1985. $(g + \delta)$ is assumed to be 0.05.*

Source: Mankiw, Romer, Weil (1992)
### TABLE V

**Tests for Conditional Convergence**

Dependent variable: log difference GDP per working-age person 1960–1985

<table>
<thead>
<tr>
<th>Sample:</th>
<th>Non-oil</th>
<th>Intermediate</th>
<th>OECD</th>
</tr>
</thead>
<tbody>
<tr>
<td>Observations:</td>
<td>98</td>
<td>75</td>
<td>22</td>
</tr>
<tr>
<td>CONSTANT</td>
<td>3.04</td>
<td>3.69</td>
<td>2.81</td>
</tr>
<tr>
<td></td>
<td>(0.83)</td>
<td>(0.91)</td>
<td>(1.19)</td>
</tr>
<tr>
<td>ln(Y60)</td>
<td>-0.289</td>
<td>-0.366</td>
<td>-0.398</td>
</tr>
<tr>
<td></td>
<td>(0.062)</td>
<td>(0.067)</td>
<td>(0.070)</td>
</tr>
<tr>
<td>ln(I/GDP)</td>
<td>0.524</td>
<td>0.538</td>
<td>0.335</td>
</tr>
<tr>
<td></td>
<td>(0.087)</td>
<td>(0.102)</td>
<td>(0.174)</td>
</tr>
<tr>
<td>ln(n + g + δ)</td>
<td>-0.505</td>
<td>-0.551</td>
<td>-0.844</td>
</tr>
<tr>
<td></td>
<td>(0.288)</td>
<td>(0.288)</td>
<td>(0.334)</td>
</tr>
<tr>
<td>ln(SCHOOL)</td>
<td>0.233</td>
<td>0.271</td>
<td>0.223</td>
</tr>
<tr>
<td></td>
<td>(0.060)</td>
<td>(0.081)</td>
<td>(0.144)</td>
</tr>
<tr>
<td>$\bar{R}^2$</td>
<td>0.46</td>
<td>0.43</td>
<td>0.65</td>
</tr>
<tr>
<td>s.e.e.</td>
<td>0.33</td>
<td>0.30</td>
<td>0.15</td>
</tr>
<tr>
<td>Implied $\lambda$</td>
<td>0.0137</td>
<td>0.0182</td>
<td>0.0203</td>
</tr>
<tr>
<td></td>
<td>(0.0019)</td>
<td>(0.0020)</td>
<td>(0.0020)</td>
</tr>
</tbody>
</table>

*Note.* Standard errors are in parentheses. Y60 is GDP per working-age person in 1960. The investment and population growth rates are averages for the period 1960–1985. $(g + \delta)$ is assumed to be 0.05. SCHOOL is the average percentage of the working-age population in secondary school for the period 1960–1985.

Source: Mankiw, Romer, Weil (1992)
The empiricals of economic growth

A. Unconditional

B. Conditional on saving and population growth

C. Conditional on saving, population growth and human capital

Figure 1

Unconditional versus Conditional Convergence

Source: Mankiw, Romer, Weil (1992)
C. Conditional on saving, population growth and human capital

Growth rate: 1960-85

Log output per working age adult: 1960

Source: Mankiw, Romer, Weil (1992)
Determinants of Growth

\[
\left( \frac{K}{L} \right)^* = A \left( \frac{s}{n + g + \delta} \right)^{1/(1-\alpha)}
\]

- Mankiw-Romer-Weil 92 condition on \( s, n, \) schooling
- But what about \( A? \)
- Perhaps differences in \( A \) are not needed to explain cross-country growth
- We will come back to this when we consider development accounting in a few lectures
Unconditional convergence:
- Within OECD countries
- Within US states, Japanese prefectures, etc.

Conditional convergence across all countries

Is convergence the dominant fact about growth?
Great Divergence

- Zooming out in time there has clearly been huge divergence
- Before 1500, most countries relatively equally poor
- Then some countries became rich and others didn’t
- Pritchett (1997): Divergence, Big Time
First Great Divergence

Not all societies with access to the Atlantic show the same pattern of growth, however. The data suggest an important interaction between medieval political institutions and access to the Atlantic: the more rapid economic growth took place in societies with relatively nonabsolutist initial institutions, most notably in Britain and the Netherlands. In contrast, countries where the monarchy was highly absolutist, such as Spain and Portugal, experienced only limited growth in the subsequent centuries, while areas lacking easy access to the Atlantic, even such nonabsolutist states as Venice and Genoa, did not experience any direct or indirect benefits from Atlantic trade.

