Precautionary Savings, Liquidity Constraints, and Self-Control

Jón Steinsson

University of California, Berkeley

Fall 2023

Three Important Ideas

- Precautionary Saving
- Liquidity Constraints
- Self-Control Problems

Certainty Equivalence

- Suppose for simplicity $\beta(1+r) = 1$
- Consumption Euler equation:

$$U'(C_t) = E_t U'(C_{t+1})$$

- With quadratic utility:

$$C_t = E_t C_{t+1}$$

- This implies certainty equivalence:
  - C_t depends only on $E_t C_{t+1}$ not $\text{var}_t(C_{t+1})$ (or any higher moments)

Certainty Equivalence

- Suppose for simplicity $\beta(1+r) = 1$
- Consumption Euler equation:

$$U'(C_t) = E_t U'(C_{t+1})$$

- With quadratic utility:

$$C_t = E_t C_{t+1}$$

- This implies certainty equivalence:
  - C_t depends only on $E_t C_{t+1}$ not $\text{var}_t(C_{t+1})$ (or any higher moments)
- Very extreme model:
  - Savings behavior unaffected by uncertainty!!
  - Consumption smoothing and intertemporal substitution only forces affecting savings (same thing for linearized or log-linearized models)

Quadratic Utility and Risk

- With quadratic utility, utility cost of given variance of consumption independent of the level of consumption
  - Amount of curvature of utility independent of level
  - Jensen's inequality term independent of the level
- But marginal utility falls with level of consumption
- Thus, with quadratic utility, consumers are willing to pay more to avoid a given amount of uncertainty (particular dollar coin toss) the richer they are
- Quadratic utility implies increasing absolute risk aversion

Common Utility Functions

- Constant Relative Risk Aversion (CRRA):

$$U(C) = \begin{cases} \frac{C^{1-\gamma}-1}{1-\gamma} & \text{if } \gamma \neq 1 \\ \log C & \text{if } \gamma = 1 \end{cases}$$

$$\text{Relative Risk Aversion } = -\frac{U''(C)C}{U'(C)} = \gamma$$

- Constant Absolute Risk Aversion (CARA):

$$U(C) = -\frac{\exp(AC)}{A}$$

$$\text{Absolute Risk Aversion } = -\frac{U''(C)}{U'(C)} = A$$

Risk Aversion in Reality

- Increasing absolute risk aversion completely unrealistic
- Implications for portfolio allocation:
  - CRRA: Constant share in risky assets
  - CARA: Constant dollar amount in risky assets
  - IARA: Decreasing dollar amount in risky assets

  as wealth increases
- In reality, richer people allocate **larger share** of wealth to risky assets
- Suggests decreasing relative risk aversion (DRRA)
  (CRRA not such a bad approximation)

See Gollier (2001, ch 2.) for more detailed discussion of various forms of risk aversion.

Precautionary Savings

- Curvature of utility almost surely falls as consumption rises:

$$U'''(C_t) > 0$$

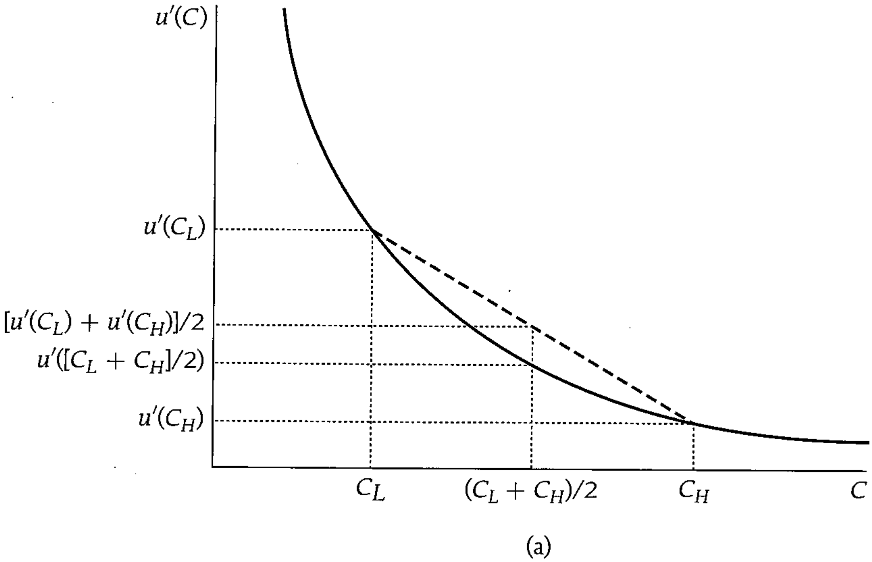
- What does this imply about savings?

