Advanced Microeconomics
(Economics 104)
Spring 2011
Strategic games I

Topics

The required readings for this part is O chapter 2 and further readings are OR 2.1-2.3. The prerequisites are the Introduction handout and O chapter 1.

- Introduction.
- Formalities.
- 2×2 examples.
- Best-response.
- Nash equilibrium.
- Dominance solvability.

Introduction

Game theory is the study of interacting decision-makers.

It is a natural generalization of the consumer theory which deals with how a utility maximizer behave in a situation in which her payoff depends on the choices of another utility maximizer.

Many fields, such as sociology, psychology and biology, study interacting decision-makers. Game theory focus on rational decision-making, which is the most appropriate model for a wide variety of economic contexts.

Next, we shall define what is a *game*, a *strategic game* and a *strategic game of perfect information*:

A game

- a multi-person (player) decision-making.

A strategic game

- a model in which each player chooses his plan of action once and for all, and these choices are made simultaneously.

A strategic game of perfect information

- a model in which each player is certain of the characteristics of all other players.

Formalities (O 2.1, OR 2.1)

A (finite) strategic game G of perfect information consists of:

a (finite) set N of players, and

for each player $i \in N$

- a (finite) non-empty set A_i of actions
- a preference relation $\succsim_i$on the set $A = \times_{i \in N} A_i$ of possible outcomes.

To clarify,

- A_i is the (finite) set of pure strategies for each player $i \in N$. An element $a_i \in A_i$ is a pure strategy (action) of player i.
- $A = A = \times_{i \in N} A_i = A_1 \times \cdots \times A_n$ is the strategy space and $a \in A$ is a strategy profile.

We will denote a strategic game G of perfect information by

$$\langle N, (A_i), (\succsim_i) \rangle$$

or by

$$\langle N, (A_i), (u_i) \rangle$$

when $\succsim_i$can be represented by a utility function $u_i : A \to \mathbb{R}$.

Examples (O 2.5, OR 2.3)

A two-player finite strategic game can be described conveniently in a bi-matrix. For example, 2×2 game

	L	R
T	A_1, A_2	B_1, B_2
B	C_1, C_2	D_1, D_2

- Player 1's actions are identified with the rows and the other player by the columns.
- The two numbers in a box formed by a specific row and column are the players' payoffs given that these actions were chosen. In the game above A_1 and B_1 are the payoffs of player 1 and player 2 respectively when player 1 is choosing strategy T and player 2 strategy L.

Applying the definition of a strategic game to the 2×2 game above yields:

- $N = 1, 2$
- $A_1 = \{T, B\}$ and $A_2 = \{L, R\}$
- $A = A_1 \times A_2 = \{(T, L), (T, R), (B, L), (B, R)\}$
- $\succsim_1$and $\succsim_2$are given by the bi-matrix.

Classical 2×2 game

– Battle of the Sexes (BoS)

	B	S
B	$2,1$	$0,0$
S	$0,0$	$1,2$

– Coordination Game

	B	S
B	$2,2$	$0,0$
S	$0,0$	$1,1$

– Prisoner Dilemma

	D	C
D	$3,3$	$0,4$
C	$4,0$	$1,1$

– Hawk-Dove

	D	H
D	$3,3$	$1,4$
H	$4,1$	$0,0$

– Matching Pennies

	H	T
H	$1,-1$	$-1,1$
T	$-1,1$	$1,-1$

Best response (O 2.8, OR 2.2)

For any list of strategies $a_{-i} \in A_{-i}$

$$B_i(a_{-i}) = \{a_i \in A_i : (a_{-i}, a_i) \succsim_i (a_{-i}, a_i') \ \forall a_i' \in A_i\}$$

is the set of players i's best actions given a_{-i}.

In words, action a_i is i's best response to a_{-i} if it is the optimal choice when i conjectures that others will play a_{-i}.

When $\succsim_i$can be represented by a utility function $u_i : A \to \mathbb{R}$

$$B_i(a_{-i}) = \{a_i \in A_i : u_i(a_{-i}, a_i) \geq u_i(a_{-i}, a_i') \forall a_i' \in A_i\}$$

Dominance (O 2.9)

An action $a_i \in A_i$ of player i is *strictly dominated* if there exists an action $a'_i \neq a_i$ such that

$$u_i(a_{-i}, a_i) < u_i(a_{-i}, a'_i)$$

for all $a_{-i} \in A_{-i}$.

An action $a_i \in A_i$ of player i is *weakly dominated* if there exists an action $a'_i \neq a_i$ such that

$$u_i(a_{-i}, a_i) \leq u_i(a_{-i}, a'_i)$$

for all $a_{-i} \in A_{-i}$ and

$$u_i(a_{-i}, a_i) < u_i(a_{-i}, a'_i)$$

for some $a_{-i} \in A_{-i}$.

One interesting result on dominated strategies is that an action of a player (in a finite strategic game) is never a best response **if and only if** it is strictly dominated.

The proof if left as an exercise.

Pure strategy Nash equilibrium (O 2.6-2.7, OR 2.2)

Nash equilibrium (NE) is a steady state of the play of a strategic game.

Formally, a NE of a strategic game $G = \langle N, (A_i), (\succsim_i) \rangle$ is a profile $a^* \in A$ of actions such that

$$(a^*_{-i}, a^*_i) \succsim_i (a^*_{-i}, a_i)$$

$\forall i \in N$ and $\forall a_i \in A_i$, or equivalently,

$$a^*_i \in B_i(a^*_{-i})$$

$\forall i \in N$.

In words, no player has a profitable deviation given the actions of the other players.

Next week, we will prove existence of Nash equilibrium using Kakutani's fixed point theorem.

Another interesting result on dominated strategies is that if we consider a game G and a game G' obtained by iterated removal of all (weakly and strictly) dominated strategies from G then

– if $a \in NE(G)$ then $a \in NE(G')$ (that is, any a which is a NE of G' is also a NE of G), and
– the converse holds for the iterated removal of strictly dominated strategies.

The 2×3 game below illustrious this result. The proof if left as an exercise.

	L	M	R
T	$3, 7$	$8, 4$	$9, 5$
C	$5, 1$	$14, 8$	$6, 9$
M	$6, 4$	$10, 2$	$8, 3$