Figures 1 and 2 illustrate the central thesis of this paper. Figure 1, panel A, shows that urbanization in Western Europe grew significantly faster than in Eastern Europe after 1500.2 Figure 1, panel B, shows that these

Source: Acemoglu, Johnson, and Robinson (2005)

Figure 1A. Western Europe, Eastern Europe, and Asia: Urbanization Rates, Weighted by Population, 1300–1850
Lack of reliable data for less developed countries in 19th century is a problem for divergence calculations.

But we can put a conservative lower bound on per capita GDP.

Argues that $250 PPP is conservative (1985 dollars).

- Lower than lowest sustained measurements on record.
- Less than enough to buy 2000 calories a day.

Backcasts current GDP per capita subject to lower bound.
DIVERGENCE, BIG TIME

Figure 1
Simulation of Divergence of Per Capita GDP, 1870–1985
(showing only selected countries)

Source: Pritchett (1997)
Table 2
Estimates of the Divergence of Per Capita Incomes Since 1870

<table>
<thead>
<tr>
<th></th>
<th>1870</th>
<th>1960</th>
<th>1990</th>
</tr>
</thead>
<tbody>
<tr>
<td>USA (P$)</td>
<td>2063</td>
<td>9895</td>
<td>18054</td>
</tr>
<tr>
<td>Poorest (P$)</td>
<td>250</td>
<td>257</td>
<td>399</td>
</tr>
<tr>
<td>Ratio of GDP per capita of richest to poorest country</td>
<td>8.7</td>
<td>38.5</td>
<td>45.2</td>
</tr>
<tr>
<td>Average of seventeen “advanced capitalist” countries from Maddison (1995)</td>
<td>1757</td>
<td>6689</td>
<td>14845</td>
</tr>
<tr>
<td>Average LDCs from PWT5.6 for 1960, 1990 (imputed for 1870)</td>
<td>740</td>
<td>1579</td>
<td>3296</td>
</tr>
<tr>
<td>Average “advanced capitalist” to average of all other countries</td>
<td>2.4</td>
<td>4.2</td>
<td>4.5</td>
</tr>
<tr>
<td>Standard deviation of natural log of per capita incomes</td>
<td>.51</td>
<td>.88</td>
<td>1.06</td>
</tr>
<tr>
<td>Standard deviation of per capita incomes</td>
<td>P$459</td>
<td>P$2,112</td>
<td>P$3,988</td>
</tr>
<tr>
<td>Average absolute income deficit from the leader</td>
<td>P$1286</td>
<td>P$7650</td>
<td>P$12,662</td>
</tr>
</tbody>
</table>

Notes: The estimates in the columns for 1870 are based on backcasting GDP per capita for each country using the methods described in the text assuming a minimum of P$250. If instead of that method, incomes in 1870 are backcast with truncation at P$250, the 1870 standard deviation is .64 (as reported in Figure 1).

Source: Pritchett (1997). 1870 estimates for LDC calculated by “smushing” distribution between lower bound and US.
Most work on convergence focuses on countries

But for welfare calculations we should focus on people

Two complications:
- Countries are of vastly different sizes
  (e.g., China more populous than all of Africa (≈50 countries))
- There is a distribution of income within countries

Attempts to calculate World Distribution of Income from 1970-2000
- Mean income level from NIPA data for each country
- Uses micro-surveys to construct distribution within country

Subtitle of paper: “Falling Poverty, and ... Convergence, Period”
Figure 1a
Growth Versus Initial Income (Unweighted)

The difference can be appreciated in Figure I. Figure Ia displays the well-known scatter plot of the growth rate between 1970 and 2000 versus the logarithm of income per capita in 1970. The correlation

Figure Ib
Growth Versus Initial Income (Population-Weighted)