Precautionary Savings

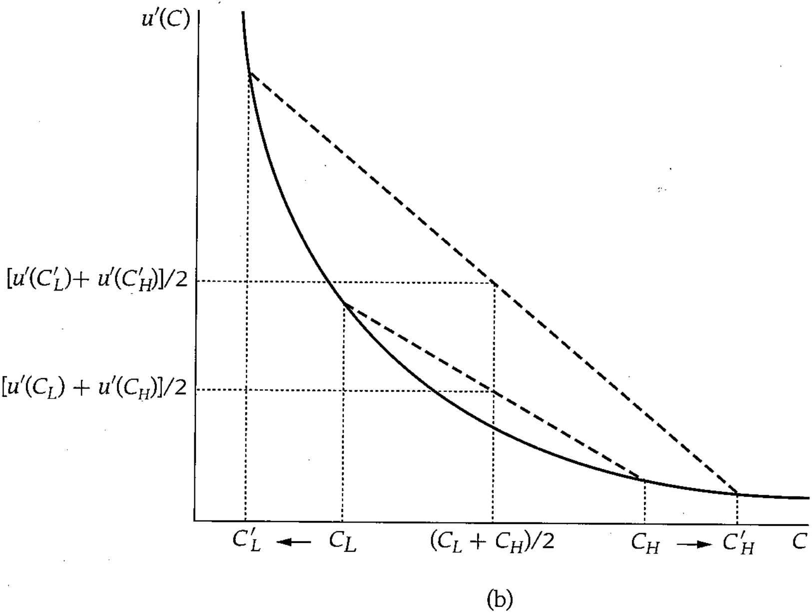
- Curvature of utility almost surely falls as consumption rises:

$$U'''(C_t) > 0$$

- What does this imply about savings?
- With $U'''(C_t) > 0$, $U'(C_t)$ is convex
- If $C_t = E_t C_{t+1}$, then
  - $U'(C_t) < E_t U'(C_{t+1})$ (since $U'(C_t) = U'(E_t C_{t+1}) < E_t U'(C_{t+1})$)
  - Marginal reduction in C_t (increase in saving) increases utility
- This extra saving relative to certainty equivalent case is called **precautionary saving**



Source: Romer (2019). 50-50 chance of C_H and C_L.



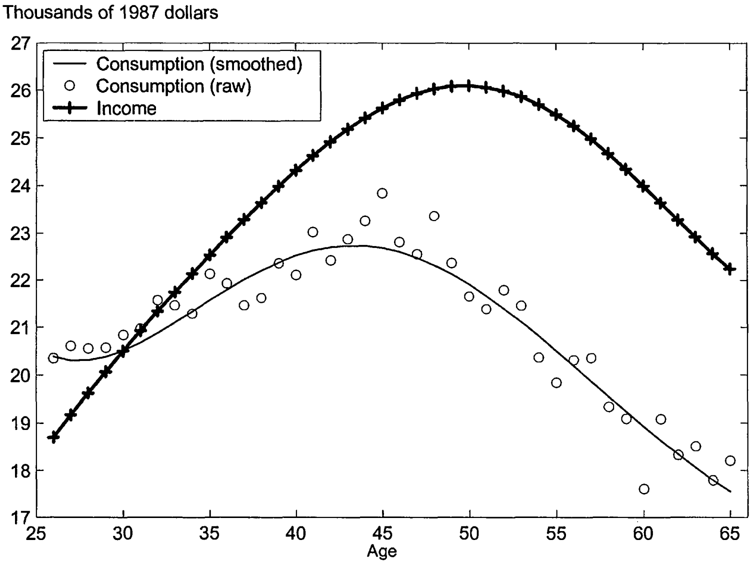
Source: Romer (2019)

Prudence

Definition: (Kimball, 1990) An agent is **prudent** if adding an uninsurable zero-mean risk to their future wealth raises their optimal savings

Proposition: (Leland, 1968) An agent is prudent if and only if the marginal utility of future consumption is convex.

Liquidity Constraints



Liquidity Constraints

- Consumption smoothing over the life-cycle likely involves substantial borrowing early in life
- Simple PIH/LCH model assumes people can borrow (unsecured) at same rate as they can save
- Highly unrealistic:
  - Most household borrowing is secured (e.g., mortgages, car loans)
  - Interest rates on car loans and even mortgages substantially higher than on savings accounts
  - Interest rates on unsecured consumer lending (i.e., credit cards) extremely high ($\sim$ 20%)
  - Limits on unsecured borrowing beyond which can't go at any rate

Liquidity Constraints

Two effects of liquidity constraints:

1. Less borrowing when they bind
2. Less borrowing even when they don't bind because they may bind in the future
   - Bad shock tomorrow may cause low consumption due to binding liquidity constraint at that point
   - Consumer saves today to "self-insure" against this future bad shock

BUFFER STOCK SAVING

- Liquidity constraints and prudence cause households facing uninsurable income risk to engage in **buffer stock saving** (i.e., self-insurance)
- Other sources of saving:
  - **Life-cycle saving** to smooth consumption over the life-cycle relative to life-cycle profile of income
  - Saving due to **patience/impatience**. If $1/\beta \neq (1 + r)$ household will tilt consumption profile (down if impatient, up if patient)
  - Saving due to **intertemporal substitution**. If interest rates are temporarily high, consumers save more to take advantage of this

How Much Buffer Stock Saving?

