Advanced Microeconomics
(Economics 104)
Spring 2011
Sample midterm questions

[1] Let G be the 2×2 strategic game given by

	L	R
T	a, b	c, d
B	e, f	g, h

and let G' (which is different from G) be the 2×2 strategic game given by

	L	R
T	a', b'	c', d'
B	e', f'	g', h'

Consider the 2×2 game $G'' = G + G'$ given by

	L	R
T	$a + a', b + b'$	$c + c', d + d'$
B	$e + e', f + f'$	$g + g', h + h'$

- Suppose that (T, L) is a pure strategy equilibrium in G and that it is also a pure strategy equilibrium in G'. Is (T, L) also an equilibrium of G''? Prove or give a counter-example.

Let p be a mixture over player 1's strategies and q be a mixture over player 2's strategies.

- Suppose that (p^*, q^*) is a completely mixed strategy equilibrium in G and that it is also a completely mixed strategy equilibrium in G'. Is (p^*, q^*) also an equilibrium of G''? Prove or give a counter-example.

[2] A soccer team has been awarded a penalty kick. The kicker (player 1) has two possible strategies: to kick the ball into the right side of the goal (R) or to kick the ball into the left side of the goal (L).

The goal keeper (player 2) has no time to determine where the ball is going before she must commit herself by jumping either to the right (R) or to the left (L) of the net.

Suppose that if the kicker makes the goal, she gets a payoff of 1 and the goal keeper a payoff of 0, and if the kicker does not make the goal she gets a payoff of 0 and the goal keeper a payoff of 1.

Also, suppose that goal keeper always stop the ball if she guesses correctly where the kicker is going to kick.

When the kicker (player 1) kicks to the left (L) and the goal keeper (player 2) jumps to the right (R) there is only probability $0 < \delta < 1$ that the kicker will score.

Thus, δ models how good the kicker is at kicking to the left side of the net when it is undefended.

The situation is given by the 2×2 strategic game with the payoff matrix

	L	R
R	$1, 0$	$0, 1$
L	$0, 1$	$\delta, 1-\delta$

where $0 < \delta < 1$.

- Find the set of all NE as a function of δ and draw the graph of best response functions.
- Explain what happens as $\delta \to 1$ and as $\delta \to 0$.

[3] Consider the two-player symmetric game

	A	B	C
A	x, x	$1, -1$	$-1, 1$
B	$-1, 1$	x, x	$1, -1$
C	$1, -1$	$-1, 1$	x, x

- Suppose $x > 1$. Find the set of all Nash equilibria. Are the equilibrium strategies ESS?
- Suppose $0 \leq x \leq 1$. Find the set of all Nash equilibria. Are the equilibrium strategies ESS?

[4] Let G be the 2×2 strategic game given by

	L	R
T	$a, -a$	$b, -b$
B	$c, -c$	$d, -d$

This game is called *strictly competitive* or *zero-sum* because for any $a \in A$ we have $u_1(a) = -u_2(a)$.

- Show that if (T, L) and (B, R) are NE of the game, then so are (T, R) and (B, L) This result called interchangeability in zero-sum games.

- Show that if $a = b = c = d$ (like in Matching Pennies) the game has a unique mixed strategy NE $(p^*, q^*) = (1/2, 1/2)$.

[5] Consider the BoS situation given by the 2×2 strategic game with the payoff matrix

	B	S
B	$\alpha, 1$	$0, 0$
S	$0, 0$	$1, \alpha$

where $\alpha \geq 0$.

- Find the set of all NE as a function of α and draw the graph of best response functions.
- Explain what happens as $\alpha = 0$, $\alpha = 1$ and $\alpha \to \infty$.

Solutions

[1] Let G be the 2×2 strategic game given by

	L	R
T	a, b	c, d
B	e, f	g, h

and let G' (which is different from G) be the 2×2 strategic game given by

	L	R
T	a', b'	c', d'
B	e', f'	g', h'

Consider the 2×2 game $G'' = G + G'$ given by

	L	R
T	$a + a', b + b'$	$c + c', d + d'$
B	$e + e', f + f'$	$g + g', h + h'$

– Suppose that (T, L) is a pure strategy equilibrium in G and that it is also a pure strategy equilibrium in G'. Hence, we know that in G

$$a \geq e \text{ and } b \geq d$$

and in G'

$$a' \geq e' \text{ and } b' \geq d'$$

Thus,

$$a + a' \geq e + e' \text{ and } b + b' \geq d + d'$$

which implies that (T, L) is also an equilibrium of G''.

Let p be a mixture over player 1's strategies and q be a mixture over player 2's strategies.

– Suppose that (p^*, q^*) is a completely mixed strategy equilibrium in G and that it is also a completely mixed strategy equilibrium in G'. Hence, we know that in G

$$p^* b + (1 - p^*) f = p^* d + (1 - p^*) h$$

and

$$q^*a + (1 - q^*)c = q^*e + (1 - q^*)g$$

and in G'

$$p^*b' + (1 - p^*)f' = p^*d' + (1 - p^*)h'$$

and

$$q^*a' + (1 - q^*)c' = q^*e' + (1 - q^*)g'$$

Thus,

$$p^*(b + b') + (1 - p^*)(f + f') = p^*(d + d') + (1 - p^*)(h + h')$$

and

$$q^*(a + a') + (1 - q^*)(c + c') = q^*(e + e') + (1 - q^*)(g + g')$$

which implies that (p^*, q^*) is also an equilibrium of G''.

Note that this result can be generalized without much difficulty to any two-player with any $n \times m$ payoff matrix.

[2] The game is given by the 2×2 strategic game with the payoff matrix

	L	R
R	$1, 0$	$0, 1$
L	$0, 1$	$\delta, 1 - \delta$

where $0 < \delta < 1$.

Let p be a mixture over player 1's strategies and q be a mixture over player 2's strategies.

- For any $0 < \delta < 1$, the game has a unique mixed strategy NE

$$(p^*, q^*) = (\delta/(1 + \delta), \delta/(1 + \delta))$$

- If $\delta \to 1$ then $(p^*, q^*) \to (1/2, 1/2)$, and if $\delta \to 0$ then $q^* \to 0$. Note that if $\delta = 0$ then the game has two pure strategies NE (R, R) and (L, R) and a continuum NE (p^*, q^*) in which player 1 mixes with any $p^* \in (0, 1)$ and $q^* = 0$.

[3] When $x > 1$, the game has three symmetric Nash equilibria in pure strategies, (A, A), (B, B), and (C, C). All equilibria are strict so A, B, and C are ESS.

- The game also has a symmetric mixed strategy Nash equilibrium in which each player's mixed strategy is $(1/3, 1/3, 1/3)$. This strategy is not an ESS. A mutant who uses any of the pure strategies obtains an expected payoff of $x/3$ against a non-mutant and x against another mutant whereas the expected payoff of a non-mutant is always $x/3$.
- When $0 \leq x \leq 1$, the game has a unique symmetric mixed strategy Nash equilibrium in which each player's mixed strategy is $(1/3, 1/3, 1/3)$. Therefore, the game does not have an ESS (same argument above).