**Distribution of Income in China**

**Figure IIa**
Distribution of Income in China

Source: Sala-i-Martin (2006)
Figure IIb
Distribution of Income in India

Source: Sala-i-Martin (2006)
DISTRIBUTION OF INCOME IN BRAZIL

Figure IIe
Distribution of Income in Brazil

Source: Sala-i-Martin (2006)
The World Distribution of Income in 1970

**Figure IIIa**

The WDI and Individual Country Distributions in 1970

Source: Sala-i-Martin (2006)
World Distribution of Income in 2000

Figure IIIb
The WDI and Individual Country Distributions in 2000

Source: Sala-i-Martin (2006)
Falling Poverty

Figure VI
Poverty Rates

Source: Sala-i-Martin (2006)
### TABLE I
Poverty Rates and Headcounts for Various Poverty Lines

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
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<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>$495</td>
<td>WB Poverty Line ($1/Day)</td>
<td>15.4%</td>
<td>14.0%</td>
<td>11.9%</td>
<td>8.8%</td>
<td>7.3%</td>
<td>6.2%</td>
<td>5.7%</td>
<td>-0.097</td>
</tr>
<tr>
<td>$570</td>
<td>$1.5/Day</td>
<td>20.2%</td>
<td>18.5%</td>
<td>15.9%</td>
<td>12.1%</td>
<td>10.0%</td>
<td>8.0%</td>
<td>7.0%</td>
<td>-0.131</td>
</tr>
<tr>
<td>$730</td>
<td>$2/Day</td>
<td>29.6%</td>
<td>27.5%</td>
<td>24.2%</td>
<td>19.3%</td>
<td>16.2%</td>
<td>12.6%</td>
<td>10.6%</td>
<td>-0.190</td>
</tr>
<tr>
<td>$1,140</td>
<td>$3/Day</td>
<td>46.6%</td>
<td>44.2%</td>
<td>40.3%</td>
<td>34.7%</td>
<td>30.7%</td>
<td>25.0%</td>
<td>21.1%</td>
<td>-0.254</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Poverty head counts (thousands)</th>
<th>Change 1970–2000</th>
</tr>
</thead>
<tbody>
<tr>
<td>Population</td>
<td>2,187,858</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
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<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>$495</td>
<td>WB Poverty Line ($1/Day)</td>
<td>533,861</td>
<td>536,379</td>
<td>498,032</td>
<td>399,527</td>
<td>362,902</td>
<td>327,943</td>
<td>321,518</td>
<td>-212,343</td>
<td></td>
</tr>
<tr>
<td>$570</td>
<td>$1.5/Day</td>
<td>699,896</td>
<td>708,825</td>
<td>665,781</td>
<td>548,533</td>
<td>495,221</td>
<td>424,626</td>
<td>398,403</td>
<td>-301,493</td>
<td></td>
</tr>
<tr>
<td>$730</td>
<td>$2/Day</td>
<td>1,028,532</td>
<td>1,052,761</td>
<td>1,008,789</td>
<td>874,115</td>
<td>798,945</td>
<td>671,069</td>
<td>600,275</td>
<td>-428,257</td>
<td></td>
</tr>
<tr>
<td>$1,140</td>
<td>$3/Day</td>
<td>1,616,772</td>
<td>1,691,184</td>
<td>1,681,712</td>
<td>1,575,415</td>
<td>1,517,778</td>
<td>1,327,635</td>
<td>1,197,080</td>
<td>-419,691</td>
<td></td>
</tr>
</tbody>
</table>

Poverty Rates are the percentages of citizens with incomes below the corresponding poverty line. Poverty head counts are constructed as the total number of people with incomes lower than the corresponding poverty line. The first poverty line (called WB poverty or $1/Day) line is the poverty line originally used by the World Bank and corresponds to $1.05/Day in 1985 prices. This corresponds to $495 per year in 1996 prices. The second poverty line is the one used by Bhalla [2002], which increases the WB by 15 percent to adjust for underreporting at the top of the distribution. This corresponds to $570 per year or, roughly, $1.5/Day. The third and fourth lines correspond to $2/Day and $3/Day in 1996 prices ($730 and $1140 per year, respectively).