Depends crucially on $\beta(1+r)$

- If $\beta(1+r) = 1$:
  - Households will eventually save themselves out of constraint
  - I.e., save enough that they will eventually never hit constraint
  - At that point, full consumption smoothing
- If $\beta(1+r) < 1$
  - Households sufficiently impatient that they don't eventually save themselves out of the constraint
  - Finite amount of buffer stock savings
  - Lack of full consumption smoothing even in the long run
- If $\beta(1+r) > 1$: Asset holdings explode in the long run

ZELDES-DEATON-CARROLL MODEL

- Households maximize

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \frac{C_t^{1-\gamma}}{1-\gamma},$$

- subject to

$$W_{t+1} = R(W_t + Y_t - C_t)$$

$$Y_t = P_t V_t$$

$$P_t = P_{t-1} N_t$$

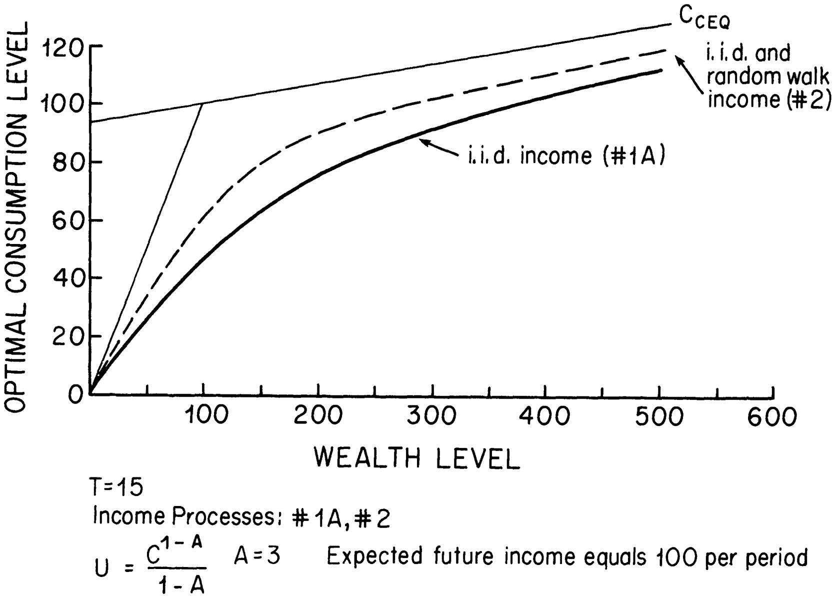
where V_t and N_t are i.i.d. log-normal random variables.

- R is given exogenously (partial equilibrium)
- Household income shocks are uninsurable
- In some versions, household faces borrowing constraint $W_t \geq \underline{W}$

See Zeldes (1989), Deaton (1991), Carroll (1992, 1997)

Zeldes-Deaton-Carroll Model

- Problem Sets 8 and 9 ask you to solve two versions of this model
- Zeldes-Deaton-Carroll argue that model helps explain:
  - High MPC out of transitory windfalls
  - That consumption tracks income over the life-cycle
    (need impatient households for this)
- Sometimes called the “buffer stock model”
- General equilibrium version called Bewely-Aiyagari-Hugget model
  (i.e., interest rate is endogenous)



Source: Zeldes (1989)

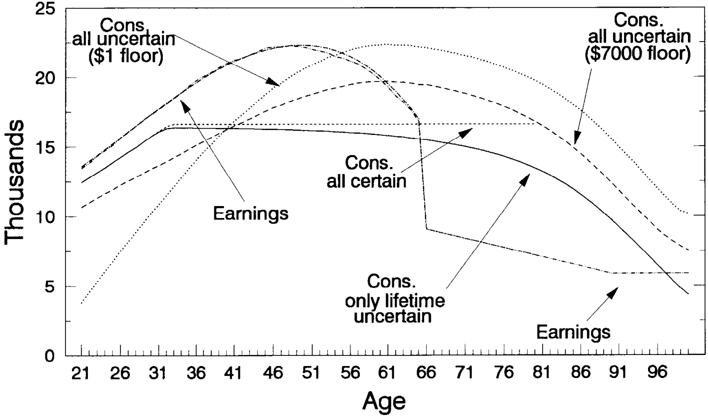
Hubbard-Skinner-Zeldes (1994)

- Basic Zeldes-Deaton-Carroll model very stylized
- Is buffer stock saving important quantitatively?
- Add:
  - Realistic life-cycle income process with retirement
  - Longevity risk
  - Health expenses
  - Taxes and government transfer programs

Figure 3a

Average Consumption and Earnings by Age

No High School Degree



gamma=3, delta=.03

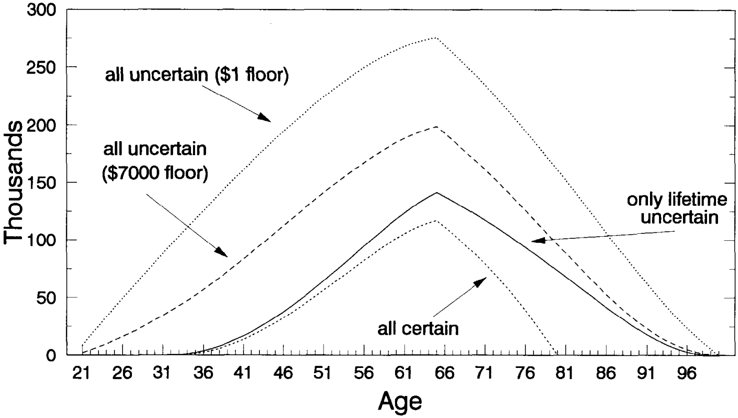
Source: Hubbard-Skinner-Zeldes (1994)

Consumption does not equal earning for young in certainty case because of medical expenses.