Source: Sala-i-Martin (2006)
### TABLE II
Poverty by Region (Original WB Poverty Line, $1.5/Day or $570/Year)

<table>
<thead>
<tr>
<th></th>
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<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>World</td>
<td>5,660,040</td>
<td>0.202</td>
<td>0.185</td>
<td>0.159</td>
<td>0.121</td>
<td>0.100</td>
<td>0.080</td>
<td>0.070</td>
<td>-0.132</td>
<td>-0.043</td>
<td>-0.059</td>
<td>-0.030</td>
</tr>
<tr>
<td>East Asia</td>
<td>1,704,242</td>
<td>0.327</td>
<td>0.278</td>
<td>0.217</td>
<td>0.130</td>
<td>0.102</td>
<td>0.083</td>
<td>0.024</td>
<td>-0.303</td>
<td>-0.110</td>
<td>-0.115</td>
<td>-0.078</td>
</tr>
<tr>
<td>South Asia</td>
<td>1,327,455</td>
<td>0.303</td>
<td>0.297</td>
<td>0.267</td>
<td>0.178</td>
<td>0.103</td>
<td>0.057</td>
<td>0.025</td>
<td>-0.277</td>
<td>-0.036</td>
<td>-0.164</td>
<td>-0.078</td>
</tr>
<tr>
<td>Africa</td>
<td>608,221</td>
<td>0.351</td>
<td>0.360</td>
<td>0.372</td>
<td>0.426</td>
<td>0.437</td>
<td>0.505</td>
<td>0.488</td>
<td>0.137</td>
<td>0.020</td>
<td>0.065</td>
<td>0.051</td>
</tr>
<tr>
<td>Latin America</td>
<td>499,716</td>
<td>0.103</td>
<td>0.056</td>
<td>0.030</td>
<td>0.036</td>
<td>0.041</td>
<td>0.038</td>
<td>0.042</td>
<td>-0.061</td>
<td>-0.074</td>
<td>0.012</td>
<td>0.001</td>
</tr>
<tr>
<td>Eastern Europe</td>
<td>436,373</td>
<td>0.013</td>
<td>0.005</td>
<td>0.004</td>
<td>0.001</td>
<td>0.010</td>
<td>0.011</td>
<td>0.004</td>
<td>-0.003</td>
<td>-0.009</td>
<td>0.001</td>
<td>0.006</td>
</tr>
<tr>
<td>MENA</td>
<td>220,026</td>
<td>0.107</td>
<td>0.092</td>
<td>0.036</td>
<td>0.016</td>
<td>0.012</td>
<td>0.007</td>
<td>0.006</td>
<td>-0.102</td>
<td>-0.071</td>
<td>-0.025</td>
<td>-0.006</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
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<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>East Asia</td>
<td>1,704,242</td>
<td>350,263</td>
<td>334,266</td>
<td>281,914</td>
<td>182,205</td>
<td>154,973</td>
<td>61,625</td>
<td>41,071</td>
<td>-309,192</td>
<td>-68,349</td>
<td>-126,941</td>
<td>-113,902</td>
</tr>
<tr>
<td>South Asia</td>
<td>1,327,455</td>
<td>211,364</td>
<td>234,070</td>
<td>236,366</td>
<td>176,536</td>
<td>113,661</td>
<td>69,582</td>
<td>33,438</td>
<td>-177,926</td>
<td>25,002</td>
<td>-122,705</td>
<td>-80,223</td>
</tr>
<tr>
<td>Africa</td>
<td>608,221</td>
<td>93,528</td>
<td>109,491</td>
<td>129,890</td>
<td>172,175</td>
<td>204,364</td>
<td>269,733</td>
<td>296,733</td>
<td>203,205</td>
<td>74,474</td>
<td>92,369</td>
<td></td>
</tr>
<tr>
<td>Latin America</td>
<td>499,716</td>
<td>27,897</td>
<td>17,014</td>
<td>10,195</td>
<td>13,836</td>
<td>17,406</td>
<td>21,012</td>
<td>-6,885</td>
<td>-17,702</td>
<td>7,211</td>
<td>3,607</td>
<td></td>
</tr>
<tr>
<td>Eastern Europe</td>
<td>436,373</td>
<td>4,590</td>
<td>1,991</td>
<td>1,418</td>
<td>369</td>
<td>1,906</td>
<td>4,238</td>
<td>4,402</td>
<td>-188</td>
<td>-3,172</td>
<td>488</td>
<td>2,496</td>
</tr>
<tr>
<td>MENA</td>
<td>220,026</td>
<td>11,250</td>
<td>10,954</td>
<td>4,991</td>
<td>2,507</td>
<td>2,101</td>
<td>1,466</td>
<td>1,264</td>
<td>-9,986</td>
<td>-6,259</td>
<td>-2,890</td>
<td>-837</td>
</tr>
</tbody>
</table>

Source: Sala-i-Martin (2006)
Falling Poverty (except in Africa)

Figure VII
Regional Poverty Rates ($1.5 a Day Line)

Source: Sala-i-Martin (2006)
HIGH GROWTH IN AFRICA SINCE 2000

Figure 1
Median Real GDP Per Capita Growth Rates in Sub-Saharan Africa, 1980–2019

Source: Archibong, Coulibaly, Okonjo-Iweala (2021)
An important aspect of the yearly evolution of the Gini coefficient is that its behavior is not monotonic. For example, we see a sudden decline in 1975 which is explained by the fact that rich countries suffered an important recession in that year due to the first oil shock, a recession that was not felt in some of the poorest and largest countries in the world. For example, in 1975 the growth rate in China was 3.6 percent and that of India was over 7 percent. Of course, when the rich suffer and the poor gain, world income inequality is reduced. Another example of a short-term reversal occurred in the late 1980s, when inequality increased for a few years before returning to its longer term downward trend. This increase in inequality can be partly explained by the large 1988 recession in China. The central point is that business cycles in the largest countries or groups of countries are associated with short-term reversals in the trend of world inequality, which implies that we should distrust empirical studies of this problem that cover very short time spans.

The rest of Table III reports the estimates of seven other inequality indexes. The main lessons are first, all indexes show a remarkably similar pattern of worldwide inequality over time. Second, inequality remained more or less constant (or possibly increased) during the 1970s. Third, inequality declined substantially during the 1980s and 1990s. The size of the decline depends a bit on the exact measure: the largest reduction occurred in the top-20 percent-to-bottom-20 percent ratio, which declined by almost 30 percent between 1979 and 2000, followed by the top-10 percent-to-

Source: Sala-i-Martin (2006)
New wave of research on income inequality since 2000

Combines data from: national accounts, tax data, household surveys, inheritance records, etc.

Tax data crucial to capture income shares at the top of the distribution

Key researchers include: Piketty, Saez, and Zucman

Have developed World Inequality Database
Figure 1. Global income inequality, 1820–2020. Interpretation. The share of global income going to top 10% highest incomes at the world level has fluctuated around 50–60% between 1820 and 2020 (50% in 1820, 60% in 1910, 56% in 1980, 61% in 2000, 55% in 2020), while the share going to the bottom 50% lowest incomes has generally been around or below 10% (14% in 1820, 7% in 1910, 5% in 1980, 6% in 2000, 7% in 2020). Global inequality has always been very large. It rose between 1820 and 1910 and shows little long-run trend between 1910 and 2020.


Source: Chancel and Piketty (2021)
FIGURE 1. Global income inequality, 1820–2020. Interpretation. The share of global income going to the top 10% highest incomes at the world level has fluctuated around 50–60% between 1820 and 2020 (50% in 1820, 60% in 1910, 56% in 1980, 61% in 2000, 55% in 2020), while the share going to the bottom 50% lowest incomes has generally been around or below 10% (14% in 1820, 7% in 1910, 5% in 1980, 6% in 2000, 7% in 2020). Global inequality has always been very large. It rose between 1820 and 1910 and shows little long-run trend between 1910 and 2020.


FIGURE 2. Global income inequality, 1820–2020: ratio T10/B50. Interpretation. Global inequality, as measured by the ratio T10/B50 between the average income of the top 10% and the average income of the bottom 50%, more than doubled between between 1820 and 1910, from less than 20 to about 40, and stabilized around 40 between 1910 and 2020. It is too early to say whether the decline in global inequality observed since 2008 will continue.