Figure 2a

Average Assets by Age

No High School Degree



gamma=3, delta=.03

Source: Hubbard-Skinner-Zeldes (1994)

Hubbard-Skinner-Zeldes (1994)

Precautionary savings and liquidity constraints:

- Yield life-cycle consumption profile the tracks income substantially
- Can contribute substantially to asset accumulation

Gourinchas-Parker 2002

- Go one step further than Hubbard-Skinner-Zeldes 94
- Estimate the preference parameters

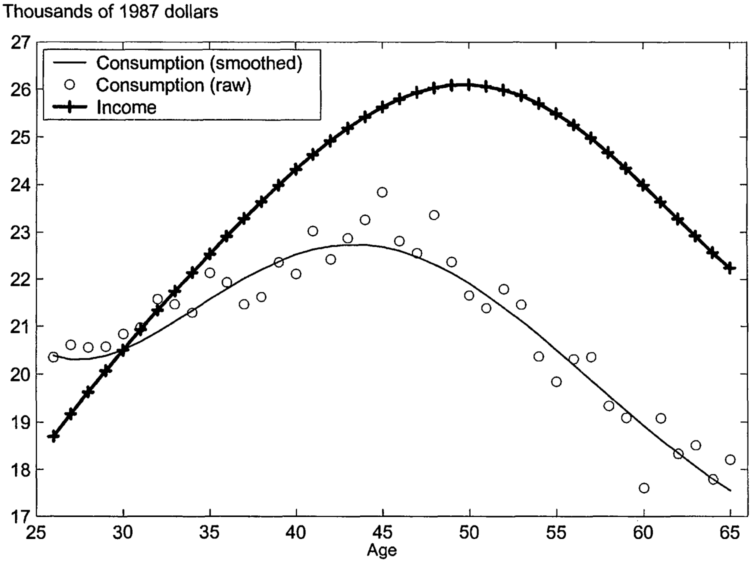


FIGURE 2.—Household consumption and income over the life cycle.

Source: Gourinchas-Parker (2002)

TABLE III

STRUCTURAL ESTIMATION RESULTS

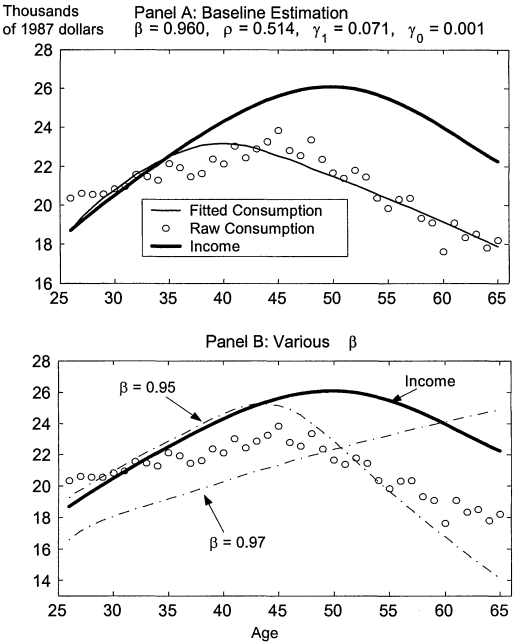
MSM Estimation	Robust Weighting	Optimal Weighting
Discount Factor (β)	0.9598	0.9569
S.E.(A)	(0.0101)	
S.E.(B)	(0.0179)	(0.0150)
Discount Rate ($\beta^{-1}-1$)(%)	4.188	4.507
S.E.(A)	(1.098)	
S.E.(B)	(1.949)	(1.641)
Risk Aversion (ρ)	0.5140	1.3969
S.E.(A)	(0.1690)	
S.E.(B)	(0.1707)	(0.1137)
Retirement Rule:		
γ_0	0.0015	$5.68\,10^{-6}$
S.E.(A)	(3.84)	
S.E.(B)	(3.85)	(16.49)
γ_1	0.0710	0.0613
S.E.(A)	(0.1215)	
S.E.(B)	(0.1244)	(0.0511)
χ^2(A)	175.25	
χ^2(B)	174.10	185.67

Note: MSM estimation for entire group. Standard errors calculated without (A) and with (B) correction for first stage estimation. Cell size is 36,691 households. The last row reports a test of the overidentifying restrictions distributed as a Chi-squared with 36 degrees of freedom. The critical value at 5% is 50.71. Efficient estimates are calculated with a weighting matrix $\widehat{\Omega}$ computed from the robust estimates.

Source: Gourinchas-Parker (2002)

Results

- Reasonable discount rate of 4%
  (Carroll-Samwick 97 had suggested larger rates were needed)
- Reasonable IES of about 2 (sensitive to weighting matrix)
- Reasonable MPC in retirement of 7% per year



Source: Gourinchas-Parker (2002)

Target Cash on Hand

- Level of cash on hand at age t that is expected to remain unchanged:

$$\bar{x}_t = E_t[x_{t+1} | x_t = \bar{x}_t]$$

- If $x_t > \bar{x}_t$ households dissave on average
- If $x_t < \bar{x}_t$ households build up assets on average

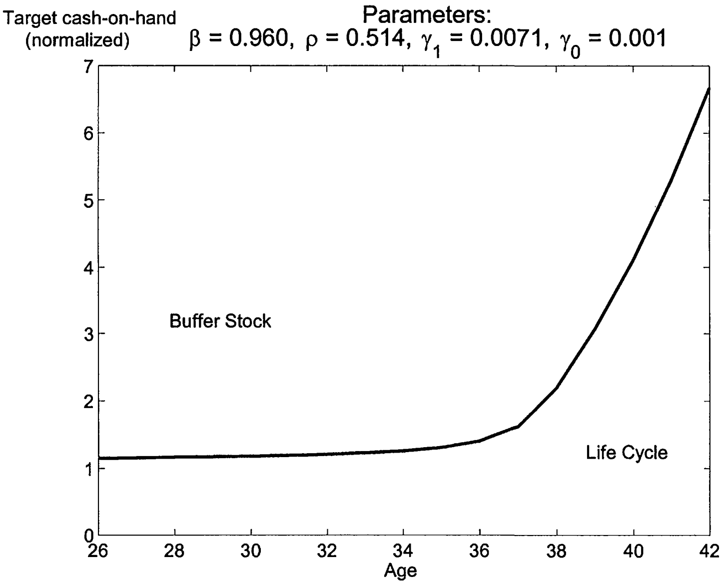


FIGURE 6.—Normalized target cash-on-hand by age.

Source: Gourinchas-Parker (2002). Cash-on-hand normalized by permanent income.

Life-Cycle Consumption Savings

- Define life-cycle consumption as consumption under complete insurance markets
- Use this concept to construct:
  - Life-cycle savings / wealth
  - Buffer-stock savings / wealth

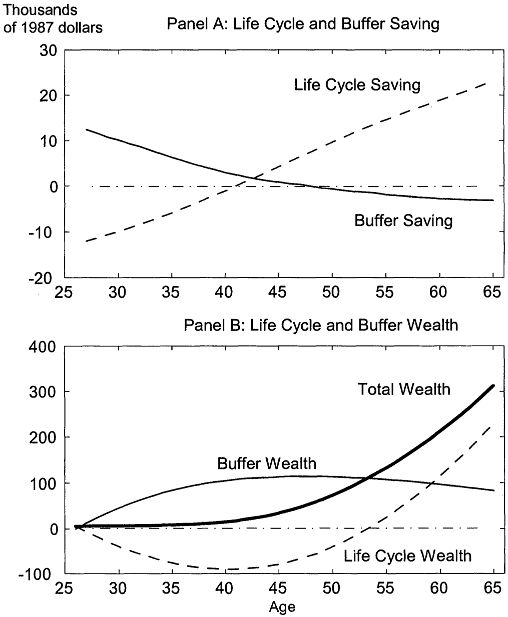


FIGURE 7.—The role of risk in saving and wealth accumulation.

Source: Gourinchas-Parker (2002)

Gourinchas-Parker 02: Summary

- Can uninsurable income risk explain comovement of consumption and income over the life-cycle?

Gourinchas-Parker 02: Summary

- Can uninsurable income risk explain comovement of consumption and income over the life-cycle?
- Yes!
- Households are impatient
  (want downward sloping consumption profiles)
- Consumption constrained by income early in life
- Households save for retirement later in life

Kaplan and Violante (2014)

- Standard incomplete markets models can't match large estimated spending responses of consumption to tax rebates
- Argue that incorporating illiquid wealth into model is key:
  - Liquid assets (-1.5% real return)
  - Illiquid asset (2.3% real return + 4% service flow)
  - Unsecured debt (6% real interest rate)
- Two types of hand-to-mouth agents:
  - Poor hand-to-mouth (no illiquid wealth, no liquid wealth)
  - Wealthy hand-to-mouth (have illiquid wealth, no liquid wealth)
- This model generates much higher rebate responses

Consumer Regret

- Common for people to display dissatisfaction with their choices
  - I am not saving enough for retirement
  - I eat too much and exercise too little
  - I spend too much time surfing the internet and work too little
- One reaction:
  - This is stupid. What you do are you actual preferences
  - What you say are some imagined idealized preferences
  - Everyone says they want to be fit and work really hard
  - But the costs of achieving these goals actually outweigh the benefits for many people