Source: Chancel and Piketty (2021)
**Figure 3.** Global income inequality, 1820–2020: Gini index. *Interpretation.* Global inequality, as measured by the global Gini coefficient, rose from about 0.6 in 1820 to about 0.7 in 1910, and then stabilized around 0.7 between 1910 and 2020. It is too early to say whether the decline in the global Gini coefficient observed since 2000 will continue.


Source: Chancel and Piketty (2021)
Global Income Inequality

Figure 4. Global income inequality, 1820–2020: between-countries versus within-countries inequality (ratio T10/B50). Interpretation. Between-country inequality, as measured by the ratio T10/B50 between the average incomes of the top 10% and the bottom 50% (assuming everybody within a country as the same income), rose between 1820 and 1980 and strongly declined since then. Within-country inequality, as measured also by the ratio T10/B50 between the average incomes of the top 10% and the bottom 50% (assuming all countries have the same average income), rose slightly between 1820 and 1910, declined between 1910 and 1980, and rose since 1980.


Source: Chancel and Piketty (2021)
What does Solow model imply about speed of convergence?

If speed of convergence is fast:
- Most countries will be close to steady state
  (already mostly converged)
- We can focus on steady state analysis

Also interesting as a possible test of the model
SPEED OF CONVERGENCE

- Start with:
  \[ \dot{k}(t) = sf(k(t)) - (n + g + \delta)k(t) \]

- So \( \dot{k}(t) \) is a function of \( k(t) \)

- Let’s write this as \( \dot{k}(k) \) (dropping dependence on \( t \) for notational simplicity)

- A first-order Taylor series approximation of \( \dot{k}(k) \) around \( k^* \) is:
  \[
  \dot{k} \simeq \left[ \frac{\partial \dot{k}(k)}{\partial k} \right]_{k=k^*} (k - k^*)
  \]

  \( (\dot{k} \) is zero at \( k^* \))

- Let’s denote \( \lambda = -\frac{\partial \dot{k}(k)}{\partial k}|_{k=k^*} \) which means we have
  \[ \dot{k}(t) \simeq -\lambda(k(t) - k^*) \]
Linear first-order differential equation:

\[ \dot{k}(t) \approx -\lambda (k(t) - k^*) \]

Solution:

\[ k(t) - k^* \approx e^{-\lambda t} [k(0) - k^*] \]

So, \( \lambda \) is rate of convergence

Half-life:

\[ 0.5 = e^{-\lambda t} \]

\[ t = -\log(0.5) / \lambda \approx 0.69 / \lambda \]
Using:

\[ \dot{k}(k) = sf(k) - (n + g + \delta)k \]

we get that

\[ \lambda = - \left[ \frac{\partial \dot{k}(k)}{\partial k} \right]_{k=k^*} = -[sf'(k^*) - (n + g + \delta)] \]

\[ = (n + g + \delta) - \frac{(n + g + \delta)k^*f'(k^*)}{f(k^*)} \]

\[ = [1 - \alpha_K(k^*)](n + g + \delta) \]

- Speed of convergence of output is the same as capital
Solow model implies that speed of convergence is

$$\lambda = [1 - \alpha_K(k^*)](n + g + \delta)$$

Rough calibration:
- Technological growth: $g = 0.02$
- Population growth: $n = 0.01$
- Depreciation: $\delta = 0.04$
- Capital share: $\alpha_K(k^*) = 1/3$

$$\lambda = \frac{2}{3}(0.01 + 0.02 + 0.05) = 0.053$$

This implies a half-life of 13 years

Very fast convergence!!
To measure speed of convergence in the data, must run convergence regressions in terms of annual growth rates

Barro and Sala-I-Martin (1991,1992) consider:

\[
\frac{1}{T} \log \left( \frac{y_{i,t}}{y_{i,t-T}} \right) = a - (1 - e^{-\beta T}) \frac{1}{T} \log y_{i,t-T} + \text{other variables}
\]

In this case, \( \beta \) is the annual rate of convergence
Table 1. Regressions for Personal Income across U.S. States, 1880–1988