Self-Control

- An alternative reaction:
  - Consumer regret is due to self-control problems
  - Arises due to present biased preferences that give rise to preference reversals

Self-Control

- An alternative reaction:
  - Consumer regret is due to self-control problems
  - Arises due to present biased preferences that give rise to preference reversals

- Consider the following choice for a worker:
  1. 15 minute break today
  2. 20 minute break tomorrow

Self-Control

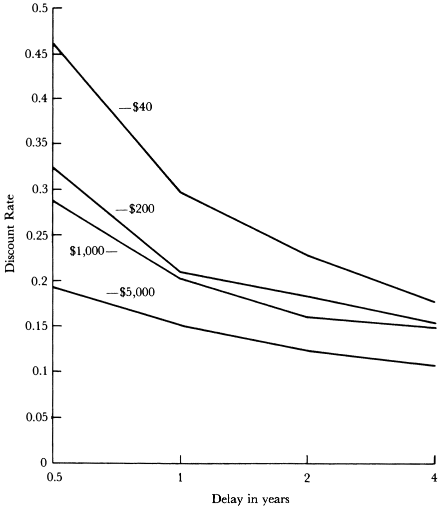
- An alternative reaction:
  - Consumer regret is due to self-control problems
  - Arises due to present biased preferences that give rise to preference reversals
- Consider the following choice for a worker:
  1. 15 minute break today
  2. 20 minute break tomorrow
- Now consider this choice:
  1. 15 minute break in 100 days
  2. 20 minute break in 101 days

Self-Control

- An alternative reaction:
  - Consumer regret is due to self-control problems
  - Arises due to present biased preferences that give rise to preference reversals
- Consider the following choice for a worker:
  1. 15 minute break today
  2. 20 minute break tomorrow
- Now consider this choice:
  1. 15 minute break in 100 days
  2. 20 minute break in 101 days
- Choosing option 1 from the first set but option 2 from the second set indicates time-discounting that is not independent of horizon

Figure 1

Discounting as a Function of Time Delay and Money Amount.



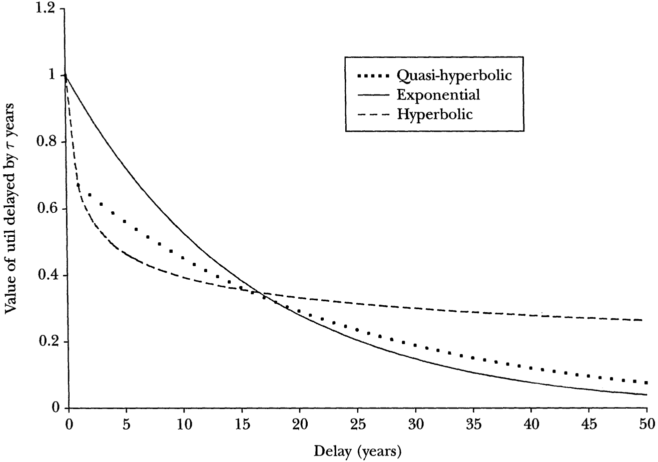
Source: Benzion et al. (1989).

Source: Loewenstein and Thaler (1989). Based on experiments where subjects are asked how much they need to be compensated to delay receiving a reward.

Hyperbolic Discounting

- Exponential discounting:
  - Discount function: β^t: 1, β, β^2, β^3, etc.
  - Discount rate independent of horizon
  - Degree of patience independent of horizon
- Hyperbolic discounting:
  - Discount function: $1/t$ or $1/(1+\alpha t)$ or $1/(1+\alpha t)^{-\gamma/\alpha}$
  - Quasi-hyperbolic discount function: $\beta\delta^t$: 1, $\beta\delta$, $\beta\delta^2$, $\beta\delta^3$, etc.
  - Non-constant rate of discounting
  - More impatient about short run than long run

Figure 1
Discount Functions



Exponential: δ^{τ}, with $\delta = 0.944$; hyperbolic: $(1 + \alpha\tau)^{-\gamma/\alpha}$, with $\alpha = 4$ and $\gamma = 1$; and quasi-hyperbolic: $\{1, \beta\delta, \beta\delta^2, \beta\delta^3, \ldots\}$, with $\beta = 0.7$ and $\delta = 0.957$.

Source: Angeletos et al. (2001)

Time-Consistent Preferences

- Suppose an agent makes a state-contingent plan at time 0 about optimal current and future actions
- But when the future arrives, the agent can reoptimize
- Will they want to change their plan?

Time-Consistent Preferences

- Suppose an agent makes a state-contingent plan at time 0 about optimal current and future actions
- But when the future arrives, the agent can reoptimize
- Will they want to change their plan?