<table>
<thead>
<tr>
<th>Period</th>
<th>Basic equation</th>
<th>Equation with regional dummies</th>
<th>Equation with regional dummies and sectoral variables&lt;sup&gt;a&lt;/sup&gt;</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>β</td>
<td>R²[σ]</td>
<td>β</td>
</tr>
<tr>
<td>1880–1900</td>
<td>0.0101</td>
<td>0.36</td>
<td>0.0224</td>
</tr>
<tr>
<td></td>
<td>(0.0022)</td>
<td>[0.0068]</td>
<td>(0.0040)</td>
</tr>
<tr>
<td>1900–20</td>
<td>0.0218</td>
<td>0.62</td>
<td>0.0209</td>
</tr>
<tr>
<td></td>
<td>(0.0032)</td>
<td>[0.0065]</td>
<td>(0.0063)</td>
</tr>
<tr>
<td>1920–30</td>
<td>−0.0149</td>
<td>0.14</td>
<td>−0.0122</td>
</tr>
<tr>
<td></td>
<td>(0.0051)</td>
<td>[0.0132]</td>
<td>(0.0074)</td>
</tr>
<tr>
<td>1930–40</td>
<td>0.0141</td>
<td>0.35</td>
<td>0.0127</td>
</tr>
<tr>
<td></td>
<td>(0.0030)</td>
<td>[0.0073]</td>
<td>(0.0051)</td>
</tr>
<tr>
<td>1940–50</td>
<td>0.0431</td>
<td>0.72</td>
<td>0.0373</td>
</tr>
<tr>
<td></td>
<td>(0.0048)</td>
<td>[0.0078]</td>
<td>(0.0053)</td>
</tr>
<tr>
<td>1950–60</td>
<td>0.0190</td>
<td>0.42</td>
<td>0.0202</td>
</tr>
<tr>
<td></td>
<td>(0.0035)</td>
<td>[0.0050]</td>
<td>(0.0052)</td>
</tr>
<tr>
<td>1960–70</td>
<td>0.0246</td>
<td>0.51</td>
<td>0.0135</td>
</tr>
<tr>
<td></td>
<td>(0.0039)</td>
<td>[0.0045]</td>
<td>(0.0043)</td>
</tr>
<tr>
<td>1970–80</td>
<td>0.0198</td>
<td>0.21</td>
<td>0.0119</td>
</tr>
<tr>
<td></td>
<td>(0.0062)</td>
<td>[0.0060]</td>
<td>(0.0069)</td>
</tr>
<tr>
<td>1980–88</td>
<td>−0.0060</td>
<td>0.00</td>
<td>−0.0005</td>
</tr>
<tr>
<td></td>
<td>(0.0130)</td>
<td>[0.0142]</td>
<td>(0.0114)</td>
</tr>
</tbody>
</table>

<sup>a</sup> The additional variables in the third column are the share of personal income originating in agriculture at the start of the period, Agry; - T, and the structural composition variable, Si, described in the text. Data for Si, are only available since 1929.

<sup>b</sup> Nine periods combined.

<sup>c</sup> The likelihood ratio test is based on the null hypothesis that the β are the same across all nine subperiods. It follows a chi-squared (χ²) distribution; the 0.05 χ² value with eight degrees of freedom is 15.5.

Source: Barro and Sala-i-Martin (1991)
## TABLE 3
### COMPARISON OF REGRESSIONS ACROSS COUNTRIES AND U.S. STATES

<table>
<thead>
<tr>
<th>Sample</th>
<th>$\hat{\beta}$</th>
<th>Additional Variables</th>
<th>$R^2$</th>
<th>$\sigma$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. 98 countries, 1960–85</td>
<td>−0.0037</td>
<td>no</td>
<td>0.04</td>
<td>0.0183</td>
</tr>
<tr>
<td></td>
<td>(0.0018)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2. 98 countries, 1960–85</td>
<td>0.0184</td>
<td>yes</td>
<td>0.52</td>
<td>0.0133</td>
</tr>
<tr>
<td></td>
<td>(0.0045)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3. 20 OECD countries, 1960–85</td>
<td>0.0095</td>
<td>no</td>
<td>0.45</td>
<td>0.0051</td>
</tr>
<tr>
<td></td>
<td>(0.0028)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4. 20 OECD countries, 1960–85</td>
<td>0.0203</td>
<td>yes</td>
<td>0.69</td>
<td>0.0046</td>
</tr>
<tr>
<td></td>
<td>(0.0068)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>5. 48 U.S. states, 1963–86</td>
<td>0.0218</td>
<td>no</td>
<td>0.38</td>
<td>0.0040</td>
</tr>
<tr>
<td></td>
<td>(0.0053)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>6. 48 U.S. states, 1963–86</td>
<td>0.0236</td>
<td>yes</td>
<td>0.61</td>
<td>0.0033</td>
</tr>
<tr>
<td></td>
<td>(0.0013)</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Source: Barro and Sala-I-Martín (1992)
Barro’s “iron law of convergence”: 2% per year
This implies a half-life of 35 years
Takes 115 years for 90% of convergence to occur
Convergence is very slow in practice!!
Convergence in basic Solow model way too fast:

$$\lambda = [1 - \alpha_K(k^*)](n + g + \delta)$$

One way to reconcile model and data is to raise the value of $\alpha_K(k^*)$

if $\alpha_K(k^*) \simeq 0.75$ then convergence will be close to 2% per year

$\alpha_K(k^*)$ is the capital share (if markets are competitive)

High $\alpha_K(k^*)$ may make sense if one includes human capital
Revisit convergence after 25 years

Absolute convergence since 2000

Why? Proximate answer: Fundamentals have converged
(i.e., $A$, $s$, $n$, etc.)

Leaves deeper question of why fundamentals have converged
Figure 1: Income convergence by decade

Notes: This figure plots, by decade, the raw scatter plots for the decade’s $\beta$-convergence regression, as well as the regression line itself.

$100 \log(GDP_{pc})_{i,t} + 10 - \log(GDP_{pc})_{i,t-1} = \alpha_t + \beta_t \log(GDP_{pc})_{i,t} + \epsilon_{i,t}$

The income measure is income per capita, adjusted for PPP, from the Penn World Tables v10.0. The sample is all countries for which data is available, excluding those with a population less than 200,000 or for whom natural resources account for $>75\%$ of their GDP. Data availability means that the number of countries is growing over time. For 2007, the period considered in 2007-2017.

Source: Kremer, Willis, You (2021)
Figure 2: Trend in income convergence, 1960-2007

(a) $\beta$-convergence.

(b) $\sigma$-convergence.

Notes: These figures show the trend in convergence from 1960 to 2007. Figure a) plots the $\beta$-convergence coefficient, for growth in the subsequent decade, over time. It is the coefficient from Equation 1 - regressing, across countries, the average growth in GDP per capita in the next decade (in %) on the log of GDP per capita, with year fixed effects, and with standard errors clustered by country. Income per capita is adjusted for PPP and comes from the Penn World Tables, v10.0. The sample is growing over time, and excludes countries with a population less than 200,000 or for whom natural resources account for more than 75% of their GDP, as in Figure 1 (neither exclusion has a meaningful effect on the trend). Figure b) plots the evolution over time of the cross-country standard deviation in GDP per capita. $\sigma$-convergence corresponds to a negative slope. Equivalent panels using balanced panels are in Figure A.5.

Source: Kremer, Willis, You (2021)
The plots show the average annual growth in GDP per capita, PPP, for the subsequent decade, averaged by income per capita quartile. Income per capita quartile is classified based on GDP per capita in that year, with the first quartile being the lowest income and the fourth quartile the highest.

Source: Kremer, Willis, You (2021)
Figure 4: Convergence in growth correlates: level in 1985 versus change 1985-2015

Notes: This figure plots β-convergence for growth six representative correlates (potential determinants of steady-state income) from 1985 (or the earliest available year) to 2015 against the baseline correlate level in 1985. We include six of the correlates which are comparable over time, for illustration: Population growth rate (%), Investment rate (% of GDP), Barro-Lee average years of education among 20-60-year-olds, Polity 2 score, government spending (% of GDP), credit by the financial sector. The sample for each figure is the complete set of countries for which the relevant data is available in 1985 and 2015.

Source: Kremer, Willis, You (2021)
Appendix
Beware the Linear Scale!

Source: Clark (2010)