- If the agent discounts future utility exponentially, then they will not change their choices even if able to reoptimize
- Their preferences are time consistent
(aka dynamically consistent)

Time-Inconsistent Preferences

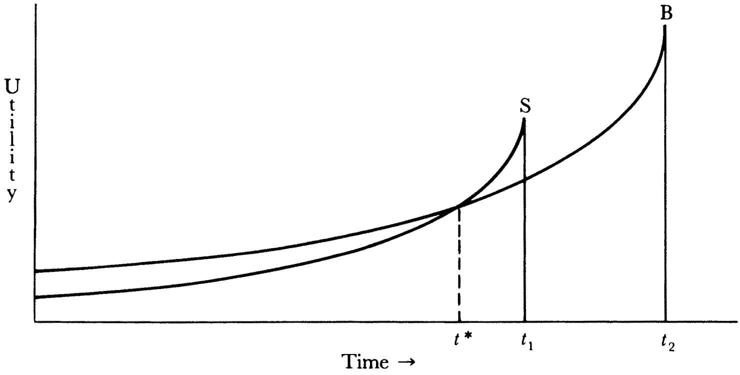
- If an agent's discount function in not exponential, then they will want to change their plan when allowed to reoptimize at a later date
- Their preferences are time inconsistent
  (aka dynamically inconsistent)
- Hyperbolic and quasi-hyperbolic preferences are time inconsistent

Preference Reversals

- Consider following choice:
  - Small reward S at time t_1
  - Bigger reward B at a later time t_2
- Suppose agent has hyperbolic preferences
- Plot present utility from each option as a function of time

Figure 2

Non-Exponential Discounting.



Source: Ainslie (1975).

Source: Loewenstein and Thaler (1989).

Instant Gratification

- Hyperbolic discounting can explain the following type of behavior:
  - On Monday: “I’ll work hard tomorrow.”
  - On Tuesday: “I’ll work hard tomorrow.”
  - Etc.
- People with hyperbolic preferences want instant gratification today but simultaneously want to make patient investments tomorrow

Intrapersonal Strategic Conflict

- Useful to think of a person as having different selves,
  one for each point in time.
- Earlier selves wish to force later selves to act patiently
- Later selves maximize their own preferences (which are different)
- Household problem becomes a game between different selves

Two Types of Hyperbolics

- Sophisticates: Understand that future selves will want to act differently
  - Want to constrain actions of future selves
  - Want access to commitment devices
- Naifs: Act under false belief that future selves will carry out current plan
  - Helps explain procrastication among other things
    (Akerlof, 1991; O'Donoghue and Rabin, 1999)

Hyperbolic Discounting and Consumption

- Key references:
  - Angeletos et al. (2001), Laibson, Repetto, Tobacman (2003), Laibson, Lee, Maxted, Repetto, Tobacman (2023, but originally ca. 2001)
- Build sophisticated life-cycle consumption savings model
- Compare model with exponential and hyperbolic discounting
- Ask whether hyperbolic discounting helps explain the data
  (Baseline case is naifs, but sophisticates give similar conclusions)

Hyperbolic Discounting and Consumption

- Model very similar to Kaplan-Violante (2014)
  (liquid assets, illiquid assets, credit card borrowing)

- Conclusions radically different

- Laibson et al. (2023) estimate: $\beta = 0.50$ and $\delta = 0.99$

- Kaplan-Violante (2014) calibrate $\beta = 0.941$

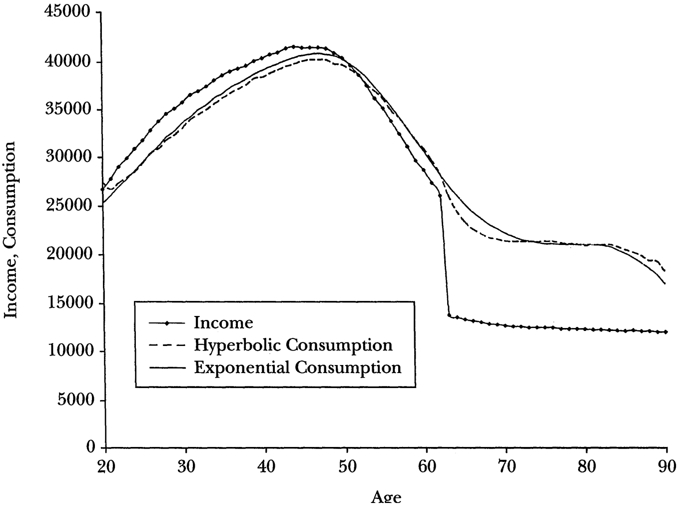
Kaplan-Violante vs. Laibson et al.

- Main tension is how to explain simultaneous:
  - Credit card borrowing at high interest
  - Accumulation of illiquid assets
- Main difference: Assumptions about interest on credit card debt
  - Laibson et al. (2023): 12% real credit card interest rate (data)
  - Kaplan-Violante (2014): 6% real credit card interest rate (calibrated)
- Exponential model can explain facts if credit card interest rate is 6% (return on illiquid asset is similarly high inclusive of service flow) but not if credit card interest rate is 12%

Kaplan-Violante vs. Laibson et al.

- Many households have revolving credit card debt and positive liquid assets.
- Kaplan-Violante:
  - Target a fraction of credit card borrowers of 26%
  - Compromise between fraction with negative net liquid wealth and fraction actually borrowing on credit cards
- Laibson et al.:
  - Target fraction of credit card borrowers of about 75% (higher for young, somewhat lower for old)
- Agents with hyperbolic discounting can simultaneously display highly patient and impatient behavior (accumulate illiquid wealth and borrow on credit cards)

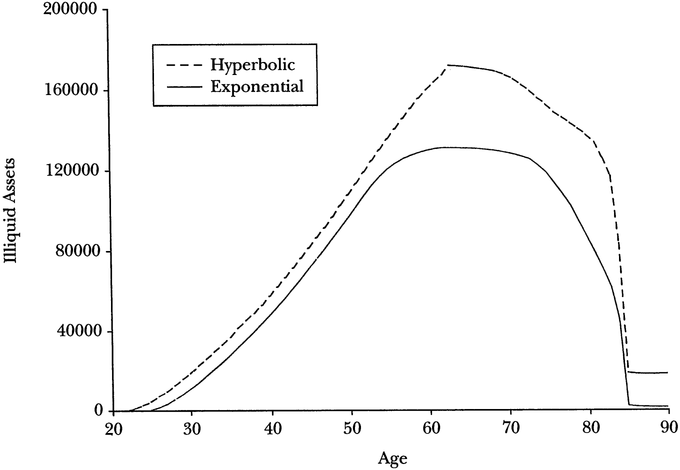
Figure 2
Simulated Mean Income and Consumption



Source: Angeletos et al. (2001).

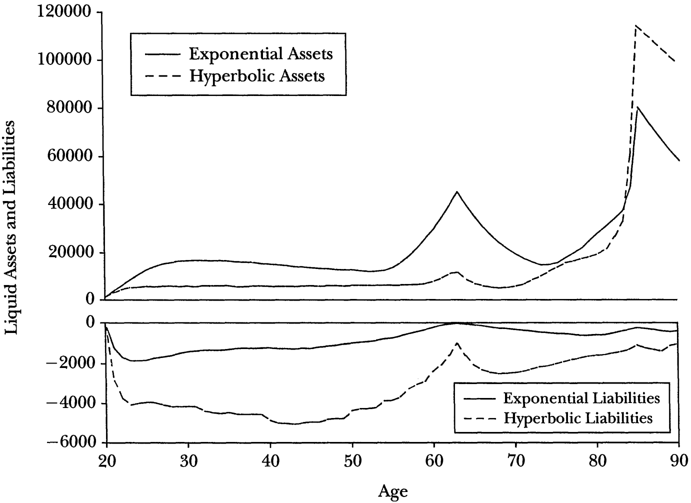
Figure 4

Mean Illiquid Assets of Households with Exponential and Hyperbolic Discount Functions



Source: Angeletos et al. (2001).

Figure 5
Mean Liquid Assets and Liabilities



Source: Angeletos et al. (2001).

Table 1

Percentage of Households with Liquid Assets Greater than One Month of Income

	Simulated Data		*Survey of Consumer Finances*		
Age Group	*Exponential*	*Hyperbolic*	*Definition 1*	*Definition 2*	*Definition 3*
ALL AGES	0.73	0.40	0.37	0.43	0.52
20–29	0.52	0.34	0.18	0.19	0.26
30–39	0.72	0.39	0.21	0.24	0.36
40–49	0.72	0.38	0.26	0.31	0.42
50–59	0.76	0.43	0.35	0.41	0.50
60–69	0.91	0.42	0.58	0.68	0.76
70+	0.77	0.46	0.62	0.71	0.78

Sources: Authors' simulations and 1995 SCF.
Notes: The table reports the fraction of households who hold more than a month's income in liquid wealth. Definition 1 includes cash, checking and savings accounts. Definition 2 includes definition 1 plus money market accounts. Definition 3 includes definition 2 plus call accounts, CDs, bonds, stocks and mutual funds.

Source: Angeletos et al. (2001).

Table 2

Share of Assets in Liquid Form

	Simulated Data		*Survey of Consumer Finances*		
Age Group	*Exponential*	*Hyperbolic*	*Definition 1*	*Definition 2*	*Definition 3*
ALL AGES	0.51	0.41	0.08	0.10	0.16
20–29	0.97	0.86	0.13	0.14	0.18
30–39	0.65	0.46	0.09	0.10	0.14
40–49	0.35	0.24	0.06	0.07	0.10
50–59	0.20	0.13	0.04	0.05	0.09
60–69	0.27	0.12	0.09	0.10	0.20
70+	0.57	0.56	0.09	0.12	0.24

Sources: 1995 SCF and authors' simulations.
Notes: Asset share is liquid assets divided by total assets—liquid assets plus illiquid assets. The three different definitions used for liquid assets are the same as in Table 1. Three complementary definitions are used for illiquid assets. Illiquid assets include money market accounts, call accounts, CDs, bonds, stocks, and mutual funds if these assets were not included in the relevant liquid asset definition. In addition, illiquid assets include IRAs, defined contribution plans, life insurance, trusts, annuities, vehicles, home equity (net of mortgage), real estate, business equity, jewelry, furniture, antiques, and home durables.

Source: Angeletos et al. (2001).